Published in final edited form as:

J Biomech. 2014 January 3; 47(1): 148–153. doi:10.1016/j.jbiomech.2013.09.020.

An analytical model to predict interstitial lubrication of cartilage in migrating contact areas

A. Moore and D.L. Burris*

Biomedical Engineering, Mechanical Engineering University of Delaware Newark, DE

Abstract

For nearly a century, articular cartilage has been known for its exceptional tribological properties. For nearly as long, there have been research efforts to elucidate the responsible mechanisms for application toward biomimetic bearing applications. It is now widely accepted that interstitial fluid pressurization is the primary mechanism responsible for the unusual lubrication and load bearing properties of cartilage. Although the biomechanics community has developed elegant mathematical theories describing the coupling of solid and fluid (biphasic) mechanics and its role in interstitial lubrication, quantitative gaps in our understanding of cartilage tribology have inhibited our ability to predict how tribological conditions and material properties impact tissue function. This paper presents an analytical model of the interstitial lubrication of biphasic materials under migrating contact conditions. Although finite element and other numerical models of cartilage mechanics exist, they typically neglect the important role of the collagen network and are limited to a specific set of input conditions, which limits general applicability. The simplified approach taken in this work aims to capture the broader underlying physics as a starting point for further model development. In agreement with existing literature, the model indicates that a large Peclet number, Pe , is necessary for effective interstitial lubrication. It also predicts that the tensile modulus must be large relative to the compressive modulus. This explains why hydrogels and other biphasic materials do not provide significant interstitial pressure under high Pe conditions. The model quantitatively agrees with *in-situ* measurements of interstitial load support and the results have interesting implications for tissue engineering and osteoarthritis problems. This paper suggests that a low tensile modulus (from chondromalacia or local collagen rupture after impact, for example) may disrupt interstitial pressurization, increase shear stresses, and activate a condition of progressive surface damage as a potential precursor of osteoarthritis.

Keywords

cartilage; contact mechanics; biphasic; interstitial lubrication; osteoarthritis

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*corresponding author: David L. Burris, Ph.D. Dept. of Mechanical Engineering, University of Delaware, Newark, DE 19716, dlbarris@udel.edu, tel (302) 831-2006.

Conflict of interest statement

The authors have no conflicts to report.

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1. Introduction

McCutchen slid cartilage against a large glass flat and was the first to propose interstitial (weeping) lubrication to explain the unusual response (McCutchen, 1959). He noted that fluid pressure, which develops under loading, reduced friction by ~10–100X while boundary lubrication with synovial fluid reduced friction by ~2X. In the joint, interstitial pressurization increases load capacity (Ateshian *et al.*, 1994), shields the matrix from stresses (Mow and Lai, 1980), signals the biochemical response (Wong *et al.*, 2003; Carter *et al.*, 2004), and reduces friction and wear (McCutchen, 1962; Soltz and Ateshian, 2000; Ateshian, 2009).

Generally speaking, interstitial fluid pressure subsides over time. Direct measurements of the fluid load fraction have shown that friction obeys the following relationship with the equilibrium friction coefficient, μ_{eq} , and the time-dependent fluid load fraction, F' (Krishnan *et al.*, 2004):

$$\mu = \mu_{eq} \cdot (1 - F') \quad \text{Eq. 1}$$

McCutchen recognized that interstitial lubrication must be restored *in-vivo* and proposed that dynamic loading and unloading was responsible (McCutchen, 1962). However, this hypothesis was rejected by direct observations of time-dependent friction during dynamic loading (Krishnan *et al.*, 2005). In 2008, it was discovered that interstitial lubrication is maintained during sliding when cartilage is self-mated (Caligaris and Ateshian, 2008). In a follow-up test, the authors demonstrated that a rigid impermeable sphere, when slid against cartilage, also maintained low friction. They proposed that fluid pressure is maintained when hydrated tissue is continually introduced into the contact; they call this the migrating contact area (**MCA**). This discovery explained how fluid pressure is maintained *in-vivo*. Based on prior modeling of biphasic cylindrical layers in rolling contact (Ateshian and Wang, 1995), Ateshian proposed that fluid load support is sustainable in MCA when the Peclet number (Pe) $\gg 1$ and negligible when $Pe \ll 1$; $Pe = V \cdot a / (H_a \cdot k)$, where V is sliding speed, a is the contact radius, H_a is aggregate modulus, and k is permeability (Ateshian, 2009).

Despite the rapid recent advancements in the field of cartilage lubrication, there remain major gaps that inhibit our ability to predict how tribological conditions and material properties impact tissue function. The state of the art provides a relationship between friction and fluid load fraction (Krishnan *et al.*, 2004; Ateshian, 2009), but there remains no analytical expression to quantitatively relate the Peclet number to the fluid load fraction for MCA sliding conditions. This paper describes and experimentally supports an analytical model that relates measureable material properties and controllable mechanical conditions to the fluid load fraction and dependent functional parameters, including contact radius, effective contact modulus, contact stress, fluid pressure, friction coefficient, and shear stress.

2. Model

2.1 Contact of a biphasic semi-infinite half-space

The force response of cartilage to deformation consists of components due to elastic stresses and those due to fluid pressure (McCutchen, 1962; Mow *et al.*, 1980). The coupling of elastic deformation and fluid flow creates a challenging non-linear contact mechanics problem. To improve the tractability of the Hertzian contact problem, we initially treat the solid and fluid mechanics independently. Although cartilage violates nearly each of Hertz's assumptions, we find that Hertz's theory provides a reasonable contact model when the

contact diameter is less than the cartilage thickness. The elastic foundation model is more appropriate in physiological conditions and the analysis follows an identical strategy.

We develop the Hertz solution over the elastic foundation solution here because we can test the Hertz solution under controllable experimental conditions. Consider the indentation of a rigid impermeable sphere into cartilage as illustrated in Figure 1. According to Hertz's theory, the **elastic force component**, F_e , is the following function of sphere radius, R , contact modulus (a material property), $E_{c0} = E/(1-\nu^2)$, and penetration depth, $\delta = a^2/R$:

$$F_e = \frac{4}{3} \cdot \frac{E}{1-\nu^2} \cdot R^{0.5} \cdot \delta^{1.5} = \frac{4}{3} \cdot E_{c0} \cdot R^{0.5} \cdot \delta^{1.5} = \frac{4}{3} \cdot \frac{E_{c0} \cdot a^3}{R} \quad \text{Eq. 2}$$

Volume-changing deformations like indentation cause interstitial fluid flow. According to Darcy's law, fluid flow through a permeable medium induces a pressure gradient:

$$\frac{dP}{dx} = \frac{V}{k} \quad \text{Eq. 3}$$

where V is the flow speed along a streamline, k is the permeability of the solid to the fluid of interest, and dP/dx is the pressure gradient along the streamline. Finite element models (Pawaskar *et al.*, 2010; Accardi *et al.*, 2011) have demonstrated that the streamlines during indentation **and** sliding approximate semi-circular arcs as shown in Figure 1. Each streamline starts at the sphere surface at a distance r from the axis of symmetry with a speed of δ in the compression direction. Conserving volume along each streamline gives the velocity as a function of starting point, r , and angle, θ

$$V(r, \theta) = \frac{\dot{\delta} \cdot r}{a - (a-r) \cdot \cos(\theta)} \quad \text{Eq. 4}$$

Assuming that the pressure outside the tissue is zero, Darcy's law can be integrated along each streamline to obtain the pressure acting on the sphere as a function r . The matrix compacts downward at a rate $\dot{\delta}$ under the contact so there is no **relative** flow at $\theta=0$. We estimate the relative flow rate by considering only the transverse component of V within the contact (*i.e.* when $0 < \theta < \pi/2$). In this case, pressure on the counter body takes the form:

$$P(r) = \int_0^{\pi/2} \frac{\dot{\delta} \cdot r \cdot (a-r) \cdot \sin(\theta)}{k \cdot (a - (a-r) \cdot \cos(\theta))} d\theta + \int_{\pi/2}^{\pi} \frac{\dot{\delta} \cdot r \cdot (a-r)}{k \cdot (a - (a-r) \cdot \cos(\theta))} d\theta \quad \text{Eq. 5}$$

Integrating the pressure distribution yields an estimate of the fluid pressure force contribution¹:

$$F_p = 1.37 \cdot \frac{\dot{\delta} \cdot a^3}{k} \cong \frac{4}{3} \cdot \frac{\dot{\delta} \cdot a^3}{k} \quad \text{Eq. 6}$$

The fluid load fraction, F' , is the primary metric of interstitial lubrication. By definition, F' is the ratio of the fluid pressure force contribution and the total applied normal force. Inserting Eqs. 2 and 6 into this definition yields:

¹The approximate constant of 4/3 was misrepresented as 2/3 in our previous paper (Bonnevie *et al.*, 2012).

$$F' = \frac{F_p}{F_p + F_e} = \frac{\frac{4}{3} \cdot \frac{\dot{\delta} \cdot a^3}{k}}{\frac{4}{3} \cdot \frac{\dot{\delta} \cdot a^3}{k} + \frac{4}{3} \cdot \frac{E_{c0} \cdot a^3}{R}} = \frac{Pe}{Pe + 1} \quad \text{Eq. 7}$$

Where $Pe \equiv \frac{\dot{\delta} \cdot R}{E_{c0} \cdot k}$ for indentation.

The mechanics of a migrating contact (MCA) are analogous to those of indentation. When the sphere in Figure 1 travels a distance, a , the tissue is consolidated by δ . Thus, the average

deformation rate is: $\dot{\delta} = \frac{V \cdot \delta}{a}$. By definition from Hertz theory, $\delta = a^2/R$, so the Peclet

number for sliding becomes: $Pe \equiv \frac{V \cdot a}{E_{c0} \cdot k}$, which is identical to that reported previously (Ateshian, 2009).

Eq. 7 suggests that $F' \rightarrow 1$ as $\dot{\delta} \rightarrow \infty$ and only holds for an infinite tensile modulus. For real materials, Eq. 7 is limited to an asymptotic limit that depends on the elastic properties and contact geometry. Soltz and Ateshian demonstrated that this asymptotic limit for F' in unconfined compression is essentially governed by the ratio of tensile modulus to compressive modulus, E^* (Soltz and Ateshian, 2000). The same mechanism applies here and is important to understand. Consider Figure 2, which illustrates the unconfined compression of a biphasic material. Assume that Poisson's ratio is 0 (Soltz and Ateshian, 2000, show that this is true to an excellent approximation) and that the deformation shown occurs instantaneously (flow cannot occur). The deformed shape conserves volume and the transverse strains are half the normal strain. Soltz and Ateshian use E_{-Y} and E_{+Y} to represent the compressive and tensile moduli, respectively. The transverse tensile stress on the matrix

is: $\sigma_+ = \frac{\sigma_-}{2} \cdot \frac{E_{+Y}}{E_{-Y}} = \frac{\sigma_-}{2} \cdot E^*$. If the interface is frictionless, fluid pressure, P , must balance the tensile stress (globally speaking) and $\sigma_- = 2P/E^*$. Therefore, the fluid load fraction is $F' = P/(P + \sigma_-) = P/(P + 2P/E^*)$. Rearranging yields:

$$F'_{\max} = \frac{E^*}{E^* + 2} \quad \text{or} \quad F'_{\max} = \frac{E^* \cdot (0.5 - \nu)}{E^* \cdot (0.5 - \nu) + 1} \quad \text{Eq. 8}$$

This is identical to the expression from Soltz and Ateshian, 2000. For a linear material, $F'_{\max} = 33\%$ which is identical to the biphasic solution (Armstrong *et al.*, 1984). For $E^* = 10$, $F'_{\max} = 83\%$. This analytical model of the asymptotic limit ($Pe = \infty$) is equivalent to the numerical solutions for unconfined compression (Cohen *et al.*, 1998; Soltz and Ateshian, 2000).

Although Eq. 8 is inapplicable for Hertzian contacts, the idea that volume conserving deformations require infinite pressure gradients as E_{+Y} approaches infinity holds equally well: $F' \rightarrow 1$ as $E^* \rightarrow \infty$. Since the general form and limit are known, a single additional reference point is needed. Numerical solutions indicate that the asymptotic limit of fluid load support is 50% when $E^* = 1$ (Agbezuge and Deresiewicz, 1974; Chen *et al.*, 2007). The general asymptotic limit for fluid load support in a Hertzian contact is:

$$F'_{\max} = \frac{E^*}{E^* + 1} \quad \text{Eq. 9}$$

The general expression for fluid load support in a Hertzian contact becomes:

$$F' = \frac{E^*}{E^* + 1} \cdot \frac{Pe}{Pe + 1} \quad \text{Eq. 10}$$

Eq. 10 is a direct quantitative link between interstitial function, measurable material properties, and controllable mechanical conditions.

3. Model demonstration and testing

3.1 Materials

Adult bovine (12–20 month) cartilage was chosen to demonstrate and test the model. A single full-thickness osteochondral plug ($\phi 12.7\text{mm} \times 10\text{mm}$) was harvested from the central aspect of the medial femoral condyle. The sample was rinsed in phosphate buffered saline (PBS). A rigid impermeable 440C stainless steel sphere with a nominal radius of 3.175 mm and an average roughness of 80 nm (measured with a Veeco NT9100) was used as the counter surface. All measurements were performed within 4 hours of extraction to maintain nominally constant material properties.

3.2 Sliding Measurements

In the interest of space, we refer the reader to Bonnevie *et al.* (Bonnevie *et al.*, 2011), where we described the apparatus and experimental methodologies in detail. Briefly, the paper describes a method for measuring the fluid load fraction and contact radius *in-situ* during MCA tribology experiments with cartilage or other biphasic materials. The tribometer makes direct measurements of sliding velocity (V), normal force (F_n), friction force (F_f), and penetration depth (δ). Sliding velocities were randomized and run in the following order: 200, 800, 300, 5000, 100, 1000, 50, 3000, 80, 500, and 2000 $\mu\text{m/s}$. In each measurement, the Z-stage was commanded to move 175 μm into the cartilage. The penetration depth is the difference between the stage motion and load cell compression, both of which are measured directly. Data were collected for the larger of 30 seconds and 5 cycles following steady state.

The performance metrics in regular font on the right of Table 1 are calculated from direct

measurements. The effective contact modulus is given by Hertz as: $E_c = \frac{3}{4} \cdot \frac{F_n}{R^{0.5} \cdot \delta^{1.5}}$, the

fluid load fraction is calculated with: $F' = \frac{F_n - F_e}{F_n} = \frac{E_c - E_{co}}{E_c}$, and contact radius is given by Hertz as: $a = \sqrt{R \cdot \delta}$. The mean contact stress is the total force divided by the contact area. The elastic force contribution, by definition, is: $F_e = F_n \cdot (1 - F')$ and the mean elastic contact stress is the elastic force divided by area. The mean shear stress is the friction force divided by area.

3.3 Results

Experimental results are shown in Table 1; only V , μ , δ , and F_n were directly measured. It is interesting to note the general trends of the direct measurements. With a 100X reduction in speed from 5000 $\mu\text{m/s}$ to 50 $\mu\text{m/s}$, the penetration depth increased by 30%, the normal force decreased by 13%, and the friction coefficient increased by nearly 600%.

Figure 3 demonstrates the strong dependence of friction on the fluid load fraction as predicted by Eq. 1. As suggested in the preceding section, the fluid load fraction approaches an asymptote as the speed approaches infinity. In this case, the fluid load fraction approaches $F'_{max} = 0.86$. Eq. 9 can be used to estimate E^* directly; rearrangement yields:

$E^* = \frac{F'_{\max}}{1 - F'_{\max}} = 6.14 \rightarrow E_{V+} = 6.14 \cdot 0.87 \text{MPa} = 5.34 \text{MPa}$. Permeability is only important at speeds that are sufficiently slow to cause significant reductions in fluid load support. Using the fluid load fraction measurement from the slowest case leaves Pe as the only

unknown in Eq. 10: $0.756 = 0.86 \cdot \frac{Pe}{Pe+1} \Rightarrow Pe = 7.27$. The Peclet number can now be used to make a single point estimation of permeability:

$$k = \frac{0.05 \text{mm/s} \cdot 0.473 \text{mm}}{7.27 \cdot 0.87 \text{N/mm}^2} = 0.0037 \text{mm}^4/\text{Ns}.$$

The same equations were used to create a model curve and the fitting parameters, k and E^* , were adjusted to obtain the least squared error. The best fit for F' yields $k = 0.0036 \text{mm}^4/(\text{Ns})$ and $E^* = 5.9$ with $R^2 = 0.95$ (Eq. 10). In addition to the high quality of fit, the material properties obtained are quite reasonable for cartilage. For example, Mow *et al.* (Mow *et al.*, 1980) report a mean permeability of $k = 0.0076 \pm 0.003 \text{mm}^4/(\text{Ns})$ for bovine cartilage and Ebara *et al.* (Ebara *et al.*, 1994) report a tensile modulus of $5.7 \pm 2.4 \text{MPa}$ for the superficial layer of the bovine glenoid. The results suggest that the model, despite its many simplifying assumptions, captures the important physics of MCA interstitial lubrication problems.

We used Eq. 10 as the basis for developing similar expressions for other dependent

functional metrics. By definition, $F' = \frac{F_n - E_e}{F_n} = \frac{E_c - E_{co}}{E_c}$ and rearrangement gives:

$$E_c = \frac{E_{co}}{1 - F'} \quad \text{Eq. 11}$$

The mean contact stress, fluid pressure, elastic contact stress, and shear stress, respectively, are:

$$\sigma = \frac{1}{\pi} \cdot \left(\frac{16}{9} \cdot \frac{F_n \cdot E_c^2}{R^2} \right)^{\frac{1}{3}} \quad \text{Eq. 12}$$

$$P = \sigma \cdot F' \quad \text{Eq. 13}$$

$$\sigma_e = \sigma \cdot (1 - F') \quad \text{Eq. 14}$$

$$\tau = \mu_{eq} \cdot \sigma_e = \mu_{eq} \cdot \sigma \cdot (1 - F') \quad \text{Eq. 15}$$

The effects of sliding speed on effective modulus, total contact stress, fluid pressure, elastic contact stress, and shear stress are illustrated in Figure 4 along with the model predictions (Eqs. 11–15) based on material properties from the fit to F' ; $R^2 > 0.95$ in each case.

4. Discussion

Analytical contact models from Hertz, Winkler, and JKR are used today to predict contact stresses, characterize material properties, and select appropriate experimental conditions (*e.g.* geometries and loads) for scaled studies of engineering problems (Johnson, 1985). This paper presents and experimentally supports an analogous model for biphasic materials.

This model, like all mathematical models, is limited because it represents a simplified variant of the physical system of interest. We have neglected numerous complexities including heterogeneity, anisotropy, and viscoelasticity among others. However, the mechanical response of cartilage agrees quite well with Hertz's theory just as the elastic response of cartilage agrees with Hooke's law within certain limits. The use of a representative contact modulus, E_c , is as limited as the use of a representative Young's modulus, E , or a representative aggregate modulus, H_a . While there is no direct evidence yet that heterogeneity, anisotropy, and viscoelasticity have important effects on interstitial lubrication, the literature provides strong evidence that biphasic structure (Ateshian, 1997) and tension-compression nonlinearity (Soltz and Ateshian, 2000) are critical elements of the interstitial lubrication phenomenon. This model has been restricted to the simplest form that is still able to quantitatively reproduce experimentally observed features. Future studies will focus on model validation and refinement.

The model is limited to small contacts ($a \ll t$); this promotes experimental testing but prevents extension to physiologically relevant problems. Because cartilage is thin, most contact problems require an elastic foundation (Winkler) approach. Applying the modeling

philosophy presented earlier to the elastic foundation problem gives: $F_e = \frac{\pi}{4} \frac{H_a \cdot a^4}{R \cdot t}$ and

$$F_p = \frac{4\pi}{9} \frac{\delta \cdot a^4}{k \cdot t}.$$

Retaining the assumption that the tensile modulus provides the primary confinement effect, the total fluid load fraction becomes: $F' = \frac{E^*}{E^* + 2} \cdot \frac{Pe}{Pe + 9/16}$, which only differs from Eq. 10 by the constants. Consider a joint with the properties from the results section, a composite radius of 20mm, a speed of 50mm/s and a force of 500N. In both models, $Pe \sim 10^5$ and the second term approaches 1. The Hertz model gives $F' = 86\%$ and $\sigma = 1.4\text{MPa}$. The Winkler model ($t = 1\text{mm}$) gives $F' = 74\%$ and $\sigma = 2.6\text{MPa}$. The Winkler model is inherently stiffer and provides larger contact stresses that are more consistent with physiological values, which range from $\sigma = 1\text{--}5\text{MPa}$ (Brand, 2005). Unlike the Hertz model, the Winkler model is sensitive to the additional confining effect of the bone, which has been neglected in the analysis above. As Ateshian and Wang showed, even linear ($E^* = 1$) biphasic materials can provide realistic fluid pressure ($F' \gg 33\%$) when the layer is thin relative to the contact radius (Ateshian and Wang, 1995). Differences in the two confinement mechanisms may be important; while tension-compression nonlinearity generates large tensile stress along the collagen axis, the substrate supported model generates high shear stress at the bone-cartilage interface.

One application of the model is prediction of mechanical effects. Park *et al.* found no evidence of interstitial lubrication with microscale contacts on cartilage and proposed the use of small contacts for controlled studies of boundary lubrication (Park *et al.*, 2004). Ateshian analyzed these results in the more recent context and indicated that interstitial lubrication was negligible because $Pe < 1$ (Ateshian, 2009). In that case, $Pe \sim 0.1$ ($V = 0.1\text{mm/s}$ and $a = 1\mu\text{m}$) and, assuming $E^* = 5$ for healthy cartilage, the model suggests 7.5% fluid load support, which is indeed small. It is actually remarkable how tolerant the interstitial lubrication mechanism is to extreme reductions in size scale and sliding speed. We routinely measure substantial interstitial load support at speeds and contact radii below 0.1mm/s and $200\mu\text{m}$, respectively ($Pe \sim 20$).

The fluid load support model can also be used to study potential effects of mechanical factors, like obesity and inactivity, which increase risk of osteoarthritis (OA). Reconsider the joint from before. If the load increases to 2,000 N or speed reduced to 1 mm/s, the model predicts no change in F' or μ because $Pe \gg 1$ ($\sim 100,000$). However, the increased force does increase the tensile stress from 1.9 to 3.9 MPa. The collagen network ruptures when tensile

stresses exceed its strength (Repo and Finlay, 1977), which ranges from 10–30MPa (Mansour, 2003). Local collagen rupture would reduce E^* , which would reduce interstitial lubrication under all conditions. Long periods of complete inactivity ($V=0$) would lessen fluid pressure over time and damage could result if joint articulation resumed in the absence of significant fluid pressure. This might explain the risk of Temporomandibular Disorders (TMDs) from third molar (wisdom tooth) extraction, a procedure that requires the jaw to be propped open an extended period of time (Huang *et al.*, 2002).

Ateshian and Wang first proposed that increased permeability from degradation could impede the stress-shielding effect of interstitial lubrication and might therefore catalyze future degradation (Ateshian and Wang, 1995). Ateshian later noted several studies in the literature citing no change in friction with various forms of degradation (Pickard *et al.*, 1998; Bell *et al.*, 2006; Northwood and Fisher, 2007; Caligaris *et al.*, 2009) and concluded that degradation must not necessarily lead to an increase in friction. Since degradation generally increases k and decreases H_a , he notes that the changes may offset one another, leaving Pe and interstitial lubrication unaffected (Ateshian, 2009). Our model supports this notion; in the physiological case, $Pe \sim 100,000$ and although a 10X increase in k alone would reduce Pe by an order of magnitude, the change has no effect on F' . The model suggests that interstitial lubrication is far more sensitive to changes in E^* than to changes in V , F , H_a , and k because E^* is much closer to 1 than Pe . In an OA joint, collagen disorganization is reported to occur before any compositional changes, resulting in reduced tensile modulus (Akizuki *et al.*, 1986; Elliott *et al.*, 1999) and a ‘softening’ condition known as chondromalacia (Pearle *et al.*, 2005). Our recent experiments showed that a very mild papain digestion had no effect on composition but caused a two-fold increase in friction ($Pe \gg 1$) that was consistent with a reduction in F' from 92% to 84% and a reduction in E^* from 11 to 5 (Baro *et al.*, 2012).

Although there is no definitive evidence in the literature that degradation impedes interstitial lubrication, we consistently find reduced interstitial lubrication with mechanical or chemical degradation using Hertzian contacts. We believe that discrepancies may be the result of one of several possibilities. Firstly, most studies use the effective friction coefficient as the only indicator of interstitial lubrication, even though it depends on μ_{eq} and F' . Krishnan *et al.*, for example, found that removal of the superficial zone removal had no effect on μ_{eff} but significantly **decreased** μ_{eq} (Krishnan *et al.*, 2004). Thus, F' (interstitial lubrication) must have decreased (Eq. 1). The decrease in μ_{eq} likely reflects compromised surface integrity, reduced shear strength, and degradation of interstitial lubrication; it should not be considered a lubrication attribute. The decrease in F' causes a reduction in stress shielding, load support, and stiffness, and manifests as arthroscopic ‘softening’. Direct fluid load support measurements are needed for comprehensive assessment of the functional consequences of degradation as suggested previously (Ateshian, 2009). Secondly, reduced E^* would not significantly impede joint lubrication if the confinement burden simply transitioned from tensile stresses in the collagen network to shear stresses at the bone-cartilage interface. This transition, while impossible in our localized contacts, will occur if E^* drops in a macroscale contact. Although this transition might have no immediate effect on interstitial lubrication, it could have significant downstream mechanical or biological effects.

A related question is whether such functional changes have biochemical consequences. Based on the hypothesis that fluid pressure and shear stress favor matrix synthesis and degradation, respectively (Wong *et al.*, 2003; Carter *et al.*, 2004), we consider the ratio $\tau^* = \tau/P$ as a metric of biochemical degradation. Based on this model, a reduction in E^* from 11 to 5 causes τ^* to increase from 0.021 to 0.050. Although the relationship between the stress state and biochemical response remains uncertain, the model suggests that OA causes a significant increase in tribological shear stress relative to fluid pressure.

In summary, we have developed an analytical model for interstitial lubrication of MCA contacts involving biphasic materials. The model has been quantitatively supported by controlled experimental measurements and suggests that the interstitial lubrication response *in-vivo* is sensitive to changes in E^* and relatively insensitive to changes in V , F , H_a , and k . It enables the design of scaled-down experiments and prediction of the functional response to changes in mechanical conditions and materials properties. It suggests mean contact stresses above 2.6MPa in a typical joint which is consistent with measured values. Additionally, it suggests that degenerative changes associated with OA will inhibit interstitial lubrication and significantly alter the stress state, either of which may signal negative cellular responses. Finally, the model provides basic insights into the design of materials that mimic cartilage function. Fibril reinforced hydrogels, for example, may provide significant interstitial lubrication and could prove invaluable for cartilage repair and replacement.

Acknowledgments

The authors acknowledge financial support from the NIH (grant P20-RR016458) for the development of the experimental and theoretical methods described in the paper.

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Figure 1.

a) Axisymmetric contact model of a rigid sphere indenting cartilage. Streamlines show likely paths of fluid flow.

Figure 2. Unconfined compression experiment exemplifying the tissue's initial conservation of volume.

Figure 3.

a) Fluid load fraction plotted versus sliding speed. The fit to the model (Eq. 10) are shown in red and reflect $k=0.0036\text{mm}^4/(\text{Ns})$ and $E^*=5.9$. b) Friction coefficient plotted versus sliding speed. Error bars represent experimental uncertainty (ISO, 1993) and are smaller than data labels.

Figure 4.

a) Effective contact modulus versus sliding speed. b) Total contact stress, fluid pressure, elastic contact stress, and shear stress plotted versus sliding speed. Experimental data are plotted as points, fits to the model (Eqs. 11–14) are shown in red and reflect $k=0.0036\text{mm}^4/(\text{Ns})$ and $E^*=5.9$. In each case, $R^2>0.95$ and error bars represent experimental uncertainty.

Direct measurements (bold) and calculated performance metrics (regular) from MCA sliding measurements of an osteochondral plug from a bovine stifle joint. The probe radius, R , is 3.175mm.

Table 1

$V(\mu\text{m/s})$	μ	$\delta(\mu\text{m})$	$F_n(\text{mN})$	$E_c(\text{MPa})$	$P'(\%)$	$a(\text{mm})$	$\sigma(\text{MPa})$	$\sigma_c(\text{MPa})$	μ_{eq}	$\tau(\text{MPa})$
4510	0.0118	54.4	182.8	6.06	85.7	0.416	0.337	0.0483	0.0825	0.00398
2870	0.0128	56.1	180.3	5.72	84.8	0.422	0.323	0.0490	0.0842	0.00413
1910	0.0133	55.7	180.9	5.80	85.0	0.451	0.326	0.0488	0.0887	0.00433
980	0.0149	58.4	176.8	5.27	83.5	0.431	0.304	0.0500	0.0904	0.00452
788	0.0176	57.4	178.4	5.47	84.1	0.407	0.312	0.0496	0.1107	0.00549
449	0.0213	58.0	177.5	5.21	83.8	0.429	0.307	0.0498	0.1312	0.00654
281	0.0278	58.8	176.2	5.35	83.3	0.432	0.301	0.0502	0.1666	0.00836
192	0.0361	62.0	171.3	4.67	81.4	0.444	0.277	0.0515	0.1941	0.01000
98	0.0510	64.9	167.0	4.25	79.6	0.454	0.258	0.0527	0.2497	0.01316
79	0.0576	66.7	164.1	4.01	78.3	0.460	0.247	0.0535	0.2656	0.01420
50	0.0702	70.4	158.4	3.57	75.6	0.473	0.226	0.0549	0.2883	0.01584
0	N/A	119.0	84.7	0.87	00.0	0.615	0.071	0.0713	N/A	N/A