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The role of series ankle elasticity in bipedal walking

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Abstract

The elastic stretch-shortening cycle of the Achilles tendon during walking can reduce the active work demands on the plantarflexor muscles in series. However, this does not explain why or when this ankle work, whether by muscle or tendon, needs to be performed during gait. We therefore employ a simple bipedal walking model to investigate how ankle work and series elasticity impact economical locomotion. Our model shows that ankle elasticity can use passive dynamics to aid push-off late in single support, redirecting the body's center-of-mass (COM) motion upward. An appropriately timed, elastic push-off helps to reduce dissipative collision losses at contralateral heelstrike, and therefore the positive work needed to offset those losses and power steady walking. Thus, the model demonstrates how elastic ankle work can reduce the total energetic demands of walking, including work required from more proximal knee and hip muscles. We found that the key requirement for using ankle elasticity to achieve economical gait is the proper ratio of ankle stiffness to foot length. Optimal combination of these parameters ensures proper timing of elastic energy release prior to contralateral heelstrike, and sufficient energy storage to redirect the COM velocity. In fact, there exist parameter combinations that theoretically yield collision-free walking, thus requiring zero active work, albeit with relatively high ankle torques. Ankle elasticity also allows the hip to power economical walking by contributing indirectly to push-off. Whether walking is powered by the ankle or hip, ankle elasticity may aid walking economy by reducing collision losses.

Introduction

Elastic energy storage and return is thought to improve economy of human locomotion by reducing the mechanical and metabolic energy demands on muscles [1], [2]. In walking, the Achilles tendon acts elastically during stance, stretching during ankle dorsiflexion and then shortening during plantarflexion [3], [4]. The calf (triceps surae) muscles simultaneously produce force in series with the tendon, and may additionally perform active shortening work during plantarflexion [3], [4]. Thus, elasticity appears to reduce active work compared

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to the same overall action performed by a muscle without a tendon [5], [6]. But that presumes a need for the stretch-shortening cycle itself. Without that cycle, the muscles might potentially be activated later in the stride to perform shortening work alone, and perhaps achieve better economy with no tendon at all. This might seem a highly unlikely possibility, but there lacks a clear explanation for the presumed advantages of the normal stretch-shortening cycle. We therefore employ a simple bipedal walking model to investigate how series elasticity at the ankle impacts economical locomotion.

Elastic energy return at the ankle contributes to a large burst of positive work at the end of stance phase in human walking [1], [3], [4], [7], [8]. Elastic energy is first stored earlier in stance, when the soleus and gastrocnemius muscle fascicles produce force but undergo relatively little displacement, particularly at slower walking speeds [3]–[6]. Without series elasticity, the same ankle moment and displacement would presumably require more active negative and positive work from the muscle fascicles, and thus greater metabolic cost [9]–[11]. But with varying amounts of elasticity, it is also quite possible that a different ankle action might be preferable. To examine that possibility, it is helpful to consider when and how work must be performed.

The net work requirement for steady state walking is zero. To maintain periodic body motion over level ground, any mechanical energy dissipation within a gait cycle requires an equal amount of positive work, not necessarily at the same time or location. Dissipation can occur actively in muscles or passively by other soft tissues in the body [12], with a major loss occurring during heelstrike collisions in the transition between pendulum-like steps [13]–[15]. In humans, energy is mainly restored by positive muscle work about the ankle and hip (e.g., [16]). However, less positive work would be required if the energy losses could be reduced.

Simple models of walking suggest collision losses may be reduced by pushing off with the trailing leg just before the collision [15], [17]. Indeed, humans perform ankle push-off with timing and work that largely offset the collision losses [15], [18]. Those losses could theoretically be reduced further with an ankle spring that resists the collision by producing a passive dorsiflexion moment and then assists push-off with a passive plantarflexion moment, as demonstrated by a dynamic walking model powered by the hips [19]. A separate model of walking with an elastic ankle orthosis shows that a passive plantarflexion push-off can indeed reduce collisions and improve walking economy [20], for a gait powered by the hips. What remains unanswered is to what degree collisions can be reduced, and how active work by the ankles should supplement an elastic stretch-shortening cycle for best economy.

In this study, we use a dynamical model to systematically investigate the biomechanical benefits of series ankle elasticity on walking economy, along with possible strategies for actively powering gait. Rather than using a detailed model of the muscle-tendon unit [3]–[6], we model the tendon as an elastic spring, with an actuator that can produce an active work loop similar to the human ankle. The model may also be powered by the hips, making it possible to consider a variety of powering strategies. We use this model to determine how active powering and passive elastic energy storage and return at the ankle should best be configured for economy of gait.

Model

We developed a dynamic walking model with series elasticity at the ankle, and used it to investigate how elasticity can affect the energetics and mechanics of locomotion (Fig. 1). This is a minimal model that yields a dynamic gait with single- and double-support phases, a passively swinging leg, and spring-like energy storage and release at the ankle (Fig. 1A). A key feature is the timing of elastic ankle push-off, which begins with stance heel lift-off—

referred to as *heel-off*. If model parameters are selected to cause heel-off to occur pre-emptively relative to opposite leg heelstrike, it can potentially reduce the collisional energy losses from that event [21]. The model will demonstrate the conditions for proper push-off timing, the amount of energy that is optimally stored and released by the ankle, and the options for powering walking by the ankle and the hip.

We added ankle elasticity to the previous “simplest walking model” [21]–[23]. This is a planar model with a point mass M for the pelvis and two legs that swing about the hip, each with length L and an infinitesimal foot mass m at its end. To this was added a massless, forward-facing foot of length l_f freely rotating about that point. Brief scuffing of the swing foot on the ground during mid-swing was ignored. As with previous dynamic walking models, heelstrike with ground was modeled as an instantaneous and perfectly inelastic collision [13], [22], [23]. We tested this model for walking gaits, meaning limit cycle behaviors without an aerial phase. We considered only long-period gaits [13], [24], [25], referring to gaits where the swing leg reverses motion prior to heelstrike (i.e., the swing hip reaches a peak flexion angle and then begins extending before foot contact). We therefore ignore short-period gaits where the heelstrike occurs while the swing hip is still flexing.

The model was actuated by active and/or passive torques at hip and ankle (Fig. 1A). For passive torques, the hip and ankle were assigned torsional stiffness k_p and k_{ank} respectively (depicted in figures as extension springs for convenience). A hip spring is helpful for modulating step frequency, by speeding the swinging of the legs in both flexion and extension [21]. The ankle spring produced only plantarflexion moment, with a slack length such that it became stretched only when the stance leg moved forward past vertical (Fig. 1B). Although both springs can affect gait economy by modulating joint motions and returning elastic energy, neither performs net positive work.

Active torques about the ankle and hip were then added to produce the positive work needed to power walking. The balance between positive work and collisional energy losses thus produced a periodic gait with a steady walking speed. Hip work was modeled with an equivalent energy gain from descending a gentle slope, which is practically equivalent to active work performed between a torso and stance leg hip [13], [21]. Ankle work was modeled with an actuator in series with the ankle spring. The actuator enhanced the spring's passive torque-displacement curve with an active work loop, produced by a vertical step increase in torque at the point of peak ankle flexion (Fig. 1B), termed a vertical work loop because of its stiffening behavior. We also investigated an alternative loop with the ankle extending at constant torque, termed a horizontal work loop. All of these methods yield limit cycles that can power walking on level ground, where positive work is needed to offset collision losses. To quantify the energetic cost of locomotion, we defined *mechanical cost of transport* (mCOT) as the total work (from net hip work and net ankle work) per unit walking distance. We defined the *optimal ankle stiffness* as that which minimized mCOT for a given set of model parameters (leg length and foot length) and gait parameters (walking speed and step length). A mCOT equal to zero indicates passive walking on level ground with no heelstrike collision losses and thus no need for active ankle or hip work.

We defined a set of nominal parameter values for the model. To non-dimensionalize the model, these are reported relative to M , L , and g (gravitational acceleration) as base units. Foot length was set to $l_f = 0.15 L$, approximating the horizontal distance from the human ankle to the first metatarsal joint, roughly where human push-off forces are located in normal gait. A nominal human gait was defined with speed $0.4 (gL)^{0.5}$ and step length $0.7 L$, respectively, equivalent to a typical walking speed of 1.25 m/s and step length of 0.7 m for a person with 1 m leg length [26].

Simplified Analysis

Prior to presenting the complete numerical simulation, we perform a brief analysis to reveal the key consequences of series ankle elasticity for walking (see Fig. 2). This analysis provides an intuitive sense for how the design parameters of the ankle/foot can be selected to reduce collision losses. The principal parameters to be related are foot length l_f and torsional ankle stiffness k_{ank} , as a function of walking speed and step length. We show that there are two main relationships that must be satisfied for an economical gait, one for heel-off timing and the other for ene relationships specifies an optimal parameter combination that minimizes collision losses for a given speed and step length. A qualitative description of these relationships is presented next.

Conditions for economy may be illustrated by focusing first on a hypothetical gait with no energy cost. We consider here an idealized approximation to the model (Fig. 2A), where the normally brief transition between stance legs is made instantaneous, with both feet starting flat on the ground and the legs separated by angle 2α . Instantaneous means that the trailing leg's ankle spring releases its energy with a perfectly elastic impulse just prior to the leading leg collision, which is also impulsive but perfectly inelastic. We assume that the corresponding swing phase timing can be adjusted through the hip spring [21]. A no-cost transition is possible if the elastic ankle spring releases sufficient energy to allow the leading leg to touch ground with no collision loss, such that the instantaneous COM velocity is perpendicular to the new stance leg. Prior to heel-off, the ankle spring k_{ank} stores energy as the stance leg rotates (with an angle τ) forward past vertical. After sufficient spring force has been developed to produce heel-off, a push-off impulse acts at the toe of the trailing foot and is directed toward the COM. The COM velocity v^- before push-off is defined by the vector subtraction of the impulse from the given post-heelstrike velocity v^+ , and by the fact that v^- and v^+ are separated by angle 2α . If these conditions are statified, there will be no heelstrike collision and no need for active work. Finally, we assume small angles and small velocities, meaning their second or higher powers are neglected. We will treat v^+ as a fixed quantity, approximating a given walking speed, and 2α as equivalent to a given step length s . Full mathematical details are provided in the Appendix.

We next consider two relationships between ankle stiffness and foot length. The first describes the timing of heel-off (Fig. 2B). Elastic ankle torque increases in proportion to forward motion ($T_{ank} = k_{ank} \theta$), and produces force through the moment arm defined by foot length. Therefore, heel-off occurs approximately when the ankle torque exceeds body weight multiplied by foot length ($T_{ank} \approx Mgl_f$), resulting in the heel-off relation

$$\frac{k_{ank}}{l_f} \approx \frac{Mg}{\theta_{HO}} \quad (\text{Eqn. 1})$$

where θ_{HO} is the leg angle (measured past vertical) at heel-off. For the impulsive push-off in the simplified analytical model, θ_{HO} is assumed to be equal to leg angle α in this idealization. This relationship (Eqn. 1) defines a line in the space relating foot length and ankle stiffness (Fig. 2B). Combinations of the two below this line cause push-off to occur late relative to contralateral heelstrike, and will do little to reduce the heelstrike collision. Combinations too far above the line will cause push-off to occur early relative to the heelstrike collision, so that during the interim some of the upward change in COM velocity will be lost to downward gravitational acceleration. The heel-off constraint (Eqn. 1) therefore defines a relatively narrow region, where ankle stiffness increases with foot length, for which the heel-off timing is appropriate.

The second relationship describes the elastic push-off energy needed to reduce the heelstrike collision (Fig. 2C). For simplicity, we artificially set the timing of heel-off to occur at stance leg angle α , regardless of the heel-off constraint (Eqn. 1). This assumes heel-off is somehow enforced if necessary, for example with an infinitely stiff elastic bumper preventing leg angles beyond α or by constraining the heel to ground and then releasing it when the leg angle α is reached. After heel-off, a reversal of ankle motion occurs, modeled as an impulsive elastic collision of the joint. This fully passive ankle reversal redirects the COM velocity in the toe-to-COM direction of the push-off impuls COM velocity equal to v^- in magnitude (i.e., energy is conserved). In order to completely avoid collision losses, the energy stored in the ankle spring must be sufficient to provide the remaining push-off work (labeled P in Fig. 2A). These constraints therefore determine the required ankle stiffness, which is a quadratic function of speed (see Appendix for derivation details). For infinitesimally short feet, ankle reversal contributes nothing, and the ankle stiffness required for zero-cost walking is determined by leg angle and the energy needed to redirect the COM upward to v^+ , along the direction of push-off. For longer feet, the necessary ankle stiffness decreases, approaching zero for l_f equal to half step length, at which point the impulsive elastic ankle reversal alone completes the entire redirection from v^- to v^+ . Zero-cost gaits therefore define a curve in length/stiffness space (for a given speed and step length, Fig. 2C). Similarly, finite-cost gaits (i.e., gaits with non-zero collisions) appear as contours offset below the zero-cost curve. Combinations of l_f and ank k_{ank} beyond the zero-cost curve would not yield walking gaits in this simplified model, because greater push-off results in an aerial phase.

The intersection of the heel-off timing and energy storage relationships specifies the optimal design of ankle stiffness and foot length for a given speed and step length. The elastic push-off energy relationship ensures there is sufficient spring energy to eliminate the heelstrike collision, and the heel-off relationship ensures that there is sufficient spring force at the appropriate time. Starting from that point, of greatest interest are combinations of stiffness and length along the radial line of the heel-off relationship. The nearer the origin, the more energy is lost at heelstrike, necessitating active powering at the hip or preferably the ankle [21].

This approximation neglects time. Its ankle reversal and elastic energy release take place over infinitesimal displacement and over infinitesimal time. This contrasts with the full computational model, where the downward impulse of gravity acts increasingly against longer push-off durations. Thus heel-off should actually precede contralateral heelstrike by a finite duration (so there is time for COM velocity redirection). The energy stored in the spring must also be greater than in the approximation, again because more work must be performed against gravity. Another expectation is that longer feet should lead to greater double support times, because the ankle/foot must undergo more spatial displacement to achieve toe-off, and hence require more time before lifting off. For a given step frequency, this will leave less time for the swing phase, necessitating a stiffer hip spring k_p . Although these approximations are not expected to be numerically accurate, they are intended to capture essential qualitative features of walking with an elastic ankle.

Computational Simulations

We performed a set of computational parameter studies to test the analytical model predictions regarding spring stiffness and foot length. We verify that ankle work can indeed reduce the energetic demands of walking by modulating heelstrike collisions, and that ankle elasticity offers an economical means of performing this work. The economy of walking depends on both ankle spring energy storage and the timing of energy return, which in turn rely on a careful balance of the model parameters (spring stiffness, foot length and spring

engagement point) with gait characteristics (speed and step length). We show that there are indeed certain model configurations capable of walking with zero cost by using elastic energy storage and return to redirect COM velocity and avoid collisions. We find that ankle-powered gait is generally more economical than hip-powered gait. In a comparison with human walking, we also observe that the optimal ankle stiffness remains roughly constant when speed and step length increase together, similar to the human's preferred relationship [23], [26].

We consider here walking gaits over a range of foot length l_f and ankle stiffness k_{ank} combinations (l_f : $0 - 0.35 L$ or about half step length; k_{ank} : $0 - 2.5 MgL$ per radian). For each combination, we solve for limit cycles corresponding to the nominal human gait. Outcome variables of interest included ankle torque and power, cost of transport and hip stiffness (a surrogate for effort required by the swing leg hip). We also examine the effect of spring rest length (i.e., engagement point) on economy, and study how the optimal spring stiffness changes with walking speed. The simulations relax a number of the analytical assumptions, most notably the instantaneous push-off, as well as small-angle and small-velocity approximations.

Simulations verify the ability of an ankle spring to produce joint kinetics roughly comparable to human (Fig. 3). Following heelstrike, the ankle spring is gradually loaded as the stance leg rotates forward like an inverted pendulum, with the elastic ankle moment increasing proportionally. If the ankle moment developed is sufficient to produce heel-off, then the energy will be released quickly with a pre-emptive push-off. The accompanying ankle power is negative during loading, and upon heel-off quickly switches sign as the ankle reverses its motion to plantarflexion and produces a burst of elastic positive work for push-off, also roughly resembling human behavior. If ankle stiffness and foot length are increased proportionally (e.g., at a fixed dimensionless ratio of about 4.9 for this model; Fig. 3B), the elastic push-off increases in power while keeping push-off timing relatively constant. Increasing the ratio of stiffness to length causes push-off to occur earlier, and decreasing causes it to occur later (Fig. 3C). These effects occur regardless of whether the gait is powered by the ankle or hip, and are consistent with the expectations of the analytical model above.

The model's cost of transport is highly dependent on the ratio between ankle stiffness and foot length. Heelstrike collision losses are greatest for very low ankle stiffness and for very short feet, where the cost approaches that of the simplest walking model without feet [22] (Fig. 4). As ankle stiffness and foot length increase proportionately, heelstrike collision losses decrease for all methods of powering. The mCOT approaches zero at high stiffness and foot length (Fig. 4), as predicted by the analytical model. A low ratio of k_{ank} to l_f is costly because it causes push-off to occur too late to redirect COM velocity prior to the leading leg collision. The ankle spring can reduce the collision magnitude somewhat by reducing the COM velocity at the time of heel impact, but this is far less helpful than pushing off. There is a contrasting reason why a high ratio of k_{ank} to l_f is also costly. It causes push-off to occur too early, so that there is time for gravity to bring the COM velocity downward, lessening the advantage gained from pre-emptive push-off. Depending on ankle stiffness, foot length, and available time, the ankle may even oscillate, an effect not generally advantageous compared to maintaining the ideal stiffness/length ratio.

There are, however, trade-offs associated with economical gait. Lower mCOT is facilitated by a relatively long foot and relatively high ankle stiffness (Fig. 5A). Longer feet necessitate greater effort to move the swing leg, as indicated by the need for increasing hip spring stiffness (Fig. 5B). That is because longer feet spend greater time in double support, so that stride duration can only be maintained by moving the swing leg faster, potentially at an

energetic cost. These lead to a relatively high ankle moment, which must be supported not only by the ankle actuator but also the entire stance leg (Fig. 5C). Also noteworthy, the optimal ankle stiffness does not correspond to maximum elastic energy storage (Fig. 5D), or maximum preemptive energy return (Fig. 5E). Rather, the optimal ankle stiffness reflects the trade-offs associated with the ideal k_{ank} to l_f ratio (Figs. 2 & 4). These trade-offs mean that achieving low mCOT in this simulation typically requires stronger actuators to support greater reaction moments, perhaps adding to structural or non-work-related costs.

We next compare gaits powered by the ankle or hip (Fig. 6). For the nominal speed and step length, both ankle and hip can successfully power walking, but with different costs. An active ankle can increase push-off work and cause redirection of the COM to occur more preemptively than an active hip, thus yielding as little as half the mCOT of hip powering. The type of ankle work loop performed also has some effect, with the vertical loop yielding better economy than the horizontal loop. This may be attributed to a higher peak force, which makes push-off more impulsive. Ankle powering is, however, rather sensitive to parameter choices for both types of work loop studied. It only yields successful gaits if push-off is timed rather precisely, and therefore requires a fairly precise ratio of ankle stiffness and foot length. It also requires a minimum ankle stiffness and foot length, without which there is insufficient energy storage to overcome collision losses. This contrasts with hip powering, which can yield periodic gaits for almost any parameter combination. It is less sensitive to timing because the hip can perform work and adds energy throughout the entire stance phase, whereas ankle push-off should be timed just before heelstrike. Hip powering is generally less economical than ankle powering, but it also gains from an elastic ankle. Without that elasticity (equivalent to zero foot length in Fig. 6), hip powering is quite costly, but decreases rapidly and converges on ankle powered walking as foot length increases. Thus, not only is hip powering more robust than ankle powering, in the sense that it can accommodate more combinations of ankle stiffness and foot length, but it can also be relatively economical provided appropriate ankle elasticity and foot length.

Another parameter not yet considered is the slack length of the ankle spring. It determines the angle at which the spring engages, and is nominally set to occur when the stance leg moves past vertical. Varying this parameter, we find that gait economy can be improved with earlier engagement (Fig. 5A), which allows for lower spring stiffness for a given walking speed and step length (Fig. 5A), resulting in more energy storage (Fig. 5D). For the nominal gait, early spring engagement (beginning at about 20% of stance phase) is about 20% more economical than late spring engagement (at about 80% of stance phase). Early spring engagement also leads to slightly lower peak ankle torque at optimal ankle stiffness (Fig. 5C), but does require higher hip spring stiffness (Fig. 5B) to achieve equivalent speed and step length. It also becomes less applicable with longer foot lengths; springs that engage too early can actually increase heelstrike collision because at the end of elastic energy return the ankle is extended so far that the COM rises above the hip-to-heel leg length, introducing an aerial falling phase before heelstrike, resulting in an uneconomical running gait. Similar to the parametric variation of foot length, varying engagement angle also indicates that the optimal ankle stiffness neither corresponds to maximum elastic energy storage nor preemptive energy return (Fig. 5D & E).

Optimal ankle stiffness also changes with speed. Keeping foot length and step length fixed, we determine the energetically optimal ankle stiffness for a given speed. The COM velocities (e.g. v^- and v^+ , Fig. 2) scale with walking speed, and so faster walking should require greater ankle stiffness for proper push-off (Fig. 2C). Simulations verify that the optimal stiffness indeed increases with speed when maintaining constant step length (Fig. 7A). A competing effect is that ankle stiffness should decrease with greater step lengths at a fixed speed, because push-off should occur at a higher leg angle (compare α with k_{ank} in

Eqn. A4), as is also verified by simulation (Fig. 7B). These competing effects largely offset each other when speed and step length increase together, as is the case with humans.

Applying a typical preferred step length for humans (increasing with speed raised to the power 0.42; [23], [26]), the optimal ankle stiffness is approximately constant (Fig. 7C), varying by less than 7% across a wide range of speeds (equivalent to 0.6 – 1.9 m/s). This finding is consistent for many reasonable foot lengths: optimal ankle stiffness varied no more than 11% across speed for any foot length between 0.09 and 0.25 L .

Discussion

We used a computational simulation to show that elastic ankle work can reduce the total mechanical work of walking, and theoretically even completely remove the need for active muscle work. The primary mechanism is the ability of the ankle to perform work that preemptively redirects COM velocity, eliminating collision energy losses. In our model, the key requirement for economy is the ratio of ankle stiffness to foot length. For a given walking speed and step length, the proper ratio enables heel lift and elastic energy release to occur just before contralateral heelstrike. To walk purely passively, the spring must also store sufficient energy to redirect the COM velocity, a condition limited by both ankle stiffness and foot length. We next interpret these findings in the context of human walking.

Our simulation suggests that series ankle elasticity can benefit walking economy by redirecting COM velocity before heelstrike. Elastic energy storage and return does not directly replace work done by muscles. Rather, it reduces heelstrike collision losses in two ways. First, it stores energy from the forward motion of the COM, reducing the kinetic energy at impact. Second and more importantly, it can produce pre-emptive heel lift-off, resulting in an elastic push-off. Just as with active push-off [21], this elastic redirection of the COM reduces collision, and thus the total amount of active positive work that must be performed to offset collisional losses. Elastic ankle push-off can even considerably improve the economy of purely hip-powered walking (Figure 6), which would otherwise be rather uneconomical [21]. Thus, elastic ankle push-off might allow humans to use positive hip work to help power gait more economically. Previous literature [3]–[6] has primarily focused on the advantages of tendon elasticity in reducing localized calf muscle work. Here, we find that elasticity is beneficial regardless of whether or not the ankles perform active push-off, and even if only the hips perform active work.

We found it theoretically possible to walk with no energetic losses. In both the computational model and its simplified analytical approximation, collisions can be eliminated entirely despite perfectly inelastic and rigid legs. Here the foot touches down with zero COM velocity along the direction of the leg, and thus no collision loss. Of course, humans do experience non-zero losses, and must perform positive work [15]. This may be because lossless walking requires relatively long foot length (Fig. 6), large ankle torque, and high ankle stiffness, all considerably greater than in humans. We have previously found human walking on long artificial feet [27], [28] to be relatively economical, but also quite uncomfortable, in part from a tendency toward knee hyperextension during the stance phase. This suggests a potential benefit of biarticular muscles such as the gastrocnemius that help counteract knee extension as ankle torque increases. Longer feet also require faster swing phases, which might be associated with higher metabolic costs to produce muscle forces for leg swing [23], [29], [30]. Finally, long feet introduce practical issues such as toe clearance during swing phase that could heighten the risk of tripping. These practical effects might partially explain why humans do not achieve the theoretical ideal of zero-cost walking.

We also found it interesting that walking was not most economical when maximizing power output with series ankle elasticity. The model's maximum rate of energy return occurs when

elastic ankle push-off is delayed until after contralateral heelstrike (Fig. 3C), consistent with prior simulation studies [20]. But this does not yield the best economy, because the timing of energy return is not sufficiently preemptive. Elastic return appears to be used by some insects, frogs, and birds to perform “power amplification” for jump performance [31]–[35], allowing for greater bursts of power than possible from muscle fascicles alone. However, as has been noted (e.g., [36]), series elasticity in human walking is distinct from that concept of “power amplification” because there is power output at push-off even if the ankle muscles perform no work during the stride. We do not consider series elasticity at the ankle to amplify power during human walking, but rather to gather kinetic energy from the body and concentrate it for push-off.

There are a number of limitations to our simple walking model compared to humans. The idealized 2-D model assumes instantaneous collisions with a point mass body. Human limbs are deformable and experience finite-time collisions that cause a slower unloading of the trailing limb, which would reduce peak powers compared to the instantaneous transition in the model, but still facilitate elastic energy return. Our model also has no knees, which are important for clearance of the swing foot [13], nor an upper body, which can also potentially contribute elastic benefits [37]. We also did not investigate costs associated with deceleration of the foot or leg at heelstrike, which appear to contribute additional metabolic costs [29], [30]. We also principally examined a single nominal gait. However, parametric variations in speed and step length (Fig. 7) suggest that qualitative findings and mechanistic interpretation are applicable to other gaits as well. Our simulation is therefore intended not to predict absolute quantities, but mainly to illustrate basic effects of series ankle elasticity.

There are certainly many physiological complexities not examined here. We focused on the mechanical work of the body's COM, which is an imperfect indicator of positive muscle work during walking [12]. We did not model muscles directly, nor did we account for costs due to force or rate of force production, which would be incurred during tendon stretch [29], [30]. However, these simplifications seem reasonable given the higher metabolic cost of positive muscle work compared to negative work or isometric contraction at the same force [7], [9]–[11]. The model also did not account for hysteresis of elastic soft tissues such as the Achilles tendon, which has been estimated to return 74–90% of its stored energy [38], [39]. We also did not examine the effects of ankle work and elasticity on other outcomes, such as stability or energy absorption abilities. Collectively, these effects could also help explain why humans experience only some of the theoretically possible benefits of ankle elasticity.

The effects of series ankle elasticity may have implications for prosthetic foot design. Conventional prosthetic feet have reduced push-off compared to the natural ankle [40], [41], which may result in greater collision losses, and thus the higher metabolic cost of walking for amputees [42]–[44]. Increased push-off could potentially lead to reductions in metabolic cost [42], [45]. Our model suggests that push-off could potentially be aided by ankle elasticity. However, it is unclear why prostheses that passively store and return energy provide little metabolic savings over less elastic prostheses [46]–[48]. Estimates of ankle stiffness for clinically prescribed prostheses cover a wide range [49], but little quantifiable evidence exists to suggest an energetically optimal stiffness [46]–[48]. Perhaps a relatively short effective foot length [50] limits the elastic benefits. Other possibilities include insufficient energy return due to hysteresis [49], [51] or viscoelastic properties of the prosthetic heel [52], or compensations due to imperfect socket interfaces [53]. In particular, the interface between a mechanical device and the deformable human body can present a number of challenges. It therefore remains undetermined whether passive elastic prostheses and other rehabilitative devices can substantially improve walking economy.

We have proposed a fundamental mechanism to explain the energy-saving benefits of elastic ankle work during walking. Ankle elasticity can passively redirect the body's COM before heelstrike, reducing mechanical collision losses and thus the total muscle work required to walk. This is another in a family of features that can reduce collision losses, such as rolling feet [27], trunk elasticity [37], pendular dynamics [2] and passive elastic soft tissue deformations [12]. Humans appear to use a number of these energy-saving mechanisms to walk more economically [3], [14], [18]. Our model suggests that, in theory, mechanical work requirements of walking could even be reduced to zero if the COM velocity were redirected to be perfectly perpendicular to the direction of force at heelstrike, avoiding collisions altogether. Thus, ankle elasticity provides a simple means to improve economy of ankle or hip powered walking, as long as ankle stiffness and foot length are favorably proportioned.

Appendix

The simplified analytical model (Fig. A1) is detailed here, in active push-off and heelstrike collision work (Fig. 2A). To facilitate calculations, we define the angle δ of push-off (with respect to vertical), directed from the toe toward the COM. In terms of leg length L , foot length l_f and stance leg angle α ,

$$\delta = \sin^{-1} \frac{(L \sin \alpha) - l_f}{\sqrt{L^2 + l_f^2 - 2Ll_f \sin \alpha}} \quad (\text{Eqn. A1})$$

The heel-off condition relates ankle stiffness with foot length and stance leg angle. It is based on the force balance for the COM, and the constraint that there be no ground reaction force under the heel at heel-off. The force balance is the component of the law of motion for the COM along the trailing leg:

$$\frac{Mv^2}{L} = Mg \cos \alpha - R_{toe} \cos(\alpha - \delta) \quad (\text{Eqn. A2})$$

where R_{toe} is the ground reaction force acting on the stance toe. At heel-off, the sum of moments about the ankle must be zero

$$R_{toe} \cos \delta = \frac{T_{ank}}{l_f} \quad (\text{Eqn. A3})$$

These two equations combine to yield

$$T_{ank} = k_{ank} \alpha = \frac{Mgl_f \left(\cos \alpha - \frac{v^2}{gL} \right) \cos \delta}{\cos(\alpha - \delta)} \approx Mgl_f \quad (\text{Eqn. A4})$$

where the ankle spring has been substituted into ankle torque. The right-hand approximation assumes small angles and small velocities, and is equivalent to Eqn. 1 in the main text, where leg angle at heel-off θ_{HO} is equal to the half angle between the legs α .

We next derive the collision work relationship, where the energy stored in the ankle spring is returned passively for push-off and reduces the heelstrike collision to zero. These quantities may be determined using a geometric diagram (Fig. 2A, Fig. A1). Two intermediate velocities can be defined to aid this calculation: the COM velocity v_r^- following reversal of ankle motion, and the velocity v_0 immediately following push-off but before the

collision. In the simplified derivation below we assume zero collision work, thus v_0 is equal to v^+ . Ankle reversal is modeled as a perfectly elastic collision following heel-off, where the ankle joint's motion is reversed from dorsiflexion to plantarflexion. The reversed velocity v_r^- must therefore have the same magnitude as v^- , and it must result from a push-off along the line of direction of toe-to-COM, but beginning from v^- . The angle between v^- and v_r^- is therefore $2\alpha - 2\delta$. The push-off impulse P refers to the release of elastic energy stored prior to heel-off, as well as any additional active work, assumed zero here. It is oriented in the toe-to-COM direction and yields an intermediate velocity v_0 . If non-zero, the collision impulse C would be oriented in the same direction as the leading leg, and change to COM velocity from v_0 to v^+ .

The following set of equations will yield ankle stiffness as a function of foot length and gait parameters. Here δ and α as determined by foot length and step length. The remaining relations are determined geometrically and using energy conservation equation. Starting with the post-heelstrike velocity v^+ , a triangle may be defined with the other vertices at v^- , but at a distance v^+ . The law of sines can then be applied and simplified to yield

$$\frac{v^+ - v^-}{\sin \delta} = \frac{2v^+ \sin \alpha}{\cos(\delta - \alpha)} \quad (\text{Eqn. A5})$$

Next we solve the energy balance relating the kinetic energy of walking to the combination of kinetic and elastic potential energy storage immediately prior to heel-off

$$M(v^+)^2 = M(v^-)^2 + k_{\text{ank}}\alpha^2 \quad (\text{Eqn. A6})$$

When simplified this yields

$$v^+ - v^- = \frac{k_{\text{ank}}\alpha^2}{M(v^+ + v^-)} \quad (\text{Eqn. A7})$$

Finally we can combine equations A5 and A7. If we treat v^+ as a fixed quantity approximately equal to walking speed v [54], and we assume $(v^+ + v^-) \approx 2v$, then this yields ankle stiffness as a function of velocity v , step length (in terms of α) and foot length (in terms of δ)

$$\frac{k_{\text{ank}}}{M} = \frac{4v^2 \sin \alpha \sin \delta}{\alpha^2 \cos(\delta - \alpha)} \quad (\text{Eqn. A8})$$

This may be verified to yield the degenerate cases. For infinitesimally short feet (as in Fig. 2C),

$$\left| \frac{k_{\text{ank}}}{M} \right|_{\delta=\alpha} \approx 4v^2. \quad (\text{Eqn. A9})$$

For long feet approaching half step length,

$$\left. \frac{k_{\text{ank}}}{M} \right|_{\delta=0} \approx 0. \quad (\text{Eqn. A10})$$

The general solution for active gaits, for which there are non-zero collisions, can also be solved for in similar fashion, although not shown here due to the length of the equations.

Symbols & Abbreviations

α	half angle between legs (when both feet are flat on the ground)
δ	angle between toe-to-COM line and vertical line
θ	leg angle (measured past vertical)
θ_{HO}	leg angle (measured past vertical) at heel-off
C	collision
COM	center-of-mass
g	gravitational acceleration
k_{ank}	torsional ankle spring stiffness
k_p	torsional pelvic spring stiffness
L	leg length
l_f	foot length
M	concentrated pelvic mass
m	infinitesimal mass of foot
mCOT	mechanical cost of transport
P	push-off
R_{toe}	ground reaction force at toe
s	step length
T_{ank}	ankle torque
v	walking velocity
v^-	velocity immediately before stance leg heel-off
v_r^-	velocity after ankle reversal
v_0	velocity immediately before contralateral heelstrike
v^+	velocity immediately after contralateral heelstrike
w^-	negative work
w^+	positive work

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Highlights

- ☐ Ankle elasticity can aid walking economy by redirecting COM, reducing collisions.
- ☐ Theoretically, zero active muscle work is required to walk with an elastic ankle.
- ☐ Economy of walking depends on the ratio between ankle stiffness and foot length.
- ☐ Proper ratio yields the optimal timing and magnitude of elastic energy storage/return.
- ☐ Ankle and hip powering strategies can both benefit from series ankle elasticity.

Fig. 1.

Dynamic walking model with elastic ankle. (A) Model has two legs supporting a point mass pelvis, and feet with infinitesimal mass. It can produce fully passive or actively powered walking gaits with pendulum-like motion. Ankle spring stores energy during single stance. Elastic energy return can then produce push-off to reduce the collision of the opposite leg with ground. (B) Ankle includes a spring with stiffness k_{ank} , modeling the human Achilles tendon. Model also includes an actuator that can optionally produce an active work loop for extension torque vs. flexion angle. Other model parameters include pelvis mass M , foot mass m , leg length L , foot length l_f , hip spring k_p , and gravitational acceleration g .

Fig. 2.

Simplified analytical model of elastic push-off and dissipative heelstrike collision as impulses, the center of mass (COM) velocity v^- before push-off and v^+ following collision may be related analytically. The idealized case with zero collision is depicted here. Heel-off is followed immediately by an elastic reversal of ankle motion and then an additional impulse P that releases elastic energy stored in the ankle spring prior to heel-off. (B) Heel-off timing. Heel-off initiates push-off, as the elastic ankle develops sufficient torque T_{ank} to overcome body weight. The trailing leg angle at heel-off θ_{HO} (measured clockwise from vertical) is determined by the ratio of ankle stiffness to foot length. If energy return precedes contralateral heelstrike, then push-off can redirect the COM and reduce collision losses. (C) Collision work. Collision loss decreases with increasing ankle stiffness, or with increasing foot length. A “zero energy cost” line defines the combinations of the two that yield a purely passive gait with no losses. These effects may be described in terms of leg angle α (proportional to half step length, $s/2$) and squared walking velocity, v^2 (see Appendix for derivations). In this idealization, when both (B) heel-off timing and (C) zero energy cost conditions are satisfied, a lossless gait is achieved.

Fig. 3.

Ankle kinetics vs. time. (A) Human ankle moment and power during stance phase are shown in dimensionless units for normal walking at 1.25 m/s. (B) Model kinetics are shown for increasing k_{ank} and l_f together at a constant ratio. This causes both ankle push-off moment and power to increase, while keeping push-off timing relatively constant. (C) Increasing the ratio of k_{ank} to l_f causes push-off to occur earlier. Gray regions denote double support (DS).

Fig. 4.

Contour plot of mechanical cost of transport (mCOT) as a function of foot length and ankle spring stiffness. Low cost gait requires a proper ratio of ankle stiffness to foot length for pre-emptive heel-off (prior to contralateral heelstrike). If the ratio is too low, then a late heel-off will occur. If the ratio is too high, then heel-off occurs too early, such that collision velocity increases again due to gravity. Alternatively, oscillatory cycles of elastic energy storage and return can occur (producing the “islands” at the left of the contour plot), also with greater collision losses. The maximum amount of energy that can be stored by the spring increases with both ankle stiffness and foot length. Coupled with appropriate heel-off timing, zero cost (collisionless) gaits can occur (denoted by ‘X’). Gait speed is 0.4 and step length is 0.7. All values are reported in dimensionless units.

Fig. 5.

Walking economy and other variables as a function of ankle stiffness, for different foot lengths (left column) and spring engagement angles (right column). (A) Mechanical cost of transport (mCOT) changes with ankle stiffness, which determines the ankle's oscillation frequency. Minimal costs decrease as foot length increases and as spring engagement becomes earlier. Vertical lines denote energetically optimal ankle stiffness for each foot length or spring engagement angle. (B) For the optimal case, hip spring stiffness increases with longer foot length and earlier spring engagement. Hip stiffness is an indicator of effort needed to move the swing leg. (C) Peak ankle torques increase with longer foot length and to a lesser extent with later spring engagement. (D) Energy stored in the spring and (E)

energy returned before heelstrike are higher for longer feet and earlier spring engagement. All models walked at fixed speed (0.4) and step length (0.7). The foot length parameter sweep was performed using the nominal (0 rad) engagement angle, when the stance leg is perpendicular to the ground. The spring engagement sweep (i.e., column) was performed with the nominal foot length (0.15). Therefore, the red dashed lines in each parameter sweep represent the same condition. Values are reported in dimensionless units.

Fig. 6.

Mechanical cost of transport (mCOT) for walking with ankle or hip powering, as a function of foot length. Shown are mCOT for the optimal ankle-powered and hip-powered gaits, for the same speed and step length (dimensionless 0.4 and 0.7, respectively, equivalent to humans at about 1.25 m/s). Ankle powering produces a more impulsive push-off, and is generally more economical than hip powering by as much as 50%. For ankle powering, a *vertical* work loop (see insets), in which torque increases at constant peak ankle flexion, was also found to be more economical than a *horizontal* work loop that maintains constant peak torque. But at long foot lengths, all costs decrease and converge toward zero. Also shown are costs for the simplest walking model with hip powering and impulsive push-off powering [21] (denoted as points on vertical axis). Active work parameters were selected for minimal mCOT.

Fig 7.

Optimal ankle stiffness as a function of speed and step length. Optimal ankle stiffness (A) increases with speed and (B) decreases with step length. (C) When speed and step length increase together along human's preferred relationship ($\text{step length} \propto \text{speed}^{0.42}$), optimal ankle stiffness is approximately constant across a wide range of gaits. All values are reported in dimensionless units. Dimensionless speed 0.2 corresponds to approximately 0.6 m/s, and 0.6 to 1.9 m/s.

Fig. A1.

Analytical model of walking with elastic ankle push-off, used to derive the heel-off condition and the heelstrike collision work. During the stance phase, elastic energy is stored in the ankle spring, slowing the COM velocity down to v^- prior to heel-off. The heel-off timing depends on the ankle torque T_{ank} and ground reaction force at the toe R_{toe} , with respect to the COM velocity v^- and the angle of push-off δ . At heel-off, the ankle spring first causes reversal of ankle motion, yielding velocity v_r^- , which is of equal magnitude to v^- , but rotated by an angle of $2\alpha - 2\delta$. The push-off P during walking is comprised of an elastic energy return (equal in magnitude to elastic energy storage) and an additional active impulsive ankle push-off. This results in an intermediate velocity v_0 followed by the impulsive heelstrike collision C that yields velocity v^+ , which begins the next walking step. The positive and negative work of these events is illustrated with blue and red shaded boxes, and the symbols w^+ and w^- , respectively.