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Analytical Quantification of the Subslab Volatile Organic Vapor Concentration from a Non-uniform Source

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Abstract

The transport of volatile organic vapors from subsurface to building involves complex processes. Since the release of the draft subsurface vapor intrusion guidance by the U.S. EPA in 2002, great progress has been made in understanding these processes in various field and modeling studies. In these studies, the importance of analyzing and predicting the subslab volatile organic vapor concentration was noted. To quantitatively predict subslab vapor concentration is, however, complicated, especially for sites located over non-uniform vapor sources. This manuscript provides a method to estimate the vapor concentration beneath the subslab using a closed-form analytical solution that can approximate full three-dimensional modeling results, but does not require the use of advanced numerical simulation. This method allows prediction of the subslab vapor concentration profile beneath the slab for various source configurations, given inputs of building slab dimension and source depth. The interaction of the influences of non-uniform source and the slab capping effect on the subslab vapor concentration is addressed.

Keywords

Analytical approximation; Vapor intrusion; Non-uniform source; Slab capping effect

1. Introduction

The risks of inhalation of indoor volatile organic compounds (VOCs) were first addressed by the U.S. EPA guidance in 1992 (EPA, 1992), and since then, the associated health and safety concerns have been discussed in numerous field studies (Fischer and Uchirin, 1996; Fitzpatrick and Fitzgerald, 2002; Folkes et al., 2010; Luo et al., 2009; McAlary et al., 2009; McHugh et al., 2012; Sanders and Hers, 2006; Turczynowicz et al., 2012) and modeling studies (Johnston and Gibson, 2010; Mills et al., 2007; Murphy and Chan, 2011; Park, 1993; Turczynowicz and Robinson, 2007; Wang et al., 2012; William et al., 2007). In 2012, the U.S. Office of Superfund Remediation and Technology Innovation (OSRTI) (EPA, 2012c) developed a vapor intrusion screening level calculator. In this calculator, the default value for the groundwater attenuation factor, defined as the predicted indoor air contaminant vapor concentration / contaminant vapor concentration in equilibrium with a groundwater source,

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was assigned a value of 0.001, and the soil gas attenuation factor, defined as the indoor air contaminant vapor concentration / subslab (or soil gas) contaminant vapor concentration, was assigned a value of 0.1. In fact, an examination of the data in the U.S. EPA vapor intrusion database shows that no single attenuation factor value is nearly adequate for describing the broad range of field results (Suuberg et al., 2011). A recent review of state guidance documents (Eklund et al., 2012) also showed large variations in recommended groundwater and soil gas screening levels between different states. These large discrepancies show a continuing need to improve the understanding of the subsurface vapor transport processes. Further, the importance of focusing more on subslab and soil gas data, rather than only on soil and groundwater characterization, was recognized in field studies (Eklund and Simon, 2007; Luo et al., 2009).

In 2012, a U.S. EPA document (EPA, 2012a) summarized and shared examples of various vapor intrusion scenarios using two sets of numerical models. The “baseline model” generally assumed a single infinite vapor source that existed at the top of the capillary fringe. In this case, the source vapor concentration was uniformly distributed beneath a domain that held a structure of interest. A relatively higher contaminant soil vapor concentration presented beneath the slab than at comparable depths outside the building footprint, which has been attributed to the slab capping effect (Shen et al., 2014). This capping effect may also extend deep into the soil, leading to higher contaminant vapor concentrations at the top of the capillary fringe beneath the building than at points away from beneath its footprint.

In the above 2012 EPA document, an “alternative model” also considered a finite contaminant source, associated with a somehow bounded subsurface vapor source (e.g., plume) that was either located underneath the building footprint or laterally separated from beneath the building footprint (Abreu and Johnson, 2005; Yao et al., 2013b). In this case, predicting the subslab vapor concentration is much more complicated.

Results from such studies are instructive, and offer more insight into real scenarios than can simple one-dimensional models. They are, however, not easily accessible to the field practitioner, in the sense that they require running a three-dimensional fluid dynamics code of some kind (Yao et al., 2012b, 2013c, e).

As distinct from these previous studies, this manuscript presents a simpler way to analyze the effect of a non-uniform contaminant source distribution on subslab vapor concentration, given an interest in a steady state vapor intrusion process. We present a new closed-form solution for the subslab vapor concentration profile in a non-uniform source scenario. This solution is applied to several different simplified scenarios. Since analytical solutions are rare for more complicated situations, we explore how superposition of the simple analytical solutions can expand their applicability to more complicated source configurations that might normally suggest use of numerical modeling at the outset.

2. Modeling Methodology

A typical vapor intrusion scenario is shown in Figure 1(a). The two-dimensional (2D) analytical model scenario is shown in Figure 1(b) with its governing equation and boundary conditions. All the parameters are shown in the Nomenclature. The soil is assumed to be homogenous (the total effective diffusivity D is constant). In the analytical model, the vapor concentration profile along the slab is the main focus. Soil gas advection is not considered in this analytical model, as advection has been seen to have limited effect on subsurface vapor concentration distribution, even though there may be a small zone near a foundation entry crack where advection contributes significantly (Abreu and Johnson, 2005; Bozkurt et al.,

2009; Pennell et al., 2009; Yao et al., 2011). A non-uniform contaminant vapor source is assumed, and is represented as consisting of three pieces as illustrated in Figure 1(b). This source can be assumed to exist at the top of the capillary fringe, and then the effect of capillary fringe resistance to vapor diffusion is for the moment ignored. The middle of the source zone has a length α , and is characterized by a vapor concentration c_τ . The left and right portions of the source zone are both taken to be at the same concentration c_l , and extend horizontally to infinity. Therefore, the y -axis is a symmetry plane as shown in Figure 1(b). When $c_\tau = c_l$, the bottom boundary would represent a uniform source. When $c_\tau > c_l = c_0 = 0$, the bottom boundary represents a finite size source centered beneath the structure. When $c_\tau > c_l$, the bottom boundary represents a non-uniform source, with a vapor hotspot beneath the structure. When $c_l > c_\tau$ the bottom boundary involves laterally separate sources displaced from beneath the structure.

In deriving the analytical solution for the subslab contaminant concentration, we have used Schwarz-Christoffel mapping. This method has been widely used in solving transient heat or mass transfer problems, as both of them are governed by the diffusion equation (Chuangchid and Krarti, 2000; Driscoll and Trefethen, 2002; Krarti et al., 1988). We have also used them as well in our previous vapor intrusion modeling work (Shen et al., 2014; Yao et al., 2012a). Generally, the derivation of the solution includes three solution planes or spaces (see Appendix): one with a known solution of the concentration field; a complex space that maps the first space; and the third one with the unknown concentration field, which can also be mapped to the second complex space. The only difference compared to earlier approaches is that, in the current derivation, the first space consisted of a known solution of the concentration field that was obtained from the analytical solution presented by Krarti *et al.* (Krarti et al., 1988). In the problem addressed by Krarti, a temperature field was of concern. The concentration field in our current scenario, is also governed by the same diffusion equation as that for heat diffusion. The detailed derivation can be found in the Appendix.

The solution for a two-dimensional vapor intrusion scenario such as that in Figure 1, is for vapor concentration (c) along the subslab ($-\beta/2 \leq x \leq \beta/2$, $y = l$):

$$c(x, y=l) = c_\tau - (c_\tau - c_0) \frac{\Theta}{\pi} + (c_l - c_\tau) \frac{2}{\pi} \arctan\left(\frac{\sin\Theta}{\Phi - \cos\Theta}\right) \quad \text{Eq. (1)}$$

where,

$$\Theta = \arccos\left[-2e^{\pi(\beta/2-x)/l} \left(\frac{e^{\pi x/l} + 1}{e^{\pi\beta/(2l)} + 1}\right)^2 + 1\right] \quad \Phi = \left[\sqrt{\frac{e^{\pi\beta/(2l)}(e^{\pi\alpha/(2l)} - 1)^2}{e^{\pi\alpha/(2l)}(e^{\pi\beta/(2l)} - 1)^2 + 1}} + \sqrt{\frac{e^{\pi\beta/(2l)}(e^{\pi\alpha/(2l)} - 1)^2}{e^{\pi\alpha/(2l)}(e^{\pi\beta/(2l)} - 1)^2 + 1}} \right]^2$$

Equation (1) is the solution for the subslab concentration with a non-uniform source. This concentration profile is a function of the non-dimensional source size α/l , the non-dimensional building size β/l , and the source strengths c_τ , c_0 , and c_l . Equation (1) can be used to obtain the subslab concentration, the maximum vapor concentration and its location beneath the slab. The concentration at the subslab center c_{ss} can be obtained by letting $x = 0$ in Equation (1):

$$c_{ss} = c_\tau - (c_\tau - c_0) \frac{\Theta}{\pi} + (c_l - c_\tau) \frac{2}{\pi} \arctan\left(\frac{\sin\Theta}{\Phi - \cos\Theta}\right) \quad \text{Eq. (2)}$$

and Θ becomes

$$\Theta = \arccos \left[-\frac{8e^{(\pi/2)(\beta/l)}}{(e^{(\pi/2)(\beta/l)} + 1)^2 + 1} \right]$$

with Φ as above.

The indoor air vapor concentration is not directly provided by this analytical model. However, the indoor air vapor concentration can be calculated by using the averaged subslab vapor concentration (Krylov and Ferguson, 1998; Olson and Corsi, 2001) or vapor concentration near the crack (Yao et al., 2013a), and combining it with an appropriate model of entry across a foundation breach (i.e., advective or diffusive).

A 2D modeling approach has been widely used in transport process modeling in non-vapor intrusion scenarios, as it captures many of the key elements of the solution in a mathematically simpler form than is possible for a full three dimensional representation (Claesson and Hagentoft, 1991b; Diallo et al., 2012; Krarti et al., 1988; Yao et al., 2012a). For accurate work, a full three-dimensional (3D) model is still preferred, and this will normally force use of a numerical approach. The subslab concentration results obtained from a 2D vapor intrusion model have nevertheless been shown to reasonably approximate the full 3D modeling results (Shen et al., 2014) in many respects. For example, Figures 2(a) and (b) compare the 2D and 3D *numerical* modeling results for the indicated boundary conditions. The form of the equations and boundary conditions relevant to the 3D numerical solution shown are those typically used in vapor intrusion modeling (Bozkurt et al., 2009), and are therefore not repeated in this manuscript. A finite element package, COMSOL, was used in solving the equations numerically.

Figure 2(a) shows the subsurface vapor concentration distribution, simulating the same scenario as shown in Figure 1(b). Figure 2(b) shows the cross section of the contaminant concentration distribution obtained using the typical 3D numerical model (Bozkurt et al., 2009; Pennell et al., 2009; Shen et al., 2013a; Shen et al., 2013b), without considering soil gas advection, which is not considered in the 2-D approximations to be examined below. The building has a footprint dimension of 10 m $\times$ 20 m. In both Figures 2(a) and (b), the source concentration hotspot ($c_s = 10$) characteristic width is 20 m, centered beneath the building slab. The concentration profiles in both figures are almost identical, which again shows that a 2D modeling approach can approximate well the 3D modeling results. Moreover, including soil gas advection by introducing a 0.01 m width perimeter crack and an indoor depressurization of 5 Pa does not significantly change the 3D concentration distribution (result not shown), as expected from earlier results by Yao *et al.* (2013a). Thus if a 2D numerical model of the problem captures well the main features of a 3D scenario, then if a reliable 2D analytical approximation can be developed, it should also capture the main features of the problem. Of course, such a 2D analytical expression should be mathematically more tractable. We believe that the 2D model developed here does so successfully, and examples are offered below.

3. Results and Discussion

3.1 A base case: the concentration profile from a non-uniform source to an open field surface

The factors that determine the subslab vapor concentration in a non-uniform source situation, such as that in Figure 1, involve two main aspects: the non-uniform source distribution and the building slab capping effect. Before considering the both factors together, let us consider a base case that describes the vapor transport from a non-uniform

vapor source through an open field, without including any building or capping structure. Here, the modeling scenario, which eliminates the slab capping effect, is shown in Figure 3(a). The concentration along the center line ($x = 0, 0 \leq y \leq l$) can be expressed by Equation (1), except now $\Phi = \exp(\pi\alpha/(2l))$, and $\Theta = \pi y/l$:

$$c(x=0, y) = c_\tau - (c_\tau - c_0)y/l + (c_1 - c_\tau) \frac{2}{\pi} \arctan\left[\frac{\sin(\pi y/l)}{e^{\pi\alpha/(2l)} - \cos(\pi y/l)}\right] \quad \text{Eq. (3)}$$

In the above equation, the vapor concentration along the center line is determined by the middle source concentrations c_τ , side source concentration c_1 , open field vapor concentration c_0 , and the middle source size α . It is obvious that this solution reduces to the linear concentration profile expected when α approaches infinite size.

Superposition of the concentration distribution solutions is also obvious. The superposition principle is well-known in working with linear partial differential equations (Arfken et al., 1985). The first two terms in Equation (3) describe the concentration which decreases linearly with elevation (see Figure 3(c)), resulting from vapor transport from a uniform source c_τ to an open field surface at c_0 . The third term describes the influence of side sources of concentration $(c_1 - c_\tau)$ at a distance α apart from each other, as shown in Figure 3(d). If $c_1 - c_\tau < 0$, the third term stands for a “sink” term. The solution expressed in Equation (3) superimposes the two hypothetical scenarios in Figures 3(c) and (d). Other examples of the use of the superposition principle can be found in solving related transport problems (Arfken et al., 1985; Claesson and Hagentoft, 1991a).

Assuming $c_0 = 0$, the vapor concentration profiles are plotted in Figure 3(b). This is not a subsurface concentration, because the slab blockage of capping effect has not been included. When $\alpha/l = 5$, the expected linear concentration profile with a large source is thus obtained, as the concentration profile is controlled by the first two terms in Equation (3). When α/l is small, especially for the cases $\alpha/l \leq 1$, non-linear concentration profiles are obtained. This kind of result is also a reminder that when probing open field soil gas concentration vertical profiles, discovery of a non-linear vertical profile is not necessarily an indication of soil non-uniformity. The variation in vapor diffusivities in a wide range of dry soils is actually not great, so other causes for non-linear profiles should be considered. In order to obtain a non-linear profile such as that in Figure 3(b), a fairly localized upwelling of contaminant-rich groundwater could be indicated. But note that the required $\alpha/l = 0.5$ defines a fairly restrictive criterion of localized upwelling that just happens to be beneath a sampling point. On the other hand, Figure 3(b) also suggests that extrapolation of concentrations from different positions and depths down to the source can give widely varying estimates of source strength, again, even for a uniform soil.

3.2 The influence of the non-uniform source on subsurface concentration

The above section has characterized the influence of the non-uniform source effect on subsurface vapor concentration. We will generalize now to consider the effect of non-uniform source in a building-capped scenario. Assume again that the contaminant vapor source concentration can be approximated using a simplified piecewise effective source vapor concentration which takes into account the source non-uniformity. In this case, the subsurface vapor concentration can be estimated using Equations (1) and (2). Again, superposition of concentration solutions has been used in obtaining Equations (1) or (2). The concept is the same as discussed above for the base case, except now the subsurface area is considered here. The first two terms of Equation 1 describe the solution for the subsurface vapor concentration for a hypothetical uniform source scenario. These two terms agree with

a previous solution for a uniform source scenario (Shen et al., 2014). The third term describes the influence of side sources ($c_l - c_d$) on the subslab vapor concentration.

For the scenario shown in Figure 1, the uncapped open field concentration c_0 is here assumed to be zero, and the middle source represents a hotspot beneath the slab $c_\tau > c_l$. The subslab center vapor concentration will in that case be the maximum vapor concentration directly beneath the slab. Equation (2) is used to calculate the subslab center vapor concentration; and results are shown in Figures 4 and 5. In both figures, the data points are the results from 2D numerical modeling (such as has already been shown in Figure 2(a)), which perfectly coincide with the corresponding analytical results, thus establishing the reliability of the analytic approximation.

In Figure 4, the hotspot size is varied; and the subslab center concentration is evaluated for three non-dimensional building slab sizes $\beta/l = 0.5, 1$ and 10 . When the hotspot size a compared to the source depth l is less than 4 , the calculated subslab vapor concentration (normalized by c_l) varies sharply with non-dimensional hotspot size a/l . When the non-dimensional middle source size a/l is larger than 4 , a limiting value is reached for the subslab vapor concentration. In this situation, the “side source” is too far away from the slab to have a significant effect on the subslab vapor concentration. Thus, this limiting subslab vapor concentration is not sensitive to the side source concentration. What is observed as the subslab concentration depends only upon the ratio of building size to source depth.

The above limiting value can be calculated by letting a/l go to infinity in Equation (2). The third term goes to zero, giving

$$\lim_{a/l \rightarrow \infty} c_{ss} = \frac{c_\tau}{\pi} \arccos \left[\frac{8e^{\pi\beta/(2l)}}{(e^{\pi\beta/(2l)} + 1)^2} - 1 \right] \quad \text{Eq. (5)}$$

This equation describes a uniform source scenario, meaning that if hotspot size $a/l \gg 4$, the subslab vapor concentration is almost the same as that calculated from a uniform source scenario. Note, though that Figure 4 shows that for capping size $\beta/l = 1$, even a modest size hotspot ($a/l = 1$) can lead to much higher subslab contaminant concentration values ($c_{ss} > c_l$) than expected based upon the source concentrations outside of the hotspot. For instance, for the scenario of a building of size 10 m located 8 m above the source, a hotspot located beneath the building that has a concentration 10 times that of the side source vapor concentration (i.e., that measured outside of the building at some distance from the foundation) can lead to a subslab vapor concentration 4 times greater than that expected based upon that exterior side source vapor concentration.

In Figure 5, the influence of non-dimensional building size is shown for certain selected source sizes. The subslab center concentration normalized by c_l is sensitive to the non-dimensional building slab size β/l for small structures or deep sources, but reaches a certain limiting value after $\beta/l > 4$. In many scenarios, it would be expected that $\beta/l < 2$. Then, both β/l and a/l determine the subslab vapor concentration, and there is no assurance that the maximum subslab concentration will reach a value consistent with either c_τ or c_l . This means that in order to accurately predict subslab concentrations, more detailed characterization of the source vapor distribution would be necessary in such cases in which there is reason to suspect non-uniform source concentration. It is clear that the existence of such hotspots beneath structures can cause great confusion, in terms of reconciling subslab concentrations with some deep soil gas concentration or groundwater contaminant concentration.

For the cases of large β/l that could be relevant for, *e.g.* large structures surrounded by paving, the subslab center concentration can reach a vapor concentration, calculated by letting β/l go to infinity in Equation (2):

$$\lim_{\beta/l \rightarrow \infty} c_{ss} = c_\tau + \frac{2}{\pi} (c_1 - c_\tau) \arctan \left[\frac{2e^{\pi\alpha/(4l)}}{e^{\pi\alpha/(2l)} - 1} \right] \quad \text{Eq. (4)}$$

c_{ss} in this case, depends on α/l and c_τ . As α/l increases, the second term quickly goes to zero, and the maximum subslab vapor concentration can approach the hotspot source vapor concentration. That means the subslab center vapor concentration can reach the hotspot source vapor concentration with relatively small middle source size. The implication is that in an area with significant impermeable cover, such as may be characteristic of urban settings, relatively small subsurface hotspots can make a significant difference in determining subslab concentrations, and sparse sampling of groundwater sources risks missing the relevant zones determining observed subslab concentrations.

On an actual vapor intrusion site, the subsurface iso-concentration contour map can be obtained by interpolation from discontinuous field sampling points using various gridding methods (Murphy and Mohsen, 2010). While taking many subsurface soil gas and groundwater samples is preferred for characterizing the subsurface source, the time and cost of sampling may often be issues. It should be noted that in the U.S. EPA vapor intrusion database, for all the reported data that included such sampling locations, about 80% of the groundwater samples were taken more than 20 m away from the building they were assigned to and 80% of the soil gas samples were taken more than 15 m away from the building they were assigned to. There were many data for which their sampling locations were not reported. The above analytical solution can help to estimate the uncertainty in predicted subslab concentrations that might be associated with source data taken at some distance from the slab.

3.3 The contaminant vapor concentration profile beneath the slab

For subslab sampling, the U.S. EPA has suggested that it be done near the center of the foundation slab (EPA, 2012b). Theoretically, assuming the vapor source is infinite and uniform, the subslab vapor concentration at the slab center (Shen et al., 2014) should represent the worst-case condition (maximum vapor concentration beneath the slab). The above modeling results can be used to evaluate the subslab vapor concentration beneath the slab for scenarios in other source configurations. In Figure 6, several different scenarios are given as examples of the predictions from Equation 1 for different scenarios. In Figure 6(a) the normalized middle vapor source size α/l is chosen to be smaller than the normalized slab size β/l , while in Figure 6(b), $\alpha/l > \beta/l$.

In both figures, the **uniform source scenarios** ($c_\tau = c_l = 1$, $c_0 = 0$) give the same concentration profiles as expected. For the **finite size source scenarios** ($c_\tau = 1$, $c_l = c_0 = 0$), the subslab concentration profile for a small source (Figure 6(a)) is less than that for a uniform source result, while that for a larger central source (Figure 6(b)) closely approximates that for a uniform source, as expected. For the **middle hotspot scenarios** ($c_\tau = 5$, $c_l = 1$, $c_0 = 0$), the maximum subslab vapor concentrations are found at the slab center and are both larger than c_l . For the **side source scenarios** ($c_\tau = 0$, $c_l = 1$, $c_0 = 0$), the subslab vapor concentration is of the same magnitude (~40 %) as the uniform source concentration (Figure 6(a)). However, in the case in which there is a large zone of low concentration beneath the structure, the subslab concentration is about 1000 times less than the result for the uniform source scenario (Figure 6(b)).

Generally speaking, the concentration variation within a subsurface source can however be even more complicated than what has been assumed above. However, the mathematical technique of superposition of the solutions can still extend to these more complicated scenarios. The basic concept is the same as described above in Figure 3. For a source that can be described as piecewise continuous, the subslab vapor concentration can still be estimated using the above analytical method. The scenario in Figure 7 is an example. Source zone A vapor concentration is 1000 ug/m^3 and 15 m in extent. Source zone B is of low concentration (100 ug/m^3) surrounding Source zone A. Zone C concentration is assumed to be zero as is the open field surface concentration. In order to calculate the vapor concentration beneath the center of the slab, the detailed steps are explained in the Supplemental Document. The described analytical approach gives an estimate of subslab center concentration of 480 ug/m^3 , which agrees perfectly with the numerical simulation of the same scenario. It is obvious that the subslab center vapor concentration is 4 times larger than Source zone B vapor concentration, so this emphasizes the importance of understanding the effects of groundwater concentration profiles, particularly localized zones of high concentration.

4. Conclusion

Using the analytical solution provided above, the vapor concentration beneath the slab and the maximum subslab vapor concentration can be estimated if details of the near structure source profile are known. Various state guidance documents have suggested different numbers of subslab sampling points (1 ~ 4) (Eklund et al., 2012). The solutions here show how different subslab profiles may arise from different subsurface situations.

The above scenarios have provided the subslab vapor concentration profiles for a source that can be represented by three pieces (center and two sides). While these calculations for predicting subslab vapor concentrations involve complicated algebraic forms, we have built all the calculations into an Excel File provided as a Supplemental Document. Utilizing the superposition technique can expand the applicability of the technique to more realistic source conditions.

Prior knowledge of the current (and future) extent of contaminant vapor source is generally not readily available during initial investigation of a site, so a default worst-case (i.e., uniform) source assumptions may be often used. But the results here warn that even small (in size) fluctuations can sometimes make a significant difference in predictions. On the other hand, if the source is well characterized allowing use of a model of the kind presented here, the soil gas profile should be much better characterizable than if a uniform source assumption is applied.

As an analytical solution, this modeling simplifies the vapor intrusion scenario and therefore has its limitations. As far as the authors know, there is no closed form analytical solution for a 2D or 3D scenario that includes soil gas advection through foundation cracks. Here, soil gas advection and also contaminant diffusion through the building slab (Patterson and Davis, 2009) have been ignored. Also, the modeling of petroleum chemicals and chemicals that are highly biodegradable was not considered in this manuscript, but results for such scenarios are being developed in separate work (Yao et al., 2013d).

The present model may be helpful for field investigators in characterizing field conditions, and can also be used to validate numerical models of the same scenarios. The hope is that it can be used in concert with field measurements, helping to understand those results as they become available, and suggesting when more measurements might be warranted. While this model does not require advanced numerical modeling effort, more complicated scenarios

involving non-homogenous soils, transient factors and so on, will still necessarily be referred to numerical models.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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Nomenclature

c	subsurface contaminant vapor concentration [–]
c₀	open field vapor concentration [–]
c₁	side vapor source strength [–]
c_{ss}	contaminant concentration beneath the center of the subslab [–]
c_τ	middle vapor source strength [–]
D	effective diffusivity including soil gas and soil water phases diffusion [m ² /s]
l	the vertical distance from the building slab to the bottom of the top soil layer [m]
α	middle source length [m]
β	building slab characteristic length [m], including outdoor pavement if any

Highlights

- An analytical estimation of the subslab vapor concentration is offered
- The method allows for non-uniform subsurface source distribution
- The parameters that determine subslab concentration are identified

(a) Vapor intrusion schematic

(b) The 2D analytical model

Figure 1.
(a): Typical vapor intrusion schematic. (b): The 2D analytical modeling scenario

Figure 2.

(a) The 2D numerical modeling result. (b) The 3D numerical modeling result

Figure 3.

The base scenario without considering the slab capping effect. (a) the modeling scenario. (b) Analytical profiles for concentration along center line. (c) The hypothetical uniform source scenario. (d) The hypothetical side sources scenario

Figure 4.
Analytical results for normalized subslab concentration when the middle source (hotspot) size varies

Figure 5.

Analytical results for normalized subslab concentration for different building capping sizes

Figure 6.

Calculated subslab vapor concentration along the building slab for four different scenarios. (a): $\alpha/l = 1$, $\beta/l = 2$. (b): $\alpha/l = 6$, $\beta/l = 2$. Source conditions: the uniform source scenario, $c_\tau = c_I = 1$, $c_0 = 0$; finite size source scenario, $c_\tau = 1$, $c_I = c_0 = 0$; non-uniform source scenario, $c_\tau = 5$, $c_I = 1$, $c_0 = 0$; side sources scenario, $c_\tau = 0$, $c_0 = 0$, and $c_I = 1$ in (a), but $c_I = 1000$ in (b).