Published in final edited form as:

IEEE Trans Biomed Eng. 2008 February ; 55(2 0 2): . doi:10.1109/TBME.2007.912674.

Magnetoacoustic Tomography with Magnetic Induction: A Rigorous Theory

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Abstract

We have proposed a new theory on mechanism of the magnetoacoustic signal generation with magnetic induction for an object with an arbitrary shape. An object under a static magnetic field emits acoustic signals when excited by a time-varying magnetic field, and that the acoustic waveform is mainly generated at the conductivity boundaries within the object. The proposed theory on the magnetoacoustic tomography with magnetic induction produced highly consistent results among computational and experimental paradigms in a two-layer sample phantom and suggests the potential applications for bioimpedance imaging.

Keywords

Electrical impedance imaging; magnetic induction; magnetoacoustic generation; tomography

I. INTRODUCTION

Medical imaging techniques are playing increasingly important roles in biomedical research and clinical diagnosis. A number of efforts have been made to improve our ability to characterize tissue properties of biological systems through the use of medical imaging. Based upon the acoustic theory [1], traditional ultrasound imaging is used to image the acoustic impedance distribution and provides high spatial resolution. However, it is still limited for soft tissues in which the acoustic impedance difference is less than 10% [2]. On the other hand, the electrical impedance of biological tissues exhibits a broad range of values and provides a good contrast for imaging the tissue properties. Electrical impedance tomography (EIT) [3] has been developed using current injection and non-invasive surface voltage detection, but is limited by its low spatial resolution and the shielding effect [4] of insulating tissues. To improve the performance of EIT, research on magnetic induction tomography (MIT) [5], which uses magnetic induction and sensing, has been pursued, although its spatial resolution is still limited. In magnetic resonance electrical impedance tomography (MREIT) [6], the magnetic field disturbance induced by the current injection is sensed by a magnetic resonance imaging system. However, MREIT is currently limited by a low signal-to-noise ratio (SNR).

In an effort to improve the spatial resolution and contrast of the existing tissue characterization techniques, the magnetoacoustic (MA) technique [7] was developed. Using this technique, an electrical current is injected onto an object under a static magnetic field, generating acoustic signals induced by the Lorentz force. To avoid current injection and the

shielding effect accompanied with current injection, a new magnetoacoustic approach, called magnetoacoustic tomography with magnetic induction (MAT-MI), was recently proposed by He and co-workers [8]-[10]. While the MAT-MI experimental results indicate its high resolution in imaging electrical conductivity variation within an object, there has been no theoretical interpretation with regard to the phenomenon of detecting conductivity boundary in MAT-MI images.

Based on the theories of magnetic eddy current induction, vibration generation and acoustic transmission, a novel theory on the mechanism of MAT-MI signal generation is proposed in this paper. The principle and the formulae of the acoustic waveform generation for an object with an arbitrary shape are provided. Here it is explained that the acoustic signal can only be generated at the conductivity boundary with corresponding amplitude and polarity, which are determined by the induced eddy current. The numerical and phantom experiments for a two-layer model were performed and the reconstructed images were also obtained using the back projection algorithm [11]. Both the collected waveform and the reconstructed image of the experimental results coincide well with the computer simulation results, suggesting the feasibility of applying the proposed MAT-MI theory for imaging conductivity differentiation.

II. METHODS

The theoretical model of MAT-MI is shown in Fig. 1. A conductive object is placed in a static magnetic field $\mathbf{B}_0 = B_0 \hat{z}$ (the bold variable refers to a vector and hatted variable denotes the unitary vector in this paper) and excited by a time-varying pulsed magnetic field $\mathbf{B}_1 = B_1(t) \hat{z}$. The pulsed magnetic field is generated by a stimulating coil at a frequency covered by the receiving transducer, and is considered to be homogeneous in the region covering the entire object. The pulsed magnetic field induces eddy currents in the conductive volume of the object, which cause vibration by interacting with the static magnetic field due to the Lorentz force. The generated acoustic signals transmit through the object and can be collected by the transducers placed around the object for image reconstruction.

As shown in Fig. 1, when an object is excited by the induced Lorentz force, the acoustic pressure p satisfies the wave equation [7]

$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = \nabla \cdot [\mathbf{J}(\mathbf{r}) \times \mathbf{B}_0], \quad (1)$$

where the vibration source is a given function of the point $\mathbf{r}(x, y, z)$ in the finite region V of the object. ∇^2 is the Laplacian operator and c is the acoustic velocity. The acoustic pressure at the observed position $\mathbf{r}'(x', y', z')$ around the object can be obtained by the integral of Green's Function [12] in a 3-dimensional free space. The source reconstruction can be accomplished using the time reversal back projection algorithm for the received acoustic waveforms [11].

In the MAT-MI system, the corresponding induced eddy current distribution $\mathbf{J}_1$ is in the xy plane and is perpendicular to the external pulsed magnetic field ($\hat{z}$ direction). The induced Lorentz force can be described as $\mathbf{J}_1 \times \mathbf{B}_0$ along the normal direction of the generated eddy current. Therefore, the 3-dimensional algorithm can be simplified to a 2-dimensional algorithm for a given magnetic excitation.

Supposing an object with a uniform conductivity σ_1 and an arbitrary shape is put into an insulating medium with conductivity $\sigma_0 = 0$. Based on Faraday's principle, the induced

electrical field $\mathbf{E}_1$ is determined by $\mathbf{B}_1$ as $\nabla \times \mathbf{E}_1 = -\mathbf{B}_1(t)/t$ and the induced eddy current $\mathbf{J}_1(\mathbf{r})$ can be described as $\mathbf{J}_1(\mathbf{r}) = \sigma_1(\mathbf{r})\mathbf{E}_1(\mathbf{r})$. So the vibration source of MAT-MI is $\nabla \cdot [\mathbf{J}(\mathbf{r}) \times \mathbf{B}_0] = \nabla \cdot \{[\sigma_1 E_1(\mathbf{r}) B_0] \hat{n}\}$ where $\hat{n}$ and E_1 are the respective normal direction and magnitude of $\mathbf{E}_1(\mathbf{r})$. Inside the object, the eddy current lines are closed loops with intensity being proportional to the conductive area it surrounds. The maximum eddy current is reached at the conductivity boundary between the conductive medium and the surrounding insulator, with a shape being similar to the cross-sectional geometry of the boundary of the conductive medium. The induced electrical field satisfies the boundary condition $\hat{n} \times (\mathbf{E}_1 - \mathbf{E}_0) = 0$ at the conductive-insulator junction.

Because the acoustic source pressure is proportional to the intensity of the vibration source, we can define it as $p_0(\mathbf{r}) = \nabla \cdot \{[\sigma_1 E_1(\mathbf{r}) B_0] \hat{n}\}$ at position $\mathbf{r}$ by ignoring the conversion coefficient. Each vibration source can be considered as a point source and the induced acoustic signal transmits along $+\hat{n}$ and $-\hat{n}$ directions [13]. The bi-directional acoustic source pressures can be presented as $p_{s\pm}(\mathbf{r}) = p_0(\mathbf{r}) \exp[j(\omega t \mp kx)]/x$, where x is the transmission distance from position $\mathbf{r}$ along the $+\hat{n}$ direction, $k = \omega/c$ is the wave number and ω is the angular frequency of the acoustic signal. Supposing there are two close points $\mathbf{T}_+$ and $\mathbf{T}_-$ along the same normal direction of the nearby eddy current loops separated by a distance of $2\Delta x$ with the point $\mathbf{r}$ in the middle, the bi-directional acoustic transmission distances from the two points to $\mathbf{r}$ are x_+ and x_- with $x_+ = -x_- = \Delta x$. Due to waveform superposition, the summed acoustic pressure along the normal direction can be obtained as $p(\mathbf{r}) = \lim_{\Delta \rightarrow 0} \{p_0(\mathbf{T}_+) \exp[j(\omega t - k\Delta x)] - p_0(\mathbf{T}_-) \exp[j(\omega t + k\Delta x)]\} / \Delta x$ when $t \rightarrow 0$. The acoustic pressure at position $\mathbf{r}$ can thus be represented as

$$p(\mathbf{r}) = \frac{\partial}{\partial \hat{n}} \nabla \cdot \{[\sigma_1 E_1(\mathbf{r}) B_0] \hat{n}\}. \quad (2)$$

From Eq. 2, we can see that the generated acoustic pressure is determined by the differential effect of the vibration source along the normal direction. The acoustic pressure generated by the conductivity variation is far higher than that by the electrical intensity variation for tissue, which indicates that the generated acoustic pressure is mainly due to the abrupt change of the induced eddy current at the conductivity boundary.

Supposing there is a two-layer cylindrical model placed in an insulator, with corresponding conductivities of σ_1 and σ_2 , their conductivity discrepancy is $\Delta\sigma = \sigma_2 - \sigma_1$. The eddy current of the inner medium can be represented as the summation of two terms [14], [15]

$$\mathbf{J}_2(\mathbf{r}_2) = \sigma_2 \mathbf{E}_2(\mathbf{r}_2) = \sigma_1 \mathbf{E}_2(\mathbf{r}_2) + \Delta\sigma \mathbf{E}_2(\mathbf{r}_2). \quad (3)$$

One is “generated” by the conductivity σ_1 with the outer medium to form a quasi-uniform homogeneous object while the other is “generated” by the conductivity discrepancy $\Delta\sigma$ only within the inner medium. This superposition principle can be applied to a piecewise homogeneous medium with multiple regions. The generated acoustic pressures of the inner and the outer media can be obtained separately as $p(\mathbf{r}_i) = \partial \nabla \cdot \{[\sigma_i E(\mathbf{r}_i) B_0] \hat{n}\} / \partial \hat{n}$, where $\sigma_i = \sigma_1$ and $\Delta\sigma$ and $\mathbf{r}_i = \mathbf{r}_1$ and $\mathbf{r}_2$ when $i = 1$ and 2 corresponding to the outer and inner media. The generated acoustic signal transmits along its normal direction and can be detected by the receiver at position $\mathbf{r}'$. Under the excitation of the propagated acoustic signal, the transducer produces a time series waveform combined with the impulse response $s(t)$ of the transducer. If the pulsed magnetic field is a step function, the induced electrical field satisfies $\nabla \times \mathbf{E}_\delta(\mathbf{r}) = -\delta(t)\hat{z}$. Considering the factors of the transducer and the pulsed magnetic field, the received acoustic waveform generated at position $\mathbf{r}_i$ is modified to

$$w(\mathbf{r}_i) = \frac{\partial}{\partial \hat{n}} \nabla \cdot \{ [\sigma_i E_\delta(\mathbf{r}_i) B_0] \hat{n} \} \otimes \frac{\partial B_1(t)}{\partial t} \otimes s(t), \quad (4)$$

where $E_\delta(\mathbf{r}_i)$ is the amplitude of $\mathbf{E}_\delta(\mathbf{r}_i)$ and $\otimes$ is the convolution operator. Therefore, the inherent characteristics of the stimulating coil and the transducer can be separated from the generated waveform. All of the induced acoustic signals transmit along their corresponding normal directions to the surrounding receivers and the detected acoustic waveform obtained by the transducer at position $\mathbf{r}'$ is the summation of all the generated acoustic signals. Supposing the acoustic transmission is in a lossless media, the received waveform can be expressed as

$$w_R(\mathbf{r}') = \iint \left[w(\mathbf{r}_i) \otimes \delta \left(t - \frac{|\mathbf{r}' - \mathbf{r}_i|}{c} \right) \right] d\mathbf{r}_i \Big|_{\hat{n} = \pm(\mathbf{r}' - \mathbf{r}_i)/|\mathbf{r}' - \mathbf{r}_i|}, \quad (5)$$

where $|\mathbf{r}_i - \mathbf{r}'|/c$ is the transmission time and it produces a time delay in waveform, $(\mathbf{r}' - \mathbf{r}_i)/|\mathbf{r}' - \mathbf{r}_i|$ denotes the unit vector between $\mathbf{r}'$ and $\mathbf{r}_i$. Each received waveform comprises information of the boundaries, including the transmission distance and the conductivity variation between adjacent regions.

In the solution to the inverse problem, the vibration source position cannot be identified precisely purely from a received waveform. It can be solved, however, using a tomographic technique based on the received waveforms surrounding the object. The magnitudes of the received acoustic waveforms $w_R(\mathbf{r}')$ are used to reconstruct the MAT-MI image by means of the back projection algorithm.

III. RESULT

To demonstrate the validity of the proposed MAT-MI theory, computer simulations were conducted in a two-layer eccentric cylindrical model with the corresponding cross section being shown in Fig. 2A(a). The entire object was immersed into distilled water ($\sigma_0 = 0$ S/m) and was considered to be acoustically homogeneous without any acoustic reflection, dispersion and attenuation. The acoustic velocity was set to 1500 m/s. The radii of the sensing surface, the outer and inner media were set to 80 mm, 50 mm and 20 mm respectively. The conductivities of the outer and the inner media were set to $\sigma_1 = 0.3$ S/m (simulating parenchyma) and $\sigma_2 = 0.6$ S/m (simulating breast cancer). The central frequency of the transducer and the frequency of the pulsed magnetic field were both set to 0.5 MHz for the purpose of comparing with the experimental results described below. The acoustic receiver was optimized to be a focus transducer with an infinitesimal aperture and the pulsed magnetic field was set to a one-cycle cosine waveform.

The generated acoustic waveforms at angles from 1° to 360° were obtained based on Eq. 5. Fig. 2A(c) shows the normalized waveform obtained at an angle of 240° . Because of the symmetry of the model, the two outer wave clusters with high amplitude and inverse vibration polarities represent the boundaries of the outer medium. The cluster at 50 mm represents the positive conductivity change from 0 S/m to 0.3 S/m while the other at -50 mm has an inverse vibration polarity for the negative conductivity change from 0.3 S/m to 0 S/m. Those of the inner medium have lower amplitude because of the lower eddy current variation. In addition, the positions of the acoustic vibration source at the corresponding boundaries are clearly displayed. With the 360 generated waveforms, the MAT-MI image was reconstructed and is shown in Fig. 2A(b). Only the conductivity boundaries are reconstructed and greater variation in eddy current results in higher acoustic amplitude. It

also shows a finite boundary width, which is introduced by the impulse response of the receiver because of the wavelength and the relative bandwidth.

The present simulation results were also compared with an experimental result. The experimental set-up was similar to those illustrated in Fig. 1. In the MAT-MI experiment, the permanent and the pulsed magnetic flux intensities were both approximately 0.1 Tesla. A planar piston transducer (V301, Panametrics, 0.5 MHz, 19 mm diameter) was used as the receiver to scan around the object with a 5 MHz sampling rate and an angle step of 2.5° . A two-layer cylindrical sample with 0% saline gel (0S/m, 30 mm diameter) as the inner medium and 0.5% saline gel (0.95S/m, 50 mm diameter) as the outer medium was placed within the distilled water. As shown in Fig. 2B(a), the received acoustic waveform closely agrees with the simulated result. Several wave clusters of the conductivity boundaries are displayed and some are blurry because of the acoustic inhomogeneity and diffraction in the experimental setup. The reconstructed image of the sample is shown in Fig. 2B(b). The boundaries are distinct and the amplitudes (brightness) of the outer medium are greater than those of the inner medium because of the higher variation of the induced eddy current. Some boundaries are not as clear due to information loss, such as waveform aliasing in acoustic superposition because of the large aperture of the receiver used in the experiment and the finite sampling number.

IV. DISCUSSION AND CONCLUSION

Figs. 2A and 2B show good consistency between the theoretical results and the experimental results. Due to the low frequency (0.5 MHz) used in the experiment, the resulting MAT-MI image shows limited spatial resolution, whereas, it can be enhanced with high frequency excitation for the spatial resolution of the boundary identification is the acoustic wavelength (0.3 mm at 5 MHz). This suggests a sub-mm spatial resolution for MAT-MI in identifying and imaging objects with conductivity inhomogeneity. Also note that the $\mathbf{B}_1$ field is assumed to be homogeneous by careful design of the stimulating coil. When small coils are used such $\mathbf{B}_1$ field may not be homogeneous and the theory would need to be revised accordingly.

In summary, we have presented a new theory with explicit formulae on the principle of MAT-MI based on the theories of eddy current induction and acoustic transmission. It is shown that the acoustic waveform is generated at the conductivity boundary with corresponding amplitude and polarity due to the abrupt change of the eddy current introduced by the conductivity variation. The proposed theory and experimental results suggest that MAT-MI provides a noninvasive means for conductivity differentiation with high spatial resolution. This property of MAT-MI may have important applications to medical imaging, such as for early detection of small cancers, which exhibit elevated conductivity, or other applications in non-invasive detection.

Acknowledgments

The authors are grateful to Xu Li for assistance in the experiment.

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FIG. 1. Illustration of the concept and theoretical model of MAT-MI. The to-be-imaged object is shaded and the directions of the static and the pulsed magnetic fields are along the $\hat{z}$ direction.

FIG. 2A.

Simulation results of the MAT-MI. (a) The cross section of the two-layer cylindrical model. The gray scale shows the conductivity values of the simulation model. (b) Reconstructed image with normalized scale and (c) normalized waveform measured at an angle of 240° .

FIG. 2B.

Experimental results. (a) Measured waveform (80 dB amplification) and (b) reconstructed image with normalized scale. The time of the waveform is transformed to transmission distance as $x = ct$. The units in the image are mm.