Effect of charge patterns along a solid-state nanopore on polyelectrolyte translocation

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We investigate the effectiveness of charge patterns along a nanopore on translocation dynamics of a flexible polyelectrolyte. We perform a three dimensional Langevin dynamics simulation of a uniformly charged flexible polyelectrolyte translocating under uniform external electric field through a solid-state nanopore. We maintain the total charge along the pore to be constant, while varying its distribution by placing alternate charged and uncharged sections of different lengths along the pore length. Longest average translocation time is observed for a pattern corresponding to an optimum section length, with a major delay in the translocation time during the pore ejection stage. This optimum section length is independent of lengths of polyelectrolyte and pore within the range studied. A theory based on the Fokker-Planck formalism is found to successfully describe the observed trends with reasonable quantitative agreement. © 2014 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4869862>]

I. INTRODUCTION

The phenomenon of polymer translocation through solid-state nanopores and biological channels is in general very complex.¹ Translocation of single DNA molecules through a solid-state nanopore under an applied electric field has been extensively studied since it has been proposed to be used as a rapid DNA sequencing technique.² With each nucleotide having a different ionic current blockade level (see Ref. 3, for example), an entire DNA may in principle be sequenced by accurately measuring ionic current trace as the DNA translocates. Controlling the stochastic nature of the process, however, is a major challenge in devising an experimental system based on this technique.

An example of the complex nature of the process is the various suggested scaling laws of the mean translocation time τ of a polymer of length N undergoing the translocation process. Sung and Park⁴ proposed two regimes of scaling behavior based on the applied potential difference for a polymer undergoing translocation through a hole, assuming the chain friction is dependent on the chain length N . For small favorable potential difference, they predicted a scaling of $\tau \sim N^{(2+\gamma)}$ while for larger driving potential difference, $\tau \sim N^{(1+\gamma)}$, with $\gamma = 1$ when hydrodynamics can be ignored. Muthukumar⁵ used a friction coefficient that is independent of N , and predicted that for a favorable potential difference and for longer chains, $\tau \sim N$. For a small electro-chemical potential difference, the scaling $\tau \sim N^2$ is recovered. This exponent was recently observed in Monte Carlo simulations of unbiased translocation by Polson and McCaffrey,⁶ with modified scaling of $\tau \sim (N - M)^2$ for cylindrical pores of length M . The scaling of $\tau \sim N^{1+\nu}$, with ν being the Flory exponent, was predicted by Kantor and Kardar⁷ for a polymer translocating through a hole in the presence of

a chemical potential difference assuming that the polymer is in equilibrium at the translocation time scales. de Haan and Slater⁸ studied non-driven translocation of polymers using Langevin dynamics simulations across a range of pore diameters and found that the scaling exponent α in $\tau \sim N^\alpha$ varies between 2.2 and 3 with variations in pore diameter. Similarly, in two-dimensional Monte Carlo simulations performed by Luo *et al.*,⁹ the scaling exponent α in the scaling equation $\tau \sim N^\alpha$ is found to depend on the pore length and polymer length, for the same applied electric field. In experiments, a number of parameters are found to affect the dynamics of a DNA translocating through a solid-state nanopore. As DNA enters the pore, the ionic current may either increase or decrease depending on the applied potential and salt concentration.¹⁰ A hybrid event is also observed for some intermediate potentials, where the current first decreases and then increases as the translocation progresses.

Typical speed of translocation measured in experiments using solid-state nanopores is of the order of 10–100 ns per nucleotide.^{11,12} This speed is too fast to be able to detect the current traces reliably. Hence, it is necessary to slow down translocation. A few strategies have been suggested to achieve slower translocation for this purpose. Experimentally, equivalent salts of different ionic sizes are found to have different effects on the translocation speed.¹² With longer pores, the interaction between pore wall and monomers of the polymer becomes increasingly significant. By introducing a parameter ϵ for the pore-polymer interaction, Muthukumar¹³ predicted the importance of this pore-polymer interaction on the mean first passage time. In the present context, ϵ is a measure of the strength of the electrostatic attraction between monomers and the nanopore wall. For large values of ϵ , these interactions become significant even for shorter pores. Two-dimensional Langevin dynamics simulations of a polymer with Lennard-Jones attractive interaction for relatively wide pores by Luo *et al.*¹⁴ show both a higher success rate and a slower

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average time of translocation with increased strength of attractive interactions. Using p and n type dopants, Luan *et al.*¹⁵ simulated translocation through a pore with a p-n junction at the center. The computed electric field is different in the p and n sections, with a reverse electric field at the pore center that helps to stretch the polymer. The resulting mean translocation time is also found to be affected due to presence of the two sections.

Effective driving force, defined as the difference between the force due to electric field and the opposing force due to electro-osmotic flow, has been predicted to be dependent on the surface charge density of the pore.¹⁶ Ding *et al.*¹⁷ performed similar numerical electro-hydrodynamic calculations based on the balance between the viscous forces and electric forces that include the effect of surface charge. They found that the surface charge density significantly affects the mean translocation time and suggest the use of surface charges to mediate translocation dynamics. The importance of pore-polymer interactions has also been highlighted in simulations of translocation into a spherical pore.¹⁸ He *et al.*¹⁹ proposed that switching the applied voltage after DNA capture can substantially increase the translocation time due to presence of charges along the pore walls. The translocation dynamics of a heterogeneous polymer shows rich sequence dependence. The mean translocation time of a multiblock polymer translocating in uncharged pores depends not only on the fraction of each block,²⁰ but also on the arrangement of these blocks.^{21,22} In charged pores,²³ an additional dependence is observed on the arrangement of charges along the pore walls.

pH is known to protonate charged amino acid residues along protein pores. Wong and Muthukumar²⁴ studied the effect of pH on translocation of NaPSS through α -Hemolysin and found that using pH gradient, the charges along the pore can be manipulated to increase the probability of successful translocation events. Site-directed mutagenesis can be used to manipulate the charge distribution along protein pores.²⁵ Manipulating charges in different regions (both the entrance region and the narrowest constriction) of an α -Hemolysin pore is found to change the rate of translocation events significantly. For solid-state nanopores, a pH sensitive coating can be used to tune the charges along the pore to control polymer translocation. This has been shown to affect translocation time both experimentally and theoretically,²⁶ emphasizing the importance of pore-polymer interactions in the translocation process.

Thus, charge distribution along the pore wall seems to have a significant effect on the translocation dynamics. In most of the studies so far, however, the pore-polymer interaction is assumed to be constant. Specifically, a uniform charge density is assumed along the pore wall. In this work, we investigate the effectiveness of charge distribution along the pore wall on translocation dynamics. Keeping the total charge along the nanopore to be constant, we decorate the nanopore with alternate charged and uncharged sections of different lengths. Using Langevin dynamics simulation, we study the translocation dynamics of a negatively charged flexible polymer through the charge-decorated nanopore. We find that the mean translocation time depends non-monotonically on the length of charged section. The slowest translocation is

observed for one particular charge distribution. Numerical calculations based on one dimensional free energy landscape capture the non-monotonic trend observed in simulations. Simulation details are discussed in Sec. II, followed by the results for translocation time distribution. A theoretical framework based on free energy landscape is discussed in Sec. IV. We conclude with a brief comparison of our simulation results with the theoretical predictions.

II. SIMULATION

Langevin dynamics simulation of a charged polymer undergoing translocation through a partly charged nanopore under an applied electric field is performed using the LAMMPS²⁷ package.

A. System

The system is made up of three components: a membrane with uncharged walls, a pore inside the membrane with patterns of charges, and a charged polymer chain. Details of each of these components are described below, with all quantities in reduced units, unless specified otherwise. A scaling of 1 unit = 12 Å is used for length. This choice is made to reflect the Kuhn length of a flexible polyelectrolyte chain such as the single stranded DNA.

The membrane is M units thick, and has a cylindrical pore of radius $r_p = 1$. The membrane walls are modelled using spherical beads of diameter $d_1 = 1$ arranged into a two dimensional grid with grid-spacing of d_1 . The pore wall is made up of smaller beads of diameter d_2 , with the ratio $d_2/d_1 = 0.25$. These beads are placed along the circumference of a cylinder, with beads approximately at a distance of $d_2/2$ from each other. (This overlapping of pore wall beads is done to make the pore cross-section more circular.) A part of membrane walls surrounding both ends of the pore is made up of beads identical to the pore wall beads. The pore is decorated with patterns of alternating charged and uncharged sections along its length. The pore length M is divided into $2N_s$ sections with each section of length $L_s = M/(2N_s)$. All the beads in an odd numbered section of the pore carry a positive fractional charge of α_p , while those in an even numbered section are uncharged. Different patterns can be generated by varying N_s . N beads of diameter d_1 are connected by bonds to form a linear homo-polymer. The equilibrium bond length is taken to be equal to d_1 . A unit negative charge is placed on each polymer bead. Hence, charged sections of the pore are attractive to the polymer.

B. Pair-wise interactions

Excluded volume interactions are modelled using a truncated Lennard-Jones potential between two beads, given by the equation,

$$U_{LJ} = \begin{cases} 4\epsilon_{LJ} \left[\left(\frac{\sigma}{r} \right)^{12} - \left(\frac{\sigma}{r} \right)^6 \right] + \epsilon_{LJ} & \text{for } r \leq 1.12\sigma \\ 0 & \text{for } r > 1.12\sigma \end{cases} \quad (1)$$

Here, r is the distance between two beads. We have taken the energy scale as $k_B T/6$, where k_B is the Boltzmann constant and T is the temperature in Kelvin. $\epsilon_{LJ} = 1$ (≈ 0.1 Kcal/mol) is the depth of the potential well, with $T = 300$ K. The potential is truncated at its minimum, corresponding to a distance of 1.12σ . These interactions exist between all the beads, with $\sigma = 1$ for interactions between polymer-polymer and polymer-membrane beads, and $\sigma = 0.625$ for pore-polymer beads. Since the positions of pore and membrane beads are fixed in the simulation, pair-wise interactions are not computed for these beads.

Electrostatic interactions between a pair of beads are modelled using the truncated Debye-Hückel potential (Eq. (2)), with a Debye length $\kappa^{-1} = 0.81$ units corresponding to a monovalent salt concentration of $0.1M$,

$$U_{DH} = \frac{C q_i q_j}{\epsilon} \frac{e^{-\kappa r}}{r} \quad \text{for } r \leq r_{c2} \quad (2)$$

with q_i and q_j corresponding to the charges on beads i and j , separated by a distance r . Although the effective dielectric constant of an electrolyte solution confined within a nanopore is unknown, we have taken $\epsilon = 80$. r_{c2} is the cutoff distance at which the electrostatic interactions are truncated. A value of $r_{c2} = 3$ is used in all the simulations (unless indicated otherwise).

Pair-wise electrostatic interactions are computed between polymer-polymer beads and pore-polymer beads. For polymer-polymer electrostatic interactions, $q_1 = q_2 = -1e$ ($e = 1.60217646 \times 10^{-19}$ C is the elementary charge). The pore-polymer electrostatic interactions are computed using $q_1 = -1e$ and $q_2 = \alpha_p e$ for pore beads belonging to a charged section. As described in Ref. 27, the energy conversion constant $C = 1$ in the present units.

The bonds between polymer beads are modelled using a harmonic potential given by Eq. (3). The equilibrium bond length between two connected polymer beads is $r_0 = 1$. The value of spring constant $K = 15480$ (≈ 10 Kcal/(mol $\text{\AA}^2$)) is large enough to prevent unrealistic bond extensions.

$$U_{harmonic} = K(r - r_0)^2. \quad (3)$$

In Langevin dynamics simulation, the above potentials are used to compute forces on each of the polymer beads. The equation of motion for each bead is

$$m \frac{d^2 r}{dt^2} = -\zeta v - \nabla(U_{harmonic} + U_{DH} + U_{LJ}) + F_r + F_{ext}, \quad (4)$$

where m is mass of the bead, ζ is the friction coefficient, and $F_r \sim \sqrt{k_B T \zeta / dt}$ is the random force due to solvent at the given temperature T , as documented extensively in the literature.²⁷ We choose the values $m = 1$ and $\zeta = 1$ in all our simulations. The force due to applied electric field E on each bead is denoted by F_{ext} . For the numerical integration of equation (4), we have used the integration time step $dt = 0.003$ units.

C. Simulation procedure

An equilibrium polymer configuration is required before starting the simulation. A straight chain aligned to the pore

axis is taken as a starting configuration to be equilibrated. The position of first bead is fixed just inside the pore while the rest of polymer beads are on the cis side of the pore. The system is then allowed to equilibrate by solving Eq. (4) for 10^5 time steps, significantly larger than the Rouse time for the polymer chain, with no electric field applied. A Verlet algorithm is used in LAMMPS to solve the equation of motion.

The polymer chain configuration obtained during the last time-step is taken as initial equilibrium polymer configuration for simulating polymer translocation under applied electric field. A uniform electric field E is specified across the pore, with no electric field on the cis and trans sides. Random velocities with a uniform distribution at the given temperature are assigned to all the polymer beads. The equation of motion is then integrated in time using Verlet algorithm until all the polymer beads are outside the pore on either side. The positions of polymer beads are stored at every 100 steps. In a successful translocation, the entire polymer is translocated to the trans side of the pore. Translocation time for a successful translocation is calculated as the time at which the last polymer bead exits the pore on trans side, starting with the initial configuration of first polymer bead just inside the cis end of the pore.

For a given set of parameter values, 2000 runs are performed, almost all of them leading to a successful translocation. The same equilibrated polymer chain is taken in each of these runs, with different initial random velocities and random forces on the polymer beads. We have also performed simulations with 2000 different equilibrium conformations as initial states. The final results are the same as long as the initial chain conformations are equilibrated. A distribution of translocation times for successful translocations is then obtained based on these runs, using which the mean successful translocation time can be computed.

III. RESULTS

A pore with a total of half the length being charged can be patterned in different ways. In the simplest configuration ($N_s = 1$), the pore is divided into two sections, each of length $M/2$, with the first section on the cis side being charged and the second section being uncharged. N_s is the number of pairs of charged and uncharged sections. Yet another pattern ($N_s = 2$) can be formed by having four sections, with each of length $M/4$. In this case, the first and third sections (from the cis side) are charged, while the second and fourth sections are uncharged. In general $2N_s$ number of sections of length $M/(2N_s)$ will have odd numbered sections from the cis side being charged, while even numbered sections are uncharged, with the same total charge in the pore. This distribution of charges inside the pore significantly affects the translocation process, as observed in the following results.

Figure 1 shows the translocation time distribution for a uniformly charged polymer with a chain length $N = 60$ units. The pore has alternate charged and uncharged sections, with varying patterns as described above. Each polymer bead carries a unit negative charge, while each pore bead in the charged sections carries a fractional positive charge of α_p . (Thus, charged sections of the pore are attractive to the

FIG. 1. Histogram of translocation time, with parameters shown in Table I. Different symbols indicate different numbers of charged sections N_s .

polymer.) A Debye length corresponding to 0.1M monovalent salt is used for screening electrostatic interactions. The first bead of the polymer is placed inside the pore to nucleate the translocation process. Under a constant applied electric field inside the pore, the polymer undergoes translocation from the cis compartment to the trans compartment. The values of parameters used in obtaining these results are given in Table I. Results using these parameters are shown in Figure 1 and are used as a base case for comparison with the rest of the results.

For smaller values of N_s ($=1,2$), the translocation time follows approximately a normal distribution, with a small standard deviation (not shown). As the number of charged sections N_s increases to 4, the distribution gets skewed, with a longer tail corresponding to a slower translocation as shown in Figure 1. The translocation time further increases at $N_s = 8$, with a non-normal distribution having a long exponential tail. However, with any further increase in the number of sections, the translocation time is significantly lower. In other words, all the curves for $N_s > 8$ fall to the left of the histogram for $N_s = 8$. This indicates that slowest translocation occurs at an optimum number of sections in a given length of pore, with corresponding optimum in the section length ($L_{opt} = 2(\times 1.12)$ units).

TABLE I. Parameter values corresponding to results in Figure 1.

Polymer length	N	$60(\times 1.12)$
Pore length	M	$32(\times 1.12)$
Debye Hückel cutoff	r_{c2}	3
Debye length	κ^{-1}	0.81
Charge on pore beads	α_p	0.1
Electric field	E	0.43119

If the charge on each pore bead is reduced to half ($\alpha_p = 0.05$) and the electric field is also halved ($E = 0.215595$), the resulting translocation time distributions show a similar behavior as seen from Figure 2(a). A broader translocation time distribution is seen for an optimum length of section ($L_{opt} = 2(\times 1.12)$ units).

The observed optimum disappears with increase in salt concentration. This can be seen from Figure 2(b), which corresponds to a salt concentration of 1M. For such a high salt concentration, the charges on the pore are considerably screened, thereby reducing the attractive nature of the pore. Hence, the effect of patterns is negligible and the histograms for different patterns almost overlap.

To check if the optimum section length is affected by the cutoff used for electrostatic interactions, simulations were performed at a different r_{c2} with the rest of the parameters as in Table I. Figure 3(a) shows the resulting histograms for different patterns. It can be seen that the slowest translocation takes place at the same number of sections. Thus, the optimum in section length is not an artifact of the cutoff distance used for electrostatic interactions. Length of the polymer chain (N) also does not seem to affect the optimum section length (Figure 3(b)). The slowest translocation is observed at the optimum number of sections, $N_s = 8$.

Increasing the pore length to twice the value in Table I results into shift of the optimum number of sections to $N_s = 16$ (Figure 4(a)). This clearly indicates that the section length is an important parameter that governs the translocation time distribution. The slowest translocation takes place at a section length of $L_{opt} = 2(\times 1.12)$. This is also verified from Figure 4(b), where the pore length is equal to twice of L_{opt} .

FIG. 2. Histogram of translocation time with (a) $\alpha_p = 0.05$, $E = 0.215595$ and (b) $\kappa^{-1} = 0.26$ (1M monovalent salt), with the rest of the parameters as shown in Table I. Different symbols indicate different numbers of charged sections N_s .

FIG. 3. Histogram of translocation time with (a) $r_{c2} = 4$ and (b) $N = 120$ and the rest of the parameters as shown in Table I. Different symbols indicate different numbers of charged sections N_s .

The optimum in this case takes place at $N_s = 1$ which also corresponds to an optimum section length of $L_{opt} = 2(\times 1.12)$.

The average translocation time obtained from the distributions is plotted as a function of length of a section $L_s = M/(2N_s)$ in Figure 5. Each curve represents the results obtained with parameters for corresponding figures as indicated in the legend. It can be seen that independent of the values of the parameters studied, the maximum in translocation time, when present, is observed at the same optimum value of $L_{opt} = 2(\times 1.12)$.

IV. DISCUSSION

As seen from the results in Sec. III, the translocation time distribution is a non-monotonic function of the charge distribution. As we spread the charges inside the pore, the translocation time first increases and then decreases. This non-monotonicity is universally observed at an optimum charge dilution, independent of the lengths of pore and the polymer, within the ranges studied. To understand this, we consider a polymer of length N , with a charge q_1 on each monomer, translocating under an applied trans-membrane po-

tential difference V_0 through a pore of length M with alternate charged and uncharged sections. The process of translocation can be divided into three stages:¹³ the pore filling stage, the translocation stage and the pore emptying stage. In each of these stages, the free energy of the system has four major components to it: pore-polymer electrostatic interaction (F_{ele}), energy due to externally applied electric field (F_{ext}), electro-chemical potential difference between the cis and trans sides (F_{μ}) and the chain entropy (F_{ent}). For a segment of the polymer with $(b - a)$ monomers inside the pore,

$$F_{ele}(a, b) = N_r \frac{q_1}{4\pi\epsilon\epsilon_0} \sum_{i=a}^b \sum_{j=1}^M q_2(j) \frac{\exp(-\kappa r(i, j))}{r(i, j)}, \quad (5)$$

where N_r accounts for interaction between a monomer bead at position i and a ring of N_r pore beads at axial position j , $r(i, j)$ is the distance between the monomer bead and the pore ring and $q_2(j)$ is the pore charge at position j along the pore axis. Note that the sum over index j is performed taking overlapping pore beads into account. The corresponding

FIG. 4. Histogram of translocation time with (a) $M = 64$, $N = 120$ and (b) $M = 4$, $N = 60$ with the rest of the parameters as shown in Table I. Different symbols indicate different numbers of charged sections N_s .

FIG. 5. Average translocation time as a function of the length of a section $L_s = M/(2N_s)$. Different symbols indicate results with parameters of the corresponding figure.

contribution due to external electric field can be written as

$$F_{ext}(a, b) = \sum_{x=a}^b q_1 V_0 \frac{x}{M}. \quad (6)$$

For every monomer that is translocated from cis side to trans side, the electro-chemical potential gradient causes free energy to change by a factor of $\mu = q_1 V_0$. Thus, for x monomers on trans side, $F_\mu(x) = \mu x$. The entropic contribution for a chain with n monomers on cis(or trans) side is given from the entropy for a polymer with one end anchored to a wall,⁵ $F_{ent}(n) = k_B T(1 - \gamma) \ln(n)$.

Using these components with appropriate limits, a free energy landscape can be constructed in terms of a translocation coordinate m . For the pore filling stage ($0 < m < M$), m monomers are inside the pore, while the rest monomers are on cis side. The free energy of the system is given by

$$F(m) = F_{ele}(0, m) + F_{ext}(0, m) + F_{ent}(N - m). \quad (7)$$

In the second stage of translocation ($M < m < N$), M monomers are inside the pore, $N - m$ on the cis side, and the rest on the trans side. In this stage,

$$F(m) = F_{ele}(0, M) + F_{ext}(0, M) + F_\mu(m - M) + F_{ent}(N - m) + F_{ent}(m - M). \quad (8)$$

For the pore emptying stage ($N < m < N + M$), $m - M$ monomers are on the trans side, while the rest are inside the pore. The free energy can be written as

$$F(m) = F_{ele}(m - N, M) + F_{ext}(m - N, M) + F_\mu(m - M) + F_{ent}(m - M). \quad (9)$$

The contribution of chain entropy F_{ent} is found to be negligible in comparison to other components within the range of parameters used and is ignored henceforth. The probability of finding a state corresponding to the translocation coordinate m at time t is governed by the Fokker-Planck equation¹

$$\frac{\partial P(m, t)}{\partial t} = \frac{\partial}{\partial m} \left(\frac{k_0}{k_B T} \frac{dF(m)}{dm} P(m, t) + k_0 \frac{\partial P(m, t)}{\partial m} \right). \quad (10)$$

FIG. 6. Cumulative translocation time distribution for a pore of length $M = 32$ and $N = 60$. Symbols represent data obtained from simulations for different numbers of charged sections N_s , while solid lines represent theoretical predictions from Eq. (11) (for $k_0 = 3125$).

Here, k_0 is a phenomenological parameter related to the effective friction coefficient per monomer on average. This parameter sets the scale for the time variable in the Fokker-Planck theory. Equation (10) can be integrated numerically to get the probability $P(m, t)$, using the appropriate free energy from Eqs. (7)–(9). The probability density for observing the translocation time τ is^{1,28,29}

$$g(\tau) = -\frac{d}{d\tau} \int_0^{N+M} P(m, \tau) d\tau. \quad (11)$$

The derivative of free energy in Eq. (10) is obtained by using a second order finite difference approximation at internal points and a first order finite difference at the endpoints ($m = 0$ and $m = N + M$). The right-hand side of Eq. (10) is then discretized using a second order finite difference scheme, with absorbing boundary conditions¹ at $m = 0$ and at $m = N + M$. Numerical integration is performed in Octave using the standard LSODE solver. A first order backward difference is used to approximate the derivative in Eq. (11) to compute the probability density function.

A comparison of the resulting translocation time distribution with that obtained in simulations for a pore of length

FIG. 7. Free energy landscape for the translocation of a polymer of length $N = 60$ translocating through a pore of length $M = 32$. (Inset) A part of the same data towards the end of translocation process.

FIG. 8. Snapshots of simulation at different times. (a) $N_s = 4$, (b) $N_s = 8$, and (c) $N_s = 16$. The chain translocates from the right-hand side to the left-hand side.

$M = 32$ is shown in Figure 6, by taking the phenomenological parameter k_0 as the only fitting parameter. As seen in Figure 6, the theory predicts that the slowest translocation time occurs for $N_s = 8$, as in the simulations. The choice of $k_0 = 3125$ in the theory enables the quantitative agreement with simulation results. However, the key result of the broadest histogram occurring at $N_s = 8$ is independent of the choice for the value of k_0 . Similar agreements are borne out for other sets of parameter values used in the present simulations as well. Such a good agreement on the nonmonotonic dependence of the translocation histogram on the number of charged sections offers an opportunity to go into the theory, and specifically the free energy landscape, in order to identify the physical reason behind this novel phenomenon.

A typical free energy landscape for the process is shown in Figure 7. The translocation process is downhill for the most part along the translocation coordinate, for all charge distributions. However, a free energy well is present towards the end of the pore emptying stage (inset of Figure 7). This free energy well exists only for a few charge distributions, with varying steepness of the well. The presence of this well results in the long exponential tail observed in the translocation time distribution. This is because the polymer gets trapped inside the well and needs to escape the free energy barrier to exit the pore. The trapping of polymer inside the free energy well can be understood as a balance between the driving force due to electric field and the opposing force due to electrostatic attraction with the pore. In the pore emptying stage, the number of polymer beads inside the pore keeps on decreasing. Thus, the total charges inside the pore keep on reducing as the pore becomes more and more empty, thereby reducing the total force due to electric field. When only a few monomers are present inside the pore, the electric driving force is very weak. Thus, if there are enough charges towards the pore end, then the driving force becomes insufficient to overcome the attractive forces. At very low number of charged sections (N_s), the pore end is neutral and hence the net attractive forces are weak. However, as N_s increases, more and more charges begin to appear near the pore end, resulting in increasing attractive forces. However, at very large values of N_s , even though charges are present near the pore end, their concentration towards the pore end decreases, thereby reduc-

ing the net attractive forces. Thus, only at intermediate values of N_s , the electrostatic attraction of the pore is strong enough to cause significant trapping of the polymer. Sample snapshots of the simulation at different times are shown in Figure 8. It can be clearly seen that for $N_s = 8$, the polymer is trapped towards the pore end for much longer times.

V. CONCLUSION

We have studied the effect of charge distributions along the pore wall on translocation kinetics of a homopolymer. The translocation is observed to be slowest at an optimum charge distribution, independent of the pore and polymer lengths. This is due to the presence of a free energy well near the pore end that results in trapping of the polymer. A simple description based on the Fokker-Planck formalism explains qualitatively this nonmonotonic behavior.

Our results emphasize the significant role of charge distributions in controlling the translocation of a charged polymer across the pore. The translocation time is significantly delayed due to the presence of a free energy well during the pore emptying stage.

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