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Stochastic Image Registration with User Constraints

Ivan Kolesov^{a,b,1}, Jehoon Lee^b, Patricio Vela^a, and Allen Tannenbaum^b

^aGeorgia Institute of Technology, Atlanta, GA

^bComprehensive Cancer Center/ECE, UAB, AL

Abstract

Constrained registration is an active area of research and is the focus of this work. This note describes a non-rigid image registration framework for incorporating landmark constraints. Points that must remain stationary are selected, the user chooses the spatial extent of the inputs, and an automatic step computes the deformable registration, respecting the constraints. Parametrization of the deformation field is by an additive composition of a similarity transformation and a set of Gaussian radial basis functions. The bases' centers, variances, and weights are determined with a global optimization approach that is introduced. This approach is based on the particle filter for performing constrained optimization; it explores a series of states defining a deformation field that is physically meaningful (i.e., invertible) and prevents chosen points from moving. Results on synthetic two dimensional images are presented.

Keywords

Non-rigid; registration; constraint; user; particle filter; implicit regularization; stochastic optimization

1. INTRODUCTION

Image registration is the task of computing a transformation that results in the best alignment of two images. A number of non-rigid registration approaches have been proposed, and a detailed literature overview can be found in Ref. 1–3. These approaches vary in the similarity metric used and the chosen optimization strategy. Further, the representation chosen for the deformation field naturally divides registration techniques into two classes: parametric models and physical models. Algorithms falling into the first category use basis functions to approximate the true deformation fields and optimize over the parameters in this representation. In the category of parametric models, Rueckert *et al.*⁴ proposed placing anchor points for B-splines uniformly within the image domain. Optimization on the B-spline parameters is done via gradient descent from coarse to fine resolution. Regularity on the deformation field is imposed by penalizing the smoothness of the deformation field globally. Physical models impose properties on the deformation field according to the type of deformation that is expected to be seen. A number of physical models have been proposed for image registration. For example, Avants *et al.*⁵ introduced

¹Send correspondence to ivan.kolesov@gatech.edu; telephone 1 678-561-4826.

the symmetric image normalization *SyN* method. The authors formulate the registration problem symmetrically, with respect to the moving and stationary image, and compute the geodesic between the images within the manifold of diffeomorphic mappings.

Regardless if intramodality/multi-modality or intrapatient/interpatient registration is performed, all approaches discussed previously rely on the key assumption that the same image structures are present in both images of interest. However, for intrapatient registration, for example, this assumption can be violated in the case of a tumor, hemorrhage or another foreign object appearing from one image to another. Thus, this image region must be treated differently during registration. In a number of applications including adaptive radiotherapy and traumatic brain injury, it is desirable to constrain certain regions or points in the image to be stationary while the surrounding areas deform non-rigidly to maximize an image similarity metric. Despite a vast literature on general image registration, relatively few methods for constrained deformable registration exist. One example is Ref. 6; the authors propose way to combine image registration with landmark registration. A functional composed of an image similarity part, landmark motions, and a regularizer is minimized using a variational approach. Thus, the constraints are soft and large deformations are expected to present problems because a gradient descent strategy is used, which is susceptible to local minima. In Ref. 7, the authors consider restricting certain regions to experience only a rigid transformation while the rest of the image is allowed to deform non-rigidly. Another example is the work of Haber *et.al*⁸ in which a constraint restricting the compression or the expansion of the deformation field is enforced. Again, a variational approach is used to compute the field. By contrast, in this work, hard constraints are placed on the landmark points' motion, and the optimization is performed with a stochastic approach allowing large deformations to be captured while satisfying the constraints. In Section 4, we test the *SyN* algorithm, display the unconstrained registration results, contrast it to the proposed, constrained approach from this note.

This paper is organized as follows. Section 2 provides the background necessary for Section 3. Section 3 presents the constrained stochastic image registration (*CSIR*) algorithm. Performance of the algorithm on 2D synthetic images is demonstrated in Section 4, and Section 5 summarizes the results and concludes.

2. REGISTRATION APPROACH

2.1 Problem Statement

Given a moving image $I_D(x)$ and a target image $I_M(x)$, registration is the task of finding a mapping between the two images with the appropriate properties. Typically, $x \in \mathbb{R}^l$ where $l = 2$ or 3 , since we consider 2D images or 3D volumes. The objective is to find a mapping $\mathcal{L}: \mathbb{R}^l \rightarrow \mathbb{R}^l$ that minimizes the distance, d , between the images I_M and I_D :

$$\min_{\mathcal{L}} d(I_M, I_D(\mathcal{L})). \quad (1)$$

We experiment with the L_2 or integrated square error (ISE),

$$d(I_M, I_D(\mathcal{L})) = \int_{\Omega} (I_M(\vec{x}) - I_D(\mathcal{L}(\vec{x})))^2 d\Omega, \quad (2)$$

to generate the results presented in Section 4; this similarity metric is appropriate when I_M , I_D were taken by the same image sensor and the same objects have similar intensities in both images.

2.2 Landmark Constraints

In addition to minimizing the similarity measure in Eq. 2, we require the deformation field to respect several constraints. First, the mapping must be bijective. The constraint

$$\det(J(\mathcal{L}(\vec{x}))) > 0, \forall \vec{x} \in S, \quad (3)$$

will ensure an orientation preserving and one-to-one transformation.⁹ The function $\det(\cdot)$ is the determinant, and $S \subset \mathbb{R}^l$ is an open subset containing the region of interest. Further, landmark constraints that prevent chosen points $x_1^{\vec{}} \dots, x_C^{\vec{}}$ from moving must also hold. Explicitly, constraints that prevent chosen points $x_1^{\vec{}} \dots, x_C^{\vec{}}$ from moving are stated as $\mathcal{L}(x_i^{\vec{}}) = x_i^{\vec{}} \forall i \in [1, \dots, C]$. Thus, the optimization problem becomes

$$\min_{\Theta} E(\mathcal{L}(\vec{x}; \Theta)) \quad (4)$$

$$\begin{aligned} s.t. \quad & \mathcal{L}(\vec{x}_1; \Theta) = \vec{x}_1 \\ & \vdots \\ & \mathcal{L}(\vec{x}_C; \Theta) = \vec{x}_C \end{aligned} \quad (5)$$

$$dJ(\vec{x}) = \det(J(\mathcal{L}(\vec{x}; \Theta))) > 0. \quad (6)$$

where $-(x; \Theta)$ is a parametrization of the deformation field, as discussed in Section 2.3, and $E(-) = d(I_M, I_D(\mathcal{L}))$ is one possible instantiation.

2.3 Parameterizing the Displacement Function

The space of non-rigid transformations is infinite-dimensional, which complicates the optimization of Eq. (1). To make the optimization tractable, we limit the degrees of freedom of the deformation field $\mathcal{L}$ by defining it to be an additive composition of three deformations. The proposed parametrization for $\mathcal{L}(\cdot)$ is :

$$\mathcal{L}(\vec{x}; \Theta) = \mathcal{L}_s(\vec{x}; \Theta_s) + \mathcal{L}_g(\vec{x}; \Theta_g) + \mathcal{L}_l(\vec{x}; \Theta_l) = \underbrace{A\vec{x} + \vec{b}}_{\text{similarity}} + \underbrace{\sum_{i=1}^N \vec{w}_i \mathcal{N}(\vec{x} | \vec{\mu}_i, \sigma_i^2 I)}_{\text{global}} + \underbrace{\sum_{j=1}^M \vec{w}_j \mathcal{N}(\vec{x} | \vec{\mu}_j, \beta^2 I)}_{\text{local}}. \quad (7)$$

One way to enforce the property that $-\mathbf{s}, -\mathbf{g}, -\mathbf{l}$ correct for increasingly finer misalignments is to perform a sequential registration starting from global, moving to local and composing the computed deformations to obtain the overall deformation:

$$\hat{\mathcal{L}}(\vec{x};\Theta)=\hat{\mathcal{L}}(\vec{x};\Theta_s,\Theta_g,\Theta_l)=(\mathcal{L}_l\circ\mathcal{L}_g\circ\mathcal{L}_s)(\vec{x}). \quad (8)$$

To enforce the landmark constraints, we change the family of functions in which the deformation must lie. This change is made by adding Gaussian functions of width σ_C^2 centered at the points $x_1, \dots, x_C$ to the original parametrization of $\hat{\mathcal{L}}$:

$$\mathcal{L}(\vec{x},\Theta)=\hat{\mathcal{L}}(\vec{x},\Theta)+\sum_{i=1}^C\vec{w}_{C_i}\mathcal{N}(\vec{x}|\vec{x}_i,\sigma_C^2I)\text{where}\vec{w}_{C_i}\text{satisfy} \quad (9)$$

$$\begin{aligned} \sum_{i=1}^C\vec{w}_{C_i}\mathcal{N}(\vec{x}_1|\vec{x}_1,\sigma_C^2I) &= \vec{x}_1 - \hat{\mathcal{L}}(\vec{x}_1;\Theta) \\ &\vdots \\ \sum_{i=1}^C\vec{w}_{C_i}\mathcal{N}(\vec{x}_C|\vec{x}_C,\sigma_C^2I) &= \vec{x}_C - \hat{\mathcal{L}}(\vec{x}_C;\Theta). \end{aligned} \quad (10)$$

The weights for these new basis functions w_{C_i} depend on the parameters Θ and satisfy $\mathcal{L}(x_i;\Theta) = x_i$ for $i \in [1, C]$. When $\mathcal{L}$ is equal to the identity, $w_{C_i} = 0$. As $\mathcal{L}$ changes from the identity at the constraint points, weights w_{C_i} are adjusted to compensate for this displacement.

The optimization is performed sequentially as follows:

$$\hat{\Theta}_s = \min_{\Theta_s} R(\Theta_s | \Theta_g, \Theta_l) \quad (11)$$

$$\hat{\Theta}_g = \min_{\Theta_g} R(\Theta_g | \Theta_s, \Theta_l) \quad (12)$$

$$\hat{\Theta}_l = \min_{\Theta_l} E(\Theta_l | \hat{\Theta}_s, \hat{\Theta}_g) + \lambda E_r(\mathcal{L}(\vec{x}; \Theta_l, \hat{\Theta}_s, \hat{\Theta}_g)) \text{s.t. } d_J(\vec{x}) > 0 \quad (13)$$

where $R(\cdot)$ is defined as

$$R(\Theta) = \begin{cases} E(\mathcal{L}(\vec{x}; \Theta)) & \text{if } r(\cdot) > 0 \\ B \cdot (\|I_M(\vec{x})\|_{L^2}^2 + \|I_D(\vec{x}; \mathcal{L}(\vec{x}; \Theta))\|_{L^2}^2) & \text{if } r(\cdot) \leq 0 \end{cases} \quad (14)$$

where $r(\Theta) = \det(J(\mathcal{L}(x; \Theta)))$.

The cost function $R(\Theta)$, in Eq. (14), imposes a constraint on the injectivity of $\mathcal{L}$. This constraint holds exactly as $B \rightarrow \infty$: the particle filter (PF) is used for optimization¹⁰ and if $r(x_n) \rightarrow 0$, then, $p(y_n|x_n) \approx 0$, which leads to the rejection of candidate particles resulting in non-physical deformation fields by setting their likelihood to zero. Also, this choice of $R(\Theta)$ ensures that the optimal deformation will satisfy the landmark constraints in Eq. (5) by

evaluating the likelihood of Θ using the deformation field $\mathcal{D}(x; \Theta)$. In Eq. (13), $E_r(\cdot)$ is a regularizing term weighted by λ . For instance, in 2D, this term is

$$E_r(\mathcal{L}) = \int_x \int_y \left[\left(\frac{\partial^2 \mathcal{L}}{\partial x^2} \right)^2 + 2 \left(\frac{\partial^2 \mathcal{L}}{\partial x \partial y} \right)^2 + \left(\frac{\partial^2 \mathcal{L}}{\partial y^2} \right)^2 \right] dx dy. \quad (15)$$

Here, too, a clear, hierarchical alignment is seen: starting from a trivial initialization of Θ_g , Θ_l , an optimal Θ_s is computed in Eq. (11), then holding Θ_s , Θ_l constant, the best Θ_g is found in Eq. (12), and finally, optimization over Θ_l is performed in Eq. (13). The difference between the proposed method and existing multi-resolution approaches is that the optimization in addition to the parametrization changes at each registration level: Eq. (11) and Eq. (12) will be solved stochastically to capture large misalignments, and for Eq. (13), a gradient descent approach is used because more degrees of freedom are in the representation of $\mathcal{D}_l$, which is necessary to capture finer registration misalignments. The deterministic registration allows more bases to be used, which would make a stochastic method too computationally intensive. Solutions to Eq. (11)–(13) are discussed in Sections 3.

3. OPTIMIZATION ALGORITHM

3.1 Stochastic Optimization of $\mathcal{L}_s$

A stochastic method for finding the similarity registration parameters is discussed in this section. In *CSIR*, optimization of the function $R(\Theta)$ is performed using a particle filter, as described in Ref. 10. The parameter vector for a similarity transformation is composed of quaternion coordinates $[w, x, y, z]$ for rotation and scale and a translation vector $[a, b, c]$:

$$\Theta_s = [w, x, y, z, a, b, c]. \quad (16)$$

The low dimensionality of Θ_s makes it possible to solve Eq. (11) using the PF method, directly. This approach allows large translations, scale changes, and rotations to be captured because the algorithm can escape local minima, which present problems for a gradient descent technique.

3.2 Stochastic Optimization of $\mathcal{L}_g$

Even after aligning the images according to Section 3.1, gross misalignments can be present between the two images. Hence, the next step is to solve Eq. (12) by computing a global, non-linear, deformation, represented by Θ_g :

$$\begin{aligned} \Theta_g &= [\vec{\mu}_1, \vec{w}_1, \sigma_1, \dots, \vec{\mu}_N, \vec{w}_N, \sigma_N] \\ \vec{\mu}_i &= [\mu_{xi}, \mu_{yi}, \mu_{zi}] \text{ and} \\ \vec{w}_i &= [w_{xi}, w_{yi}, w_{zi}]. \end{aligned} \quad (17)$$

A stochastic approach is preferred to robustly capture large deformations. This is done by grouping the elements of the parameter vector into zones as

$$\Theta_g = [\vec{\theta}_1, \dots, \vec{\theta}_N] \text{ where } \vec{\theta}_i = [\vec{\mu}_i, \vec{w}_i, \sigma_i] \text{ for } i \in [1, N] \quad (18)$$

and Eq. (12) is optimized with respect to a single zone $\vec{\theta}_i$ at a time:

$$\vec{\theta}_i^* = \underset{\vec{\theta}_i}{\operatorname{argmin}} R(\vec{\theta}_i | \Theta_{-i}, \hat{\Theta}_s, \Theta_l), \quad (19)$$

where $\Theta_g = [\Theta_{-i}, \vec{\theta}_i]$. The optimization in Eq. (19) is accomplished over the lower dimensional space with the PF, as per Ref. 10. The overall optimization strategy is shown in Fig. 1.

Each particle in the PF corresponds to a value of Θ . Given a value of Θ , the w_i for $i \in [1, \dots, C]$ can be computed such that the landmarks constraints are satisfied by solving the system of linear equations in Eq. 10. Then, the likelihood of $\mathcal{L}(\cdot)$ defined by Θ can be evaluated.

3.3 Gradient Descent Optimization of $\mathcal{L}_l$

A larger number of bases is required to capture fine misalignments and stochastic optimization becomes computationally intensive with negligible benefit compared to a gradient descent approach since it is assumed that rough alignment has been achieved after the *similarity* and *global* registration phases have been completed. Hence, in this last level of registration, we use a larger number of Gaussian bases in the representation of $\mathcal{L}$ and find the optimal parameters with a deterministic approach. The bases are distributed according to a uniform grid covering the entire domain, a common standard deviation β is set by the user, and the optimal weights w_j are computed.

Whereas the registration steps in Section 3.1–3.1 only required an evaluation of the cost function $R(\Theta)$, in the last step, a descent direction must be established. We use the gradient projection approach¹¹ to ensure that the constraints are satisfied throughout the optimization. Starting from a feasible setting for Θ , a new feasible point (with a lower value for $E(\mathcal{L}; \vec{\Theta})$) is obtained by computing the gradient direction with respect Θ of the expression in Eq. 13 and projecting it onto the set of constraints.

4. RESULTS

Evaluation of the proposed approach is performed first using the synthetic example in Fig. 2, which aims to simulate a scenario from oncological imaging data. There are two sources for the image differences: the first is a deformation that existing tissue experiences causing the misalignment in the bottom left of the image and the second is the appearance of a structure previously not present in the image, an appendage in the top right corner. These differences must be treated separately. Since the appendage shown in Fig. 2(b) simulates a previously non-existent tissue, there is no corresponding image data in Fig. 2(a) that is a true match. Thus, using the SyN⁵ approach, with no constraints, the registration algorithm attempts to incorrectly match the images, as shown in Fig. 3. Two images are overlaid and displayed

according to a checker-board pattern to show initial misalignment and final registration results.

Now, we demonstrate the effect of incorporating landmark constraints, as described in Sec. 2.2. A set of six points are selected in Fig. 4(a) and these points are constrained to remain stationary. Intentionally, for illustration, the effect of each input is chosen to be local; by selecting more points or a larger area of influence, the appendage in the top, right corner of the image can be further constrained. As seen in Fig. 4(b), the deformation in the bottom, left of the image is computed, and the fixed points remain in place.

5. CONCLUSION

This work presents a stochastic image registration methodology that enforces hard landmark constraints while performing intensity based, deformable registration. Additionally, a gradient descent step is performed to capture fine misalignments while respecting the landmark constraints. Results are shown for a phantom image and demonstrate the ability of the algorithm to capture large deformations. Future work includes incorporating new constraints based on image regions into the proposed framework.

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REFERENCES

1. Zitova B, Flusser J. Image registration methods, a survey. *Image and Vision Computing*. 2003; 21:977–1000.
2. Pluim JPW, Maintz JBA, Viergever MA. Mutual-information-based registration of medical images: a survey. *Medical Imaging, IEEE Transactions on*. 2003 Jul.22:986–1004.
3. Holden M. A Review of Geometric Transformations for Nonrigid Body Registration. *Medical Imaging, IEEE Transactions on*. 2007 Dec.27:111–128.
4. Rueckert D, Sonoda L, Hayes C, Hill D, Leach M, Hawkes D. Nonrigid registration using free-form deformations: Application to breast mr images. *IEEE Transactions on Medical Imaging*. 1999; 18:712–721. [PubMed: 10534053]
5. Avants B, Duda J, Kim J, Zhang H, Pluta J, Gee J, Whyte J. Multivariate analysis of structural and diffusion imaging in traumatic brain injury. *Academic Radiology*. 2008; 15(11):1360–1375. [PubMed: 18995188]
6. Fischer B, Modersitzki J. Combining landmark and intensity driven registrations. *Proceedings in Applied Mathematics and Mechanics*. 2003; 3(1):32–35.
7. Loeckx, D.; Maes, F.; Vandermeulen, D.; Suetens, P. International Conference on Medical Image Computing and Computer Assisted Intervention (MICCAI), Proceedings of the], Lecture Notes in Computer Science. Vol. 78. Springer Berlin: Heidelberg; 2004. Nonrigid Image Registration Using Free-Form Deformations with a Local Rigidity Constraint; p. 639-646.3216 ch.
8. Haber E, Horesh R, Modersitzki J. Numerical optimization for constrained image registration. *Numerical Linear Algebra with Applications*. 2010; 17(2–3):343–359.
9. Meisters G, Olech C. Locally one-to-one mappings and a classical theorem on schlicht functions. *Duke Mathematics Journal*. 1963; 30:63–80.

10. Kolesov, I.; Lee, J.; Vela, P.; Tannenbaum, A. Image Processing: Algorithms and Systems XI. San Francisco: SPIE Electronic Imaging; 2013 Feb. A stochastic approach for non-rigid image registration.
11. Rosen J. The gradient projection method for nonlinear programming. part ii. nonlinear constraints. Journal of the Society for Industrial and Applied Mathematics. 1961; 9(4):514–532.

Figure 1.
The CSIR registration algorithm described in Section 3.2.

(a) Target Image.

(b) Moving Image.

Figure 2.

This is a test example that simulates a tumor (appendage in top right corner of Fig. 2(b)) appearing some time between two scans of a patient skull. Additionally, there is misalignment in the bottom left corner of Fig. 2(b) compared to Fig. 2(a).

Figure 3.

Registration results using the SyN^5 algorithm on the synthetic image pair from Fig. 2.

Without any constraints, the appendage in the upper right is artificially warped to fit the model data despite not having any correspondence.

Figure 4.

Registration results for the *CSIR* algorithm proposed in this note. In Fig. 4(a), the initial misalignment and landmark points, which are constrained to remain stationary, are shown in red. The registration result using landmark constraints is in Fig. 4(b). Although the constrained points do not move, there is deformation of the appendage around them. The effect of each user constraint is intentionally made local and the number of constraints is few to demonstrate the effect on the deformation field.