Andreev reflection, a tool to investigate vortex dynamics and quantum turbulence in $^3\text{He-B}$

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Edited by Katepalli R. Sreenivasan, New York University, New York, NY, and approved October 18, 2013 (received for review July 15, 2013)

Andreev reflection of quasiparticle excitations provides a sensitive and passive probe of flow in superfluid $^3\text{He-B}$. It is particularly useful for studying complex flows generated by vortex rings and vortex tangles (quantum turbulence). We describe the reflection process and discuss the results of numerical simulations of Andreev reflection from vortex rings and from quantum turbulence. We present measurements of vortices generated by a vibrating grid resonator at very low temperatures. The Andreev reflection is measured using an array of vibrating wire sensors. At low grid velocities, ballistic vortex rings are produced. At higher grid velocities, the rings collide and reconnect to produce quantum turbulence. We discuss spatial correlations of the fluctuating vortex signals measured by the different sensor wires. These reveal detailed information about the formation of quantum turbulence and about the underlying vortex dynamics.

vortex reconnection | Kolmogorov spectrum | Richardson cascade | quantized vortices

Quantum turbulence consists of a tangle of quantized vortex lines (1, 2) that interact via their self-induced flow. This results in complex dynamics that may support structures with a large range of length scales. At very low temperatures there is no normal fluid component and no associated viscosity. Conceptually, these are very simple conditions in which to study turbulence. Because the flow is entirely determined by the quantized vortex lines, the problem essentially reduces to a study of vortex dynamics.

Quantum turbulence in superfluid ^4He has been studied for many decades (3–5). There has been a great deal of renewed interest in quantum turbulence in recent years owing to several factors: quantum turbulence was discovered in superfluid ^3He (6–8), techniques were developed to extend the study of quantum turbulence in superfluid ^4He to very low temperatures (9, 10), imaging techniques were developed to visualize superfluid turbulence at higher temperatures (11–13) (see the review in ref. 14), mechanical resonator techniques were developed for quantum turbulence (15–18), and quantum turbulence was studied in dilute gases (19, 20); there were many important theoretical developments, for example refs. 21–26, and numerical methods were enormously improved (27–29) to complement and better interpret experimental results.

In this article, we discuss quantum turbulence generated by vibrating grids in superfluid $^3\text{He-B}$ at low temperatures, where the normal fluid component is very small. The primary thermal excitations in ^3He are quasiparticle and quasihole excitations (30). At low temperatures, the excitation mean free path between collisions vastly exceeds the experimental dimensions. In this ballistic regime, excitations move independently and normally scatter only with the container walls. The normal fluid concept no longer applies because there is no collective motion of excitations.

In addition to normal scattering, excitations can also undergo Andreev reflection. Andreev scattering produces nearly perfect retroreflection of excitations with very little momentum transfer: Quasiparticles become quasiholes, and vice versa. This provides an ideal probe for vortices and quantum turbulence at low temperatures.

Andreev Scattering from Superfluid Flow

The dispersion curve for excitations is shown in Fig. 1A. The excitation group velocity $v_g = dE/dp$ is parallel to the momentum

for quasiparticles that have $p > p_F$, where p_F is the Fermi momentum and is antiparallel to the momentum for quasiholes that have $p < p_F$. The dispersion curve is tied to the reference frame of the superfluid. So, according to an observer moving with velocity $-\mathbf{v}$ with respect to the superfluid, equivalent to a superfluid flow with velocity $\mathbf{v}$ in the laboratory frame, the curve will be tilted by the Galilean transformation

$$E(\mathbf{p}) \rightarrow E(\mathbf{p}) + \mathbf{p} \cdot \mathbf{v}, \quad [1]$$

as shown in Fig. 1B. Excitation states with momenta along the flow direction are shifted to higher energies and those with momenta against the flow are shifted to lower energies.

The energy shift owing to the flow acts as an effective potential:

$$U_{\text{eff}}(\mathbf{r}) = \Delta + \mathbf{p} \cdot \mathbf{v}(\mathbf{r}), \quad [2]$$

where Δ is the superfluid energy gap. In most situations, quasiparticles move much faster than vortex lines, so the effective potential is approximately time-independent. In this case, the quasiparticle excitations move with constant energy E and cannot propagate into regions where $E < U_{\text{eff}}$ (31).

Consider a particle excitation moving to the right in a stationary fluid, for example, the particle excitation shown on the right-hand branch of Fig. 1A, which is incident on a region in which the superfluid is moving to the right (Fig. 1B). As the local flow increases, the excitation experiences a force $\dot{\mathbf{p}} = -\nabla(\mathbf{p} \cdot \mathbf{v})$, which decreases its momentum toward p_F and reduces its group velocity. The excitation, which moves with constant energy, is pushed toward the minimum of the rising dispersion curve. If the flow is sufficiently large ($U_{\text{eff}} > E$) the excitation is pushed around the minimum, becoming a quasihole with a reversed group velocity. The excitation is retro-reflected and the quasihole retraces the path of the incoming quasiparticle. Note that Fig. 1 is not drawn to scale: At low temperatures all thermal excitations have momenta very close to p_F , so a very small momentum transfer, $\sim 10^{-4}p_F$, is required to reverse the group velocity. This is the essential feature of Andreev reflection (32), which provides a passive probe of superfluid flow.

Andreev Scattering from Vortices

The circulating superfluid flow velocity around a vortex is given by (1)

$$\mathbf{v}(r) = \frac{\kappa}{2\pi r} \hat{\mathbf{e}}, \quad [3]$$

where $\kappa = h/2m_3$ is the circulation quantum and m_3 is the bare mass of a ^3He atom. The flow presents a local energy barrier to excitations with the appropriate momentum direction. This gives rise to a large cross-section for Andreev reflection.

Author contributions: S.N.F., Y.A.S., and V.T. designed research; S.N.F., M.J.J., Y.A.S., and V.T. performed research; S.N.F., M.J.J., Y.A.S., and V.T. analyzed data; and S.N.F., Y.A.S., and V.T. wrote the paper.

The authors declare no conflict of interest.

This article is a PNAS Direct Submission.

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Fig. 1. (A) Dispersion curve for the quasiparticle excitations in a stationary superfluid. Quasiparticles (filled circles) have a group velocity parallel to the momentum and quasiholes (open circles) have group velocity antiparallel to the momentum. (B) The dispersion curve for a moving superfluid is tilted. Excitations incident on a region of flow are Andreev-reflected if they have insufficient energy to proceed.

Consider excitations approaching from the left of a vortex, shown in Fig. 2. Far from the vortex, the superfluid is stationary and the dispersion curves are untilted. The circulating superfluid velocity is largest near the core and tilts the dispersion curve depending on the flow direction. In Fig. 2 the dispersion curve is tilted downward for quasiparticles on the upper side of the vortex so these excitations are unimpeded. However, the dispersion curve for quasiholes is tilted to higher energies, so the flow presents an energy barrier. Quasiholes with energies below the barrier are Andreev-reflected. On the lower side of the vortex the flow direction is reversed, so here all of the incident quasiholes are transmitted but low-energy quasiparticles are Andreev-reflected.

Below, we calculate the transmission probabilities for a straight vortex line. We neglect the narrow core region of the vortex and the small probability of “overbarrier” reflection.

The flux of excitations incident on the vortex line in Fig. 2 (the number of incident particles or holes per unit area per unit time) in one dimension can be written as (33)

$$\begin{aligned} \langle nv_g \rangle_i &= \int_0^\infty v_g(E) G(E) f(E) dE \\ &= G(p_F) k_B T \exp(-\Delta/k_B T), \end{aligned} \quad [4]$$

where $v_g(E)$ is the group velocity of excitations with energy E , $G(E)$ is the density of states, $f(E)$ is the Fermi distribution function, which we can approximate to the Boltzmann distribution at low temperatures, and $G(p_F)$ is the density of momentum states at the Fermi surface.

The maximum tilt of the dispersion curve occurs at the distance of closest approach b (the impact parameter) and is given by $|\mathbf{p} \cdot \mathbf{v}| = p_F \hbar / (2m_3 b)$. The tilt is positive for excitations with momenta along the flow direction. The transmitted flux is thus given by

$$\begin{aligned} \langle nv_g \rangle_t^+ &= \int_{\Delta + p_F \hbar / 2m_3 b}^\infty v_g(E) G(E) f(E) dE \\ &= G(p_F) k_B T \exp[-(\Delta + p_F \hbar / 2m_3 b) / k_B T], \\ \langle nv_g \rangle_t^- &= G(p_F) k_B T \exp(-\Delta / k_B T), \end{aligned} \quad [5]$$

where the + and – superscripts correspond to the two sides of the dispersion curve with $\mathbf{p}_F \cdot \mathbf{v}(\mathbf{r}) > 0$ and $\mathbf{p}_F \cdot \mathbf{v}(\mathbf{r}) < 0$, respectively. The fraction of the excitation flux transmitted (the transmission probability) is obtained after dividing by the incident flux given in Eq. 4:

$$\begin{aligned} f_t^+(b) &= \exp(-p_F \hbar / 2m_3 b k_B T), \\ f_t^-(b) &= 1. \end{aligned} \quad [6]$$

The total fraction of excitations that are Andreev-reflected by the vortex is therefore

$$\begin{aligned} f_r(b) &= 1 - \frac{f_t^+(b) + f_t^-(b)}{2} \\ &= \frac{1 - \exp(-p_F \hbar / 2m_3 b k_B T)}{2}. \end{aligned} \quad [7]$$

The reflected flux as a function of the impact parameter is shown in Fig. 2 for typical experimental conditions. The flux is symmetric about the vortex core at $b = 0$ and has a maximum $f_r(0) = 0.5$, corresponding to the total reflection of one component (quasiholes or quasiparticles).

A typical thermal excitation with energy $E \sim \Delta + kT$ will be reflected if $\mathbf{p}_F \cdot \mathbf{v}(\mathbf{r}) > k_B T$. This condition is satisfied for excitations incident on one side of the vortex with impact parameters less than

$$b_0 = \frac{p_F \hbar}{2m_3 k_B T}. \quad [8]$$

We define the Andreev scattering length K as the scattering cross-section per unit length of vortex line. The parameter b_0 sets the scale of the Andreev scattering length, $K \sim b_0$, but its precise value will depend on the geometry. At the lowest accessible temperatures, $T \approx 100 \mu\text{K}$ at 0 bar pressure, $b_0 = 6.3 \mu\text{m}$. This is much larger than the vortex core size, which is of order of the superfluid coherence length ($\xi_0 \sim 50 \text{ nm}$ at zero pressure and $\xi_0 \sim 10 \text{ nm}$ at melting pressure). The large cross-section combined with low momentum transfer make Andreev reflection an ideal probe for vortices. In general, the amount of Andreev scattering is dependent on the geometry of the vortex line and on the presence of surrounding vortices, as discussed below.

Andreev Reflection from a Vortex Tangle

As a first approximation, we can estimate the amount of Andreev reflection from a vortex tangle as follows (34). Consider a thin slab of superfluid, with unit cross-section and thickness Δx , containing a homogeneous vortex tangle of line density (length of line per unit volume) L . The length of vortex line in this volume is $L\Delta x$. Thermal excitations will be Andreev-reflected, on average, if they approach a vortex core to within a distance of $\sim b_0$. We can represent the vortices as tubes of diameter $\sim b_0$ such that the probability Δp that an incident excitation will be Andreev-

Fig. 2. Andreev reflections from a vortex line. (Upper) Dispersion curves for excitations incident on the upper side of the vortex line. (Upper Left and Right) The untilted curves far from the vortex. (Upper Center) The tilted dispersion curve due to the flow. On the top side of the vortex, quasiparticles pass unimpeded but low-energy quasiholes are Andreev-reflected. On the bottom side of the vortex, quasiholes pass, whereas low-energy quasiparticles are reflected. (Lower Right) The fraction of excitations reflected as a function of the impact parameter b , for 4.3 bar pressure and temperature $T = 0.17T_c$, where T_c is the superfluid transition temperature.

reflected is equal to the projected area of the tubes onto the face of the slab, $\Delta p \sim b_0 L \Delta x$.

Now consider a homogenous tangle of thickness d . The probability of an excitation's being Andreev-reflected per unit distance traveled through the tangle is $\Delta p / \Delta x$, so the flux of excitations transmitted through the tangle will decay exponentially with distance. The total fraction of excitations that are Andreev-reflected by the tangle is therefore

$$f = 1 - \exp(-d/\lambda), \quad [9]$$

where the decay length is

$$\lambda = \frac{\Delta x}{\Delta p} \sim \frac{2m_3 k_B T}{\hbar p_F L}. \quad [10]$$

The fraction f of thermal excitations that are Andreev-reflected may be determined experimentally as described below. Rearranging Eqs. 9 and 10 allows us to infer the corresponding vortex line density as

$$L \sim -\frac{2m_3 k_B T}{\hbar p_F d} \ln(1-f). \quad [11]$$

The above model only serves to give us a very rough estimate of the vortex line density. A more precise calculation requires a knowledge of the complex flow field of the vortices.

Numerical Simulations of Andreev Scattering

The semiclassical approach is used to calculate quasiparticle trajectories through a complex network of vortices. This is applicable because the interaction term $\mathbf{p} \cdot \mathbf{v}$ in Eq. 1 varies on a scale that is typically much larger than the coherence length ξ_0 . Following Greaves and Leggett (35), a thermal excitation can be considered as a compact object with an effective mass m^* , position $\mathbf{r}(t)$, and momentum $\mathbf{p}(t)$. The effective mass contains pressure-dependent Fermi liquid corrections, measured by Greywall (36). At low and moderate pressures $m^* \sim 3m_3$. The semiclassical Hamiltonian equations of quasiparticle motion can be written as (37)

$$\frac{d\mathbf{r}}{dt} = \frac{\epsilon_p}{\sqrt{\epsilon_p^2 + \Delta^2}} \frac{\mathbf{p}}{m^*} + \mathbf{v}, \quad \frac{d\mathbf{p}}{dt} = -\frac{\partial}{\partial \mathbf{r}} (\mathbf{p} \cdot \mathbf{v}), \quad [12]$$

where $\epsilon_p = p^2/(2m^*) - \epsilon_F$ is the "kinetic" energy of a quasiparticle relative to the Fermi energy, ϵ_F . The flow field $\mathbf{v}(\mathbf{r}, t)$, which defines the motion of vortices at low temperatures, is obtained by the Biot-Savart integral over all vortex lines.

A qualitative analysis of Andreev scattering from collective flows was first made for a 2D random system of point vortices, each moving in the flow field produced by all other vortices, known as the "Onsager point vortex gas." By simulating quasiparticle trajectories through such systems, it was discovered (38, 39) that the collective flow can have a significant effect on Andreev reflection. The total amount of Andreev reflection is usually less than the sum of the Andreev reflections from each isolated vortex (38, 39). This phenomenon was called "partial screening."

To help interpret experimental results we require theoretical and numerical analysis of Andreev scattering by 3D vortex configurations and vortex tangles. A simple vortex structure of particular interest is the quantized vortex ring (40). A vortex ring propagates owing to its self-induced flow. The velocity of a vortex ring with diameter d is given by (1)

$$v_{\text{ring}} = \frac{\kappa}{2\pi d} \left[\ln\left(\frac{4d}{\xi}\right) - \frac{1}{2} \right], \quad [13]$$

where $\xi \sim \xi_0$ is the radius of the vortex core.

Using the approach described above we have numerically calculated the cross-section for Andreev scattering for various vortex configurations. The cross-section is defined as

$$\sigma = \dot{N}_r / \langle n v_g \rangle_i, \quad [14]$$

where $\dot{N}_r$ is the total number of quasiparticles Andreev-reflected per unit time. The value of $\dot{N}_r$ is found from the numerical solution of Eq. 12 together with the equations governing the vortex dynamics.

The cross-section for Andreev scattering for an isolated vortex ring depends on the orientation (40). It is convenient to define a mean cross-section $\langle \sigma \rangle$ averaged over all possible orientations of a ring. Our numerical results for $T = 0.1 T_c$ can be approximated as (40)

$$\begin{aligned} \langle \sigma \rangle &\approx K_I \pi d - \beta \quad \text{for } d \gtrsim d_s, \\ \langle \sigma \rangle &\approx K_s \pi d \quad \text{for } d \lesssim d_s, \end{aligned} \quad [15]$$

where $d_s = 4.8 \mu\text{m}$, $K_I = 10 \mu\text{m}$, $K_s = 6.8 \mu\text{m}$, and $\beta = 5.1 \times 10^{-11} \text{ m}^2$. As expected, the cross-over length d_s and the Andreev scattering length for small rings K_s are both similar to b_0 given by Eq. 8. The parameter K_I , also comparable to b_0 , can be identified as the Andreev scattering length for large rings with $d \gg d_s$, but for intermediate rings this identification is invalidated by the small offset β . Partial screening is more important for smaller rings because the flow field contributions from different segments of the ring result in better cancellation, leading to less Andreev reflection.

For a "gas" of vortex rings partial screening plays an additional role. For higher ring densities the total cross-section for Andreev reflection is found to be significantly less than the sum of the cross-sections for individual rings. Moreover, for systems of vortex rings with the same total line length the scattering cross-section increases with the size of rings (40).

In general, partial screening has two contributions. The first contribution arises from the modification of the flow field of a vortex by other neighboring vortices. A second contribution arises from the effects of large line curvatures, as found in small rings or in high-frequency Kelvin waves. Kelvin waves are helical disturbances of vortex lines. Energy may be transferred to high frequencies by a "Kelvin wave cascade" (41), where it dissipates by quasiparticle emission. Numerical calculations show that, for typical vortex tangles, the probability distribution function of curvature has a tail indicating a significant contribution from Kelvin waves. For large/dense vortex configurations there is an additional "geometrical screening" where vortices located downstream with respect to the flux of quasiparticles will be partially obscured by vortices in front.

We have performed extensive numerical simulations for excitations incident on a 1-mm cubic box of quantum turbulence with a wide range of vortex line densities. Preliminary analysis reveals several distinct regimes characterizing the total Andreev reflection as a function of the line density. In very dilute tangles screening effects associated with the line curvature are relatively weak, and the fraction of reflected quasiparticles, which grows linearly with L , is similar to that of a system of randomly oriented straight vortex filaments. For moderate line densities the reflected fraction is also approximately linear with L but with a significantly smaller coefficient of growth. In denser tangles with $L \sim 2 \times 10^8 \text{ m}^{-2}$ the fraction of Andreev-reflected quasiparticles is found to be almost constant and completely dominated by screening effects.

Vortex Measurements with Mechanical Resonators

Andreev reflection from vortices has been measured directly using sophisticated quasiparticle beam techniques (34). Below we describe a simpler, and more versatile, technique that allows us to obtain information about the time and spatial evolution of vortices.

Thermal excitations can be probed by small, high-quality-factor mechanical resonators such as vibrating wires (42) or quartz tuning forks (43). The frequency width Δf_2 of the resonance is proportional to the linear damping force per unit velocity. The thermal damping in $^3\text{He-B}$ at low temperatures arises from

scattering with ballistic thermal excitations. This produces a frequency width that can be written as (44)

$$\Delta f_2^T = \frac{\gamma' \langle n v_g \rangle d p_F^2}{2 \pi m k_B T}, \quad [16]$$

where $\langle n v_g \rangle$ is the flux of thermal excitations incident on the wire (fork), m is the mass per unit length of the wire (fork prong) of diameter (width) d , and γ' is a dimensionless geometrical constant. Measurements give values of $\gamma' \approx 0.28$ for vibrating wires (45) and $\gamma' \approx 0.62$ for quartz tuning forks (46).

Andreev reflection from surrounding vortices will reduce the incident flux of excitations on the resonator, which leads to a proportionate reduction in the damping. The fraction of incident excitations that are reflected by the vortices is thus given by the fractional reduction in the thermal damping:

$$f = \frac{\Delta f_2^T - \Delta f_2^V}{\Delta f_2^T}, \quad [17]$$

where Δf_2^T is the thermal damping, measured in the absence of vortices, and Δf_2^V is the thermal damping measured in the presence of surrounding vortices. The vortex line density for dilute tangles may then be estimated using Eq. 11. For denser tangles, where screening effects are dominant, Eq. 11 will significantly underestimate the line density.

In many situations, we only require a qualitative measure of vortex lines. For this purpose, we can define a “vortex signal” S as the product of the fractional Andreev reflection and temperature:

$$S = fT. \quad [18]$$

This gives a temperature-independent measure of the amount of vortices surrounding the wire, independent of any assumptions or approximations concerning the dynamics or the geometry of the surrounding vortices.

In practice, the resonators are usually driven on resonance and the resulting signal is measured continuously, from which we can infer the resonant width as a function of time $\Delta f_2(t)$. Because the excitations are ballistic, there are no temperature gradients in the cell, so the thermal damping in the absence of vortices Δf_2^T may be measured simultaneously using a remote “thermometer” resonator, which is also used to determine the temperature T (45). To increase the signal-to-noise ratio, the resonators may be driven at higher velocities where the thermal damping becomes nonlinear. The nonlinear thermal damping is well understood

Fig. 3. The experimental arrangement showing the grid, the sensor wires, and the thermometer wire located behind a screen. (Inset) An electron microscope image of the grid mesh.

Fig. 4. The vortex signal from each of the four sensor wires as a function of time while the grid is driven at 7.2 mm/s for a period of 110 s.

(33, 47, 48) and can be calibrated for each wire to infer the low-velocity damping $\Delta f_2(t)$.

Vortex rings and turbulence have a negligible direct effect on the wire response, as measured at the lowest achievable temperatures in the virtual absence of thermal excitations. However, small features associated with local vortex production from remnant vortices have been observed (49).

Experimental Arrangement

All of the measurements presented below were taken in superfluid $^3\text{He-B}$ at a pressure of 4.3 bar, over a temperature range from $T = 0.17T_c$ to $T = 0.19T_c$. The superfluid is cooled by a Lancaster-style nuclear cooling stage (50, 51) mounted on the Lancaster advanced nuclear cooling refrigerator (52). The experimental arrangement is illustrated in Fig. 3.

The turbulence was generated using the vibrating grid resonator labeled grid 1 in Fig. 3. Three sensor wires, labeled 1, 2, and 3 in the figure, were positioned directly behind grid 1 at distances of 1.47, 2.37, and 3.49 mm, respectively. A fourth sensor, wire 4, was placed roughly 5.8 mm away from grid 1 and is located behind a second grid, grid 2.

Each vibrating grid resonator is formed from a 125- μm -diameter Ta wire, bent into a 5-mm square frame, which supports a 5.1×3.5 -mm copper mesh. The grid mesh is glued to the Ta wire over a thin layer of diluted GE varnish for electrical insulation. The inset to Fig. 3 shows an electron microscope image of the mesh used for grid 1. The copper mesh is quite thin, $\sim 1 \mu\text{m}$. It has a periodicity of $34.5 \mu\text{m}$ and the square holes have side lengths $22.6 \mu\text{m}$. The grid resonates at a frequency ($\sim 1,340$ Hz for grid 1) determined by its effective mass and the stiffness of the Ta wire legs. The grid used in earlier experiments (53–59), which we refer to as the “earlier grid,” had a similar resonant frequency. The earlier grid mesh had the same periodicity and a similar overall size, 5.1×2.8 mm, but it had a much rougher surface (58) and was much thicker, $\sim 11 \mu\text{m}$.

Grid 2 was made using the earlier grid mesh material. Unfortunately, grid 2 had a large intrinsic damping that led to very significant heating at velocities required to produce turbulence. The results presented here, therefore, focus on vortices generated by grid 1.

Each vibrating wire resonator is formed by a 2.5-mm-diameter loop of a 4.5- μm -diameter NbTi wire (42). The four sensor wires were positioned such that the top of the wire loops are approximately aligned with the center of the mesh of grid 1 along the axis of motion. The thermometer wire, used to infer the thermal quasiparticle flux in the absence of vorticity, is located about 3 mm to the side of the grid and is placed behind a screen.

Previous experiments (60) suggest that turbulence tends to be produced preferentially along the axis of motion, so we do not expect much turbulence to the side of the grid. The screen is a further precaution to minimize the turbulence in the vicinity of the thermometer wire.

The vibrating grid resonator is operated using the same techniques for vibrating wire resonators (42). The grid is driven by applying an alternating current I through the Ta wire in a vertical applied magnetic field, $B \sim 100$ mT. The resulting Lorentz force is

$$F = BID, \quad [19]$$

where $D = 5$ mm is the length of the Ta wire crossbar. As the grid moves, a Faraday voltage is induced with amplitude

$$V = BDv, \quad [20]$$

where v is the velocity amplitude of the top of the grid (the crossbar). The Ta wire is superconducting, so background voltages are very small. The background voltages are measured in zero field and are subtracted from the measured voltage to leave only the Faraday contribution. The current is supplied by a function generator that outputs a voltage across an appropriately chosen series resistor. The Faraday voltage is preamplified by a low-temperature transformer and measured with a phase-sensitive lock-in amplifier.

A typical measurement proceeds as follows. We produce vortices by driving grid 1 on resonance for a period of time. We simultaneously monitor the damping on the four sensor wires and the thermometer wire. Using Eq. 17 for each sensor wire, we infer the fraction of incident thermal excitations that are Andreev-reflected by the surrounding vortices. We then find the vortex signal S , Eq. 18, which gives us a qualitative measure of the local density of vortices. For dilute vortex tangles we can also estimate the local vortex line density L using Eq. 11.

Fig. 4 shows a typical measurement where the grid is driven to a velocity amplitude of 7.2 mm/s for a period of 110 s. The vortex signal is shown for each of the four sensor wires. We can define three distinct regimes: (i) the onset of the vortex signal, (ii) the steady state, which displays significant fluctuations, and (iii) the decay of the vortex signal after stopping the grid motion. Below we first focus on the information obtained from the steady-state regime and then compare it with observations for the onset and decay regimes.

Fig. 5. Cross-correlations between signals from sensor wires 1 and 2 (A), wires 2 and 3 (B), wires 1 and 3 (C) at various grid velocities. The signals arise from ballistic vortex rings emitted by the grid. The cross-correlation peak shows a time delay corresponding to the velocity of the vortex rings.

Fig. 6. The mean velocity of vortices propagating between different pairs of sensor wires, determined from the corresponding cross-correlation function, versus the grid velocity amplitude. For low grid velocities, the velocity corresponds to that of ballistic vortex rings. For grid velocities above 2.7 mm/s the velocity corresponds to the drift velocity of quantum turbulence.

Spatial Correlations of Vortex Rings

From previous measurements it is known that a vibrating grid in $^3\text{He-B}$ produces small vortex rings. The rings propagate away ballistically at low grid velocities, so the observed vortex signal decays quickly. As the grid is driven to higher velocities, the flux of rings increases. Eventually, the rings collide and recombine. Recombination may produce larger rings that travel more slowly. These have a greater chance of further recombinations resulting in the production of a vortex tangle (quantum turbulence). The above scenario was first inferred from measurements of the decay of the vortex signal (54) and was confirmed by numerical simulations (56). We can obtain more detailed information from the steady-state regime by studying fluctuations, as discussed below.

The steady-state vortex signal exhibits substantial fluctuations in the turbulent regime (59). Fluctuations from vortex rings emitted at low grid velocities are much smaller. In the current experiments we have enhanced the signal-to-noise ratio sufficiently to allow us to study these in more detail.

Fig. 5 shows cross-correlations between the vortex ring signals from the first three sensors, 1, 2, and 3. The cross-correlation is defined as

$$R_{a,b} = \frac{\langle \delta S_a(t) \delta S_b(t - \Delta t) \rangle}{\sqrt{\langle \delta S_a^2 \rangle} \sqrt{\langle \delta S_b^2 \rangle}}, \quad [21]$$

where Δt is the delay time, δS_a and δS_b are the fluctuations in the vortex signals from wires a and b , respectively, where $a, b = 1, 2, 3$. Fig. 5A shows the cross-correlations between sensor wires 1 and 2, Fig. 5B shows the cross-correlations between sensor wires 2 and 3, and Fig. 5C shows the cross-correlations between sensor wires 1 and 3. The signal from the fourth sensor is too small to give useful results.

Fluctuations in the signals from vortex rings arise from fluctuations in the size and density distribution of the rings in the vicinity of the sensor. These fluctuations will propagate outward with the vortex rings. So, a sensor far from the grid will experience a given fluctuation at later times compared with a near sensor. This is clearly observed in the data shown in Fig. 5. The peak of the cross-correlation for neighboring wires ~ 1 mm apart ($R_{1,2}$ and $R_{2,3}$ in Fig. 5A and B, respectively) occurs at a delay time of $\Delta t \sim -0.1$ s, whereas the peak of the cross-correlation for the wires ~ 2 mm apart ($R_{1,3}$ in Fig. 5C) occurs at a delay time of $\Delta t \sim -0.2$ s. This indicates that the vortex rings propagate away from the grid with

Fig. 7. The cross-correlation function $R_{1,2}$ for sensor wires 1 and 2 for various grid velocities as indicated. At low grid velocity the narrow peak is due to vortex rings. At higher grid velocities quantum turbulence forms, with slower-moving structures giving cross-correlations at much longer time delays (discussed in the text).

a mean velocity of about 10 mm/s. More precise values for the mean ring velocities, calculated as the wire separation divided by the delay time of the cross-correlation peak, are shown in Fig. 6. The inferred mean ring speed $v = (7 \pm 3)$ mm/s seems to be independent of the grid velocity within experimental uncertainties.

Such a high velocity can only be achieved by small, independent vortex rings (a vortex tangle evolves much more slowly, as discussed below). The self-induced velocity of a vortex ring is given by Eq. 13. This implies a mean ring diameter of $d \approx (10 \pm 5)$ μm , consistent with earlier estimates based on measurements of the decay time of the vortex signal (55) and on measurements of the spatial decay of the rings at higher temperatures owing to mutual friction (57). However, Eq. 13 is strictly only applicable to circular rings. In practice, Kelvin waves excited on the rings will tend to slow the rings (61), so the inferred ring diameter may be slightly overestimated.

Within the experimental uncertainty, the inferred ring speeds in Fig. 6 are independent of which sensors are used, indicating that the rings travel at a nearly constant velocity. The rings should dissipate owing to mutual friction with the thermal excitations. Dissipation arises from normal scattering of thermal excitations with bound excitations in the vortex core (62). As the rings dissipate they shrink and hence travel faster according to Eq. 13. However, previous measurements (57) indicate that dissipation is not significant at these low temperatures, so the rings propagate at constant velocity to the cell walls, where they are absorbed.

The cross-correlation peaks in Fig. 5 have half-widths of around 1 s. The width of the cross-correlation peak roughly coincides with the frequency spectrum of the fluctuations (the dominant fluctuations are below 1 Hz) (59). This “intrinsic broadening” must reflect the ring production process, but the details are not yet known. In principle, one would also expect to see an “extrinsic broadening” of the correlation peaks owing to the distribution of ring speeds, corresponding to different ring diameters. Extrinsic broadening should cause the correlations to stretch with the distance traveled. So, for instance, the extrinsic broadening of $R_{1,3}$ should be roughly twice as large as that of $R_{1,2}$ and $R_{2,3}$. Because we see no evidence of this, we infer that the intrinsic broadening dominates and that there are few vortex rings with diameters greater than ~ 20 μm (larger-diameter rings would produce a measurable extrinsic broadening).

Spatial Correlations of Quantum Turbulence

Fig. 7 shows cross-correlation function $R_{1,2}$ for wires 1 and 2 for quantum turbulence at various grid velocities. For reference, the plot includes a dataset for vortex rings emitted at a lower grid velocity. As the grid velocity increases, vortex rings are emitted at a higher rate and ring collisions become more frequent. On collision, larger rings may be generated along with high-frequency Kelvin waves (56). The larger rings move more slowly and have a greater probability of further collisions with the smaller rings, thus acting as the seeds for quantum turbulence. Computer simulations (56) show that these large “seed” rings have very irregular shapes and a broad range of sizes. This should produce a spreading of the cross-correlation toward larger negative time delays because the rings take longer to pass from one sensor to another. This is observed in the measurements shown in Fig. 7. At a grid velocity of 2.7 mm/s, the fast, narrow vortex ring peak is superimposed on a broad tail, which we associate with the large, irregular seed rings.

At slightly larger grid velocities (ring production rates) simulations (56) show that the larger seed rings become tangled, resulting in quantum turbulence. In Fig. 7 we see that for grid velocities above ~ 3 mm/s the narrow peak from the vortex rings is completely absorbed into the much broader turbulence peak. At first sight, this seems to suggest that almost all of the small primary vortex rings generated by the grid are absorbed by the vortex tangle before they reach the sensor wires. The cross-correlation function is normalized by the size of the fluctuations (Eq. 21). The fluctuations are much larger when quantum turbulence is generated, and thus the relative size of the ring signal shrinks.

At higher grid velocities the long tail in the cross-correlation transforms into a well-defined peak, shown in Fig. 7. We associate this peak with a well-developed vortex tangle (quantum turbulence). The existence of such a tangle was first inferred from the decay behavior of the vortex signals (55) and was later observed in dedicated computer simulations (56). The peak is quite broad but is clearly shifted to negative time delays. This suggests that the vortex tangle has some outward drift velocity. On increasing grid velocity, the cross-correlation peak gains height, narrows, and shifts to less-negative time delays. The inferred drift velocities are shown in Fig. 6.

Signal Onset and the Turbulent Front Velocity

We compare the cross-correlation data with the vortex signal at the onset of turbulence shown in Fig. 4. The vortex signal from each sensor wire rises quickly when the grid is driven. This

Fig. 8. The vortex signal detected by the nearest sensor, wire 1 as a function of time after stopping the grid. Each curve corresponds to a different initial grid velocity as indicated. At the lowest grid velocity, the signal is due to vortex rings. At higher grid velocities there is a superposition of vortex rings and quantum turbulence.

corresponds to the arrival of the primary vortex rings emitted by the grid. Unfortunately, the measurement time constant is too slow to resolve the transit time of the rings; otherwise, this would have given an independent measure of their speed.

For the nearest wire, wire 1, the initial rise is followed by a plateau and then a second rise. We associate the second rise with the onset of the turbulence. The second rise occurs at later times for the sensor wires further from the grid. We conclude that the turbulence initiates close to the grid where the flux of primary vortex rings is largest. The turbulence then spreads outward. The second rise of the vortex signal from each of the sensor wires is quite sharp, which suggests that the leading edge of the turbulence, the “turbulent front,” is well-defined.

We can infer the velocity of the turbulent front by dividing the distance between each sensor and the grid by the corresponding onset time for the turbulent signal. For the data shown we infer that the front moves at a roughly constant velocity of ~ 0.35 mm/s. This is roughly twice smaller than the drift velocity of the turbulence shown in Fig. 6.

For wires 2 and 3 the initial rise owing to vortex rings is followed by a transient dip in the vortex signal. This suggests that the developing turbulence close to the grid absorbs some of the primary vortex rings. The dip is more pronounced on wire 3 because this wire is further from the grid, so the primary rings have to traverse a larger region of turbulence. However, the dip is absent for wire 4, which is located behind the second grid. The holes in the second grid are only ~ 23 μ m wide, so this will preferentially absorb the larger rings, as will the intervening turbulence. So, we attribute the fast signal on wire 4 to smaller primary vortex rings, which are less affected by the developing turbulence.

Decay of the Vortex Signal

Fig. 8 shows the decay of the vortex signal for wire 1 after turning off the drive to grid 1. At the lowest grid velocity, the signal corresponds to vortex rings. The vortex rings travel quickly, and hence the signal decays rapidly when the grid is stopped.

At higher grid velocities the rings collide and recombine to generate turbulence. The turbulence decays on much longer time scales, so we expect the vortex signal to decay slowly, as observed for measurements on the earlier grid (55). However, at higher initial grid velocities Fig. 8 shows both fast and slow components to the decay. This implies a superposition of primary vortex rings and a vortex tangle, so not all of the primary vortex rings are absorbed by the intervening turbulence. This is also evident from the data shown in Fig. 4, where wire 4 detects a fast vortex ring signal at the same time as the nearer wires detect turbulence. Coexistence of quantum turbulence and primary vortex rings was observed in measurements with the earlier grid, but only very close to the transition region (54).

In the turbulent regime, the vortex line densities inferred from Eq. 11 are roughly twice larger for the earlier grid. Hence, one would expect the turbulence from the earlier grid to be more efficient at absorbing the primary vortex rings. The maximum inferred vortex line densities from Eq. 11 are of order 10^8 m $^{-2}$ for the

earlier grid. However, as discussed above, numerical simulations show that Eq. 11 may seriously underestimate the line density for such dense tangles. Based on estimates of the ring transmission probability, using arguments similar to those given above for Andreev reflection, we estimate for the earlier measurements (55) at high grid velocity the true vortex line density close to the grid may have been at least an order of magnitude higher.

Comparing the relative sizes of the vortex signals at high grid velocities, we find that the current grid produces a much larger and more open vortex tangle compared with the earlier grid. Consequently, it has a higher ring transmission probability, which explains the supposition of fast and slow components to the decaying vortex signals in Fig. 8. Further work is needed to understand how the properties of the quantum turbulence depend on the properties of the vibrating grid.

Summary

Andreev reflection offers an ideal tool for studying vortex dynamics and quantum turbulence in superfluid $^3\text{He-B}$ at very low temperatures. Vortices have a large cross-section for the Andreev reflection of thermal quasiparticle excitations. The momentum transfer is very small, so the reflection has negligible effect on the vortex dynamics. Mechanical resonators measure the local flux of thermal excitations, which thus provide convenient local probes of Andreev reflection. The principles of the reflection process are well understood. For dilute tangles the amount of Andreev reflection is proportional to the vortex line density. For dense tangles the reflection is sensitive to collective flows. The latter property might prove useful for further probing the nature of large-scale flow in quantum turbulence.

A vibrating grid resonator is a very convenient device to generate vortices in superfluid $^3\text{He-B}$. It produces vortex rings that disperse ballistically at low grid velocities. The rings recombine to generate vortex tangles (quantum turbulence) at higher grid velocities. Using an array of sensors, we gain information about the spatial and time evolution of the turbulence. We gain useful information from the fluctuations in vortex signals measured simultaneously at different locations. The cross-correlation functions provide information on vortex dynamics within the quantum turbulence. The fluctuating vortex signals depend on the superfluid flow over a range of length scales and may be sensitive to large-scale collective flows, which are expected from analogies with classical turbulence (63–65). However, further work is required to gain a quantitative understanding of Andreev reflection from the complex collective flows found in quantum turbulence.

ACKNOWLEDGMENTS. We thank A. Stokes and M. G. Ward for excellent technical support and C. F. Barenghi, D. I. Bradley, A. M. Guénault, R. P. Haley, P. V. E. McClintock, G. R. Pickett, N. Suramishvili, and M. Tsubota for useful discussions. This research is supported by the United Kingdom Engineering and Physical Sciences Research Council (EPSRC UK), the Leverhulme Trust, and the European FP7 Program MICROKELVIN Project 228464.

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