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Design and Modeling of a 3-D Magnetic Actuator for Magnetic Microbead Manipulation

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Abstract

This paper presents the design, implementation, modeling, and analyses of a hexapole magnetic actuator that is capable of 3-D manipulation of a magnetic microbead. The magnetic actuator employs six sharp-tipped magnetic poles placed in hexapole configuration, six actuating coils, and a magnetic yoke. The magnetic poles concentrate the magnetic flux generated by the coils to the workspace, resulting in a high magnetic field with a large field gradient for magnetic force generation on the magnetic microbead. A lumped-parameter magnetic force model is then established to characterize nonlinearity of the magnetic force exerting on the magnetic microbead with respect to the applied currents to the coils and the position dependence of the magnetic force in the workspace. The force generation capability of the designed system is then explored using the force model. Moreover, an inverse force model is derived and its effect on the magnetic actuation capability is investigated. The inverse force model facilitates the implementation of a feedback control law to stabilize and control the motion of a magnetic microbead. Experimental results in terms of the magnetic force in relation to stable motion control of a magnetic microbead are used to validate the force model.

Keywords

Magnetic circuit; magnetic force model; magnetic micromanipulation; 3-D magnetic actuation

I. Introduction

Micromanipulation using magnetic microbeads has become an important biological technique in the study of biophysics and cell mechanics. In general, magnetic forces in the relevant biological range are generated on magnetic microbeads using magnetic actuators. The magnetic beads are then employed as measurement probes to study the mechanical properties of a variety of biological samples. For example, Haber and Wirtz [1] developed an electromagnetic instrument and utilized magnetic microbeads for DNA micromanipulation. Alenghat *et al.* [2] employed a single electromagnet to apply magnetic forces to cell cytoskeleton using ligand-coated magnetic microbeads. de Vries *et al.* [3] implemented a multipolar system to manipulate magnetic microbeads inside live cells. In

addition, many other magnetic micromanipulators have been developed and employed to probe cell membranes [4], [5], manipulate macromolecules [6]–[9], and characterize intracellular properties [10]–[12].

In order to produce magnetic forces of adequate magnitude (at least a few picoNewtons) for biological manipulation on microscopic magnetic beads, the magnetic actuators usually employ the design with electromagnets and sharp-tipped poles to generate a high magnetic field with a large field gradient. The electromagnets are utilized to produce magnetic flux, and the sharp-tipped poles, each of which is magnetically connected to an electromagnet, are used to conduct and concentrate the produced magnetic flux to the pole tip. As the flux strongly diverges outward from the tip, a high magnetic field with a large field gradient is produced in the proximity of the pole tip, resulting in an applicable magnetic force on the magnetic microbead for biological manipulation.

Due to the noncontact nature of the magnetic forces, multiple degrees of freedom actuation can be readily achieved without mechanical cascading [13]–[15]. In fact, the actuator's ability to generate magnetic force is directly related to the number of magnetic poles employed. Therefore, according to the number of their magnetic poles, the magnetic actuators can be categorized into distinct groups. The single-pole actuators [2], [10], [16], [17] are straightforward to implement, but can only generate attractive magnetic forces toward the pole tip. With two opposing magnetic poles, magnetic forces in both forward and backward directions can be generated along one axis [1], [11]. Three in-plane poles placed 120° apart [3] or four poles in quadrupole configuration [18], [19] have been employed for 2-D force application. To generate magnetic forces in 3-D directions, six-pole actuators [9], [20] have been designed and implemented.

In order to enable controllable force application using the magnetic actuator, it is essential to establish a force model that relates the applied currents to the actuating coils of the electromagnets to the resulting magnetic forces on the magnetic microbead. Moreover, an effective force model can also facilitate the implementation of feedback control for advanced manipulation of the magnetic microbead. It is, however, very difficult to obtain such a model for multipolar magnetic actuators due to at least two reasons. First, it is challenging to model the magnetic field. The magnetic field is produced by all the magnetic poles, whereas these poles are mutually magnetized. In other words, the magnetic field produced by one magnetic pole will magnetize the other poles, and vice versa. Second, the magnetic force is due to the gradient of the magnetic field, which further complicates the derivation of the force model. Consequently, the force model is usually determined through experimental calibration [11], [17] and/or finite element analysis [3], [5]. The two methods work very well with single-pole and two-pole actuators. It is, however, very difficult to apply these methods to establish the force model for 2-D and 3-D actuators, because the magnetic force is generally a nonlinear function of multiple variables, e.g., in the case of 3-D actuation using six poles, the force model involves three position variables and six current variables.

We have developed a systematic approach to analyze the magnetic field generated by multipolar magnetic actuators and the magnetic force exerted on the magnetic microbead. This approach has been successfully applied to establish a lumped-parameter force model for a quadrupole magnetic actuator [18], and a model-based feedback control law has been established to realize stabilization and manipulation of the magnetic microbead [19] in a 2-D plane. In this paper, we present the design, implementation, and modeling of a hexapole magnetic actuator that is capable of 3-D manipulation of a magnetic microbead. The actuator employs six magnetic poles to allow back-and-forth actuation in three orthogonal directions. The force model of the 3-D magnetic actuator is then derived by following the

previously developed approach. It accurately characterizes the nonlinearity with respect to the applied currents to the coils and the position dependence of the magnetic force. It further enables us to explore the force generation capability of the developed magnetic actuator. In addition, an inverse force model is derived. Using the inverse force model, we can directly determine the six input currents to the coils in order to generate the desired 3-D force. The derived inverse force model also facilitates the implementation of a feedback control law to stabilize and control the motion of a magnetic microbead suspended in water. Experimental results in terms of magnetic force in relation to stable motion control of a magnetic microbead are, therefore, used to validate the force model.

This paper is organized as follows. Section II introduces the design and implementation of the 3-D magnetic actuator. The magnetic force model and the actuator's capability of force generation are presented in Section III. Section IV describes the derivation of the inverse force model and its effect on the magnetic actuation capability. Section V presents the experimental results to verify the accuracy of the force model. Conclusion is made in Section VI.

II. Actuator Design and Implementation

A. Design Concept

The design concept of the 3-D magnetic actuator is illustrated in Fig. 1(a). It consists of six sharp-tipped magnetic poles that concentrate the magnetic flux into the workspace. Each magnetic pole is actuated by an individual coil. All the coils and poles are then magnetically connected by a magnetic yoke. The yoke completes the magnetic circuit and increases the efficiency of magnetic field generation. The six pole tips enclose the workspace, wherein the specimen and the magnetic microbead are placed.

It is worth noting that the six poles are not placed in the same plane, but in two layers, as shown in Fig. 1(b), which illustrates the zoomed-in view from the side of the actuator. In particular, the three dark-colored poles as drawn in Fig. 1(a) are placed on a plane below the specimen and the three light-colored poles are on a plane above the specimen. The specimen, which is inside a cell chamber formed by two coverslips, is located in the workspace. More details of the cell chamber will be introduced in Section II-B.

As each magnetic pole can only generate attractive magnetic forces toward the pole tip, a pair of two opposing magnetic poles is usually placed along one axis to generate magnetic actuation in both forward and backward directions. Therefore, to realize 3-D magnetic force actuation, six magnetic poles can be placed in the configuration as shown in Fig. 2(a), in which the solid dots indicate the locations of the six magnetic-pole tips. In this configuration [see Fig. 2(a)], the two magnetic poles along the z -axis would, however, block the optical path when integrated to an inverted microscope. This problem can be easily solved by applying a coordinate rotation to the arrangement of the six magnetic poles. After the rotation, the hexapole configuration is shown in Fig. 2(b). The six poles are placed on two parallel horizontal planes, and the optical path is free of blockage. This hexapole configuration [see Fig. 2(b)] is, therefore, adopted.

To facilitate the modeling and control analyses, two coordinate frames are defined as shown in Fig. 2. They are the actuation coordinate frame $\{O; x_a, y_a, z_a\}$ and the measurement coordinate frame $\{O; x_m, y_m, z_m\}$. The indices of the six poles are also labeled in the figure. The six poles are located at $[\pm\ell\ 0\ 0]$, $[0\ \pm\ell\ 0]$, and $[0\ 0\ \pm\ell]$ in the actuation coordinate frame, where ℓ is the distance from the workspace center to each magnetic pole. The relationship between the two coordinate frames can be expressed by the rotational matrix

$${}^a_m\mathbf{R} = \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & 0 & \frac{1}{\sqrt{3}} \end{bmatrix}.$$

Left-multiplying ${}^a_m\mathbf{R}$ to a vector will transform a vector from the measurement coordinate frame to the actuation coordinate frame.

When an electric current is applied to a coil, magnetic flux is generated and permeated through the magnetic pole to the pole tip. Then the flux strongly diverges outward from the tip, and a high magnetic field with a large field gradient is produced in the proximity of the pole tip, resulting in an applicable magnetic force for biological manipulation. By using six magnetic poles working together, magnetic forces in arbitrary 3-D directions can be generated.

B. Implementation

The assembly process of the hexapole magnetic actuator is illustrated in Fig. 3. The magnetic poles are manufactured using high-permeability magnetic foils of 178 μm thick. The foil material is nickel–iron–molybdenum alloy, which offers high permeability with minimum hysteresis loss, and its saturation induction is around 2.1 T. The foils are machined to form the desired geometry of the magnetic poles using wire electrical discharge machining, and each of the machined poles has a round tip whose radius is around 40 μm . Three magnetic poles are glued onto a No. 1 coverslip (thickness 0.13–0.15 mm) using low-viscosity epoxy, as shown in Fig. 3(a), and their pole tips form an equilateral triangle whose side length is about 840 μm . The other three poles of the same thickness are fixed on a 1.2-mm thick glass slide, as shown in Fig. 3(b), and their pole tips also form an equilateral triangle of the same size. The two layers of magnetic poles are then fixed face-to-face, as shown in Fig. 3(c). The distance between the two layers of poles is controlled by the height of the spacers [see Fig. 3(b)], which is 508 μm . Therefore, the hexapole configuration as shown in Fig. 2 is realized and the distance from the workspace center to each magnetic pole is $\ell = 594 \mu\text{m}$. During experiments, a sealed chamber, which is formed by stacking one piece of No. 1 coverslip, a seal ring whose thickness is 120 μm , and one more piece of No. 1 coverslip, is inserted in the gap. Water solution with suspended magnetic microbeads is sealed inside the chamber, and controlled magnetic forces can be applied to the beads that are inside the actuator's workspace.

The magnetic yoke has the shape of a square loop and it has six protrusions, on which six actuating coils are assembled. The yoke is made of cold-rolled steel. The coils are wound by hand using American wire gauge #24 magnetic wire (diameter = 0.51 mm) for 50 turns. With the magnetic yoke and actuating coils assembled, the whole setup of the actuator is shown in Fig. 3(d).

The actuating coils are driven by six linear power amplifiers (BTA-18V-6A, Precision Micro Dynamics). These linear power amplifiers are configured to work in current mode, in which the output current is proportional to the input voltage with the proportional gain of 0.3 A/V.

The actuator is then placed on an inverted microscope (Nikno TE2000-U), as shown in Fig. 4. Since the setup is less than 10 mm thick, it can fit between a high-numerical-aperture (NA) objective lens and a high-NA condenser of the microscope.

III. Magnetic Force Model

In this section, a lumped-parameter magnetic force model, which characterizes the nonlinearity and the position dependence of the magnetic force exerted on a magnetic microbead, is derived for the designed 3-D magnetic actuator. Since the resulting magnetic force, both in magnitude and in direction, is position dependent in the workspace, it is important to analyze the force field so as to enable controllable force application. Therefore, the force generation capability of the developed actuator is analyzed and presented following the theoretical derivation of the force model.

A. Theoretical Derivation

The fundamental concept of the lumped-parameter magnetic force model is the assumption of “magnetic charges” [24]. This assumption is based on the fact that the sharp-tipped magnetic pole generates a magnetic field that looks to the magnetic microbead in the specified workspace as though it is generated by a point source. In this model, the magnetic field generated by each magnetic pole is approximated by the field of a point magnetic charge associated with that pole, and the total magnetic field produced by the system can be obtained by applying the principle of superposition. In addition, the sum of the charges is always equal to zero so there is no net magnetic charge in the system. Therefore, this model can be considered as an extension of the magnetic dipole model [25]: it is a magnetic hexapole model.

According to the magnetic hexapole model, the total magnetic field is given by

$$\mathbf{B} = \sum_{j=1}^6 k_m \frac{q_j}{r_j^2} \mathbf{u}_j \quad (1)$$

where $\mathbf{B}$ is the magnetic induction vector, $k_m = \mu_0/4\pi = 1.0 \times 10^{-7} \text{ N/A}^2$, μ_0 is the magnetic permeability of vacuum, q is the magnetic charge that is defined by $q = \Phi/\mu_0$, Φ is the magnetic flux, r is the distance from the magnetic charge to the magnetic particle, and $\mathbf{u}$ is the unit directional vector originated from the magnetic charge to the magnetic particle. In this paper, vectors and matrices are indicated in bold.

Equation (1) establishes the relationship between the magnetic field and the magnetic charges. The relationship between the magnetic charges and the input currents to the coils can be determined through magnetic circuit analysis. The magnetic circuit of the hexapole magnetic actuator is shown in Fig. 5. In this figure, $\mathfrak{F} = N_c I$ is the magnetomotive force generated by a coil with N_c turns and current I . $\mathfrak{R}_p$ and $\mathfrak{R}_y$ are the lumped reluctances of a magnetic pole and one-sixth of the yoke, respectively. $\mathfrak{R}_a$ is the lumped reluctance from the pole tip to the workspace center in the air. Because the yoke and the poles are made of materials much more magnetic permeable than air, $\mathfrak{R}_a$ is significantly larger than $\mathfrak{R}_p$ and $\mathfrak{R}_y$ in this circuit. Thus, this circuit can be greatly simplified by neglecting the effects of $\mathfrak{R}_p$ and $\mathfrak{R}_y$.

The magnetic flux through each magnetic pole can be determined using Hopkinson's law, which is the magnetic counterpart of electrical Ohm's law, according to the magnetic circuit. Then, the magnetic charges can be determined from the magnetic flux [15], as in

$$\mathbf{Q} = \frac{N_c}{\mu_0 \mathfrak{R}_a} \mathbf{K}_I \mathbf{I} \quad (2)$$

where $\mathbf{Q} = [q_1 \ q_2 \ \cdots \ q_6]^T$ is the vector of magnetic charges, $\mathbf{I} = [I_1 \ I_2 \ \cdots \ I_6]^T$ is the vector of input currents to the coils, and $\mathbf{K}_I$ is the magnetic flux distribution matrix. In particular, $\mathbf{K}_I$ is a 6×6 matrix with diagonal terms equal to $\frac{5}{6}$ and all other terms equal to $\frac{1}{6}$ for the circuit shown in Fig. 5.

The distribution of the magnetic field inside the workspace of the hexapole actuator can, therefore, be directly calculated using (1) and (2). The only unknown variable in the equations is the reluctance $\mathfrak{R}_a$. One way to determine the accurate value of $\mathfrak{R}_a$ is by comparing the magnetic field calculated using (1) with the field determined using finite element software [18], and the value of $\mathfrak{R}_a$ is determined to be 2.8×10^9 A/Wb (details not shown). Alternatively, $\mathfrak{R}_a$ and some other parameters are lumped into a single parameter $[k_{\hat{\mathbf{I}}}]$ in (4) in the force model, and the lumped parameter can then be determined using experiments. The latter approach is adopted in this paper.

The calculated magnetic field inside the workspace of the hexapole actuator when an electric current of 1 A is applied to the coil associated with pole #5 [P5 in Fig. 2(b)] is drawn in Fig. 6. In particular, the 3-D field is plotted in two planes: the $x_m - y_m$ plane and the $x_m - z_m$ plane.

It can be easily seen in (1) that the magnetic field is linearly related to the applied currents to the coils and is position dependent. However, due to the gradient nature of the magnetic force, the magnetic force becomes nonlinearly related to the input currents as derived later.

The relationship between the magnetic field and the gradient magnetic force applied to a superparamagnetic microbead can be described by [26]

$$\mathbf{F} = \frac{1}{2} \nabla (\mathbf{m} \cdot \mathbf{B}) \quad (3)$$

where $\mathbf{m} = (3V/\mu_0)(\mu - \mu_0)/(\mu + 2\mu_0)\mathbf{B}$ is the effective magnetization of the magnetic bead, μ is its permeability, and V is its volume.

By substituting the derived magnetic field into (3), the magnetic force equation can be obtained and organized into a quadratic form, and further into a normalized form, as shown in

$$\hat{\mathbf{F}} = \frac{\mathbf{F}}{k_{\hat{\mathbf{I}}}} = \hat{\mathbf{I}}^T \mathbf{N} \hat{\mathbf{I}} \quad (4)$$

where $\hat{\mathbf{F}} = \mathbf{F}/k_{\hat{\mathbf{I}}}$ is the normalized 3×1 magnetic force vector that is dimensionless, $\mathbf{F}$ is the magnetic force vector, $k_{\hat{\mathbf{I}}}$ is the lumped force gain that has the unit of Newton, $\hat{\mathbf{I}} = \mathbf{I}/I_{\max}$ is the normalized 6×1 input current vector that is also dimensionless, and $\mathbf{N}$ is a 6×6 position dependent matrix, each item of which is a 3×1 dimensionless vector. The details on how matrix $\mathbf{N}$ is computed can be found in [18].

Specifically, the properties of the magnetic microbead, the properties of the magnetic circuit, as well as the maximum input current to the coils are all lumped into a single parameter: the force gain $k_{\hat{\mathbf{I}}}$. The currents are normalized so that $\hat{I}_i (i = 1 \cdots 6)$ is always within $[-1, 1]$, and the position vector is also normalized so that the six magnetic charges are located at unit distance to the workspace center in the normalized coordinate frame.

In this lumped-parameter force model, the nonlinearity of the magnetic force with respect to the applied currents to the coils is explicitly expressed by the quadratic formula and the position dependence of the magnetic force is characterized by the position-dependent matrix $\mathbf{N}$.

B. Magnetic Force Generation Capability

The derived lumped-parameter analytical force model enables us to perform a thorough analysis of the force generation capability of the developed actuator within the workspace. This analysis provides us a better understanding of the position dependence of the actuation. It also lays the foundation for the implementation of feedback control and advanced manipulation of the magnetic microbead. In this section, the normalized force model $\hat{\mathbf{F}} = \hat{\mathbf{I}}^T \mathbf{N} \hat{\mathbf{I}}$ is fully investigated and all the analyses are performed in the normalized actuation coordinate frame, in which the six magnetic charges are located at $[+1 \ 0 \ 0]$, $[0 \ \pm 1 \ 0]$, and $[0 \ 0 \ \pm 1]$.

Due to the complexity of the force model, it is very difficult to analytically characterize the magnetic actuator's force generation capability, so numerical simulation is employed instead. According to (4), the resulting forces due to all possible combinations of the input currents to the six coils are calculated at any location in the workspace. The calculated forces associated with a spatial location are recorded as vectors in a 3-D force space. The end points of these vectors fill up a closed volume, the envelope of which is then used to characterize the force generation capability of the designed 3-D magnetic actuator.

Fig. 7 shows the force envelopes at three different locations. Each plot in Fig. 7 is drawn in the force space and the three axes are $\hat{F}_x$, $\hat{F}_y$, and $\hat{F}_z$, respectively. Each point on the envelope surface indicates the maximal force that can be generated along the direction of the force vector. At the workspace center $[0, 0, 0]$, the envelope is symmetric and the zero force point resides at the center of the volume confined by the envelope. As the location deviates from the center, the force envelope deforms and appears to be stretched toward the closest magnetic pole. It is worth noting that the zero force point is always inside the envelope, indicating that the magnetic actuator is able to generate forces in any direction. However, due to the fact that $\mathbf{N}$ is position dependent, it is not surprising that the force envelope varies significantly in size and shape when the magnetic microbead is placed at different locations. The size of the envelope dictates the force generation capability of the actuator at the associated location, whereas the shape characterizes force generation anisotropy.

By comparing all the force vectors that constitute the force envelope, two quantitative measures are defined for each location within the workspace: 1) force generation anisotropy Γ , the magnitude ratio between the smallest and the largest force vectors; and 2) minimum envelope force $\hat{F}_{\min}$, the magnitude of the smallest force.

Γ has the largest value of 0.58 at the workspace center and decreases while moving away from the center and approaching any of the six poles. Fig. 8 shows three contour surfaces of Γ . The innermost surface corresponds to $\Gamma = 0.25$, the middle surface $\Gamma = 0.10$, and the outermost one $\Gamma = 0.04$. The two outer surfaces are deliberately cut open to show the innermost contour surface. The anisotropy measure is guaranteed to be greater than 0.25 when placing the magnetic microbead inside the innermost contour surface. When Γ is too small, it may not be practically possible to generate the desired force in the weakest force direction accurately, due to various uncertainties of the system, such as assembly errors and fluctuations of the commanded currents. It is, therefore, important to limit the workspace to be within the region with a Γ value not too small. For example, if we constrain $\Gamma > 0.1$ for accurate force application, the workspace should be within a sphere with a radius of about

0.3, as shown in Fig. 8. Again, it is worth noting that the distance is defined in the normalized actuation coordinate frame.

The minimum envelope force $\hat{F}_{\min}$ also decreases as the microbead moves away from the workspace center toward any of the six poles. The contour surface for $\hat{F}_{\min}=3$ is drawn in Fig. 9. The value 3 is chosen because it is half of the maximum force that can be generated when applying the inverse force model, which is discussed in the next section.

IV. Inverse Force Model

The force model enables us to calculate the magnetic force exerted on a magnetic microbead when the input currents applied to the coils are given, whereas an inverse force model is necessary to determine the input currents to produce desired magnetic forces for practical force application using the actuator. Mathematically, the inverse force model is the solution to (4). However, it is not straightforward to obtain the analytical solution to (4) due to two reasons. First, the matrix $\mathbf{N}$ is position dependent and very complex. Second, there are six unknown variables in current vector $\hat{\mathbf{I}}$ whereas only three independent equations in (4). It is, therefore, an underdetermined problem with a 3-D solution space.

To obtain a unique solution to (4), three constraints are introduced and an inverse force model is derived accordingly. Due to the three constraints, the achievable magnetic forces as well as the size of the force envelopes are reduced. Therefore, the analyses similar to those presented in Section III-B are performed to characterize the restriction introduced by the proposed inverse force model in the second part of this section.

A. Theoretical Derivation

One constraint

$$\hat{I}_1 + \hat{I}_2 + \hat{I}_3 + \hat{I}_4 + \hat{I}_5 + \hat{I}_6 = 0 \quad (5)$$

can be applied to simplify the force model. This constraint is a consequence of the underlying constraint that the summation of the total magnetic charges is always zero. A scenario that helps to recognize this constraint is to examine the resulting magnetic field when the same amount current is applied to each of the six coils. The magnetic charges associated with each pole should be equal and are ideally zero, since the sum of all magnetic charges in the system is always zero. Thus, the strength of the resulting magnetic field is zero in the workspace, leading to the fact that the arithmetic mean or the sum of the six input currents does not help generate the wanted magnetic field.

This constraint leads to a simpler force model. Especially, the magnetic force model has a simple and decoupled form at the center of the workspace, i.e.,

$$\begin{cases} {}^a\hat{F}_{cx} = 6 \left(\hat{I}_1^2 - \hat{I}_2^2 \right) \\ {}^a\hat{F}_{cy} = 6 \left(\hat{I}_3^2 - \hat{I}_4^2 \right) \\ {}^a\hat{F}_{cz} = 6 \left(\hat{I}_5^2 - \hat{I}_6^2 \right) \end{cases} \quad (6)$$

It should be noted that (6) is derived in the normalized actuation coordinate frame.

It can be easily verified that the normalized forces are within the range of $[-6, 6]$ from (6) at the workspace center.

Although (6) is still underdetermined, a linear solution can be easily derived, and is given by

$$\hat{\mathbf{I}} = \mathbf{D}^a \hat{\mathbf{F}}_c + \mathbf{E} \quad (7)$$

where ${}^a\hat{\mathbf{F}}_c = [{}^a\hat{F}_{cx} \ {}^a\hat{F}_{cy} \ {}^a\hat{F}_{cz}]^T$ is the normalized force vector at the center of the workspace presented in the normalized actuation coordinate frame. $\mathbf{D}$ and $\mathbf{E}$ are constant matrices

$$\mathbf{D} = \begin{bmatrix} \frac{1}{8} & \frac{-1}{8} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{8} & \frac{-1}{8} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{16} & \frac{1}{16} \end{bmatrix}^T$$

$$\mathbf{E} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{-2}{3} & \frac{-2}{3} \end{bmatrix}^T.$$

This solution (7) satisfies the constraint (5) and relates the six unknown currents to three variables in ${}^a\hat{\mathbf{F}}_c$. Thus, three constraints are introduced by (7). By substituting (7) to the force model (4), we get

$${}^a\hat{\mathbf{F}} = (\mathbf{D}^a \hat{\mathbf{F}}_c + \mathbf{E})^T \mathbf{N} (\mathbf{D}^a \hat{\mathbf{F}}_c + \mathbf{E}). \quad (8)$$

The constrained force model (8) is no longer underdetermined and a unique inverse solution of ${}^a\hat{\mathbf{F}}_c$ can be obtained. Then, according to (7), the currents $\hat{\mathbf{I}}$ can be determined.

The constrained force model (8) is then approximated by first-order Taylor series expansion in 2-D about variables ${}^a\hat{\mathbf{F}}_c$ and ${}^a\mathbf{P} = [{}^a\hat{x} \ {}^a\hat{y} \ {}^a\hat{z}]^T$ (which is defined in the normalized actuation coordinate frame), near ${}^a\hat{\mathbf{F}}_c = [0 \ 0 \ 0]$ and ${}^a\mathbf{P} = [0 \ 0 \ 0]^T$, to serve two purposes: first, the constrained model (8) is nonlinear with respect to ${}^a\hat{\mathbf{F}}_c$. Thus, numerical analysis using iterative methods is required to solve the intermediate variable ${}^a\hat{\mathbf{F}}_c$ from the desired force ${}^a\hat{\mathbf{F}}$, whereas iterations are not desired in real-time implementations. This first-order Taylor expansion establishes the best linear approximation of the relationship between ${}^a\hat{\mathbf{F}}_c$ and ${}^a\hat{\mathbf{F}}$. Second, the expansion also linearizes the position dependence of the magnetic force in the region close to the center of the workspace. In this way, the force model is greatly simplified. The simplified result is given by

$${}^a\hat{\mathbf{F}} \approx {}^a\hat{\mathbf{F}}_c + \mathbf{U}^a\mathbf{P} \quad (9)$$

where $\mathbf{U} = \text{diag}(8, 8, 32)$ is a diagonal matrix.

Finally, the simplified inverse force model can be derived by substituting (9) into (7), i.e.,

$$\hat{\mathbf{I}} \approx \mathbf{D} ({}^a\hat{\mathbf{F}} - \mathbf{U}^a\mathbf{P}) + \mathbf{E}. \quad (10)$$

On the conditions that the magnetic microbead is not far from the workspace center and that the desired force is not very large, (10) is considered to be a simple yet accurate inverse force model. When the conditions are not satisfied, we need to solve (8) for accurate solutions.

B. Magnetic Force Generation Capability When the Simplified Inverse Force Model is Applied

Constraints are introduced in the derivation of the inverse force model to yield a unique solution to (4). These constraints limit the achievable magnetic forces and reduce the size of the force envelopes. To characterize their effects, numerical analyses similar to those presented in Section III-B are performed.

Fig. 10 shows the force envelopes at three different locations when the currents are calculated according to the constraints (7). The volumes enclosed by these envelopes are smaller than those shown in Fig. 7. Moreover, it can be anticipated that the zero force point no longer resides inside the envelopes when the location is not in close proximity to the workspace center (when at $[0, 0, 0.3]$). To characterize these differences, the contour surfaces of the force generation anisotropy and the minimum envelope force are generated and presented as follows.

The contour surfaces of the force generation anisotropy are shown in Fig. 11. The three surfaces correspond to $\Gamma = 0.25$, $\Gamma = 0.10$, and $\Gamma = 0.04$, respectively, from inside to outside. The regions enclosed by these surfaces are all reduced in size when compared with those shown in Fig. 8. Moreover, a degeneration boundary, which is very close to the $\Gamma = 0.04$ surface, can be found. When the magnetic bead goes beyond this boundary, the force generation capability of the magnetic actuator degenerates when using the simplified inverse force model, namely magnetic forces in some directions cannot be produced. As a result, stable position feedback control cannot be established beyond the degeneration boundary. The anisotropy value can, thus, be considered as a measure of the controllability. The larger the value of Γ , the easier it is to establish stable feedback control.

The contour surface of the minimum envelope force $\hat{F}_{\min}=3$ is drawn in Fig. 12. It is not surprising that the region confined by this surface is smaller than that shown in Fig. 9.

According to the previous analyses, the derived inverse force model does not fully utilize the capability of the developed actuator. It, nevertheless, provides a simple linear formula to calculate the required currents in order to produce the desired magnetic force with adequate precision. Deriving an inverse force model that can employ the full potential of the actuator would be a meaningful mathematical problem to be dealt with.

V. Experimental Validation of the Force Model

Direct measurement of the force exerted on the magnetic microbead is very challenging, mainly due to the following two reasons. First, the workspace is very small ($\sim 1\text{--}100\ \mu\text{m}$) and the force range is very small ($\sim 1\text{--}100\ \text{pN}$). Second, motion stability of the magnetic microbead is impossible if the bead is not mechanically anchored or actively controlled. Therefore, a feedback control law is developed according to the derived inverse force model to achieve motion stability of the magnetic microbead. The design of the feedback control law is, however, beyond the scope of this paper. The control results, nonetheless, are used to verify the accuracy of the presented force model.

The developed actuator is assembled on an inverted microscope (Nikon TE2000-U) with a $60\times$ dry objective lens (CFI Super Plan Fluor ELWD $60\times\text{C}$). To realize feedback motion control, the measurement of the 3-D motion of the magnetic microbead is desired, and is acquired by a computer vision system, which employs a high-speed CMOS camera (Mikrotron MC 1310). By analyzing the in-focus and off-focus images of the microbead, its 3-D location can be determined and subnanometer measurement resolution can be achieved [18], [19]. The lateral measurement range is $200\ \mu\text{m}$, which is limited by the camera field of

view, and the axial range is up to 18 μm , majorly limited by the depth of focus of the objective lens. The control experiments are performed at a sampling rate of 200 frames/s. During the control process, the position of the magnetic bead is tracked by the visual measurement system, and the digital controller adjusts the magnetic force exerted on the magnetic bead by varying the currents applied to the coils according to the inverse force model, so that the bead can stay at its desired position.

Magnetic microbead of 2.8- μm diameter (M280, Dynal) is controlled to raster scan a 3-D grid of locations in water, as shown in Fig. 13. In this section, the position and the force are discussed in the measurement coordinate frame [as shown in Fig. 2(b)] and the superscript m on the left of these variables is omitted for simplicity. The grid is $5 \times 5 \times 3$ in size, and each lateral step is 20 μm and the axial step is 5 μm . The axial scan range is not as large as the horizontal ranges because it is limited by the axial measurement range of the visual measurement system. The maximum input current to the coils $I_{\max}$ is set to be 1.2 A. The bead is controlled to stay at each location for 10 s before moving to the next one. Fig. 13 is plotted by drawing the position data of the microbead for the last 5 s at each location after all the transient responses have been vanished.

The magnetic microbead always undergoes Brownian motion in water. Its motion can be described by the Langevin equation [23]. The motion equation in x -direction is given by

$$m\ddot{x}(t) + \gamma \dot{x}(t) = F_T(t) + F_x(t) \quad (11)$$

where m is the mass of the magnetic bead, γ is the damping coefficient of the bead in water, $F_T(t)$ is the random thermal force, also called Langevin force, and $F_x(t)$ is the applied magnetic force in x direction. In this equation, the inertia force $m\ddot{x}(t)$ is negligible when compared with the drag force $\gamma \dot{x}(t)$ for the microscopic particle, and the mean value of the random thermal force is equal to zero.

Since the bead is controlled to stay at each location for 10 s, after the transient response vanished, the mean value of the damping force goes to zero. According to (11), the mean value of the applied magnetic force also equals zero. The steady-state currents applied to the coils during this experiment are shown in Fig. 14. As $I_1 + I_2$ is a constant according to the constraint (7), only I_1 is plotted to save space. The same applies to I_3 and I_5 . It can be seen from this figure that the currents commanded by the feedback control system vary according to the set points. They should, however, always generate zero magnetic force as discussed earlier. By substituting the applied currents and the corresponding positions of the magnetic probe to the force model, the forces in three directions are calculated and they are plotted in Fig. 15. As shown in this figure, the average forces in three directions are very close to zero, which agrees well with our analysis. This result, therefore, partially verifies the accuracy of the presented force model. It is worth noting that the fluctuations in the applied currents and the magnetic forces are due to the controller's action to compensate the effect of the thermal forces.

With the establishment of a stable feedback control, the force gain $k_{\hat{T}}$ of the magnetic actuator can be calibrated using the input–output relationship of the control loop. It can be derived that the loop gain of the feedback system is proportional to $k_{\hat{T}}$. In fact, the loop gain is the product of the digital control gain programmed in the computer and the force gain $k_{\hat{T}}$. When the control set point is kept constant, the thermal force $F_T(t)$ can be considered as the input, and the probe's motion as the output. The thermal force is known as a white noise with a flat power spectrum density (PSD). Therefore, according to the PSD of the controlled Brownian motion of the probe, the control gain and thus $k_{\hat{T}}$ can be calibrated [19]. Its value

is estimated to be 0.53 pN when the maximum input current to the coils $I_{\max}$ is 1.2 A. The force range of the magnetic actuator can then be obtained by multiplying $k_{\hat{r}}$ to the force envelopes as shown in Figs. 7 and 10.

VI. Conclusion

The design, implementation, modeling, and analyses of a 3-D magnetic manipulator have been presented in this paper. The magnetic actuator employs six sharp-tipped magnetic poles placed in hexapole configuration and is capable of generating magnetic forces on a magnetic microbead in arbitrary 3-D directions. To establish the relationship between the applied currents to the coils and the resulting magnetic force, an analytical magnetic force model was derived. The force model accurately characterizes the nonlinearity and the position dependence of the relationship and enables us to investigate the force generation capability of the magnetic actuator. Moreover, an inverse force model was derived. The force generation capability when applying the inverse force model was also analyzed.

With the derived magnetic force model and inverse model, the developed magnetic micromanipulator can be employed to noninvasively apply controlled forces to a magnetic microbead in arbitrary 3-D directions. The microbead can be employed to probe biological samples. The developed micromanipulator, thus, has the potential to provide us a better understanding of the 3-D mechanical properties of biological samples.

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Fig. 1.

(a) Design concept (top view) of the 3-D magnetic actuator. The three dark-colored poles are placed on a plane below the specimen and the three light-colored poles are on a plane above the specimen. (b) The zoomed-in view from the side of the actuator.

Fig. 2.

(a) Six pole tips placed on three orthogonal axes, two on each axis, (b) six pole tips placed on two parallel planes after a coordinate rotation is performed. The solid dots indicate the locations of the magnetic-pole tips.

Fig. 3.

Assembly process of the 3-D magnetic actuator. (a) Three poles on the top layer, (b) three poles on the bottom layer, (c) two layers assembled, and (d) whole setup with the magnetic yoke and coils.

Fig. 4.
Picture of the hexapole magnetic actuator assembled on an inverted microscope.

Fig. 5.
Magnetic circuit of the hexapole magnetic actuator.

Fig. 6.

The calculated magnetic field inside the workspace of the hexapole actuator when current is applied to the coil associated with pole #5 (P5). (a) Field in the $x_m - y_m$ plane. (b) Same field in the $x_m - z_m$ plane. The six magnetic poles are also drawn in scale in the plots. In particular, the three dark-colored poles are above the specimen, while the other three light-colored ones are below.

Fig. 7.
Force envelopes at different locations inside the workspace.

Fig. 8.
Contour surfaces of the force generation anisotropy of the hexapole magnetic actuator.

Fig. 9.

Contour surface of the minimum envelope force $\hat{F}_{\min}=3$.

Fig. 10.

Achievable force envelopes at different locations when currents are calculated according to the inverse force model.

Fig. 11.

Contour surfaces of the force generation anisotropy of the hexapole magnetic actuator when currents calculated according to the simplified inverse force model are applied.

Fig. 12.

Contour surface of the minimum envelope force $\hat{F}_{\min}=3$ when currents calculated according to the inverse force model are applied.

Fig. 13.

3-D grid trajectory. (Figure is drawn in the measurement coordinate frame. The data points are intentionally drawn with large marker for display purpose. The real Brownian motion is restricted within ± 210 nm.)

Fig. 14.
The steady-state currents applied to coils 1, 3, and 5.

Fig. 15.

Calculated steady-state magnetic forces using the presented force model according to the applied currents shown in Fig. 14 and the probe's location as in Fig. 13.