Published in final edited form as:

Magn Reson Med. 2014 January ; 71(1): 105–117. doi:10.1002/mrm.24636.

High resolution, large dynamic range field map estimation

Joseph Dagher^{1,*}, Timothy Reese², and Ali Bilgin^{1,3}

¹Department of Electrical and Computer Engineering, University of Arizona

²Athinoula A. Martinos Center for Biomedical Imaging, Department of Radiology, Massachusetts General Hospital, Harvard Medical School

³Department of Biomedical Engineering and the BIO5 Institute, University of Arizona

Abstract

Purpose—We present a theory and a corresponding method to compute high resolution field maps over a large dynamic range.

Theory and Methods—We derive a closed-form expression for the error in the field map value when computed from two echoes. We formulate an optimization problem to choose three echo times which result in a pair of maximally distinct error distributions. We use standard field mapping sequences at the prescribed echo times. We then design a corresponding estimation algorithm which takes advantage of the optimized echo times to disambiguate the field offset value.

Results—We validate our method using high resolution images of a phantom at 7T. The resulting field maps demonstrate robust mapping over both a large dynamic range, and in low SNR regions. We also present high resolution offset maps in vivo using both, GRE and MESE sequences. Even though the proposed echo time spacings are larger than the well known phase aliasing cutoff, the resulting field maps exhibit a large dynamic range without the use of phase unwrapping or spatial regularization techniques.

Conclusion—We demonstrate a novel 3-echo field map estimation method which overcomes the traditional noise-dynamic range trade-off.

Keywords

MR Phase; field map estimation; field inhomogeneity; phase unwrapping

INTRODUCTION

Accurate field maps are of critical importance in numerous MR applications, including functional MRI, MR thermometry, MR spectroscopy, and various quantitative MR methods (1, 2, 3, 4, 5, 6). In this article, we say our “field maps” result from any deviation from the desired magnetic field that is caused by B₀-field inhomogeneity, chemical shift or susceptibility-induced inhomogeneity. Common approaches to computing the field maps consist of acquiring multiple scans with different echo times (1, 7, 8).

A fundamental accuracy-dynamic range trade-off exists with multi-echo field mapping: a large difference between the echo times ΔTE reduces the maximum detectable field offset value and causes phase wrapping while a short ΔTE yields unreliable measurements with high variance. To overcome the resulting inaccuracies, a class of spatio-temporal post-

*Corresponding author: jdagher@email.arizona.edu.

processing algorithms, termed “phase unwrapping,” has been proposed (9, 10, 11, 12, 13). An inherent assumption with these methods imposes a smoothness constraint on the measured data. This reduces the spatial resolution of the measurements (8). Furthermore, the efficiency of phase unwrapping algorithms often depends on the accuracy of the initial phase estimate (10), thus requiring expert user intervention. Finally, phase unwrapping methods are computationally expensive particularly in two-dimensions (2D) and higher where the unwrapping problem becomes NP hard (10, 11). Other existing methods adopt a more statistical approach but also rely on spatial regularization for robust mapping (8, 13, 14). Such smoothness assumptions may not be realistic as they do not account for small or abrupt variations common in MR images. This is particularly important in quantitative imaging where physiological information is extracted from field maps. Ultimately, the performance of all these methods is subject to the trade-off inherent in the choice of the echo time spacing (8).

In this paper, we propose a novel paradigm for field map estimation that is not subject to this echo time spacing trade-off. In such a paradigm, the following important properties arise: (i) reducing ΔTE is not required to increase the dynamic range, (ii) using large ΔTE is no longer limited by wrapping artifacts and, (iii) no spatial regularization or phase unwrapping are needed. The first two properties (i) and (ii) result in reduced phase uncertainty and increased robustness, while property (iii) implies an increase in the resulting spatial resolution at a drastically reduced computation time. Using theoretical formulation, we show that three echo times can be optimized to yield low noise, high dynamic range field maps, over a spectral dynamic range of interest. Our approach designs the echo times before image acquisition so as to maximally disambiguate the phase in estimation. The echo time optimization requires knowledge of the worst case Signal-to-Noise Ratio (SNR) and largest field offset value in the region of interest. We validate our results in a phantom, and present high-resolution large dynamic range field maps in vivo.

THEORY

Field mapping trade-offs

Consider the measurement obtained using a spin-warp sequence at echo time TE (3, 14, 15)

$$g(x, y; TE) = \rho(x, y; TE) \exp \{ i 2\pi \Delta B(x, y) TE \} + w(x, y). \quad [1]$$

In this equation, $\rho(x, y; TE)$ is the spin density at TE, $\Delta B(x, y)$ is the field value (in Hz) at location (x, y) , and $w(x, y)$ is the additive complex-valued Gaussian noise, $w(x, y) = w_R(x, y) + i w_I(x, y)$, with $w_R, w_I \sim \mathcal{N}(0, \sigma_w)$. Hereafter, we drop the pixel subscripts (x, y) with the understanding that the remaining analysis applies separately to each voxel in the image. It is clear from [1] that we can compute ΔB from the slope of the phase of g as a function of TE. However, instead of the actual phase, we are restricted to work with the numerically computed angle, namely,

$$\psi_k = 2\pi \Delta B (TE_0 + k \Delta TE) + \Omega_k + 2\pi r_k, \quad [2]$$

where ΔTE is the echo spacing, k is the echo index, $k = 0, \dots, K-1$, TE_0 is the echo time at $k = 0$, and Ω_k is the radian phase contribution of the additive noise term at time index k . Note that the phase noise Random Variable (RV) Ω_k is a function of the echo time. The integer r_k in [2] is the phase wrapping term, which is non-zero when the sum of the first two terms on the right side of [2] falls outside $[-\pi, \pi)$. In that case, r_k forces the total phase back into the observed range $[-\pi, \pi)$.

It is clear from [2] that estimating the slope of ψ_k is affected by two sources of error: noise and phase wrapping. The contribution of each source to the overall error is determined by the value of ΔB with respect to a cutoff given by

$$\Delta B_{\text{cutoff}} = \frac{1}{2\Delta TE}. \quad [3]$$

The choice of ΔTE defines three regimes. In **Regime I**, ΔB in a given voxel is larger than ΔB_{cutoff} and phase aliasing (wrapping) dominates the error in that voxel (8). In **Regime II**, ΔB is much smaller than ΔB_{cutoff} and noise dominates in [2]. Most field mapping methods choose moderate values for ΔTE (**Regime III**) in an attempt to trade-off Regimes I and II. However, in addition to noise and phase-wrapping, Regime III is also amenable to noise-driven phase wrapping errors (10, 16). Decoupling these errors requires the use of complex spatial regularization and phase unwrapping post-processing methods (8, 9, 13, 14). This ultimately reduces the spatial precision of the resulting field maps (8). Furthermore, the robustness of these post-processing methods quickly decreases with the number of captured echoes K . There, the total scan time is obviously the limiting factor.

Three-echo methods have recently been proposed in the literature. Windischberger *et.al.* (17) used irregular echo spacing of 0.5ms (Regime II) and 1.5ms (Regime I) at 3T to demonstrate field maps over a limited dynamic range of $\pm 333\text{Hz}$. Assuming a maximum of a single phase wrap between echo pairs, the authors in (17) applied linear fitting to determine the best fit (in least squares sense) from seven possible phase wrap scenarios. Then, median and Gaussian filtering was applied to average errors inherent in the linear fitting process. Aksit *et.al.* (18) proposed another 3-echo method which computes an estimate of the phase twice, once using a very short ΔTE (Regime II) and another using a much longer ΔTE (Regime I). This method attempts to unwrap the long echo pair estimate using information from the short echo pair estimate.

In this paper, we quantify the ambiguity (error) space associated with a field estimate obtained with any given echo pair. Then, we show that by using an optimal choice of three echoes, the overall ambiguity space can be engineered to possess specific distinct features. We then design a simple field map estimation routine which takes advantage of such an engineered ambiguity space. We will show that this approach overcomes the trade-offs of Regimes I-III.

Novel paradigm: Phase Ambiguity functions

We propose first a novel paradigm for quantifying the uncertainty in the field map estimated $\widehat{\Delta B}_{0,1}$ when computed from two echoes, TE_0 and TE_1 . Namely,

$$\widehat{\Delta B}_{0,1} = \frac{\angle g(TE_1) - \angle g(TE_0)}{2\pi \Delta TE_{0,1}} \quad [4]$$

where $\Delta TE_{0,1} = TE_1 - TE_0$ is the echo spacing. In the absence of phase wrapping, it has been shown that $\widehat{\Delta B}_{0,1}$ is the Maximum Likelihood (ML) field map estimate for methods utilizing 2 echo acquisitions (8). We can easily show that the overall error in this estimate can be written as:

$$\mathcal{E}_{0,1} = \widehat{\Delta B}_{0,1} - \Delta B = \frac{n_{0,1} + \Delta \Omega_{0,1} / 2\pi}{\Delta TE_{0,1}} \quad [5]$$

where $\Delta\Omega_{0,1} = \Omega_1 - \Omega_0$ is the difference between the phase noise random variables in [2], and $\mathbf{n}_{0,1}$ denotes a phase wrapping integer which is non-zero whenever $2\pi\Delta B\Delta TE_{0,1} + \Delta\Omega_{0,1}$ falls outside $[-\pi, \pi)$. The probability density function (PDF) function of $\mathcal{E}_{0,1}$ completely describes the error in the estimate. This PDF gives rise to an uncertainty (ambiguity) about the original phase slope value, which we label as the Phase Ambiguity (PA) function.

Assuming that, over a range of field map values of interest, all possible values for $\mathbf{n}_{0,1}$ are equally likely, we can show that the PA function of $\mathcal{E}_{0,1}$ can be written as,

$$\chi_1 = \alpha_1 f_{\Omega_1}(2\pi\Delta TE_{0,1}\mathcal{E}) * f_{\Omega_0}(2\pi\Delta TE_{0,1}\mathcal{E}) * \sum_p \delta\left(\mathcal{E} - \frac{p}{\Delta TE_{0,1}}\right) \quad [6]$$

where $\mathcal{E}$ is the observed error in Hz, $f_{\Omega_k}(\omega)$ is the PDF of Ω_k and $*$ denotes the convolution operator. α_1 is a PDF scaling parameter over the domain of χ_1 . Following derivations similar to (19), we can show that $f_{\Omega_k}(\omega)$ can be written as:

$$f_{\Omega_k}(\omega) = \frac{\exp\left(-\lambda_k^2/2\right)}{2\pi} \left\{ 1 + \lambda_k \sqrt{\frac{\pi}{2}} \cos \omega \exp\left(\lambda_k^2 \cos^2 \omega / 2\right) \left[1 + \operatorname{erf}\left(\frac{\lambda_k \cos \omega}{\sqrt{2}}\right) \right] \right\} \quad [7]$$

where

$$\lambda_k = 0.65 \text{SNR}_0 \exp\left(-\frac{TE_k - TE_0}{T2^*}\right) \quad [8]$$

and SNR_0 is the magnitude-domain SNR at TE_0 , $\text{SNR}_0 = \rho^{(TE_0)}/\sigma_w$. It is important to note that we are only interested in a limited sub-domain of the PA function corresponding to $[-2\delta\mathcal{B}_{\max}, 2\delta\mathcal{B}_{\max}]$, where $\delta\mathcal{B}_{\max}$ is the maximum field value expected over a region of interest. Hereafter we refer to the quantity $2\delta\mathcal{B}_{\max}$ as the dynamic range of the field map.

Figures 1(a)-1(c) show example PA functions for acquisitions at an echo spacing $\Delta TE_{0,1}$ of 1.5ms, 0.5ms and 1.0ms, respectively. The PA functions are plotted on a log scale. The horizontal axis denotes the error in the field map estimate ($\mathcal{E}_{0,1}$ in Hz), while the vertical axis represents the likelihood of observing that error. The dynamic range of interest here is $2\delta\mathcal{B}_{\max} = 900\text{Hz}$ with a noise level corresponding to $\text{SNR}_0 = 10$. The minimum decay constant $T2^*$ is set to 12ms. We clearly see that the PA function in Figure 1(a) is dominated by wrapping errors, as can be seen from the multiple peaks. This is expected in Regime I where field values are larger than the cutoff ΔB_{cutoff} . On the other extreme, a very wide single lobe in Figure 1(b) implies a noise-limited performance, characteristic of Regime II. Finally, a moderate choice of $\Delta TE_{0,1}$ in Figure 1(c) displays a PA function with error contributions from both phase wrapping and noise, as well as noise-induced phase wrapping.

Proposed solution: MAGPI

The trade-offs in field mapping Regimes I-III can be overcome using the Phase Ambiguity paradigm introduced above. Specifically, we show here that the underlying ambiguity can be resolved by using a careful choice of three echoes.

Repeating the same analysis above using echo pairs 0 and 2, we obtain Phase Ambiguity χ_2 where,

$$\mathcal{E}_{0,2} \triangleq \widehat{\Delta B}_{0,2} - \Delta B = \frac{n_{0,2} + \Delta\Omega_{0,2}/2\pi}{\Delta TE_{0,2}} \quad [9]$$

and

$$\chi_2 = \alpha_2 f_{\Omega_2}(2\pi \Delta TE_{0,2} \mathcal{E}) * f_{\Omega_0}(2\pi \Delta TE_{0,2} \mathcal{E}) * \sum_q \delta \left(\mathcal{E} - \frac{q}{\Delta TE_{0,2}} \right) \quad [10]$$

where $\mathcal{E}$ is the error variable in Hz, $f_{\Omega_k}(\omega)$ is as given in [7], $\Delta \Omega_{0,2} \triangleq \Omega_2 - \Omega_0$, $\Delta TE_{0,2} \triangleq TE_2 - TE_0$, $\mathbf{n}_{0,2} = 0, \pm 1, \pm 2, \dots$, and α_2 is a PDF scaling parameter over the range $[-2\delta \mathcal{B}_{\max}, 2\delta \mathcal{B}_{\max}]$.

We immediately point out two important properties from [6] and [10]:

Property PA-1—Irrespective of the choice of echo times, the Phase Ambiguity functions will always have a non-zero peak at $\mathcal{E}_{0,1} = \mathcal{E}_{0,2} = 0$. That is, the original true solution is a common occurrence in both functions, irrespective of the value of the echoes and other parameters, such as SNR, T2*, etc.

Property PA-2—If both PA functions were identical, then this implies that knowledge of $\mathcal{E}_{0,2}$ does not reduce the ambiguity (or increase the information) about the true value of $\Delta \mathcal{B}$, with respect to $\mathcal{E}_{0,1}$. And vice versa.

It is clear from these 2 properties that we would want χ_1 and χ_2 to be maximally distinct. This ensures that the ambiguity remaining after measuring echoes TE_0 , TE_1 and TE_2 is as close to the real value as possible. Therefore, we advocate choosing echo times so that the resulting χ_1 and χ_2 are as distant as possible, over $[-2\delta \mathcal{B}_{\max}, 2\delta \mathcal{B}_{\max}]$. We denote the distance between χ_1 and χ_2 over this domain as $\mathcal{D}(\chi_1, \chi_2)$.

Our proposed echo time optimization routine then solves the following problem:

$$\begin{aligned} [TE_0^{\text{opt}}, TE_1^{\text{opt}}, TE_2^{\text{opt}}] = \arg \max_{[TE_0, TE_1, TE_2]} \mathcal{D}(\chi_1, \chi_2) \\ \text{such that, } [TE_0, TE_1, TE_2] \in \mathcal{C}, \end{aligned} \quad [11]$$

where $\mathcal{C}$ is the set of allowable echo times that could be utilized with the pulse sequence of choice. This formulation constitutes the basis of our novel joint acquisition/estimation method, which we refer to hereafter as, **Maximum AmbiGuity distance for Phase Imaging (MAGPI)**.

Various distance measures $\mathcal{D}$ could be considered in order to maximize the distinction between χ_1 and χ_2 (20, 21). In this paper, we choose a geometric measure based on maximizing the minimum distance between respective peaks from χ_1 and χ_2 . However, this measure alone would attempt to decrease the echo time spacings, which reduces the peaks' spacing but also broaden the peaks, as we have seen in Figure 1. The latter is undesirable since we also need to minimize the overlap between the center peak of χ_1 and that of χ_2 , corresponding to the noisy wrap-free solution. The overall objective function $\mathcal{D}$ thus combines these two geometrical metrics, namely, $\mathcal{D} = \{\text{minimum distance between } (\chi_1, \chi_2) \text{ peaks}\} - \{\text{total width of central peaks}\}$.

We apply MAGPI, with our choice of $\mathcal{D}$, to our acquisition scenario of Figure 1. We optimize echo times to operate over a dynamic range of 900Hz, a worst case SNR₀ of 10 and T2* values as low as 12ms. The resulting optimal echo times were: $TE_0 = 2\text{ms}$, $TE_1 = 3.24\text{ms}$ and $TE_2 = 4.05\text{ms}$ ($\Delta TE_{0,1} = 1.24\text{ms}$ and $\Delta TE_{0,2} = 2.05\text{ms}$). The corresponding PA functions are shown in Figure 1(d). We note that both echo pairs have a large spacing with

respect to $1/2 \times 900$, thus operating in a wrap-dominated regime (Regime I). Nevertheless, the overlap pattern of the error distributions is engineered to disambiguate phase wrapping (due to minimal overlap of peaks) and reduce noise variance (due to large echo time spacings).

Corresponding estimation method: CL2D

The echoes optimized by [11] guarantee that the resulting PA functions are maximally distinct, in the sense defined by the metric $\mathcal{D}$ above. We present here a corresponding estimation method which takes advantage of the designed ambiguity space in order to recover the original field map value.

Similar to the distance metric $\mathcal{D}$, the estimation algorithm proposed here is based on an intuitive geometrical approach. Specifically, we first point out that the angle given by [4] corresponds simply to one sample realization from the PA function χ_1 . We illustrate this in Figure 2 with the magenta “x” marker, where the horizontal axis denotes all possible field map values. This sample could have resulted from any one of the periodic peaks in χ_1 . Thus, replicating this sample by the inherent period of $1/\Delta TE_{0,1}$ results in the set of all possible solutions to [5] (magenta “o” marker), for this particular noise realization. Any one of these replicated values could in fact belong to the wrap-free peak of χ_1 . This ambiguity is readily resolved using the second echo pair (TE_0, TE_2): we repeat the same steps for $\widehat{\Delta B}_{0,2}$ and its corresponding χ_2 and obtain another set of values, as shown in Figure 2 in red. Given that the optimizer designed the two PA functions to be maximally distant, we know that the two values from these sets that are closest to one another (in $\mathcal{L}^2$ -distance sense) correspond to the two wrap-free peaks from χ_1 and χ_2 . This is illustrated in Figure 2. Finally, a field map estimate can be easily recovered by, for example, averaging these two closest values. We summarize this Closest-L2 Distance (CL2D) estimation method in Table 1

Note that, instead of the simple averaging at the final step, other estimation methods could be proposed. In this paper, we use a simple weighted averaging procedure, as indicated in Table 1, thereby restricting our attention to the advantages obtained with our novel MAGPI echo optimization, in spite of using a simple estimation method.

Finally, we should emphasize that the proposed CL2D estimation method is applied independently one voxel at a time.

Methods

Echo time optimization

The optimization routine of [11] is run offline, once. We can see from [7], [8] and [11] that the only prior information needed to compute the optimal echo times with MAGPI are the maximum dynamic range of interest $2\delta B_{\max}$, the minimum SNR_0 and the minimum expected $T2^*$ in the regions of interest. We solve the optimization problem using an Adaptive Simulated Annealing (ASA) routine (22, 23).

Phantom

We report results obtained at 7T with a cylindrical water phantom containing oil and air running in tubes along its long axis. The data was acquired on a 7T Siemens Trio scanner (Siemens Medical Solutions, Erlangen, Germany). The acquisition parameters were as follows: Field of View (FOV) = $120 \times 120 \text{ mm}^2$, slice thickness 1.5mm, matrix size 256×256 , flip angle 45° , 8 slices, TR= 185ms, with fat saturation turned off, and automatic shimming

turned on. The acquisition thus generates images with high spatial resolution, corresponding to $0.46 \times 0.46 \times 1.5\text{mm}^3$ voxels.

Subjects and MRI protocol

We also demonstrate the utility of our method *in vivo*. The ankle and brain of healthy volunteers were imaged after approval was obtained from our Institutional Review Board (IRB) and informed consent was given by the subjects.

We acquire field offset maps in a subject's ankle using a Siemens 3T Trio. The offset map was obtained using GRE acquisitions at different echo times. All acquisitions employed the following parameters: FOV = $300 \times 300\text{mm}^2$, slice thickness 3mm, matrix size 256×256 , flip angle 55° , 8 slices, TR= 100ms, with both fat saturation and automatic shimming turned off. The voxel size is thus $1.1 \times 1.1 \times 3\text{mm}^3$.

We also acquire field maps in a volunteer's brain using the 7T Siemens Trio. We use both, GRE acquisitions as well as Multi-Echo Gradient-Echo (MEGE) acquisitions. Both sequences acquire high resolution images, corresponding to approximately $0.33 \times 0.33 \times 1.5\text{mm}^3$ voxels. The parameters for the GRE acquisitions were: FOV = $210 \times 210\text{mm}^2$, slice thickness 1.5mm, matrix size 640×640 , flip angle 45° , 8 slices, TR= 180ms, with both fat saturation and automatic shimming turned on. The parameters for the MEGE acquisitions were: FOV = $146 \times 210\text{mm}^2$, read out axis set along the shorter dimension, slice thickness 1.5mm, matrix size 420×640 , flip angle 55° , 8 slices, TR= 100ms, with both fat saturation and automatic shimming turned on.

Results

We illustrate in Figure 3 a basic numerical example highlighting the properties of each of the acquisitions discussed in Figure 1. The example simulates GRE measurements taken at three echo times in two voxels with field offset values of 400Hz (Figure 3(a)) and 10Hz (Figure 3(b)). The noise level corresponds to an initial SNR of $\text{SNR}_0 = 10$ and $T2^*$ of 12ms. We first simulate 3 echoes acquired at an echo spacing of 1.5ms ($\Delta B_{\text{cutoff}} \approx 333\text{Hz}$, black), 0.5ms ($\Delta B_{\text{cutoff}} = 1\text{KHz}$, magenta) and 1.0ms ($\Delta B_{\text{cutoff}} = 500\text{Hz}$, blue). We then compute the angle at each echo time and perform a phase unwrapping operation across the three time samples. The solid lines are the Least Squares (LS) linear fit to the unwrapped angle data. The trade-offs in each of the regimes are consistent with the trade-offs discussed earlier. Our proposed method samples the phase irregularly at large echo time spacings of 1.24ms ($\Delta B_{\text{cutoff}} \approx 403\text{Hz}$) and 2.36ms ($\Delta B_{\text{cutoff}} \approx 244\text{Hz}$), as optimized by MAGPI for $2\delta B_{\text{max}} = 900\text{Hz}$, a worst case SNR_0 of 10 and minimum $T2^*$ of 12ms (Figure 1(d)). The red line shows the result of the simple estimation procedure CL2D. As we can see, our method consistently achieves the correct estimate in this example, for both large and small values of ΔB .

For validation with real data, we present high resolution field maps estimated inside the water/oil phantom of Figure 4(a). Since we did not suppress fat signals with the GRE sequence, we expect the field map to contain offsets due to both, chemical shifts and B0 field inhomogeneity. At 7T, fat is offset by about 980Hz, thereby implying a dynamic range of 1960Hz. This requires sampling at echo spacings less than 0.51 ms. Figures 4(b) and 4(c) show the field map obtained with 16 regularly spaced echoes, with $\Delta\text{TE} = 0.25\text{ms}$. The method in Figure 4(b), labeled DFT-16, estimates the slope in each voxel using the Discrete Fourier Transform (DFT) along with temporal zero-padding, while in Figure 4(c) the slope is estimated directly in temporal domain using an LS fit (LS-16) (24). We also show a field map obtained using the traditional 2-echo method in Figure 4(d) (labeled ML-2), computed according to [4], with $\Delta\text{TE} = 0.25\text{ms}$. For further comparison, we present in Figure 4(e) the

field map obtained using the 3-point method proposed by Aksit *et.al.* (18). This Long Echo/Short Echo (LeSe-3) method attempts to balance the advantages of each regime. A short echo spacing, ΔTE_{se} (Regime II) is used to unwrap the phase from the long echo spacing, ΔTE_{le} (Regime I). Here we used $\Delta TE_{se} = 0.25\text{ms}$ and $\Delta TE_{le} = 15 \times \Delta TE_{se}$, as suggested in (18). Finally, we show the result obtained with our method, using the 3 echoes $TE_0 = 5\text{ms}$, $TE_1 = 6.32\text{ms}$, $TE_2 = 6.76\text{ms}$ (or $\Delta TE_1 = 1.32\text{ms}$ and $\Delta TE_2 = 1.76\text{ms}$), as prescribed by the optimizer in [11]. MAGPI picked these optimal echo times for a worst case SNR_0 of 20, a minimum expected $T2^*$ of 15ms, and a maximum expected dynamic range of 2200Hz.

Next we evaluate the performance of our method in vivo. We first consider images taken of a volunteer's ankle at 3T. Since it is difficult to perform complete fat suppression and extensive shimming in extremities, it is often common to capture images with both fat suppression and shimming turned off (25, 26). Separation of the chemical shift and inhomogeneity effects is then done in post-processing using the acquired complex images (27). The resulting field maps thus display the compounded effects of B0-field inhomogeneity and chemical shift. Figure 5(a) shows a high-resolution sagittal slice acquired with a GRE sequence at $TE = 6\text{ms}$. Figures 5(b), 5(c), 5(d) show field offset maps estimated using DFT-16, LeSe-3 and MAGPI, respectively. The echo times used by each method are indicated in the figure. MAGPI used echo times optimized for a minimum SNR_0 of 15, a maximum expected $2\delta B_{\max}$ of 1000Hz and minimum $T2^*$ of 15ms.

Finally, we present high resolution field maps in the brain at 7T. Figures 6(a) and 6(b) show two GRE axial images in two different slices. All field map methods here use only 3 echoes, with either GRE or MEGE sequences.

Figures 6(c) and 6(e) display the field maps estimated with the GRE sequences. Figure 6(c) uses the Least-Squares fit method (LS-3) with echoes captured regularly at 4, 4.5, and 5ms. Figure 6(e) shows the result obtained with our proposed method. MAGPI used the echo times of 4, 5, 6.5ms, or $\Delta TE_1 = 1\text{ms}$ and $\Delta TE_2 = 2.5\text{ms}$, optimized for a minimum SNR_0 of 18, a maximum expected $2\delta B_{\max}$ of 1700Hz and a minimum $T2^*$ of 15ms.

Field maps generated using the MEGE sequence are shown in Figure 6(d) (LS-3) and Figure 6(f) (MAGPI). MEGE sequences are often preferred in field map imaging over GRE sequences for being faster at acquiring the same number echoes with similar parameters. However, MEGE sequences have an inherent constraint on the minimum echo spacing that could be accommodated. With our sequence parameters here, this minimum possible echo spacing is 1.2ms. Therefore, regularly spaced echo time methods, such as LS-3 would use echoes 4, 5.2, and 6.4ms. The resulting field map is shown in Figure 6(d). The field map obtained with our MAGPI method is shown in Figure 6(f). The MAGPI echoes used were optimized to yield optimal field maps over a dynamic range of 1600Hz in regions with minimum SNR_0 of 18, and minimum $T2^*$ of 15ms. Furthermore, the set $\mathcal{C}$ in [11] constrained consecutive echoes to be separated by at least 1.2ms, resulting in $TE_1 = 4\text{ms}$, $TE_0 = 5.2\text{ms}$, $TE_2 = 7\text{ms}$.

Discussion

We presented phantom data as well as in vivo results showing robust high-resolution field maps obtained without the traditional trade-offs of Regimes I-III. It is important to emphasize the salient aspects of our proposed method which consist of:

- A. quantifying the ambiguity in the field map estimate (PA function) for a given echo time spacing ([6], [7] and [10]),

- B. optimizing 3 echo times (2 echo spacings) for a given SNR_0 , $T2^*$ and $\delta B_{\max}$ of interest, so that the two resulting PA functions are maximally distinct [11],
- C. designing a corresponding estimation routine which takes advantage of the engineered ambiguity space above in order to recover the original field map value.

We make the following important observations about each of the points above:

(A.1) We have validated the theoretical noise PDF [7], and PA functions ([6], [10]) using numerical simulations and phantom data. The histograms perfectly match our theoretical predictions for various scenarios. We skip this demonstration here for space constraints.

(B.1) Our optimizer chooses the echo times based on worst case imaging scenarios, corresponding to a maximum dynamic range and minimum SNR_0 and $T2^*$ of interest. In our work, we simply picked these values based on prior knowledge of the magnetic field strength, the sequence parameters and the material of interest.

(B.2) Table 2 lists optimal echo time prescriptions for various imaging scenarios commonly encountered at 3T. It is clear that, at high SNRs, the MAGPI echo time spacings do not drastically change with $T2^*$ or SNR_0 . However, at low SNRs, the optimal echo spacings are more sensitive to the minimum expected $T2^*$ and SNR_0 . Furthermore, note that the echo time spacings get smaller at low $T2^*$ and SNR_0 , in order to avoid noise-induced phase wrapping. Our formulation is thus particularly important in the latter imaging scenarios, which are encountered in high resolution rapid field mapping.

(B.3) We should note that there could exist more intuitive search routines than our numerical optimization algorithm. However, such an intuitive search would depend on both the distance measures chosen and the proposed estimation algorithm. In this paper, the focus was on the overall novel framework and a general numerical method which could generate echo times suitable for arbitrary scenarios of interest.

(C.1) In addition to our proposed CL2D method, there exists multiple estimation algorithms that could recover the original field map value from the optimized MAGPI acquisitions. However, such methods should take into account the unique pattern of the designed PA function in order to disambiguate the errors. We illustrate this in Figure 7 where estimation methods from the literature are used in conjunction with echoes optimized for the scenario of Figure 1(d). As we can see, only CL2D utilized the feature of the designed ambiguity space while the remaining methods still suffered from the noise-phase wrapping ambiguities.

(C.2) Our estimation method have not used any form of spatial regularization/processing. This makes CL2D and MAGPI highly suitable for high resolution field mapping (8). We therefore restricted our attention to voxel-per-voxel estimation methods which constitute the bases of commonly used techniques. Any additional spatial regularization could be applied to all methods presented here.

We validated the performance of our field map estimation method using the phantom of Figure 4. We used the 16-point methods of DFT-16 (Figure 4(b)) and LS-16 (Figure 4(c)) as a “gold standard” in this case. Despite the fact that these field maps were obtained from the noise-limited Regime II, we expect improved robustness with 16 echoes in static phantoms with long $T2^*$ decay. Obviously, such long acquisitions are sub-optimal in practical applications, due to subject motion, $T2^*$ decay effects, and long acquisition times. In comparison, the commonly used two-point method (Figure 4(d)), suffers from severe noise amplification. This behavior is justified by the acquisition’s Phase Ambiguity function, shown in Figure 8(a). We immediately observe its single peak with a wide lobe extending

over hundreds of Hertz, characteristic of a noise-limited regime. The LeSe-3 method (Figure 4(e)) also exhibits large variations, despite using 3 echoes. To understand its performance, we also refer to the PA functions of LeSe-3, shown in Figure 8(b). We note the significant overlap between the PA function of the short echo spacing pair ΔTE_{se} and the PA function of the long echo spacing pair ΔTE_{le} . As explained in **Property PA-2** in Theory, such an acquisition strategy leads to an inherent ambiguity about the true field value. Irrespective of the estimation algorithm used, such ambiguity could not be resolved without making additional assumptions about the spatial behavior of the field map (smoothing or regularization). MAGPI, on the other hand, uses echo times which result in maximally distant PA functions, as shown in Figure 8(c). The resulting high-resolution field map in Figure 4(f) looks visually very similar to those obtained with DFT-16 and LS-16, displaying robust and large dynamic range estimates, without the use of any spatial processing.

We inspect these phantom results numerically in Figure 9 by considering profiles along 2 different rows. The location of the rows is indicated by the dotted lines in Figure 4(a). The profile in Figure 9(a) plots the field map through 2 oil rings and the water background, while the profile in Figure 9(b) shows the field map in water only. Figure 9(c) is a magnified profile through one oil ring. These plots demonstrate the efficiency and robustness of the MAGPI field estimates and their similarity to field maps obtained with 16 point methods. In comparison, LeSe-3 and ML-2 struggle to achieve the desired field map values, especially in water (Figure 9(b)) where the performance of MAGPI is particularly impressive. The fact that MAGPI produces high resolution ($0.46 \times 0.46 \times 1.5\text{mm}^3$) field maps comparable to those obtained with 16-point methods, at 18% of the acquisition time, is noteworthy.

We also should emphasize that the smallest echo time spacing used with MAGPI in our phantom experiment would traditionally have caused aliasing for field values larger than $\Delta B_{\text{cutoff}} \approx 379\text{Hz}$. Our method was able to disambiguate field values up to a maximum value of 1100Hz using the engineered structure of the PA functions. This implies an impressive gain in dynamic range of a factor of 2.9.

We also demonstrated the efficacy of our proposed method in vivo. Comparing the field map obtained in the ankle at 3T using DFT-16 (Figure 5(b)) with that using MAGPI (Figure 5(d)), we can see that our method is able to obtain a similar field offset map without noise or phase wrapping artifacts. It is important to point out here that MAGPI's field offset map appears to display finer structure than that obtained with DFT-16. That is attributed to slight subject motion, which we detected with the 16-echo scan. This highlights one important practical advantage of acquisitions with smaller number of echoes. In comparison, the LeSe-3 method (Figure 5(c)) was unable to attain the quality achieved with MAGPI nor DFT-16. Note that we chose a slightly larger ΔTE_{se} for the LeSe-3 method in order to increase its robustness (reduce the lobe width of its PA function). This reduces the noise-driven errors (white arrows) as compared to LeSe-3's performance in the phantom. However, this comes at the expense of increased wrapping artifacts (yellow arrows), due to additional peaks in PA functions.

Recall that the field maps in Figure 5 display the compounded effects of B0-field inhomogeneity and chemical shift. It is beyond the scope of our work here to decouple these two effects. We expect, however, for any separation method which takes into account phase information to benefit from our joint acquisition/estimation strategy, which removes inherent ambiguities in phase measurements.

We also generated field maps in the brain at 7T using GRE sequences. We can clearly see from Figure 6(e) that our proposed method achieves more robust estimates as compared to methods with the same amount of acquisition time, such as LS-3 (Figure 6(c)). The field

map obtained with MAGPI displays high resolution features at a markedly reduced noise variance, as we can clearly see from the magnified image section. Our method also exhibits robust estimation for both large and low field map values across the brain, without spatial processing.

The second 7T experiment consisted of extracting field maps from a 3-echo MEGE sequence, often used for its faster acquisition time. The 1.2ms minimum echo spacing constraint of this sequence imposes a maximum dynamic range of $\pm 417\text{Hz}$ in field maps obtained with regularly spaced echoes. This is seen in Figure 6(d) where LS-3 suffers from wrapping artifacts (white arrow and sinus areas). Even though the long echo spacings in Figure 6(d) reduced noise contributions as compared with the short echo spacings used with GRE and LS-3 (Figure 6(c)), they significantly increased wrapping errors.

On the other hand, our imaging paradigm is readily suited for this additional echo time constraint, as incorporated in $\mathcal{C}$. The optimal echo times $\text{TE}_1 = 4\text{ms}$, $\text{TE}_0 = 5.2\text{ms}$, and $\text{TE}_2 = 7\text{ms}$ are designed to yield field maps over the dynamic range of 1600Hz and a worst case SNR of 18. We note the location of the reference echo, TE_0 . Our optimizer in this case automatically determines that this arrangement is optimal as it reduces the value of the last echo time TE_2 , and in turn T_2^* decay effects. The resulting field map in Figure 6(f) displays properties similar to what we observed and validated in the phantom study: low variance in smooth regions and large dynamic range mapping up to $\pm 780\text{Hz}$. This is a significant 91% gain in dynamic range, as compared to sequences limited by the 1.2ms minimum echo spacing constraint. In this example, hardware limitations constrain the dynamic range achievable even with 16-point methods. MAGPI was immune to this constraint and becomes an essential tool in such applications.

There are limitations to MAGPI's capabilities. In cases of extremely low SNR, or extremely large echo time spacing constraints, MAGPI may not be able to obtain optimal echoes arbitrarily large dynamic range. In such instances, the geometrical distance measure employed here may not be able to attain a robust solution. The onus then is placed on the pulse sequence to alleviate one of the constraints. Nevertheless, we should mention that we have not yet explored the theoretical limit of our method, as it may be possible to use more suitable distance measures $\mathcal{D}$ under such extreme conditions. Additional future work includes investigating statistical distance measures, instead of our proposed geometrical measure, and corresponding statistical estimation algorithms, in lieu of the simple weighted averaging procedure.

In conclusion, we demonstrated here a new paradigm for estimating off-resonance maps using 3 echoes only. Our proposed echo time optimizer is able to explore search spaces corresponding to large echo time spacing, which was never considered before to the best of the authors' knowledge. Our proposed method is able to simultaneously combine in 3 echoes the advantages of large echo spacings without their inhibiting wrapping constraints. The MAGPI scheme selects the echo times and characterizes the resulting phase errors so as to be maximally disambiguated in estimation. MAGPI overcomes many of the traditional constraints on field maps, as demonstrated throughout this paper. Specifically, in our proposed paradigm: (i) reducing ΔTE is not required to increase the dynamic range, (ii) using large ΔTE is no longer limited by wrapping artifacts and, (iii) no spatial regularization or phase unwrapping are needed.

Acknowledgments

This work has been supported by NIH grant T32EB013180.

References

1. Jezzard P, Balaban R. Correction for Geometric Distortion in Echo Planar Images from B0 Field Variations. *Magnetic Resonance In Medicine*. Jul.1995 34:65–73. [PubMed: 7674900]
2. Yu H, Reeder S, Shimakawa A, Brittain J, Pelc N. Robust field map estimation in a Dixon water-fat separation algorithm with short echo time increments. *International Society for Magnetic Resonance in Medicine*. 2004; 11:345.
3. Liu G, Ogawa S. EPI image reconstruction with correction of distortion and signal losses. *Journal of Magnetic Resonance Imaging*. 2006; 24(3):683–689. [PubMed: 16892198]
4. Rieke V, Butts Pauly K. MR thermometry. *Journal of Magnetic Resonance Imaging*. 2008; 27(2): 376–390. [PubMed: 18219673]
5. Kressler B, de Rochefort L, Liu T, Spincemaille P, Jiang Q, Wang Y. Nonlinear regularization for per voxel estimation of magnetic susceptibility distributions from MRI field maps. *IEEE Transactions on Medical Imaging*. 2010; 29(2):273–281. [PubMed: 19502123]
6. Holland D, Kuperman JM, Dale AM. Efficient correction of inhomogeneous static magnetic field-induced distortion in Echo Planar Imaging. *NeuroImage*. Mar.2010 50:175–183. [PubMed: 19944768]
7. Reber P, Wong E, Buxton R. Correction of off resonance-related distortion in echo-planar imaging using EPI-based field maps. *Magnetic Resonance In Medicine*. 1998; 39(2):328–330. [PubMed: 9469719]
8. Funai A, Fessler J, Yeo D, Olafsson V, Noll D. Regularized field map estimation in MRI. *IEEE Transactions on Medical Imaging*. 2008; 27(10):1484–1494. [PubMed: 18815100]
9. Moon-Ho Song S, Napel S, Pelc N, Glover G. Phase unwrapping of MR phase images using Poisson equation. *IEEE Transactions on Image Processing*. 1995; 4(5):667–676. [PubMed: 18290015]
10. Ghiglia, DC.; Pritt, MD. *Two-Dimensional Phase Unwrapping: Theory, Algorithms, and Software*. Wiley-Interscience; 1998.
11. Bioucas-Dias J, Valadao G. Phase Unwrapping via Graph Cuts. *IEEE Transactions on Image Processing*. 2007; 16(3):698–709. [PubMed: 17357730]
12. Jenkinson M. Fast, automated, N-dimensional phase-unwrapping algorithm. *Magnetic Resonance In Medicine*. 2003; 49(1):193–197. [PubMed: 12509838]
13. Zhou K, Zaitsev M, Bao S. Reliable two-dimensional phase unwrapping method using region growing and local linear estimation. *Magnetic Resonance In Medicine*. 2009; 62(4):1085–1090. [PubMed: 19572389]
14. Nguyen H, Sutton B, Morrison R Jr, Do M. Joint Estimation and Correction of Geometric Distortions for EPI functional MRI using Harmonic Retrieval. *IEEE Transactions on Medical Imaging*. 2009; 28(3):423–434. [PubMed: 19244014]
15. Liang, Z-P.; Lauterbur, PC. *Principles of Magnetic Resonance Imaging: A Signal Processing Perspective*. Wiley-IEEE Press; 2000.
16. Frey B, Koetter R, Petrovic N. Very loopy belief propagation for unwrapping phase images. *Neural Information Processing Systems*. 2001:737–743.
17. Windischberger C, Robinson S, Rauscher A, Barth M, Moser E. Robust field map generation using a triple-echo acquisition. *Journal of Magnetic Resonance Imaging*. Oct.2004 20:730–734. [PubMed: 15390143]
18. Aksit, P.; Derbyshire, J.; Prince, J. Three-point method for fast and robust field mapping for EPI geometric distortion correction. 4th IEEE International Symposium on Biomedical Imaging; 2007. p. 141-144.
19. Hagmann W, Habermann J. On the phase error distribution of an open loop phase estimator. *IEEE International Conference on Communications*. 1988; 2:1031–1037.
20. Cha SH. Comprehensive survey on distance/similarity measures between probability density functions. *International Journal of Mathematical Models and Methods in Applied Sciences*. 2007; 1(4):300–307.
21. Papoulis, A. *Probability, Random Variables and Stochastic Processes*. McGraw-Hill College; 1991.
22. Ingber AL. Adaptive simulated annealing (asa): Lessons learned. *Control and Cybernetics*. 1996; 25(1):33–54.

23. Oliveira, HA.; Ingber, L.; Petraglia, A.; Petraglia, MR.; Machado, MAS. Stochastic Global Optimization and Its Applications with Fuzzy Adaptive Simulated Annealing. Springer; 2012. ch. 4.
24. Chen N, Wyrwicz A. Correction for EPI distortions using multi-echo gradient-echo imaging. Magnetic Resonance In Medicine. 1999; 41(6):1206–1213. [PubMed: 10371453]
25. Glover GH, Schneider E. Three-point Dixon technique for true water/fat decomposition with B0 inhomogeneity correction. Magnetic Resonance In Medicine. Apr.1991 18:371–383. [PubMed: 2046518]
26. Rybicki FJ, Chung T, Reid J, Jaramillo D, Mulkern RV, Ma J. Fast three-point dixon MR imaging using low-resolution images for phase correction: a comparison with chemical shift selective fat suppression for pediatric musculoskeletal imaging. American Journal of Roentgenology. Nov.2001 177:1019–1023. [PubMed: 11641161]
27. Reeder SB, Wen Z, Yu H, Pineda AR, Gold GE, Markl M, Pelc NJ. Multicoil Dixon chemical species separation with an iterative least-squares estimation method. Magnetic Resonance In Medicine. Jan.2004 51:35–45. [PubMed: 14705043]

Figure 1.

Phase Ambiguity functions for the prescribed echo time spacings of (a) 1.5ms (b) 0.5ms and (c) 1.0ms over the dynamic range of interest, $[-2 \times 450 : 2 \times 450]$ Hz, in a voxel with an SNR_0 of 10 and a decay constant T_2^* of 12ms. Regime I is limited by phase aliasing, as can be seen from the multiple peaks. Regime II has a wide single lobe, thus implying a noise-limited performance. Regime III attempts to trade-off Regime I and Regime II. (d) Phase Ambiguity functions at the echo time differences chosen by MAGPI: χ_1 for $\Delta\text{TE}_1 = 1.24\text{ms}$, and χ_2 for $\Delta\text{TE}_2 = 2.05\text{ms}$.

Figure 2.

A demonstration of the estimation algorithm described in Table 1. Note that the horizontal axis correspond to the possible field map values $[-\delta\mathcal{B}_{\max} : \delta\mathcal{B}_{\max}]$, and not the error. The estimation method takes advantage of the prior-information that the optimizer guarantees maximum separation between the peaks. The closest solutions would inherently belong to the peaks corresponding to the true field value, in this example 400Hz.

Figure 3.

Example illustrating the trade-off in the choice of the echo time steps, as represented by Regime I ($\Delta TE = 1.5\text{ms}$, black), Regime II ($\Delta TE = 0.5\text{ms}$, magenta) and Regime III ($\Delta TE = 1.0\text{ms}$, blue). The original linear phase (green line) has a slope of (a) 400Hz and (b) 10Hz . In each of these traditional regimes, data is sampled regularly at 3 time intervals. The slope is estimated using a Least Squares fit. Regime I achieves the best results when the original slope is low, while Regime II produces a good estimate for large slope values. In Regime III, attempts to balance both regimes. Our proposed method MAGPI (red) samples the phase irregularly at 1.24ms and 2.05ms . With a simple estimation procedure (CL2D), MAGPI achieves in this example the best performance for both the large and small slope values.

Figure 4.

(a) Axial GRE image of a water/oil/air phantom. The tubes containing oil (yellow arrows) are expected to yield a field offset value around 980Hz at 7T. Field maps as estimated using (b) 16 point DFT method (c) 16-point Least Squares fitting method (d) the well-known 2-point method (e) a 3-point method proposed by Aksit *et.al.* and (f) MAGPI. Our method used $TE_0 = 5\text{ms}$, $TE_1 = 6.32\text{ms}$, $TE_2 = 6.76\text{ms}$ (or $\Delta TE_1 = 1.32\text{ms}$ and $\Delta TE_2 = 1.76\text{ms}$), as prescribed by the optimizer in [11]. MAGPI optimized the echo times for a worst case SNR_0 of 20, a minimum expected $T2^*$ of 15ms, and a maximum expected dynamic range of 2200Hz.

Figure 5.

(a) High-resolution GRE image of a volunteer's ankle taken at 3T, at TE= 6ms. Estimate of the field offset maps as obtained with (b) the 16-point DFT method, with echo times separated by 0.75ms, (b) the LeSe-3 method, at echo times of 5, 6 and 15ms, (c) our 3-point MAGPI method at TE= 5, 6.6, 7.4ms ($\Delta TE_1 = 1.6$ ms and $TE_2 = 2.4$ ms). The MAGPI echo times were optimized to detect a maximum offset value of 500Hz in voxels with SNRs of more than 15. Note that MAGPI obtains offset maps consistent with the DFT-16 method, albeit at a slightly higher spatial resolution, due to reduced motion artifacts. MAGPI's estimates are far more robust than traditional voxel-based methods, such as LeSe-3, which suffer from wrapping (yellow arrows) and noise amplification (white arrows).

Figure 6.

Results from the 7T brain scan, where we estimate field maps at a spatial resolution of $0.33 \times 0.33 \times 1.5$ (mm³). Two example GRE slices are shown in (a) and (b), both taken at TE= 5ms. The second row displays field maps obtained with a Least-Squares fitting method (LS-3), using 3 echoes. In the third row we show the field maps obtained with our method, MAGPI. In (c) and (e) we estimate field maps from GRE acquisitions, taken at 3 different echo times. The echoes used by LS-3 in (c) were 4, 4.5, 5ms while MAGPI used the optimized echo times of 4, 5, 6.5ms, corresponding to $\Delta TE_1 = 1.0$ ms and $\Delta TE_2 = 2.5$ ms. Note the impressive noise performance of MAGPI, without the use of any spatial smoothing. This is particularly apparent in the enlarged section of the field maps, where the map is scaled between $[-300, 300]$ Hz for better visualization. In (d) and (f), we generate field maps from a MEGE sequence with 3 echoes. The MEGE sequence has an inherent constraint on the minimum echo time spacing of 1.2ms. Thus, we see that in (d) the LS-3 method, with echoes captured at 4, 5.2, 6.4ms, suffers from phase wrapping in regions where field map values exceed ± 417 Hz. On the other hand, MAGPI in (f) produced field maps without any wrapping artifacts, using the optimized echo times $TE_1 = 4$ ms, $TE_0 = 5.2$ ms, $TE_2 = 7$ ms. The high resolution field map obtained with MAGPI (f) has a dynamic range that is 91% larger than methods with similar echo spacing constraint (d).

Figure 7.

Estimation performance of methods from the literature when used along with MAGPI echoes. The original phase (green line) has a slope of 380Hz, in a voxel with SNR_0 of 10 and $T2^*$ of 12ms. Methods “Windischberger+MAGPI” and “Aksit+MAGPI” use the 3 echoes proposed by our optimizer in Figure 1(d), along with the estimation algorithm proposed in (17) and (18), respectively. Clearly, in this case, our proposed simple estimation routine (CL2D) is the only method which takes into account the ambiguity space designed by MAGPI in order to resolve the correct phase.

Figure 8.

Phase Ambiguity functions for the phantom acquisitions of Figure 4 corresponding to (a) the ML-2 echoes with $\Delta TE = 0.25\text{ms}$, (b) the LeSe echoes with $\Delta TE_{se} = 0.25\text{ms}$ and $\Delta TE_{ie} = 3.75\text{ms}$, (c) MAGPI with $\Delta TE_1 = 1.32\text{ms}$ and $\Delta TE_2 = 1.76\text{ms}$.

Figure 9.

Profiles through the water/oil phantom at the rows indicated by the dashed line in Figure 4(a). We show in (c) a magnified portion of the profile in (a), which highlights the similarity of the DFT-16, LS-16 and MAGPI methods. Note also the drastically improved robustness with MAGPI from the profile through water (b), as compared to ML-2 and LeSe-3. This challenging high resolution ($0.46 \times 0.46\text{mm}^2$) mapping shows that we are able to obtain robust, large dynamic range field maps at a fraction (18%) of the time needed by 16 echo methods. The gain in dynamic range compared to methods using the same minimum echo time spacing as MAGPI is a factor of 2.9.

Table 1

Closest L2-Distance (CL2D) Estimation

Step 1	For echo pair (0, 1), compute the angle at location (x, y). The result yields $\hat{\Delta B}_{0,1}$ in [5]. As shown with the magenta “x” marker in Figure 2, the resulting value constitutes one realization from $\mathcal{Z}_1$. Repeat the same procedure for echo pair (0, 2) (red “x” marker in Figure 2).
Step 2	Replicate $\hat{\Delta B}_{0,1}$ and $\hat{\Delta B}_{0,2}$ by $1 / \Delta TE_{0,1}$ and $1 / \Delta TE_{0,2}$, respectively, over the dynamic range of interest. This is shown with the “o” markers in Figure 2. The red and magenta samples thus constitute a set of values from each PA function that would solve each of [5] and [9], respectively.
Step 3	The solutions that are closest, in the L^2 sense, to one another correspond to samples from the peaks centered around the original field value. This utilizes the important prior information that all the peaks are guaranteed by the optimizer to be maximally distant.
Step 4	Finally, we estimate the field map as the weighted average of these closest 2 samples. The weighting would take into account the respective echo times (later echoes would have a lower weight, to account for T2* effects).

Table 2

Optimal echo spacing pairs ($\Delta TE_{0,1}$, $\Delta TE_{0,2}$), in ms, for different SNR/T2* combinations, as chosen by MAGPI for a $\delta B_{\max} = 340\text{Hz}$. All scenarios assume a reference echo at $TE_0 = 5\text{ms}$.

$\begin{array}{c} \text{SNR}_0 \\ \text{T2}^* \end{array}$	13	14	17	30	31
12 ms	(1.43, 2.57)	(1.6, 2.65)	(2.55, 3.83)	(3.74, 5)	(3.74, 5)
15 ms	(1.43, 2.64)	(1.56, 2.67)	(2.55, 3.83)	(3.74, 5)	(3.74, 5)