Published in final edited form as:

Commun Stat Theory Methods. 2013 ; 42(6): 566–567. doi:10.1080/03610926.2012.718848.

Corrigendum for The Distribution of Family Sizes Under a Time-Homogeneous Birth and Death Process

Corrigendum for The Distribution of Family Sizes Under a Time-Homogeneous Birth and Death Process

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Regretably, there are a number of significant errors in Moschopoulos and Shpak, (2010). *Communications in Statistics—Theory and Methods* 39:1761–1775. Most are typographic errors that have the potential to cause confusion, and in some instances, have carried through to several equations.

1. There should be negative exponents in Eq. (1.2):

$$p_n(t) = e^{-\lambda t} (1 - e^{-\lambda t})^{n-1}. \quad (1.2)$$

2. In the equation defining Q_n immediately above Eq. (2.5), there is a missing negative exponent in the denominator $\exp[-\omega t]$; it should be:

$$Q_n = \int_0^\infty \frac{\rho \omega \exp[-(\omega + \rho)t]}{1 - \theta \exp[-\omega t]} \left(\frac{1 - \exp[-\omega t]}{1 - \theta \exp[-\omega t]} \right)^{n-1} dt.$$

3. The coefficient of $1 - \theta$ in Eq. (2.5) (and those derived from it) cancels with the change of variable, and should not appear in the equations. Note that this did not lead to errors in the numerics because of normalization. The equations should be:

$$Q_n = \rho \int_0^1 y^{n-1} (1-y)^{\rho/\omega} (1-\theta y)^{-\rho/\omega-1} dy. \quad (2.5)$$

$$q_n = \frac{\rho}{\omega} \int_0^1 y^{n-1} (1-y)^{\rho/\omega} (1-\theta y)^{-\rho/\omega} dy. \quad (2.6)$$

$${}_2F_1(a, b, c, \theta) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(c)_k} \frac{\theta^k}{k!} \quad (2.7a)$$

$$Q_n = \rho \Gamma \left(1 + \frac{\rho}{\omega} \right) A \sum_{k=0}^{\infty} \left(1 + \frac{\rho}{\omega} \right)_k B \frac{\theta^k}{k!}. \quad (2.8)$$

Furthermore, there is an error in the position of the bracket relative to powers of θ in (2.9), which should be

$$Q_n = \rho \Gamma \left(1 + \frac{\rho}{\omega} \right) n^{-1-\frac{\rho}{\omega}} \left[1 - \frac{\frac{\rho}{\omega} \left(1 + \frac{\rho}{\omega} \right)}{2n} + \frac{\frac{\rho}{\omega} \left(1 + \frac{\rho}{\omega} \right) \left(3 \left(\frac{\rho}{\omega} \right)^2 + \frac{\rho}{\omega} \right)}{24n^2} \right] \times \sum_{k=0}^{\infty} \left(1 + \frac{\rho}{\omega} \right)_k \left[1 - \frac{k}{n} \left(1 + \frac{\rho}{\omega} \right) + \frac{\varphi(\rho, \omega, k)}{n^2} \right] \frac{\theta^k}{k!} \times (1 + O(n^{-3})). \quad (2.9)$$

4. The factor $1 - \theta$ is extraneous to the asymptotic 2.11–14 as well. Note the typo in the paragraph immediately before (2.11), i.e., we have $\theta > 1$ rather than $\theta < 1$ in this instance. Moreover, there are a number of other errors with the indices and signs (most significantly, the equations should have θ^{-1} in place of θ).

$$Q_n = \rho \int_0^{1/\theta} y^{n-1} (1-y)^{\rho/\omega} (1-\theta y)^{-\rho/\omega-1} dy. \quad (2.11)$$

$$Q_n = \rho \theta^{-n} \int_0^1 z^{n-1} \left(1 - \frac{z}{\theta} \right)^{\rho/\omega} (1-z)^{-\rho/\omega-1} dz. \quad (2.12)$$

Note that powers of θ should be negative, since $\theta > 1$

$$Q_n = \rho \theta^{-n} \Gamma \left(-\frac{\rho}{\omega} \right) A \sum_{k=0}^{\infty} \left(-\frac{\rho}{\omega} \right)_k B \frac{\theta^{-k}}{k!}. \quad (2.13)$$

For the above distribution, the coefficients A and B are

$$A = \frac{\Gamma(n)}{\Gamma(n - \frac{\rho}{\omega})}, \quad B(\rho, \omega, k, n) = \frac{(n)_k}{(n - \frac{\rho}{\omega})_k} = \frac{\Gamma(n+k) \Gamma(n - \frac{\rho}{\omega})}{\Gamma(n) \Gamma(n+k - \frac{\rho}{\omega})}.$$

As in (2.9), there is also an error in the position of the bracket relative to the powers of θ in the expansion

$$Q_n = \rho \theta^{-n} \Gamma \left(-\frac{\rho}{\omega} \right) n^{\frac{\rho}{\omega}} \times \left\{ 1 - \frac{\frac{\rho}{\omega} \left(\frac{\rho}{\omega} + 1 \right)}{2n} + \frac{\left(\frac{\rho}{\omega} - 1 \right) \left(\frac{\rho}{\omega} - 2 \right) \left[3 \left(1 + \frac{\rho}{\omega} \right)^2 - \frac{\rho}{\omega} - 1 \right]}{24n^2} \right\} \times \sum_{k=0}^{\infty} \left(-\frac{\rho}{\omega} \right)_k \left[1 + \frac{k}{n} \left(\frac{\rho}{\omega} + 1 \right) + \frac{\varphi(\rho, \omega, k)}{n^2} \right] \frac{\theta^{-k}}{k!} + O(n^{-3}). \quad (2.14)$$

The asymptotic approximation to this distribution, calculated for $n \rightarrow \infty$, is:

$$Q_n \approx \rho \theta^{-n} (1 - \theta^{-1})^{\rho/\omega} \Gamma \left(-\frac{\rho}{\omega} \right) n^{\rho/\omega}. \quad (2.15)$$