Directly observed relations in complete galaxy samples and the predictions of redshift–distance power laws

(comparative cosmology/luminosity function)

I. E. SEGAL

Massachusetts Institute of Technology, Cambridge, MA 02139

Contributed by I. E. Segal, June 10, 1986

ABSTRACT Directly observed relations in complete galaxy samples (apparent magnitude or diameter vs. redshift) are compared with the predictions of redshift–distance power laws. The predictions are obtained by an objective, nonparametric, statistically uniform, and fully reproducible procedure. In all cases the linear law fits even more poorly than a cubic law, and the optimal law is approximately quadratic. Even a 1.2 power law is conspicuously better-fitting than a linear law. The results of the present study in terms of directly measured quantities are consistent with and confirm earlier studies in terms of theoretical quantities such as absolute magnitudes and diameters. They show that there is no positive evidence for the Hubble law in manifestly fair galaxy samples and that the law can be reconciled with the data in complete samples only, if at all, by the adjunction of a tissue of ancillary hypotheses, none of which is capable of direct observational substantiation.

The Hubble law, from which the physical notion of an “expanding universe”* derives, states that the redshifts of galaxies are proportional to their distances. The distances are however too great to be measurable by parallaxes, and there is no other model-independent direct method. The basic empirical content of the law lies rather in its implied relation between apparent magnitude, or other directly measured quantity whose variation with distance is *a priori* determined (such as the apparent diameter), and redshift. Thus if the redshift z varies as the p th power of the distance r , then the inverse square law for apparent brightness B implies that the apparent magnitude m ($= -2.5 \log B + C$, C = constant, on the conventional “Pogson” scale) has the form

$$m = (5/p) \log z + M, \quad [1]$$

where M is the “absolute magnitude,” definable as the apparent magnitude at a standard distance. Here I follow general practice in neglecting a variety of sources of “noise” that appear to be relatively inconsequential. Large complete galaxy samples are presently confined to redshifts $\leq 1/100$ th of the largest observed redshifts, so that the inverse-square law may be expected to be an adequate approximation, and there is in any case no model-independent alternative.

If M were directly measurable, as m and z are, or at least constant, it would be extremely straightforward to check the Hubble law, which in the present notation states that $p = 1$. However a model-independent or direct measurement of M is impossible, and analysis has depended on the assumption that its statistical distribution is independent of distance, and hence by implication of z , at least in the range of complete galaxy samples. Failing such an assumption, and thus permitting absolute magnitudes to be correlated with z , it is evident that the empirical value of p is indeterminate and that

the concept of a redshift–distance exponent would be without scientific meaning. As in ref. 1, I call this assumption “luminosity uniformity” (LU).

The distribution of M , known as the “luminosity function” (LF) of the (properly, *ab initio* specified) objects under observation, is not at all known *a priori* but can only be estimated from observations. Herein lies the rub, for irrespective of the value of p , Eq. 1 determines M and thereby an apparent LF in any given sample. A seeming inconsistency may appear in the form of a large correlation between M and z , in apparent contradiction with the LU assumption. But the *observational cutoff*—i.e., the necessary limits on the apparent magnitude or diameter that specify completeness of the sample—can readily give rise to such spurious correlations.

In consequence, the rigorous testing of the Hubble law has typically involved somewhat sophisticated modern statistical considerations regarding the absolute magnitude and related quantities, which are p - and hence theory-dependent. While this is well understood in the framework of the general theory of testing of statistical hypotheses, acceptance or rejection of theories is seen as a slightly abstract matter of probability levels involving theory-dependent quantities, such as the cited correlation.

AN ULTIMATE COMPARISON: THE $\langle m|z \rangle$ RELATION

My aim here is rather to report a check of maximal directness and conclusiveness: that of the apparent magnitude or diameter to redshift relation, *as directly observed*, in comparison with the *corresponding predictions* of assumed redshift–distance laws. These are the observed relations that gave rise to the hypothesis that the redshift was basically a function of distance, and they remain the crucial ones for cosmological testing. Naturally their analysis involves the LU hypothesis and requires a (necessarily p -dependent) determination of the relevant LF. These determinations were here made on the basis of a theoretically optimal procedure (2) in an entirely uniform way, using the same number of adjustable parameters in all cases. The procedure is a nonparametric form of maximum-likelihood estimation resulting in sufficient, efficient, and unbiased statistics, in the terms of statistical theory.

The results are summarized in Fig. 1. The abscissa is a hypothetical value of p (plotted as p^{-1} , the more relevant parameter in the $\langle m|z \rangle$ relation). The ordinate is the overall deviation (both rms and maximum are shown) between the *directly measured* $\langle m|z \rangle$ relation and the corresponding prediction, according to the value of p in question. The observed

Abbreviations: LU, luminosity uniformity; LF, luminosity function.

*This is a somewhat nebulous concept from a mathematical physical point of view; precisely what it is expanding into, and the relations between cosmic and locally empirically observable time and energy, remain murky.

The publication costs of this article were defrayed in part by page charge payment. This article must therefore be hereby marked “advertisement” in accordance with 18 U.S.C. §1734 solely to indicate this fact.

FIG. 1. rms and maximum deviations as a function of the redshift-distance exponent p . The deviations are between the $\langle m/z \rangle$ relation as observed in each of four complete galaxy samples and as predicted assuming exponent p .

$\langle m/z \rangle$ relation is represented by the average apparent magnitude or diameter (the latter as likewise reduced to a Pogson scale: $m_\theta = \text{constant} - 5 \log \theta$) in each of 10 disjoint redshift bins, arranged in order of increasing redshift, as a function of the (geometric) mean redshift. The predicted relation was obtained by calculating from the estimated LF $F_p(M)$ the expected (mean) magnitude at each redshift in the observed sample:

$$\langle m/z \rangle = \frac{\int m F_p(m - 5p^{-1} \log z + C) dm}{\int F_p(m - 5p^{-1} \log z + C) dm}, \quad [2]$$

where the integration is over the range in which $m - 5p^{-1} \log z + C \leq m_{\text{lim}}$, the limiting magnitude (or Pogson-reduced diameter) of the sample. These were then averaged in each of the same 10 redshift bins.

The data used were as follows. First, galaxies with redshifts cz less than 500 km s^{-1} were deleted from all samples, in accordance with our standard practice, to minimize noise. Second, the completeness limits used are published ones (although not necessarily by the observers where more conservative ones have been used). The samples are otherwise as follows: A, ref. 3, $m_{\text{lim}} = 13.8$, 1006 galaxies; B, angular diameters based on ref. 4, as reported in ref. 3, with $m(\theta)_{\text{lim}} = 19$, $m(\theta)$ being defined as $20 - 2.5 \log D$, where D is the mean diameter in minutes; C, ref. 5, $m_{\text{lim}} = 12.2$, 501 galaxies; D, ref. 6, $m_{\text{lim}} = 12.4$, 290 galaxies. These appear to constitute the major large complete representative published galaxy samples.

It is clear from Fig. 1 that, in all four samples, the linear law fits conspicuously more poorly than even a 1.2 power law, despite the slight difference in the values of p . As p increases

the fit continues to improve until achieving optimality at approximately the quadratic law. Thereafter the fit gradually declines. The linear-law deviations are of the order of twice those from a quadratic law and 50% greater than those from a cubic law, overall.[†]

Since the Doppler theory of the redshift and thereby the primeval explosion scenario for the origin of the universe are founded on the linear law, and are widely used in the literature, it has seemed appropriate to check very carefully these apparently unequivocal results by analysis of a large number of subsamples of greater conservatism or homogeneity. These analyses included subsamples with brighter completeness limits, in higher redshift ranges, in high galactic latitudes, and of spirals and ellipticals separately. The results regarding the comparative fits of redshift-distance laws were entirely in accord with those shown in Fig. 1. For historical perspective and a possible indication of future trends, an early catalog (7) and a recent sample of Seyfert galaxies (8) were subjected to the same analysis, with similar results.

To confirm the intrinsic neutrality of ROBUST, independently of statistical theory, and check on the possibility of a

[†]In an article (10) entitled "The galaxy luminosity function and the redshift-distance controversy (A Review), Salpeter and Hoffman discuss "the controversy of $p = 1$ versus $p = 2$ " and conclude that the particular aspects they discuss favor the linear law. The "controversy" here is one between objective fact, comprehensively and rigorously documented, and preconceived opinion, rationalized by speculative use of selected observations. This is logically circular, and no positive evidence for the linear law is added in truly direct measurements of manifestly fair samples. The case for $p = 1$ has always been purely theoretical, apart from "observations by [Hubble's] associates and successors that must be seen in the context of a natural tendency to seek the validation and development of a previously indicated hypothesis, rather than to explore possible alternatives" (11). Thus if there is a controversy, it is social rather than scientific; to be scientific, statements must at least be capable of empirical and/or logical falsification, in principle.

More specifically, ref. 10 claims "to emphasize the direct observational nature of the question," "presenting the observational data in 'almost raw' form," "using only the most naive . . . statistical analysis." Unfortunately, the treatment is interwoven with informal ancillary hypotheses, an appeal to "majority opinion" rather than "a minority view," extreme extrapolations wholly lacking in statistical control. In it, two important samples (used above) are neglected and there is no overall analysis of the magnitude-redshift relations in the two samples emphasized. There is no response to comprehensive statistical studies deriving objective probabilities for the deviations of the linear-law predictions from actual observations, finding that these are extremely small, even when allowance is made for four different types of local perturbations proposed as explanatory of the deviations. There is also no response to the close fit of the quadratic-law predictions to the observations, which is inexplicable on the assumption of the linear law; all the more so when large-scale perturbations to the "Hubble flow" are postulated.

For several reasons, including the need for brevity in this journal, the question of the determination from directly observed quantities of the redshift-distance exponent (assuming such a concept is valid) will not be further discussed here. Certainly astronomy is an observationally very difficult and specialized subject, and one must respect the intuitions of observers. But Hubble and Tolman were reserved in their endorsement of the Doppler theory of the redshift, stating only that it seemed the most reasonable explanation in the absence of "new physics." The proposal of the chronometric theory that the redshift represents the difference between the energy appropriate in the Einstein universe and the locally observable (Minkowski) energy represents just such new physics. Moreover these observational difficulties surely confer no right on theorists to represent model-dependent hypotheses as directly observational, irrespective of the strength of their belief in them. On the contrary, these difficulties only increase the need for the greatest objectivity and the most efficient statistical methods.

I thank Salpeter and Hoffman for a communication including their originally submitted version of ref. 10. The published article kindly refers to my comments. To avoid possible misunderstanding, I note that it disregards statistical analyses and other comments that I promptly conveyed to them.

programming error, computer simulations were made to show that if the linear law were correct, the procedure used here would establish it in preference to other power laws. Specifically, samples were constructed in accordance with the linear law, having the sample redshifts and limiting magnitude, by random drawings from hypothetical but empirically reasonable LFs. Such samples were then analyzed assuming successively $p = 1, 2, 3$. The results were highly favorable to $p = 1$ and unfavorable to $p = 3$. For example, for sample D, the respective rms deviations, on the same basis as earlier, were 0.12, 0.25, and 0.38; the maximum deviations were 0.20, 0.52, 0.92, respectively, in great contrast to the results for the sample as actually observed. The LF used was the $p = 1$ estimate for the sample.

CONCLUSION

There is no positive indication for the linear law in the apparent magnitude or diameter to redshift relation in complete, and therefore fair, galaxy samples. Indeed, it is very strongly contraindicated. The quadratic law is approximately optimal among redshift-distance power laws. It has substantial predictive power regarding directly observed galaxy quantities.

DISCLOSURE OF INTEREST

The results presented here use only published data and formulas. They are believed to be fully accurate and repro-

ducible on the basis of the information and references provided here. The reader should nevertheless be informed that I have a special interest in the quadratic law. Specifically, it is predicted at low redshifts by a non-Doppler cosmology that I have proposed (9).

1. Segal, I. E. (1983) *Astron. Astrophys.* **123**, 151–158.
2. Nicoll, J. F. & Segal, I. E. (1983) *Astron. Astrophys.* **118**, 180–188.
3. Huchra, J., Davis, M., Latham, D. & Tonry, J. (1983) *Astrophys. J.* **52**, Suppl. 89–120.
4. Nilson, P. (1973) *Uppsala Astron. Obs. Ann.* **6** (Uppsala General Catalog).
5. Sandage, A. & Tammann, G. A. (1981), *A Revised Shapley-Ames Catalog of Bright Galaxies* (Carnegie Institute, Washington, DC).
6. Visvanathan, N. (1979) *Astrophys. J.* **228**, 81–94.
7. de Vaucouleurs, G. & de Vaucouleurs, A. (1964) *Reference Catalogue of Bright Galaxies*, Univ. of Texas Press, Austin.
8. Cheng, F., Danese, L., De Zotti, G. & Franceschini, A. (1985) *Mon. Not. R. Astron. Soc.* **212**, 857–871.
9. Segal, I. E. (1976) *Mathematical Cosmology and Extragalactic Astronomy* (Academic, New York).
10. Salpeter, E. E. & Hoffman, G. L., Jr. (1986) *Proc. Natl. Acad. Sci. USA* **83**, 3056–3063.
11. Nicoll, J. F. & Segal, I. E. (1982) *Astron. Astrophys.* **115**, 398–403.