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Multi-Coil Magnetic Field Modeling

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Abstract

The performance of multi-coil (MC) magnetic field modeling is compared to dedicated wire patterns for the generation of spherical harmonic (SH) shapes as these are the workhorse for spatial encoding and magnetic field homogenization in MR imaging and spectroscopy. To this end, an example 48 channel MC setup is analyzed and shown to be capable of generating all first through fourth order SH shapes over small and large regions-of-interest relevant for MR investigations. The MC efficiency for the generation of linear gradient fields shares the same order of magnitude with classic and state-of-the-art SH gradient coils. MC field modeling becomes progressively more efficient with the synthesis of more complex field shapes that require the combination of multiple SH terms. The possibility of a region-specific optimization of both magnetic field shapes and generation performance with the MC approach are discussed with emphasis on the possible trade-off between the field accuracy and generation efficiency.

MC shimming has been shown previously to outperform current SH shimming. Along with the efficiency gains of MC shimming shown here, the MC concept has the potential to 1) replace conventional shim systems that are based on sets of dedicated SH coils and 2) allow optimal object-specific shim solutions similar to object-specific RF coils.

Keywords

magnetic fields; modeling; efficiency; accuracy; spherical harmonic functions

INTRODUCTION

Generation of Magnetic Field Shapes with Dedicated Coils

To date, magnetic field shapes resembling spherical harmonic (SH) functions are the workhorse for MR and specialized wire patterns are used for their generation with dedicated single [1; 2; 3] or composite coils [4]. X, Y and Z gradients, corresponding to first order SH functions, are employed for spatial encoding and complemented by higher order SH terms for the homogenization of magnetic field distributions, so-called magnetic field shimming. Gradient coil design aims at the generation of accurate field distributions at maximal strength and efficiency while minimizing the coil's inductance to allow fast current

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switching, i.e. maximum slew rates. Although variable shape gradient systems have been presented for special applications [5; 6; 7], gradient systems are typically constructed on a cylindrical surface to surround the subject and the RF coil(s). Methods like the target field approach [8] and improvements of the gradient coil design such as self-shielding [9] or 3D current geometries [10] replaced the earlier relatively simple Golay-type gradient systems [2; 3; 11; 12] with advanced designs. The coil patterns for the generation of higher order SH fields are still dominated by the designs described by Romeo and Hoult [2] and only minor modifications have been applied to account for the limited radial space in MR scanners [13].

Conventional coil systems are optimized over a predefined spatial range, e.g. a ‘diameter spherical volume’ (DSV), that is chosen large enough to cover the range of targeted subject sizes and potential placement variations. Once built, the magnetic field amplitude generated by an SH coil is a linear function of the applied coil current and independent of the considered region-of-interest (ROI) within the DSV. Magnetic field shapes to be generated, e.g. for shimming, are approximated by the available SH terms and generated by a weighted superposition of SH coil fields. The orthogonality of the employed basis fields played an important role in the early days of MR as, in principle, the serial optimization of individual terms is possible. However, SH functions are only strictly orthogonal in centered, spherical volumes which rarely coincide with anatomical, clinical or functional areas-of-interest. In addition, SH field shapes generated by SH coils commonly contain imperfections, i.e. deviations from their exact, theoretical shape which impacts their orthogonality and renders the serial adjustment of individual terms difficult. To date, magnetic field imperfections are therefore typically mapped either in 3 dimensions with MRI methods or along selected projections (FASTMAP [14] and its derivatives) and converted to shim fields via least-squares optimization. These analytical approaches can be fully automated to provide robust and user-independent magnetic field homogeneity.

Magnetic Field Generation with a Set of Small, Generic Coils

It has been shown recently that small, generic coils can form a magnetic field modeling system capable of generating simple and complex magnetic field shapes in a flexible and experiment-specific fashion [15]. Romeo and Hoult achieved magnetic field modeling with orthogonal basis shapes [2]. The multi-coil (MC) approach demonstrated that successful field modeling is possible even without orthogonal basis functions when least-squares methods are applied to decompose a desired magnetic field into the set of available basis shapes. No generic DSV needs to be predefined and performance parameters such as field accuracy or generation efficiency can be chosen and optimized for specific ROIs on a subject- or MR application-specific basis. Magnetic fields can even be synthesized and optimized on a per-slice-basis, thereby allowing the minimization of the necessary MC currents [15] or the application of slice-specific correction fields for dynamic shimming [16; 17; 18]. The limited susceptibility to the details of the MC geometry such as the number of individual basis coils or their placement has been used in previous MC designs to minimize [17] or avoid [16; 18] interactions with the RF system.

Performance Assessment of Multi-Coil Magnetic Field Modeling

SH-shaped magnetic fields are applied for decades by the MR community and the generation of individual SH terms by dedicated wire patterns is well-established. The MC approach for magnetic field modeling is still in its infancy. To date, a comprehensive description of its performance characteristics is lacking and so is the comparison to conventional coil systems that are based on dedicated wire patterns. The current work sets out to fill this gap.

The first MC publication introduced the methodological, i.e. theoretical and experimental, framework of the MC approach and described the basic concepts [15]. The following work demonstrated the benefits of static and dynamically updated MC fields for magnetic field shimming in the mouse [16], the rat [18] and the human brain [17]. The current publication builds on previous work by evaluating the capabilities and limitations of the MC magnetic field synthesis in more detail. The analysis of the characteristics of the magnetic field generation with the MC technique is provided for an example MC setup and compared to conventional coil systems that apply dedicated wire patterns. Emphasis is placed on the tradeoff between the efficiency of the field generation and the achievable field accuracy.

Preliminary results of this work have been published in abstract form [19].

METHODS

The evaluation of the performance of a field modeling system requires the definition of magnetic field shapes to be produced. In this study, the decision was made to analyze the properties of the MC approach for the generation of individual SH terms (or combinations thereof) due to their key role in MR and to allow the comparison of performance measures with dedicated SH coils. Note that typical MR experiments apply multiple SH shapes together. The consideration of individual performances is therefore somewhat hypothetical and must not be overstated. The properties of MC and SH coil systems were subsequently analyzed for the generation of shim fields in the mouse brain, i.e. a real-world MR problem.

More specifically, the selected MC setup (Fig. 1B) was compared to two types of conventional coil SH systems: The basic SH wire patterns described by Romeo and Hoult ([2], Fig. 1A), hereafter referred to as 'conventional', and a set of state-of-the-art SH wire patterns [20]. The conventional wire patterns were included as they set the standard of SH coil design for several decades and as similar coils are still used in today's MR scanners for the generation of higher SH order shim fields. Furthermore, the Romeo and Hoult designs are in the public domain, whereas the details of most modern designs are proprietary. As such, conventional SH wire patterns can serve as a performance reference for the indirect comparison of the chosen MC design with any other coil system. In addition, an example comparison has been done with selected, modern X and Z gradient coil designs as described in [20].

Performance Analysis: Field Accuracy and Generation Efficiency

Magnetic fields were calculated on a $83 \times 83 \times 83$ grid at $250 \mu\text{m}$ isotropic resolution for dedicated SH wire patterns and individual MC basis fields by integration of Biot-Savart's law. The accuracy of the field distributions generated by dedicated SH coils or the MC approach was quantified over specific ROIs with respect to shape through the $1-R^2$ value [21] and in absolute terms as average deviation (in %) of the generated field from the target field, normalized by the maximal amplitude within the chosen ROI [15]. Frequency offsets were removed before the analysis as they could be considered as simple frequency shifts in the MR sequence. Notably, magnetic field offsets can readily be provided by the MC approach which has been demonstrated for dynamic MC shimming [16; 17; 18].

Coils and coil systems for the generation of magnetic fields are typically characterized by a series of technical and performance parameters. These include, among many others, the size of the coil (diameter and length), the shape of the wire pattern, the wire length and the concomitant conductor mass, the resulting inductance and the resistance. The most relevant performance characteristics are the accuracy of the synthesized field shape over a given ROI, the achievable switching time (or alternatively its slew rate) and the coils' efficiency in generating a given magnetic field. Coil efficiency and field generation efficiency are

interchangeably used for SH coil systems, since every SH term is generated by a dedicated, fixed (single or composite) coil. However, this concept does not apply to the MC approach, since more than one field shape can be generated by the same coil setup. The efficiency for the generation of magnetic field shapes with a given coil system has therefore been defined as provided field amplitude per applied current and coil wire length

$$efficiency = \frac{A}{\sum_{j=1}^{coils} L_j \cdot I_j} \quad (1)$$

with the field amplitude A that is generated over a given ROI when a set of coil elements of lengths L_j is driven by coil-specific currents I_j . A SH wire pattern is a special case of this definition with coils = 1. The wire lengths were calculated by numerical integration of the basic wire paths, however, supporting and interconnecting wires were neglected in all cases. Note that for a grid of identical coils all wire lengths L_j are the same. Efficiency as defined in equation (1) has $\text{Hz} \cdot \text{cm}^{-n} \cdot \text{A}^{-1} \cdot \text{m}^{-1}$ units for SH fields. The conversion of the considered Larmor frequency for protons to the magnetic field strength in Tesla can be achieved by division through the ^1H gyromagnetic ratio. Further experimental details such as energy deposition or the generation of hot spots as described in [22] were not considered nor were geometric properties such as the thickness of the coil system e.g. the required multi-layer structure for a set of physical SH coils.

Multi-Coil Magnetic Field Modeling

The MC field synthesis aims at minimizing the difference Ω of the target field distribution T and the sum of all individual coil fields MC that are generated by a set of coil-specific currents I_j for the considered ROI voxels i in the least-squares sense (Eq. 2).

$$\Omega(I_j) = \sum_{i=1}^{voxels} \left(T_i - \sum_{j=1}^{coils} MC_{ij}(I_j) \right)^2 + cw \sum_{j=1}^{coils} \sqrt{I_j^2} \quad (2)$$

The efficiency of field-generating coil systems is a function of the wire length and the applied currents (Eq. 1), and it is desirable for a given MC setup (at fixed wire length) to minimize the applied currents for the generation of a target field distribution. An additional term has therefore been added to the right side of Eq. 2 to minimize the overall, magnitude current requirement. Emphasis of the current minimization with respect to the quality of the regular field synthesis is achieved by a current weighting factor cw . This approach allows the flexible trade-off between field accuracy and generation efficiency. Since the accuracy of the MC field synthesis is reduced with the inclusion of additional terms in the cost-function e.g. for current minimization, the level of acceptable field imperfection has to be defined on an application-specific basis. The Lorentz forces that act on a coil and the energy deposition that leads to coil heating depend on the coil current. Although not considered here, the minimization of individual and overall currents also reduces potential problems related to forces/torques and coil heating. Note that for dedicated SH coils such an approach is only possible during the design process. Once built, the relationship between applied current and the generated field amplitude is linear and the efficiency is an inherent, constant property of the coil.

The wire length of both dedicated SH coils and MC setups scales linearly with the cylinder size and field generation efficiency diminishes as

$$efficiency \propto \frac{1}{R^{N+2}} \quad (3)$$

with increasing cylinder radius R and order N of the SH shape to be generated. The results presented here were calculated for an example cylinder radius of 15 mm (Fig. 1), however, they can be readily rescaled to any other cylinder dimension via equation 3. For instance, an X gradient field ($N=1$, $M=1$) generated by a wire pattern with a radius of 50 cm requires an efficiency rescaling by a factor of $(15 \text{ mm} / 50 \text{ cm})^{1+2} = 2.7 \cdot 10^{-5}$, and an efficiency rescaling of $(15 \text{ mm} / 45 \text{ cm})^{3+2} = 4.1 \cdot 10^{-8}$ is necessary to derive the efficiency for the generation of a Z2X ($N=3$, $M=1$) term for a coil system with a 45 cm radius. Note that the relative efficiencies between SH wire patterns and the MC approach presented in this paper are inherently consistent as they were calculated for the identical former dimension and SH shape (i.e. SH order N). As such, all efficiency comparisons are independent of the chosen example former size and are generally applicable.

Coil Setups and Simulation Details

An example MC setup consisting of 48 circular coils was selected that provided good coverage over the range of considered ROIs and sufficient modeling flexibility for the targeted SH and shim field shapes (Fig. 1B). The coils were distributed on a regular grid in 6 equidistantly spaced rings of 8 coils. Note that coils from neighboring rows were rotated by half the angle between two neighboring coils within a row, i.e. $45/2$ degrees = 22.5 degrees, to allow a more dense packing. Each of the 10.5 mm diameter coils had a wire length of 33 mm which added up to a total wire length of the MC setup of 1583 mm. Notably, the similarity of the selected MC design to a previous MC implementation [16] allowed the relation of theoretical results derived here to experimental results from previous studies and, more importantly, the theoretical application of both MC- and SH-based field modeling to shimming of magnetic fields in the mouse brain under realistic conditions.

SH wire patterns for the generation of the first and second SH orders as well as the Z3 and Z4 terms were taken from [2]. Wire patterns for the remaining third and fourth order SH terms were provided by Agilent Technologies (Oxford/UK). They had been designed along the lines of [2] and numerically optimized as trade-off between accuracy, efficiency and production feasibility of the magnetic field terms. In addition, the properties of state-of-the-art X and Z gradient coils were assessed for the wire patterns presented in [20]. All first through fourth order SH wire patterns were arranged on the same 30-mm diameter cylindrical surface, i.e. they were matched in size with the MC setup, and only half of the tesseral SH functions ($M=0$) were considered due to the rotational symmetries of the SH shapes and the selected MC setup.

A DSV at one third of the coil diameter was chosen by Romeo and Hoult [2], but current coil designs commonly apply larger optimization volumes for both first and higher SH orders. The performance of the different coil systems and methods was therefore assessed over a spherical volume at the radius of a third of the cylinder radius, here 5 mm (Fig. 2, I) and a spherical volume at 1/2 the cylinder radius or 7.5 mm (Fig. 2, II). The spherical ROIs at 1/3 and 1/2 of the cylinder radius consisted of 33371 and 112947 voxels, respectively.

MC magnetic field modeling allows the ROI-specific optimization of magnetic field shapes and their generation properties for the MR application at hand. Applications to employ this principle include dynamic shimming [16; 17; 18], anatomy-specific spatial selection [23], conventional [24] and algebraic [25] MC-MRI. Such applications are only useful in practice, however, if the field shapes can be realized in all relevant ROIs. To this end, the MC field modeling capability and the concomitant generation efficiencies were also determined for a stack of circular axial slices at 1/3 of the cylinder radius to resemble the geometries of MR imaging and spectroscopic imaging sequences. The stack height covered a -20..20 mm range along the B_0 direction (i.e. the z-axis) corresponding to 2/3's of the cylinder radius (Fig. 2, III & IV). Immediately neighboring slices were included in the field optimization and

analysis of every such slice ($3 \cdot 1245 = 3735$ voxels) to assure volumetric ROIs and to account for the 3-dimensional character of magnetic fields including through-plane components [16; 26].

The cylinder diameter of the coil systems analyzed in this study were similar to the MC design applied previously for magnetic field shimming of the mouse brain [16] and, in principle, both SH- and MC-based coil systems could be applied to this task. The efficiencies for the generation of correction fields for magnetic field shimming were therefore evaluated for static whole brain shimming of the 7 mouse brains considered in above study. For comparability, MC shimming was applied to B_0 homogeneity levels equivalent to those achievable by first, second and third order SH shimming by means of identical standard deviation of the magnetic field distribution after shimming.

All magnetic field simulations and performance analyses were done with customized Matlab software (MathWorks, Natick/MA, USA).

RESULTS

Spherical Harmonic Field Generation with Conventional Wire Patterns

All SH coils provided the corresponding SH shape and most fields were found to be highly accurate over a spherical ROI at $1/3$ of the cylinder radius (Table 1). Some terms such as Z2X ($N=3$, $M=1$) and Z3X ($N=4$, $M=1$), however, were significantly contaminated with a combination of X and X3, and ZX, respectively. Such imperfection is not necessarily problematic in practice, as long as the artificial components can be described and therefore compensated by other available SH coils. The mean error was therefore reevaluated after the removal of other artificial SH components up to the order of the considered SH term. Neglecting artificial SH terms from the fields created by Z2X and Z3X SH coils, both coils provide the desired SH term.

The generation of SH fields was also analyzed for a set of axial slices with a radius of $1/3$ of the cylinder radius. These slices were placed at the isocenter of the coil system, at 5 mm and 10 mm offsets corresponding to $1/3$ and $2/3$'s of the cylinder radius, respectively (compare figure 2). SH fields were found to be very accurate in the centered slice that was part of the DSV for which the wire patterns have been optimized (Table 1). Both the field amplitude and the shape accuracy deteriorate quickly outside the DSV. This must not be misinterpreted as a flaw, since areas outside the DSV have not been considered in the coil optimization and, therefore the field behavior in these areas cannot be expected to be accurate. SH efficiencies in axial slices are given relative to the efficiency for the generation of the same SH term in a centered sphere at $1/3$ of the cylinder radius. Raw errors are reported for the original field shapes as generated. Compensated errors summarize the remaining imperfections after removal of the other, available SH terms. Even though most spherical harmonic imperfections were removed perfectly in Table 1, it should be noted that this may not be possible in reality due to the limited range of the higher order SH terms. Remember that the term 'slice' refers to the considered 2-dimensional slice itself plus one additional slice on each side. As such, all analyses considered narrow 3-dimensional volumes and all SH terms could be reasonably determined. The limited range of through-slice amplitude variations should be kept in mind when field efficiencies and average errors are considered, e.g. for Z ($N=1$, $M=0$).

Spherical Harmonic Field Generation with State-of-the-Art Wire Patterns

The efficiency of the considered state-of-the-art X gradient coil exceeded the conventional Romeo and Hoult design by 15%, however, the efficiency of the Z gradient coil was reduced by 27% compared to the conventional 2-loop design (Table 1, X*, Z*). Notably, these

gradient coils were optimized over a cylindrical region-of-interest with diameter and length corresponding to 53% of the cylinder diameter (Fig. 3g in [20]). The generated field terms therefore proved to be highly accurate even over the largest region-of-interest considered here.

Spherical Harmonic Field Generation with Multi-Coil Field Modeling

The performance of the first-through-fourth order SH field generation with the MC field modeling was determined for the example MC setup shown in figure 1B. The field accuracy was adjusted to an average error of 1% for all fields and field generation efficiencies were calculated for the same SH terms that were previously provided by dedicated SH wire patterns (Table 2). Note that the average error of the MC field generation corresponds to the raw error of the SH field generation as the additional correction of remaining imperfections is neither necessary nor possible with the MC approach.

All individual SH terms could be successfully approximated with the MC approach and MC efficiencies were found to be of the same order of magnitude as dedicated SH wire patterns. More specifically, the averaged efficiency for the generation of the term-specific SH fields in centered spheres of 1/3 and 1/2 of the cylinder diameter with the MC approach was 63% and 43%, respectively, compared to their generation with individual SH wire patterns (compare Table 2). For circular slice-shaped volumes positioned at the isocenter, and at offsets of 1/3 and 2/3's of the cylinder diameter, average efficiencies of 117%, 114% and 115%, respectively, were found relative to individual Romeo and Hoult designs. Notably, MC/SH efficiency ratios largely varied between 3% and 296%. While MC field modeling is capable of providing magnetic fields over large ROIs, relative efficiency gains can be realized when ROIs can be limited to smaller volumes, e.g. as the stack of individual slices considered here.

The maximal field amplitude that can be achieved with the MC approach is a function of the acceptable imperfection of the shape to be generated. This dependence is shown in figure 3 for a selection of first through fourth order SH terms when generated with the example MC setup in a centered spherical ROI at 1/3 of the cylinder radius assuming a current limit of 1 A per MC channel. The general ability of the MC approach to generate the considered shapes can be appreciated from the negligible errors for small field amplitudes for which the currents in the optimization procedure are effectively unconstrained. The contribution of each MC coil becomes progressively restricted to its primary spatial component for the generation of larger SH amplitudes and the accuracy of the synthesized field starts to diminish. The error of the generation of Z2X, for instance, remains below 1% over an amplitude range that covers more than an order of magnitude. Eventually, however, the field synthesis becomes truly amplitude limited and the quality of the results degrades rapidly. Note that this analysis is based on single-turn coils and a 1 A current limitation. As such, maximal field amplitudes in an experimental realization can be increased multifold when coils with multiple turns are used and amplifiers with higher dynamic range are applied.

Similarly, accuracy can be traded for efficiency with MC field modeling via the current weighting factor c_w of equation 2. Examples of this functional dependency are shown in figure 4 for the SH shapes Z, Z2X and Z3. Their accurate generation at small weighting factors c_w (solid line) is reflected by the negligible error in this regime. Increased weighting of the current cost-function minimizes the necessary overall current sum for the generation of a given field shape and thereby leads to considerable efficiency gains (dashed line). The concomitant constriction of the MC system, however, diminishes the achievable field accuracy as reflected by progressively increasing mean errors at higher current weighting. Achievable efficiency gains can exceed several orders of magnitude if the best field accuracy is not necessary. The accuracy of the Z gradient shape generated with the MC

approach starts to diminish even for small current weightings (solid line), but the slope is small and the mean error stays well below 1% even with a cw increase of 3 orders of magnitude from 10^{-2} to 10^0 . The corresponding reduction of overall coil currents from 48 A to 16 A over the displayed range of cw 's reflects an efficiency gain of a factor of 3. Current sum and mean error of the field optimization for MC modeling depend on the details of the applied MC setup with respect to the considered target field shape. Even though they are correlated and follow opposite tendencies, their detailed features can differ significantly (e.g. Z vs. Z3) and they therefore have to be calculated independently. MC modeling allows the synthesis of magnetic fields in the center of the MC setup and in its periphery. For example, linear X gradients could be generated in horizontal, circular slices with a radius of a third of the cylinder radius ($-5..5$ mm) over a vertical range spanning 4/3 times the cylinder radius ($-10..10$ mm)(Fig. 5A, black). MC-fields were adjusted to a 1% error level, but higher precision could be achieved in all slices. For comparison, the conventional SH coil from [2] showed excellent field accuracy within the range covering 1/3 of the cylinder radius ($-5..5$ mm) corresponding to the DSV it was optimized for (Fig. 5A, gray). The field accuracy deteriorated outside this range, however, this is not unexpected and must not be misinterpreted, since these areas were not considered in the design process of the coil. The MC efficiency for the generation of X gradients throughout the considered stack of slices was found similar to the Romeo and Hoult design, but appeared less homogeneous (Fig. 5B, Table 2).

Overall, the MC efficiency for the generation of individual first order SH terms was found to be somewhat reduced compared to dedicated SH coils (Table 2, Fig. 5). Considering all first through fourth order SH shapes and ROIs considered in this study, the efficiencies for the MC approach and the SH wire patterns were comparable.

Generation of Combined Spherical Harmonic Magnetic Field Shapes

The MC efficiency becomes progressively more competitive for the synthesis of magnetic fields that required more than one SH term as, for example, encountered in MRI of oblique planes and shimming. The efficiency of global (static) shimming of the brains of 7 mice from [16] with the MC approach to homogeneity levels achievable by first order SH shimming were found to be comparable to the efficiency that can be expected from a conventional first order SH coil system (Fig. 6). MC shimming became progressively more efficient compared to conventional SH shimming, when higher SH terms were included for improved shim outcome. As such, the MC efficiency for global, static shimming of the mouse brain to the homogeneity levels achievable with second and third order SH shimming were 1.6 and 3.4 times as high, respectively, than for SH coil systems.

DISCUSSION

The characteristics of the MC approach for magnetic field modeling were evaluated in this study. Zero through fourth order SH magnetic field shapes which are the workhorse in NMR were successfully generated in a variety of ROIs relevant for MR imaging, spectroscopy and shim applications. The results were compared to the performances of dedicated SH coils and coil systems, namely the basic designs described by Romeo and Hoult [2] and a set of modern X and Z gradient coils [20].

The ROI-specific synthesis of magnetic field shapes with the MC approach has been shown to allow the flexible optimization of field accuracy or generation efficiency (Eq. 2, Fig. 4). Although the achievable accuracies and the related efficiencies for the generation of a linear X gradient in a stack of axial slabs varied for the presented MC setup as a function of the slice position, 1% errors were always achieved (Fig. 5). Similarly, the considered SH terms could be provided at higher precision in almost all ROIs when no current limitations were

applied. The only exception was the X4 shape for which a minimal error of 1.2% was found, primarily dictated by the chosen MC setup geometry. As such, MC field errors averaged over all considered SH terms of 0.09%, 0.35%, 0.07%, 0.02% and 0.01% were achieved within the centered spheres of 1/3 and 1/2 of the cylinder radius, a centered slice and slices at off-center positions of 1/3 and 2/3's of the cylinder diameter, respectively.

The experimental realization of a 48-channel MC setup at similar geometry (diameter 32 mm) and 30 turns per coil along with a dynamic current range of ± 1 A [16] has been demonstrated to provide gradient amplitudes of 40 kHz/cm (94 mT/m) and to allow slice-selective RF excitation as well as imaging applications [15; 24]. For applications like arterial spin labeling or signal spoiling, perfect linearity is not required and the reduced demands on accuracy can be translated into improved coil efficiency, i.e. higher magnetic field amplitudes and/or reduced MC currents. This feature is particularly relevant when strong field amplitudes are to be generated with limited amplifier currents (Fig. 4, [24]). The example MC setup analyzed in this study has not been able to provide linear gradient fields over large ROIs at accuracy and efficiency levels comparable to dedicated SH wire patterns and, consequently, it seems unlikely that similar MC designs have the potential to replace SH gradient coils on regular MR scanners. Especially, the MC efficiency for the generation of Z shapes in large volumes is not comparable to dedicated wire patterns. In the same vein, the suitability of the MC approach for shimming of MR scanners seems limited, e.g. compared to passive shimming (not considered in this study). However, the development and implementation of object-specific MC setups for the generation of strong gradient fields e.g. for spine or wrist MRI seems straight-forward with the presented methodology. Also, the MC approach is still in its infancy and the potential of algebraic MC-MRI based on non-SH field shapes still needs to be explored further [25]. The capability of the MC approach to generate magnetic field shapes in peripheral areas close to the MC cylinder or along the magnet bore might be used for peripheral applications such as spin tagging or to shorten the coil setup itself. Since fields can even be synthesized outside the MC setup (data not shown), one-sided applications such as skin imaging [27] or magnetic field shimming in specific body parts that are difficult to access otherwise (e.g. the spine) might be possible. Notably, the low inductances concomitant to the small individual MC coils (mouse setup 10 PH [16], human setup 680 PH [17]) facilitate rapid current alterations which is a prerequisite for fast MR methods such as dynamic MC shimming or MRI. MC-MRI on the basis of switched MC fields has been demonstrated already [24; 25] and no significant coil-to-coil interactions have been observed. The comprehensive analysis of potential dynamic effects related to the fast current switching or the details of the MC geometry, however, has to be part of future research.

The MC approach for magnetic field modeling relies on the availability of a broad repertoire of field shapes within the considered target ROI. A set of 48 coils has been chosen in this paper similar to previous implementations [16], but MC applications have also been demonstrated with significantly simpler setups. For instance, 24 coils were sufficient to realize an MR imaging sequence based on MC-generated field gradients along with basic shimming capabilities [15]. In general, the more coils are available, the higher the likelihood that the available shapes comprise the spatial features of a given target field. If the relevant spatial feature of an MC basis field is accompanied by other shape contributions that are not required, then these components have to be canceled by separate fields from other coils which effectively lowers the field generation efficiency. Although MC field modeling does not critically rely on the details of the applied MC geometry, reasonable coil dimensioning and placement are necessary. For instance, the curviness of the MC efficiency for the generation of X gradient fields in a stack of axial slices (Fig. 5) results from the geometric details of the MC array with respect to the ROI shape and position. The central dip can be avoided by including a gap between the center rows in a regular MC matrix design [15] or

by breaking up the symmetry of the MC setup along the z-axis, e.g. by spiraling the coils around the cylindrical former (data not shown).

To date, MC coils are still wound in circular shapes from standard magnet wire although other shapes might be more beneficial. Due to the vector product in Biot-Savart's law, only currents orthogonal to the scanners' B_0 direction contribute MR-relevant field components. Therefore, in principle, maximal efficiency and modeling flexibility are expected from a large matrix of (infinitesimally) short horizontal wire pieces as the cancellation of field components from opposite parts of the same coil are prevented. The use of horizontal pieces also minimizes ineffective and therefore lossy current components parallel to the scanner B_0 field (under the assumption that these currents are not simply shifted to the supporting circuitry). The theoretical demonstration of this idealized MC design is straight-forward with the methods presented in this paper. The technical feasibility of driving all these pieces with individual currents and moreover the successful implementation, calibration and use of such MC system for the generation of MR-relevant field amplitudes is yet to be demonstrated. More importantly, a large part of the expense concomitant with the installation of MC field modeling systems lays with the amplifiers (as opposed to SH shimming) and economic reasons are expected to eventually limit the number of basis fields.

The efficiencies for the generation of magnetic fields were compared in this study at identical geometric constraints. It has been shown that the usefulness of the MC approach is not primarily based on the proximity of the MC coils to the ROI, but a result of the MC field modeling approach. In particular, the ability to restrain the field optimization to specific ROIs for the MR application at hand allows improvements in both accuracy of the field modeling and the efficiency at which magnetic fields can be realized compared to the consideration of large DSVs. The relationship of field efficiency and cylinder radius is the same for all coil systems and wire patterns (eq. 1) and, therefore relative MC and SH efficiencies are independent of the specific cylinder diameter chosen in this study. Moreover, MC magnetic field modeling is not limited to miniature setups and dynamic MC shimming has been shown already to outperform state-of-the-art dynamic shim updating (DSU) with zero through third order SH terms for magnetic field shimming of the human brain at 7 Tesla [28].

The study aimed at a comparison of the basic properties of the different field modeling approaches and the physical extension of the wire patterns has been neglected for both conventional SH coils as well as the MC approach. In reality, however, limitations in space, heat conductivity or the variations in former diameter, e.g. due to the stacking of SH coils, cannot be neglected. Modern SH coil systems are the result of advanced design strategies that do not only take basic parameters such the accuracy and efficiency into account, but also include further characteristics such as mechanical properties or heat dissipation [22]. Mechanical forces or coil heating have not been a problem with the low-current MC applications so far and the consideration of the large variety of potential experimental realizations for SH or MC wire patterns and coil setups was outside the scope of the current study.

The accuracy of SH fields that are generated by dedicated SH wire patterns is determined by the coil optimization process. In practice, potential manufacturing imperfections or positioning errors of the applied SH coils with respect to each other can cause additional imperfections. These artificial field components, either inherent to the coil or induced by experimental imperfections, can be compensated with other available SH terms when the systems' cross-term matrix is considered [26]. As such, the dominant X contamination generated by the Z2X wire pattern in this study can be compensated. In the same vein, the field generated with the analyzed Z3X coil showed large ZX impurities. Their removal

reduced the attainable field errors from 714% to 0.1% and from 10.9% to 0.2% within a center sphere at 1/3 of the cylinder radius, respectively (Table 1). The X3 coil, however, contained field imperfections that were above fourth SH order which precluded the generation of an X3 shape at 1% error or below. Although the consideration of cross-terms is a viable option for magnetic field shimming, it is not easily applicable for first order gradients due to their large amplitude range along with their highly dynamic application in MR imaging sequences. In practice, a tilt or a rotation of an SH gradient coil cannot be corrected by other SH terms and might lead to subsequent MR artifacts.

Potential manufacturing imperfections are readily taken into account with MC field modeling, since the method neither relies on the details of its basis shapes nor on their orthogonality.

The thorough calibration of experimental MC fields, however, is an essential prerequisite for a successful implementation, since errors add up and typically every field modeling analysis – even for simple target field shapes – involves all available MC basis terms. While iterative adjustments have been used for SH-based shimming [29], a similar approach is not practical with the MC technique. Instead, high accuracy calibrations of the experimental MC basis fields have been shown to enable the accurate generation of magnetic field shapes for MR imaging and B_0 shimming in a single step.

Individual SH terms have been used as target distributions in this study to demonstrate that the MC approach can synthesize a broad spectrum of field shapes with the same setup. MC and SH coil performances for the generation of individual SH terms, however, have only limited meaning, since SH shapes are rarely applied individually in experimental reality. Instead, all available SH shapes are applied together e.g. for magnetic field shimming and, therefore, the combined efficiency for the application at hand is the more relevant performance measure. Field shimming of the mouse brain has been applied in this study to demonstrate relative efficiency gains of MC- compared to SH-based shimming for the generation of high-level shim fields (Fig. 6). Note that current SH shimming suffers from limitations to resemble the shape of the field imperfections observed *in vivo* (and can be further impaired if the available dynamic range is insufficient). MC magnetic field modeling has been shown previously to outperform SH approaches for the homogenization of magnetic fields in human and animal subjects *in vivo* due to improved field shaping [16; 17; 18]. Along with the efficiency gains of MC shimming shown here, the MC concept has the potential to replace conventional, large-bore shim systems that are based on sets of SH coils.

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Research highlights for the manuscript entitled 'Multi-Coil Magnetic Field Modeling' by Juchem et al.:

- Methodology for the evaluation and optimization of multi-coil designs is presented
- Multi-coil field modeling allows flexible trade between efficiency and accuracy
- Multi-coil efficiency for linear field gradients compares to conventional coils
- Multi-coil field synthesis becomes more efficient for more complex field shapes
- Multi-coil concept has the potential to replace spherical harmonics for shimming

FIGURE 1.

Coil setups for the generation of magnetic fields compared in this study. A: Conventional SH coils consist of dedicated wire patterns for the generation of magnetic field shapes resembling SH functions (X gradient ($N=1$, $M=1$), from [1]). The radius of the sphere inside the coil corresponds to $1/3$ of the wire patterns' cylinder radius (compare figure 2, case I). B: Nonorthogonal basis fields generated from individual, generic coils are combined with the MC approach to synthesize the desired field shape. An example MC setup consisting of 6 rows of 8 coils, i.e. 48 coils total, has been analyzed in this study and its field modeling properties have been compared to the performances of conventional SH wire patterns.

FIGURE 2.

A) The performance of magnetic field generating systems was analyzed for various regions-of-interest, namely centered spherical volumes at $1/3$ (corresponding to [2], case I) and $1/2$ of the cylinder radius (case II), and individual circular, axial slices of the same radii (cases III and IV). B) Besides centered slices, the field modeling capacity was tested over a continuous stack of slices spanning the $-2/3..2/3$ range of the cylinder radius. Note that the immediate neighbors were included in any slice analysis to assure 3D volumes (example indicated in white).

FIGURE 3.

Achievable field accuracy for the generation of SH field shapes by MC magnetic field synthesis as a function of the dynamic amplitude range. Higher field amplitudes can be achieved with the same coil setup and current limitations if increased field imperfections are accepted. This functional dependence allows a flexible tradeoff between amplitude range and accuracy.

FIGURE 4.

Required current as a function of field accuracy for the generation of SH field shapes with the MC field modeling. The successful synthesis of Z, Z2X and Z3 shapes can be appreciated by the negligible errors when coil currents are effectively unconstrained (solid line at weighting factor 10^{-2}). Significant efficiency gains corresponding to a reduced sum over all coil currents (dashed line) can be realized with the MC approach if the best field accuracy is not necessary.

FIGURE 5.

Performance of MC field modeling (black) and a dedicated SH wire pattern (gray, from [2]) for the generation of a linear X gradient shape over a stack of horizontal slices at 1/3 of the coils' cylinder radius. The conventional SH coil showed excellent field accuracy within the range covering 1/3 of the cylinder radius (−5..5 mm) corresponding to the DSV it was optimized for. The deterioration of the field accuracy outside this range was expected, since these areas were not considered in the design process of the coil. MC magnetic field modeling allowed the generation of X gradient shapes throughout the entire stack of slices with an error level adjusted to 1%. The MC efficiency was similar to the conventional design, but appeared less homogeneous.

FIGURE 6.

Efficiency analysis for the generation of shim fields with the MC setup and SH coils. Global MC shimming has been applied to 7 mouse brains to B_0 homogeneity levels equivalent to those achievable by first, second and third order SH shimming (ie. identical SD of the residual field imperfections). In analogy to equation (1), efficiency has been defined as the inverse product of the current requirement and wire length (in $[A^{-1} m^{-1}]$). The efficiency of first order SH shimming is comparable to the efficiency of MC shimming. MC shimming becomes progressively more efficient compared to the conventional SH approach, when higher order SH terms are included for improved shim outcome.

PERFORMANCE ANALYSIS SPHERICAL HARMONICS

TABLE 1

Performance analysis of conventional first through fourth order SH wire patterns [2] and modern gradient coils [20] (X^* , Z^*) for the generation of SH field shapes in spherical volumes at 1/3 and 1/2 of the cylinder radius and selected, circular center and off-center axial slices. The field accuracy was accessed with respect to shape by the $1-R^2$ value and as average error in percent. All SH coils provided the corresponding SH shape and most fields were highly accurate over a spherical ROI at 1/3 of the cylinder radius. The mean error over the spherical ROIs was reevaluated after the compensation of artificial field components that could be modeled with other SH terms up to the SH order of the considered term. SH amplitudes in axial slices and relative efficiencies are given relative to the corresponding values in a centered sphere at 1/3 of the cylinder radius.

	X	Z	X*	Z*	X2-Y2	ZX	Z2	X3	Z(X2-Y2)	Z2X	Z3	X4	ZC3	Z2C2	Z3X	Z4
SH order N / term M	1 / 1	1 / 0	1 / 1	1 / 0	2 / 2	2 / 1	2 / 0	3 / 3	3 / 2	3 / 1	3 / 0	4 / 4	4 / 3	4 / 2	4 / 1	4 / 0
total wire length [mm]	512	189	6303	4521	712	709	1320	609	239	937	943	588	2192	1798	1511	1885
amplitude [Hz·cm ⁻ⁿ ·A ⁻¹]	1739	1525	24530	26714	488	563	2895	101	136	259	1368	418	22.6	115	425	461
efficiency [Hz·cm ⁻ⁿ ·A ⁻¹ ·m ⁻¹]	3394	8088	3892	5909	686	795	2194	166	570	276	1451	7.11	10.3	63.7	281	245
center sphere, radius 1/3 of cylinder	0.00002	< 1e-5	< 1e-5	< 1e-5	0.00002	0.00001	< 1e-5	0.00630	0.00170	0.99918	0.00062	0.00275	0.00012	0.00069	0.28659	0.00065
raw error [%]	0.1	0.1	0.1	0.2	0.1	0.1	< 0.1	1.3	0.8	714	0.5	0.8	0.2	0.4	10.9	1.1
compensated error [%]	0.1	0.1	0.1	0.2	0.1	0.1	< 0.1	1.3	0.8	0.1	0.1	0.8	0.2	0.2	0.2	< 0.1
relative efficiency	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
center sphere, radius 1/2 of cylinder	0.00044	0.00011	0.00004	0.00002	0.00063	0.00029	0.00011	0.03164	0.00861	0.99515	0.00052	0.01515	0.00080	0.00206	0.07459	0.00104
raw error [%]	0.6	0.3		0.4	0.6	0.3	0.2	2.7	1.7	315	0.4	1.9	0.4	0.7	5.2	0.6
compensated error [%]	0.6	0.3	0.3	0.4	0.6	0.3	0.2	2.7	1.7	0.5	0.3	1.9	0.4	0.6	0.6	0.2
relative efficiency	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.07	1.06	1.27	1.00	0.95	0.98	1.00	0.99	0.99
center slice	0.1	0.3	< 0.1	0.2	0.1	0.2	0.1	1.5	1.2	1478	0.5	0.9	0.3	0.8	8.2	1.2
compensated error [%]	0.1	0.3	< 0.1	0.2	0.1	0.2	0.1	1.5	1.2	7.0	< 0.1	0.9	0.3	0.4	0.2	0.2
relative efficiency	0.89	0.89	0.93	0.89	1.18	1.28	1.13	0.50	0.53	0.10	0.85	1.26	1.04	0.59	0.60	0.79
off-center slice at 1/3 cyl. radius	0.6	0.5	0.3	0.6	1.2	0.4	0.3	13.2	3.8	351	0.7	5.4	0.7	0.6	15.2	0.6
raw error [%]	0.1	0.1	0.3	0.6	0.3	0.1	0.1	1.4	0.7	0.1	0.4	0.2	0.1	0.1	0.7	0.1
compensated error [%]																
relative efficiency	1.40	0.59	0.85	0.78	1.60	2.61	1.18	-0.30	-0.53	1.30	-0.40	0.34	-0.03	-2.18	-1.71	-1.33
off-center slice at 2/3 cyl. radius	14.2	2.3	1.0	1.2	16.6	7.8	0.8	34.0	19.8	92.3	3.3	14.2	4.4	11.4	8.5	3.3
raw error [%]	14.2	2.3	1.0	1.2	16.6	7.8	0.7	34.0	19.8	92.3	3.3	14.2	4.4	11.4	8.5	1.7
compensated error [%]																

¹ Reversed polarity of the desired field shape is reflected as negative efficiency.

PERFORMANCE ANALYSIS MULTI-COIL

TABLE 2

Performance analysis of MC magnetic field modeling for the generation of first through fourth order SH field shapes in spherical volumes at 1/3 and 1/2 of the MC cylinder radius and in selected, circular center and off-center axial slices. The field efficiencies for all SH shapes were calculated for a centered spherical ROI with a radius of 1/3 of the cylinder radius and the efficiencies of all other ROIs were expressed relative to them. The efficiency ratio MC/SH relates the efficiency of the MC approach to the reference efficiency of the corresponding conventional SH coil for a centered spherical volume of 1/3 of the cylinder radius similar to Table 1. The relative efficiency MC vs. SH coil for a specific ROI can be obtained by division of the relative efficiencies, e.g. $2.28/1.00 = 2.28$ for the generation of Z2 in a center slice. MC field modeling provided accurate shapes in variable radius centered spheres, but also in ROIs with significant off-center positions. Note that a 1% mean error could not be reached for the generation of an X4 shape over a centered spherical ROI of half the cylinder radius and that the reported efficiencies refer to a 2% error. Although MC is capable of providing magnetic fields over large ROIs, significant efficiency gains can be realized when ROIs can be limited to smaller volumes such as individual slices.

	X	Z	X2-Y2	ZX	Z2	X3	Z(X2-Y2)	Z2X	Z3	X4	ZC3	Z2C2	Z3X	Z4
SH order N / term M	1 / 1	1 / 0	2 / 2	2 / 1	2 / 0	3 / 3	3 / 2	3 / 1	3 / 0	4 / 4	4 / 3	4 / 2	4 / 1	4 / 0
total wire length [mm]	1583	1583	1583	1583	1583	1583	1583	1583	1583	1583	1583	1583	1583	1583
center sphere, radius 1/3 of cyl.														
efficiency [Hz·cm ⁻ⁿ ·A ⁻¹ ·m ⁻¹]	1952	2049	397	820	634	50.0	101	691	241	7.73	1.74	37.8	34	229
efficiency ratio MC/SH _{ref}	0.58	0.25	0.58	1.03	0.29	0.30	0.18	2.50	0.17	1.09	0.17	0.59	0.12	0.94
center sphere, radius 1/2 of cyl.														
relative efficiency	0.90	0.83	0.82	1.05	0.67	0.71	0.45	0.19	0.90	0.76*	0.15	1.38	0.75	1.00
efficiency ratio MC/SH _{ref}	0.52	0.21	0.47	1.08	0.19	0.22	0.08	0.46	0.15	0.83*	0.03	0.82	0.09	0.94
center slice														
relative efficiency	1.46	0.97	3.35	0.92	7.89	3.54	0.95	1.17	1.07	2.72	3.64	2.00	0.57	1.12
efficiency ratio MC/SH _{ref}	0.84	0.24	1.94	0.95	2.28	1.07	0.17	2.92	0.18	2.96	0.62	1.19	0.07	1.05
off-center slice at 1/3 cyl. radius														
relative efficiency	1.66	3.68	2.00	2.36	2.69	1.12	2.93	0.40	2.31	0.97	7.59	3.73	2.03	1.98
efficiency ratio MC/SH _{ref}	0.96	0.93	1.16	2.44	0.78	0.34	0.52	1.01	0.38	1.06	1.28	2.21	0.25	1.85
off-center slice at 2/3 cyl. radius														
relative efficiency	1.73	2.48	3.42	1.05	1.70	3.43	2.73	0.38	2.95	2.57	14.59	0.80	2.49	2.17
efficiency ratio MC/SH _{ref}	0.99	0.63	1.98	1.09	0.49	1.03	0.48	0.94	0.49	2.80	2.46	0.48	0.30	2.03