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Analytical Study of Two Pin-Loaded Holes in Unidirectional Fiber-Reinforced Composites

The objective of this paper is to investigate the effects of geometrical parameters such as the edge distance-to-hole diameter ratio $\{e/d\}$, plate width-to-hole diameter ratio $\{w/d\}$, and the distance between two holes-to-hole diameter ratio $\{l/d\}$ on stress distribution in a unidirectional composite laminate with two serial pin-loaded holes, analytically and numerically. It is assumed that all short and long fibers lie in one direction while loaded by a force p_o at infinity. To derive differential equations based on a shear lag model, a hexagonal fiber-array model is considered. The resulting pin loads on composite plate are modeled through a series of spring elements accounting for pin elasticity. The analytical solutions are, moreover, compared with the detailed 3D finite element values. A close match is observed between the two methods. The presence of the pins on shear stress distribution in the laminate is also examined for various pin diameters. [DOI: 10.1115/1.4007226]

1 Introduction

Composite materials are used extensively in aircraft structures. The main driver is their weight-saving potential, but composites also benefit from good fatigue properties and corrosion resistance. Designing useful structures means that several parts must be joined together. This can be done by adhesive joining, mechanical joining, or hybrid joining which is a combination of the first two techniques. The use of adhesive joints in heavily loaded structures is often restricted by the low out-of-plane strength of the composite. Mechanical joining, i.e., using bolts or rivets, is the most important method in the aerospace industry. Although it is the preferred joining technique in many cases, it is still associated with difficulties. The presence of a hole in a laminated plate subjected to external loading introduces a disturbance in the stress field. Stress concentrations are generated in the vicinity of the hole. Inserting a fastener into the hole and reacting load through the contact between the fastener and the hole surface will make the stress concentrations even more severe. Because of this, bolted and pin joints are weak spots in a composite structure and must be properly designed to achieve an efficient structure. Due to the importance of this problem, many authors have tried to investigate the stress distribution around a pin-loaded hole through experimental and/or analytical methods, using different assumptions. The complex function method was used by De Jong [1] in an early work where he used an assumed stress distribution from the pin, thus transforming the contact problem into a boundary value problem. Waszczak and Cruse [2], Zhang and Ueng [3], studied the bolt load on strength and failure of a composite plate using the method of complex functions. A more general analysis was conducted by Hyer et al. [4,5] in which the complex function method was used for both the pin and the plate, thus accounting for deformation of both parts. The contact problem was solved by means of the collocation method where the slip and no-slip regions were identified in an iterative process. Liu et al. [6,7] have investigated the effect of plate thickness on the load-displacement behavior of glass fabric/phenolic composite double-lap with single-pin joints.

Results showed that thick composites with small pins and thin composites with great pins had worse efficiencies for joint rigidity and joint strength than those having similar dimensions between pin diameter and plate thickness. Li et al. [8] improved the bearing strength in carbon fiber reinforced epoxy composite laminates due to a significant increase of peak load by using a modified steered pattern. The 2D finite element method has been used extensively for this problem [9–17]. In most cases, the analyses are straightforward and will not be discussed further. Experimental results are a large part of the literature published, and effects of clearance (Kelly and Hallstrom [18]) and Geometry parameters (Icten and Sayman [19], Okutan [20], Mevlut Tercan and Aktas [21]) are investigated. In previous work [22], the effect of fiber arrangement in fiber-reinforced composite with a pin load hole investigated. The stress concentrations in fiber-reinforced composites is usually effected by many factors such as fiber spacing, the characteristics of fiber/matrix interface, ratio of fiber to matrix elastic moduli, and so on. Hence, it becomes important to evaluate the effects of these factors on the stress concentrations around fiber breaks in any fibrous composite.

In this paper, the effect of pin load is examined around two pins in series used to join two composite plates. Each plate is loaded at infinity. The fiber arrangement in each laminate is assumed to be hexagonal. The resulting pin load on the composite plate is modeled through a series of spring elements accounting for pin elasticity. This model is used for both analytical and finite element solutions. In addition, the effects of geometric parameters such as edge distance to pin hole diameter e , pin hole diameters, and center to center distance of the two pins $\{l\}$ are examined on stress distribution in the joint. The material used for the laminate is assumed to be graphite-epoxy.

2 Derivation of Field Equations

In order to derive the field equations, we consider a multilayered composite laminate with two holes of diameters d and d^* , as shown in Fig. 1. The center to center distance of the two pin holes is taken to be l . Furthermore, fibers were modeled to have hexagonal cross sections with side s , as shown in Fig. 2. The distance between any two successive fibers along the laminate width is shown by δ (see Fig. 2). The area of each fiber is A_f . In addition, it is assumed that all fibers take only an extensional load, while each

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Fig. 1 Division of the laminated into three regions (top view)

Fig. 2 Fibers in a hexagonal arrangement of fibers

matrix bay sustains only pure shear (Shear-Lag theory). This is a good assumption for most composites with phenolic resins or a weak extensional stiffness. All fibers behave as linear elastic up to the point of fracture. Let x and y correspond to principal axes of the plate. Load p_0 is acting on all fibers along x direction, as the composite plate is loaded at infinity. As shown in Fig. 2, M and Ψ , correspond to the total number of fibers along y and z directions respectively (starting from value of 1). Also, s_1 and s_2 correspond to the last broken fibers at the bottom edge of pins one and two, respectively. One can show that volume fraction of fiber, v_f , may be expressed as follows:

$$v_f = \frac{\left\{ M\Psi + \left(\frac{\Psi-1}{2} \right) \right\} A_f}{\left\{ M\Delta + (M-1)\delta \right\} \left\{ 2s + \frac{\sqrt{3}}{2} (\Psi-1)(\Delta + \delta) \right\}} \quad (1)$$

Where according to Fig. 2, Δ is measured between any two flat surfaces of a hexagon, and δ is the distance between any two opposite flat surfaces of any two neighboring fibers.

To write equilibrium equations, fibers are grouped into five categories as follows (see Fig. 2):

- Group I are those surrounded by six fibers (hexagonal arrangement)
- Group II are those surrounded by five fibers
- Group III are those surrounded by four fibers (edge fibers with i ranging from 1 to M)
- Group IV are those surrounded by three fibers (edge fibers with j ranging from 1 to Ψ)
- Group V are those surrounded by two fibers (corner fibers)

To derive equilibrium equations, the laminated plate is divided into three regions one, two, and three, as shown in Fig. 1. According to Fig. 1, parameter e corresponds to the distance between the center of the pin hole to the free edge of the laminate. For fibers in region three, the equilibrium equations for group I fibers result into (see Ref. [22]):

$$\frac{dp_{i,j}}{dx} dx + s \{ \tau_{i+1,j} - \tau_{i-1,j} + \tau_{i,j+1} - \tau_{i,j-1} - \tau_{i,j+1} - \tau_{i+1,j+1} \} dx = 0 \quad (2)$$

Where $p_{i,j}$ and $\tau_{i,j}$ represent the normal load in each fiber and shear stress in the neighboring matrix bays, respectively. Also, x represents the axial coordinate along the direction of fibers; now, we introduce the following nondimensional terms as (see Ref. [22] and Fig. 2);

$$\xi = \sqrt{\frac{G_m}{E_f A_f}} x, \quad U_{i,j} = \sqrt{\frac{E_f A_f G_m}{p_0^2}} u_{i,j}, \quad \Omega = \frac{s}{\delta} \quad (3)$$

Here, G_m represents matrix shear modulus of the matrix, p_0 is the applied load at infinity, E_f and $u_{i,j}$ correspond to the elastic modulus of fibers and their axial displacement, respectively. Using Eq. (4), one may write the equilibrium Eq. (3) into a nondimensional form as:

$$\frac{d^2 U_{i,j}}{d\xi^2} + \Omega \{ U_{i+1,j} + U_{i-1,j} + U_{i,j+1} + U_{i,j-1} + U_{i,j+1} + U_{i+1,j+1} - 6U_{i,j} \} = 0 \quad (4)$$

For other fibers categorized as groups II to V (in region III), similar expressions can be written (see Ref. [22]). All fibers grouped in region I and II are considered as short fibers. For such fibers, similar expressions (as above) may be written. These equations must be highlighted for region one by a superscript “*” and similar for region two by a superscript “**,” standing for short fibers. Nondimensional equilibrium equations may be written in a matrix notation as:

$$[U''] - [L][U] = [0] \quad (5)$$

While

$$[U]^T = [U_{1,1}, U_{1,2}, \dots, U_{M-1,\Psi}, U_{M,\Psi}] \quad (6)$$

In Eq. (5), $[L]$ is the coefficient matrix and $[U'']$ corresponds to the second derivative of U with respect to ξ . Also:

$$U'' = \frac{d^2 U}{d\xi^2} \quad (7)$$

Hence, the solution to differential-difference Eq. (5) may be written for each region as it will follow (please note that from this point on, each asterisk used on a parameter corresponds to variation of that parameter in that region).

Region one:

In this region, each fiber, surrounded by the laminate free edge and the neighboring pin hole, is considered as a short fiber. This is specifically true when e in Fig. 1 gets smaller. In such a case, the solution to Eq. (5) may be written in terms of eigenvalues λ_k and eigen-vectors $[R]^k$ as:

$$U_{i,j}^* = \sum_{k=1}^N D_k [R]_{i+m(j-1)}^k e^{\lambda_k \xi} + \sum_{k=1}^N E_k [R]_{i+m(j-1)}^k e^{-\lambda_k \xi} \quad (8)$$

In the above equations, $[R]_{i+m(j-1)}^k$ is considered as a value associated with the $\{i+m(j-1)\}$ row of the k th eigen-vector.

Region two:

In this region, each fiber cut by any two successive pin holes is considered as a short fiber. This postulation becomes specifically true when dimension l in Fig. 1 gets smaller. Hence, solving Eq. (5), one can write:

$$U_{ij}^{**} = \sum_{k=1}^N F_k [R]_{i+m(j-1)}^k e^{\lambda_k \xi} + \sum_{k=1}^N H_k [R]_{i+m(j-1)}^k e^{-\lambda_k \xi} \quad (9)$$

Region Three:

The filaments embedded in this region are considered as long fibers. Hence, to fulfill the bondness condition on loads at infinity (its values in each fiber must be finite as ξ approaches infinity), only the negative values of eigenvalues (and their corresponding eigen-vectors) are accepted. As a result, one may write:

$$U_{ij} = \sum_{k=1}^N C_k [R]_{i+m(j-1)}^k e^{\lambda_k \xi} + \xi \quad (10)$$

As a result;

$$P_{ij} = \frac{dU_{ij}}{d\xi} \quad (11)$$

In the above equations, D_k , E_k , F_k , H_k , and C_k , are constants yet to be defined from boundary conditions.

3 Boundary and Continuity Conditions

To solve for the constants introduced in expressions for loads and displacements, the following boundary conditions and continuity equations are used.

- (1) On the free edge of all fibers in region one of the laminate, one may write (see Ref. [22]):

$$(P_{ij}^*)_{\xi_{ij}=-\mu_{ij}} = 0 \quad (12)$$

- (2) Based on the hexagonal arrangement of fibers, a force balance in nondimensional form on any cut fibers in direct contact with the pin results into (see Ref. [22]):

$$P_{ij}^* + \Omega Z [U_{i-1,j}^* - U_{ij}^*] \tan \theta_{ij} + \frac{3}{4} \Omega Z [U_{i,j+1}^* - U_{ij}^*] \times \tan \theta_{ij} + \frac{3}{4} \Omega Z [U_{i,j-1}^* - U_{ij}^*] \tan \theta_{ij} + \Gamma U_{ij}^* = 0 \quad (13)$$

The angle θ_{ij} locates any fiber with respect to the x or ξ axis passing through center of the hole. Here, the pin has been modeled as a linear spring support with stiffness k where in the above equations, Z and Γ are defined as:

$$Z = \Delta \sqrt{\frac{G_m}{E_f A_f}}, \quad \Gamma = \frac{k}{\sqrt{G_m E_f A_f}} \quad (14)$$

Using Eq. (13), one may write a similar expression for short fibers of region two which are in contact with the second pin.

- (3) Due to the nature of loading, all short fibers located on the left half of each pin hole experience a free surface. A force balance on these edges results into the following equation for region three (see Ref. [22]):

$$P_{ij} + \Omega Z [U_{i-1,j} - U_{ij}] \tan \theta_{ij} + \frac{3}{4} \Omega Z [U_{i,j+1} - U_{ij}] \tan \theta_{ij} + \frac{3}{4} \Omega Z [U_{i,j-1} - U_{ij}] \tan \theta_{ij} = 0 \quad (15)$$

- (4) The amount of displacement and stress at the boundary between regions one and two is equal. One can use the above boundary conditions and continuity equations to solve for the $5 \times M \times \Psi$ unknowns present in equilibrium

equations. It is worth to mention that S_t , S_c , and S_{yx} correspond to tensile stress concentration, compressive stress concentration, and nondimensional shear stress within the laminate and are defines as follows:

$$S_t = \frac{P_{ij}}{P_o}, \quad S_c = -\frac{P_{ij}}{P_o}, \quad S_{yx} = \{U_{ij} - U_{i-1,j}\} \quad (16)$$

4 Discussion of Results

In order to investigate the effect of geometric parameters on stress variation within the laminate, the values listed in Table 1 were used for fibers and matrix. To check the accuracy of the results based on the analytical solution, the laminate was modeled in a finite element (FE) environment. This model is shown in Fig. 3. The results are obtained from commercial computer code ANSYS 12.0. The finite element results on peak tensile stresses within the fibers are then compared to those of the analytical solution. The results are based on typical values of $d/D = 10$ (crack), and $r = 4$, $M = 16$ and $\Psi = 3$. Table 2 shows these results. Comparison of the predicted shear stresses and average normal stresses based on analytical results to finite element analysis demonstrates that analytical model can be used to rapidly estimate the average normal stress distribution in the various constituents.

Based on the results, it is observed that stress distribution in the neighborhoods of any pin hole is affected by several parameters, among which one may point out to plate width, diameter of each pin (hole), dimensional ratio of e/d , and the distance between any two successive pins. Parameter r corresponds to the number of broken fibers in each layer, by the right pin hole and q is the similar parameter associated with the left pin. The horizontal distance between the two successive pins is defined by a dimensionless parameter, $\eta = 2l/(d + d^*)$. In order to investigate the effect of pin joint on tensile and compressive stresses, as well as peak shear stresses developed within the laminate, The results on Table 3 are for $M = 15$, $r = q = 5$, $e/d = 4$, and $\eta = 2$. These results are compared to those obtained through deduced equations in this study.

Table 1 Dimensions and mechanical properties of selected materials

E_f (GPa)	E_m (GPa)	Δ (m)	δ (m)	M	Ψ
250	4.2	0.00107	0.00107	15	3

Fig. 3 Finite element model of the laminated plate (spring elements are not shown)

Table 2 Comparison of finite element results to those of the analytical solution

Maximum tensile stress concentration	
FEM at right pin edge	3.02
Analytical solution at right pin edge	3.17
FEM at left pin edge	5.12
Analytical solution at left pin edge	5.27

Table 3 The effect of Γ on tensile and compressive stress concentration factors behind the right pin

	St		Sc				St
Γ	5	6	7	9	10		11
40	3.58	4.50	1.30	1.30	4.50		3.58
20	3.58	4.45	1.32	1.32	4.45		3.58
8	3.59	4.35	1.38	1.38	4.35		3.59
4	3.60	4.23	1.44	1.44	4.23		3.60

Table 4 The effect of equal broken fibers on tensile stress concentration factors at edge of right pin

$e/d=1$							$e/d=2$					
$r=q$	2	4	6	8	10	12	2	4	6	8	10	12
η												
2	3.27	2.76	2.93	3.41	4.20	5.81	4.62	3.23	3.17	3.59	4.42	6.19
3	1.75	1.80	2.25	2.73	3.28	4.20	2.95	2.50	2.77	3.19	3.75	4.72
4	1.27	1.69	2.20	2.59	2.96	3.52	2.37	2.51	2.85	3.14	3.45	3.99
5	1.13	1.77	2.28	2.59	2.83	3.20	2.24	2.73	3.02	3.18	3.32	3.63
η												
$e/d=3$							$e/d=4$					
2	4.49	3.02	3.07	3.58	4.46	6.26	4.14	2.85	3.02	3.58	4.48	6.28
3	3.09	2.51	2.84	3.29	3.86	4.82	2.95	2.47	2.86	3.34	3.90	4.85
4	2.62	2.63	2.98	3.28	3.57	4.08	2.60	2.65	3.03	3.33	3.61	4.10
5	2.58	2.91	3.19	3.33	3.43	3.71	2.62	2.97	3.26	3.39	3.47	3.73

Table 5 The effect of equal broken fibers on tensile stress concentration factors at the edge of the left pin

$e/d=1$							$e/d=2$					
$r=q$	2	4	6	8	10	12	2	4	6	8	10	12
η												
2	9.91	7.56	6.60	6.43	7.08	9.57	8.33	5.99	5.25	5.27	6.12	8.81
3	8.35	6.13	5.52	5.65	6.56	9.22	7.25	5.23	4.85	5.17	6.24	9.04
4	7.30	5.44	5.09	5.40	6.42	9.13	6.50	4.86	4.71	5.16	6.30	9.08
5	6.59	5.07	4.90	5.31	6.40	9.13	5.97	4.67	4.66	5.18	6.34	9.11
η												
	$e/d=3$						$e/d=4$					
2	7.27	5.22	4.70	4.90	5.89	8.69	6.55	4.80	4.43	4.75	5.83	8.67
3	6.48	4.73	4.53	4.99	6.16	9.01	5.94	4.43	4.37	4.92	6.13	9.01
4	5.90	4.51	4.52	5.07	6.26	9.07	5.47	4.30	4.41	5.03	6.25	9.07
5	5.50	4.42	4.54	5.13	6.32	9.10	5.14	4.26	4.47	5.11	6.32	9.10

Due to the nature of the loading, compressive stresses are produced within the laminate, behind the pin, in region one and two, while tensile stresses are developed in regions one, two, and three.

Generally speaking, stress concentration stems from the existence of excessive shear stress in the matrix and/or bearing stresses produced in the laminate. In order to investigate the effect of pin stiffness (support stiffness) on stress concentration within the laminate, parameter Γ was allowed to vary from 20 to 4. For typical data used in this analysis, the two foregoing values correspond to $k = 226$ MN/m and 45.2 MN/m, respectively. Examination of the results in Table 3 indicates that in any typical fiber behind the pin (region two), the tensile or compressive stress concentrations (S_t or S_c), do not seem to be much dependent on Γ (or k).

The values in Tables 4 and 5, are specifically written for fiber with $i = v_1 + 1$ (v_1 is last broken fiber by right pin), while j ranges from 1 to 3. As realized, the tensile stress concentration values are highly dependent on e/d ratio. Holding r and η as a constant, an increase in e/d ratio, except $\{e/d = 1\}$ will increase values of stress

Fig. 4 Compressive stress distribution around the left pin hole

Fig. 5 Compressive stress distribution around the right pin hole for $r=6$ and $\eta=3$

concentration at the edge of the right pin hole. The values in Table 5 are specifically written for fiber with $i = v_2 + 1$ (v_2 is last broken fiber by left pin), while j ranges from 1 to 3. The tensile stress concentration is seen to be much higher at the left pin compared to that of the right pin. According to this table, stress concentration decreases with an increase in η ratio.

Figure 4 depicts compressive stress distribution around the left pin hole, for a case in which $r=q$. According to this figure, as well as Table 4, the amount of load taken by the fibers in the vicinity of the pin holes depends on several parameters such as pin radii (shown in terms of r or q), distance between the two pin holes, and dimensional ratio e/d . Figure 5 represents the effect of broken fibers by left pin q , on compressive stress distribution around the right pin hole, for a case in which $r=6$. Based on results observed with an increase broken fiber by the left pin, compressive stress around the right pin is reduced.

Figure 6 represents the effect of broken fibers by the second pin q , on compressive stress distribution around the first pin hole (right pin), for a case in which $r=6$. Figures 6 and 7 depict the effect of dimensionless center to center distance of the two pin holes on compressive stress distribution around of right and left pin for $r=q=6$. Comparison of the results shows compressive stresses are less sensitive to variation of η for fibers further away from the x -axis. Moreover, an increase in η ratio causes a decrease in maximum compressive stresses within the laminate behind the pin.

Based on results observed with an increase broken fiber by left pin, compressive stress around the first pin is reduced. Figure 8

Fig. 6 Compressive stress distribution around the right pin hole for $r = 6$

Fig. 9 Maximum tensile stress concentration around the right pin hole for $\eta = 3$ and $e/d = 2$

Fig. 7 Compressive stress distribution around the left pin hole for $r = 6$ and $e/d = 2$

Fig. 10 The effect of broken fibers and distance between two pin on dimensionless shear stress in point a

Fig. 8 Maximum tensile stress concentration around the left pin hole for $\eta = 3$ and $e/d = 2$

Fig. 11 The effect of broken fibers and distance between two pin on dimensionless shear stress in point c

represents a variation of maximum tensile stress concentration at the left hole edge as the number of broken fibers r and q are changed. For larger values of q , stress concentrations values (at the left pin hole) become almost independent of r . Figure 9

depicts the effect of broken fibers on the stress concentration factor at the first hole edge. As it appears, for any constant value of q , an increase in size of the right pin hole (increasing value of r), will result in a tensile stress concentration in the laminate. For any

constant value of r , a decrease in size of the left hole will result in a smaller tensile stress concentration around the right hole.

Figures 10 and 11, represent the effects of equal size pin holes (equal values of r and q) and center to center distance of the two pin holes on nondimensional shear stress, S_{yx} , occurring at points a , b , c , and d (see also Fig. 1), respectively. The amount of shear stress at points a and c are higher than points b and d .

5 Conclusions

According to the results obtained on two pins in a series used to join two composite unidirectional laminates loaded at infinity, one may conclude that:

- (1) The higher compressive stress in the laminate occurs in the vicinity of the left pin as opposed to the other.
- (2) The e/d ratio has a significant effect on tensile stress developed within the laminate.
- (3) The tensile stress concentration in the pin further away from the free laminate edge is more pronounced compared to the other.
- (4) Except for the point in front of the left pin hole (point d), maximum shear stress reduces as the number of broken fibers r and q ($r = q$), are increased.
- (5) Holding the first pin diameter fixed, an increase in the second pin diameter will increase the tensile stress concentration in each laminate.

Nomenclature

A_f = cross-sectional area of the circular fibers
 d, d^* = pin hole diameter
 e = distance from laminate edge to right pinhole center
 E_f = fiber elastic modulus
 G_m = matrix shear modulus
 k = pin stiffness
 l = the center to center distance of the two pin holes
 L = coefficient matrix
 M = total of fiber in each layer
 N = total of fiber
 p_0 = normal load applied to the laminate at infinity
 p, p^*, p^{**} = localized load in each fiber
 R = eigen-vector
 r, q = total number of broken fiber in each layer
 s = cross sectional area of each fiber
 S_t = tensile stress concentration
 S_c = compressive stress concentration
 S_{yx} = no dimensional shear stress
 u, u^*, u^{**} = fiber displacement
 U, U^*, U^{**} = no dimensional fiber displacement
 x = axial coordinate along direction of fibers
 Ψ = the number of layer
 δ = fiber spacing
 λ = eigenvalue

η = the no dimensional center to center distance of the two pin holes
 ξ = nondimensional coordinate along direction of circular fibers
 τ = shear stress

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