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Three-dimensional Fourier Encoding of Simultaneously Excited Slices: Generalized Acquisition and Reconstruction Framework

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Abstract

Purpose—Simultaneous multi-slice (SMS) acquisitions have recently received much attention as a means of increasing single-shot imaging speed. SMS acquisitions combine the advantages of single-shot sampling and acceleration along the slice dimension which was previously limited to 3D volumetric acquisitions. A 2D description of SMS sampling and reconstruction has become established in the literature. Here, we present a more general 3D Fourier encoding and reconstruction formalism for SMS acquisitions that can easily be applied to non-Cartesian SMS acquisitions.

Theory and Methods—A “SMS 3D” k-space is defined in which the field of view along the slice select direction is equal to the number of excited slices times their separation. In this picture, SMS acceleration can be viewed as an under sampling of SMS 3D k-space that can be freely distributed between the in-plane and slice directions, as both are effective phase encoding directions.

Results—Use of the SMS 3D k-space picture is demonstrated in phantom and *in-vivo* brain acquisitions including data obtained with blipped-CAIPI sampling. SMS SENSE reconstruction is demonstrated as well as non-Cartesian SMS imaging using blipped spiral trajectories.

Conclusion—The full framework of reconstruction methods can be applied to SMS acquisitions by employing a 3D k-space approach. The blipped-CAIPI method can be viewed as a special case of undersampling an SMS 3D k-space. The extension of SMS methods to non-Cartesian 3D sampling and reconstruction is straightforward.

Introduction

Single-shot two-dimensional Echo Planar Imaging (2D EPI) (1) is widely used in functional MRI and diffusion-weighted imaging (DWI). With EPI, an imaging volume is covered by sequentially exciting and acquiring 2D slices. Reducing the resulting volume acquisition times (TR) of typically a few seconds is of great interest in fMRI since it can increase the statistical power (2) and allows better separation of physiological noise components in the time series (3). Single-shot imaging can also reduce motion sensitivity. One common means to increase acquisition speed is with parallel imaging (4–6). Parallel imaging techniques

allow omitting k-space samples by exploiting the encoding properties of spatial sensitivity variations in multi-coil arrays. In single-shot 2D sequences such as EPI the use of parallel imaging can shorten the readout train, which reduces susceptibility artifacts and shortens the minimum echo time (TE) (7–9). However, parallel imaging allows only marginal reductions in the effective volume TR, as this is largely determined by the minimum TE needed for fMRI contrast or by the duration of diffusion gradients.

Simultaneous multi-slice (SMS) acquisitions have recently received much attention for increasing imaging speed in single-shot EPI. SMS methods combine the advantages of single-shot k-space acquisition with acceleration along two independent encoding dimensions. In its basic implementation, SMS imaging is typically achieved by simultaneous excitation of multiple slices using multi-band RF pulses followed by 2D gradient encoding (10, 11). The slice images are aliased and their separation can be achieved during image reconstruction by exploiting the spatial encoding inherent to coil arrays with sensitivity variation along the slice direction using SENSE (5) or GRAPPA (6) techniques as originally proposed by Larkman *et al.* (10). Following the advances in receiver coil design, a series of studies recently demonstrated the potential of SMS for single-shot acquisitions with EPI (11, 12) or spiral readouts (13).

Slice separation in SMS imaging can be improved by extending the concepts of the CAIPIRINHA (Controlled Aliasing In Parallel Imaging Results In Higher Acceleration) technique (14) to single-shot trajectories. This was first demonstrated for EPI by Nunes *et al.* (15) who applied additional phase encoding blips along the slice direction during an otherwise unchanged EPI readout train. The additional slice dependent phase accumulated during the signal readout produces a shift, along the in-plane phase encoding direction, of the simultaneous slice images with respect to each other. While this method alone is not sufficient to disentangle the aliased slices without receiver coil sensitivity information, it distributes the slice aliasing more evenly in image space, analogous to the CAIPIRINHA method for multi-slice and volumetric sampling (14, 16). This ultimately reduces the g-factor penalty in a subsequent parallel imaging reconstruction, and allows a greater number of simultaneously acquired slices.

Setsompop *et al.* introduced the “blipped-CAIPI” technique (17) to overcome the inevitable signal dephasing due to strong through-slice gradients in the approach of Nunes *et al.* The slice shifts in blipped-CAIPI are achieved by small, alternating z-gradient blips during the EPI readout train, which only cause minor dephasing. These blips impose a slice position dependent phase difference for consecutive k-space lines, which again causes off-center slices to be shifted along the phase encoding direction of the reconstructed images. Blip moments are customized for the number and separation of simultaneously acquired slices. This approach has significantly improved SMS image quality.

Although the 2D framework of overlapping slices provides an accurate description of Cartesian SMS imaging, the introduction of z-gradient blips suggests that a description using a 3D encoding space may be beneficial. The utility of a 3D framework was first noted in previous work (18) that showed that the Fourier encoding nature of the multiband RF excitation could be used to describe SMS imaging as a form of 3D encoding. A 3D

framework treats the blips along z as phase encoding gradients equivalent to the second phase encoding dimension as in commonly used 3D volumetric acquisitions. Adapting a 3D framework will allow for a more cohesive description of all SMS imaging modalities, including those using RF phase modulation, such as Phase Offset Multi-Planar (POMP) imaging (19) and CAIPIRINHA, single-shot blipped-CAIPI by z -gradient modulation, as well as non-Cartesian sampling such as spiral. Of note, under sampling of non-Cartesian trajectories generally does not result in coherent aliasing artifacts and spatial shifts in image space and cannot be easily described using a 2D framework.

We propose a generalized 3D Fourier encoding and reconstruction formalism for SMS acquisitions and reconstructions. The conventional definition of the field-of-view (FOV) in the slice direction is modified to reflect an imaging geometry that consists of N slices of thickness d equidistantly spaced with separation Δz . A “SMS 3D k-space” can then be defined as the Fourier encoding space for a 3D imaging volume consisting of N contiguous samples along z with a z -resolution of Δz symmetrically placed around the excited slices. By employing the SMS 3D k-space, the existing 3D encoding and reconstruction methods can be applied to SMS acquisitions. We begin by showing how a set of N equidistant spaced slices can be separated using a fully sampled SMS 3D k-space without any coil sensitivity information. Although this has no acceleration advantage over single-slice methods, it serves as an important reference from which SMS undersampling can be accurately defined. We then show how blipped-CAIPI schemes can be represented as an undersampled SMS 3Dk-space, requiring coil sensitivity information for reconstruction. We also show that SMS CAIPIRINHA encoding using phase modulated multiband RF excitation can be incorporated in the SMS 3D k-space framework. Finally, it is demonstrated that generalized 3D SENSE (20) can be used to reconstruct blipped-CAIPI-EPI data and that non-Cartesian “blipped spiral” sampling can be implemented for a fast single shot SMS acquisition.

Theory

Nyquist k-space sampling of simultaneously excited slices

In this section we will define the model of a SMS 3D k-space including the concept of a “fully sampled” SMS acquisition. For image reconstruction in MRI, Nyquist sampling on a Cartesian grid is defined by the k-space increment Δk , the total number of k-space samples N , and the maximum k-space value $k_{\max}$:

$$\Delta k = \frac{1}{\text{FOV}} \quad \text{with} \quad \text{FOV} = Nd \quad \text{and} \quad k_{\max} = \Delta k \frac{N}{2} \quad [1]$$

The FOV is given as the number of samples N times the resolution d . N linearly independent complex k-space samples along one direction are needed to reconstruct a complex image with N pixels along that dimension. For the purpose of this work we restrict our analysis to the case of one-dimensional k-space encoding in the z -direction (orthogonal to the slice plane). A closer look at the basic sampling theorem underlying Eq. [1], see Ref. (21) for example, reveals that it is derived from a more general expression, which states that, given a finite continuous spin density in image space, it follows that a finite set of N discrete samples of the spin density can be reconstructed from a set of N discrete samples in Fourier

space. The discretization of the spin density is described by a comb sampling function

$U(z) = \sum_{q=N/2}^{\frac{N}{2}-1} \delta(z - q\Delta z)$ which is a sum of N δ -functions separated by Δz . The two sets of discrete samples in image and Fourier space are related via a discrete Fourier transform pair with the k-space increment Δk_z , the slice separation Δz , the number of samples N , and the field-of-view mutually connected by:

$$\Delta k_z \Delta z = \frac{1}{N} \text{ and } \text{FOV}_z = N \Delta z \quad [2]$$

Applied to the case of simultaneous multiple slices it follows that the “SMS Field of View” FOV_z is given by the number of slices N times the center-to-center distance, the “SMS z-resolution,” Δz between neighboring slices. The k-space increment has to be set accordingly:

$$\text{FOV}_z = N \Delta z \text{ and } \Delta k_z = \frac{1}{\text{FOV}_z} \text{ and } k_z^{\max} = \Delta k_z \frac{N}{2} \quad [3]$$

By using a k-space sampling pattern that fulfills Eq. [1] for contiguous slices or Eq. [3] for separated slices, it is assured that a discrete representation of the underlying physical spin density can be reconstructed by applying an inverse discrete Fourier transform to the discrete k-space samples. A SMS 3D k-space is then composed of the conventional planar 2D k-space in the x- and y-directions and the SMS k-space in the slice direction. A fully sampled SMS 3D k-space would then consist of N fully sampled k_x - k_y planes at N different k_z -coordinates given by Eq. [3]. Furthermore, with an SMS 3D k-space defined, the extension to fully sampled and accelerated non-Cartesian SMS acquisitions with encoding in all three dimensions is straightforward.

Based on this definition of a fully sampled SMS 3D k-space, one can define the degree of undersampling that is required for acceleration as in commonly described SMS imaging. As customary in parallel acceleration, the undersampling factor R is given by the ratio of total number of samples in the fully sampled SMS 3D k-space to the number of actually acquired samples. Undersampling can be freely distributed in any dimension taking into account the sensitivity profile of the receive coil array used for signal reception. Consequently, undersampled encoding strategies will ultimately suffer from a $1/R$ SNR loss plus additional g-factor losses compared to a fully sampled SMS 3D k-space. Compared to fully encoded, single slice acquisitions, however, the $1/R$ SNR loss due to undersampling may be partially or completely compensated by the N SNR gain due to the SMS excitation.

Figure 1 gives a pictorial interpretation of the SMS 3D encoding concept as defined by Eq. [3] and illustrates its relationship to conventional volumetric 3D encoding. The example is for $N = 4$ simultaneously excited slices with slice separation $\Delta z = 25$ mm and slice thickness $d = 2$ mm, displayed as yz-projections in Fig. 1b and xy-slices in Fig. 1c. These slices were synthesized from a volumetric FLASH image acquired using a multiband RF pulse instead of a slab-select pulse. The scan details can be found below in the Methods section. A fully sampled 3D encoded scan with a FOV_{ZVOL} of 200 mm at 1 mm spatial resolution in the slice direction was acquired. The corresponding k-space increment for the volumetric 3D k-space is therefore given as $\Delta k_{\text{ZVOL}} = 5 \text{ m}^{-1}$.

From Eq. [3] it follows that $k_{ZSMS} = 10 \text{ m}^{-1}$ and $N = 4$ k-space samples in the slice direction are necessary for fully sampled SMS 3D encoding. This SMS 3D k-space corresponds to $N = 4$ samples along the slice select direction with a z-resolution of 25 mm symmetrically positioned around the excited slices. The z-resolution of the SMS displayed by the red dotted lines in Fig. 1b. The volumetric scan geometry was chosen so that the SMS 3D k-space increment in the slice direction was an integer multiple of the volumetric 3D k-space. The circles in Fig. 1d show a pictorial representation of the complete initial volumetric 3D k-space in the k_y - k_z plane, with the samples required for SMS 3D k-space highlighted in red. The SMS 3D k-space is shown in Fig. 1e and was obtained by taking the subset of red circles in Fig. 1d.

Reconstruction of the fully sampled SMS 3D k-space using a 3D FFT results in $N = 4$ slices displayed in Fig. 1f. These are virtually identical to the result in Fig. 1c. Although the nominal SMS z-resolution is still 25 mm, due to the multiband excitation it is easy to see that only the thinner excited slices contribute to the signal. The slice selectivity of the multiband excitation does not affect the imaging geometry and the SMS 3D k-space.

Blipped-CAIPI and Undersampled SMS 3D Fourier encoding

In this section we show that the blipped-CAIPI technique employed by Setsompop *et al.* represents an undersampled SMS 3D k-space. The aim of blipped-CAIPI is to reduce the g-factor penalty in an SMS single-shot EPI reconstruction that relies on coil sensitivities to separate slices. In this method the simultaneously excited slices are shifted relative to each other, thereby distributing aliasing more evenly in image space. It employs the fact that in Cartesian imaging a shift in image space corresponds to a phase ramp in k-space. Nunes *et al.* recognized that the necessary position dependent phase shifts could be achieved by adding gradient blips in the slice direction during the phase encoding blips in an EPI train. Given N simultaneously excited slices, the shift between neighboring slices can be freely chosen and is given as the fraction $1/L$ of the FOV in the phase encoding direction y :

$$\text{shift} = \frac{\text{FOV}_y}{L} \text{ where } L \leq N \quad [4]$$

A full FOV_y shift corresponds to a phase of 2π . Therefore, the relative phase between slices must be $2\pi/L$. The necessary gradient moment A_{blip} creating this phase difference between slices separated by Δz is given by

$$A_{\text{blip}}(L, \Delta z) = \frac{2\pi}{\gamma \Delta z L} \quad [5]$$

with the gyromagnetic ratio γ . The k-space increment due to the slice blip can be calculated by multiplying A_{blip} by $\frac{\gamma}{2\pi}$

$$\Delta k_z^{\text{blip}} = \frac{\gamma}{2\pi} \frac{2\pi}{\gamma \Delta z L} = \frac{1}{L \Delta z} \quad [6]$$

For $L = N$, it follows that the blipped-CAIPI k-space increment Δk_z^{blip} is equal to the k-space increment Δk_z necessary for full sampling in the slice direction as defined in Eq. [3]. If $L < N$ the acceleration factor along the slice direction can be defined as $R_{\text{slice}} = N/L$. The k-space increment for a blipped-CAIPI acquisition is then $\Delta k_z^{\text{blip}} = R_{\text{slice}} \Delta k_z$ and the slice direction is effectively undersampled. The optimal choice of the relative slice shifts depends on the slice separation and the receive coil's sensitivity variation in the slice direction.

In Figure 2 three blipped-CAIPI acquisitions are transformed to a pictorial SMS 3D k-space description with circles representing fully sampled k-space lines along one dimension (read). The y-axis corresponds to the normal phase encoding direction orthogonal to the read and slice directions. The gray circles represent skipped phase encoding lines. The SMS 3D k-space pattern for $N = 3$ slices with a relative FOV_y shift of $1/(L = 3)$ between neighboring slices is shown in Fig. 2a. A negative pre-blip of the same area as the actual blip shifts the first line to $-k_z$ and the two following positive blip moments are cancelled by a negative blip of twice the blip size bringing the k_z -component back to $-k_z$. The SMS 3D k-space can be regarded as being fully sampled in the z-direction since missing k-space samples can be entirely reconstructed by restoring missing points along k_y .

In Fig. 2b the SMS 3D k-space pattern for $N = 5$ slices with a relative FOV_y shift of $1/(L = 5)$, displayed as blue dots, is compared to a relative FOV_y shift of $1/(L = 3)$, displayed as red dots. The SMS 3D k-space for $L = N = 5$ can be regarded as fully sampled in the slice direction since every k_z -plane contains samples. For $L = 3$ the blip k-space increment Δk_z^{blip} is larger than the SMS 3D k-space increment for full sampling Δk_z , resulting in an effective undersampling of the slice encoding direction by $N/L = 5/3$. In the case where the z-direction is undersampled, the coil sensitivity variation in the slice direction is needed to restore samples in the missing k_z -planes in a subsequent parallel acceleration reconstruction.

An alternative blip pattern for $N = 4$ slices is displayed in Fig. 2c. Instead of one large rewinder blip at the end of the blip train, the k_z -component is decreased stepwise resulting in a zig-zag sampling pattern. The minimal echo spacing time in an EPI train is no longer affected by the large moment of the rewinder blip, reducing the echo spacing by 10–15%. As we will show later, the zig-zag sampling pattern can also have advantageous reconstruction properties ultimately resulting in a reduced g-factor penalty.

Phase correction of SMS signal prior to reconstruction

In the case of gradient encoding along the slice-direction, as in blipped-CAIPI, a phase correction is required if the multiband excitation is shifted relative to the gradient isocenter. The phase correction is identical to the phase shift of the k-space data for off-center slab excitation in the case of 3D Fourier encoding. Figure 3 displays the scenario of two simultaneous slices acquired with two samples along k_z . The blue line represents the phase modulation for the $k = -k_z$ sample that is imposed by the z-blips on the spins prior to signal acquisition. It spans a period of 2π over the entire FOV relative to the magnet's isocenter. Any offset z_{offset} of the simultaneous slices relative to the isocenter adds an additional phase to the acquired signal. This phase depends on z_{offset} and on the k-space position. For Cartesian sampling along the slice direction the effect of this additional phase is a shift in

the image domain. The required phase correction can be obtained using the Fourier shift theorem:

$$\varphi_{\text{offset}}(k_z, z_{\text{offset}}) = e^{-i2\pi k_z z_{\text{offset}}} \quad [7]$$

After the phase correction the reconstruction process is identical to a standard volumetric 3D reconstruction in the isocenter.

Encoding of simultaneously excited slices using phase-modulated RF pulses

The SMS 3D k-space framework can also be applied to describe the use of phase modulated RF pulses as a means of phase encoding in SMS imaging. RF phase modulation for SMS imaging was originally presented in Ref. (19) as a method known as POMP imaging. In this approach, k_z phase encoding is achieved by altering the phase offsets at the individual slice locations by applying the corresponding phase offsets to the frequency bands of a multiband RF pulse. Here, RF phase encoding replaces the phase encoding that is achieved by the z-gradient as in blipped-CAIPI. The k_x - k_y encoding is performed using standard 2D gradient encoding (single-shot or multi-shot).

A multiband RF pulse that excites N slices simultaneously with $M = Nk_z$ phase encoding increments can be written as a sum of N individual RF pulses characterized by a resonance frequency f and phase offset Φ

$$RF_{MB}^m = \sum_{n=1}^N RF(f_n, \Phi_n^m) \quad [8]$$

where $m (= 1 \dots M)$ is the RF phase encoding index and n is the slice index. Full RF phase encoding of N simultaneously excited slices hence requires $M = N$ different pulses with phase RF_{MB}^m modulation

$$\Phi_n^m = 2\pi k_z^m z_n \quad [9]$$

with $k_z^m = -k_z^{max} + m\Delta k_z$ and $z_n = n \cdot z + z_{\text{offset}}$ representing the slice locations relative to the isocenter as a function of the slice separation z plus an arbitrary slice offset z_{offset} . Figure 4 shows an example of phase modulated RF encoding of four SMS slices with subsequent 3D FFT reconstruction. In case of RF-encoding the phase offsets of SMS groups can be precisely defined. Therefore, the phase correction from the previous section (for gradient encoding) is not necessary. Note that the same SMS 3D k-space description can be used for the original CAIPIRINHA method (14) that also used RF phase modulation methods.

Methods

All experiments were performed on a 3T Tim Trio system (Siemens Healthcare, Erlangen Germany) equipped with a 12-channel and a 32-channel head coil array. Data were acquired from four healthy subjects after obtaining informed consent using an institutionally approved protocol.

Simultaneous multi-slice excitation

SMS excitation was performed using a pulse created from the complex summation of N standard windowed sinc pulses with time bandwidth product 3 and 5.12 ms duration. Each sinc pulse was frequency shifted to match the desired slice location f_n (n is the slice number and f_n is measured in Hz) by application of a phase ramp proportional to the slice selection gradient G_{slice} and the relative distance z :

$$f_n = \frac{\gamma}{2\pi} n \Delta z G_{\text{slice}} \quad [10]$$

FLASH sequence with SMS excitation and conventional 3D readout

In order to validate the proposed 3D SMS k-space formalism, we acquired a fully encoded 3D FLASH scan covering four simultaneously excited slices ($z = 25$ mm, slice thickness $d = 2$ mm) with a conventional 3D readout covering the entire volume spanned by the multiple slices. Both the 12-channel and the 32-channel head coils were used for signal reception. The imaging parameters for the 3D FLASH sequence were: FOV = $256 \times 256 \times 200$ mm³, resolution = $128 \times 128 \times 200$, bandwidth = 390 Hz/pixel, TR/TE = 30/7 ms, and flip angle of 15 degrees. From this scan, a fully sampled SMS 3D k-space, an undersampled SMS 3D k-space using a blipped-CAIPI pattern, and an undersampled SMS 3D k-space using a zig-zag pattern were generated by selecting the corresponding k-space samples.

Blipped-CAIPI EPI sequence

In a standard EPI sequence, the slice selective excitation pulse was replaced with a multi-band excitation pulse as described above. Gradient blips in the slice direction with variable blip moments were added to the EPI readout train in order to traverse the 3D SMS k-space. SMS acquisitions with $N = 5$, $d = 3$ mm, and $z = 15$ mm were performed. The imaging parameters for the five simultaneous slices were: FOV = $192 \times 192 \times 75$ mm³, resolution = $96 \times 96 \times 5$, CAIPIRINHA FOV_y shift fraction 1/3 ($L = 3$), no partial Fourier, bandwidth = 2004 Hz/pixel, TR/TE = fully relaxed/35ms, effective echo spacing of 0.69 ms, and flip angle of 90 degrees. Corresponding single slice acquisitions with identical in-plane parameters were performed for reference. The 32-channel head coil array was used for signal reception.

Blipped spiral sequence

Non-Cartesian SMS imaging was also demonstrated using three spiral trajectories. The first spiral trajectory was a standard multi-shot “stack of spirals” shown in Figure 5 at hat consisted of 16 segments with eight segments for each of two k_z planes in order to ensure identical T2* and off-resonance weighting along the k_z -direction. The trajectory was designed for full SMS 3D sampling of two simultaneous slices with slice gap 40 mm: FOV = $224 \times 224 \times 80$ mm³, matrix size = $96 \times 94 \times 2$, TR/TE = 200/8 ms, approximately 10ms per segment. For better visibility only eight segments are plotted in Figure 5a.

The other two spiral trajectories had blips in the slice direction analogous to the blipped-CAIPI EPI trajectory. First, basic planar spirals were designed to make optimal use of the

maximum slew rate (135 T/m/s) by solving the differential equation describing the evolution of the slew rate vector as proposed in Ref. (22). Gradient blip waveforms were then designed by employing a linear programming algorithm for optimal gradient waveforms (23) given a required k-space increment. Once the x- and y-gradient waveforms of the planar spirals were known, a z-gradient blip with the correct moment was played out at each full rotation in the xy-plane while continuing the x- and y-gradients. Fully sampled and two-fold undersampled SMS 3D blipped spiral trajectories were then designed for $N = 2$. For full sampling of two simultaneous slices the initial planar spiral had to be two-fold oversampled (compared to full sampling of a single slice) in order to ensure Nyquist sampling of each k_z -plane. Figure 5b shows the two-fold undersampled trajectory for a target resolution of $64 \times 64 \times 2$ and $\text{FOV} = 224 \times 224 \times 80 \text{ mm}^3$. This trajectory is analogous to a blipped-CAIPI EPI sequence with $L = 2$. The corresponding gradient waveforms are plotted in Figure 5c with the inset figure indicating the exact positions of the blips relative to the spiral gradient waveform. The total sampling time was approximately 40 ms for the fully sampled spiral and 20ms for two-fold undersampling. Data were acquired continuously including during the z-gradient blips.

3D reconstruction of undersampled SMS data

Since the SMS3D imaging framework is entirely derived from standard 3D k-space formulation, existing 3D reconstruction methods that have been developed for conventional imaging geometries can be used without modification. This includes conventional FFT reconstruction for fully sampled acquisitions and 3D GRAPPA and SENSE in case of accelerated parallel imaging. For the purpose of this work, which focuses on theoretical and basic aspects of the SMS reconstruction problem, we used a general reconstruction formulation by solving the inverse problem using the method of conjugate gradients and a nuFFT gridding algorithm (<http://www.eecs.umich.edu/~fessler/irt/irt>), referred to as generalized 3D SENSE (20). Importantly, the nuFFT-based reconstruction adds no additional information to the reconstruction problem and in the case of Cartesian sampling is virtually equivalent to a SENSE type reconstruction where the coil sensitivity maps are explicitly known. It was chosen out of convenience and because of the great flexibility it provides for reconstructing non-Cartesian sampling patterns like spirals. However, for the experiments that did not require parallel imaging techniques we used a conventional 3D FFT reconstruction. A detailed description of the nuFFT reconstruction formulation can be found in Ref. (24) and details of the particular implementation for undersampled non-Cartesian 3D reconstruction problems can be found in Refs. (25) and (26). Image reconstruction was performed off-line for both EPI and spiral data. Coil sensitivity maps were derived from a multi-slice gradient echo prescan covering the entire FOV spanned by the SMS acquisitions (resolution = $2 \times 2 \times 2 \text{ mm}^3$, TE = 2.46 ms, TR = 500 ms, flip angle of 25 degrees).

Results

3D FLASH phantom data

Figure 6a shows the inverse 3D FFT reconstruction of the fully sampled SMS 3D k-space of two simultaneously excited slices (slice gap $z = 25 \text{ mm}$, thickness $d = 3 \text{ mm}$) acquired with the single-channel body coil and the 3D FLASH sequence. Prior to the reconstruction, the

phase correction given by Eq. [7] was applied with $z_{\text{offset}} = \frac{25 \text{ mm}}{2} = 1.25 \text{ mm}$ (for two slices placed symmetrically around isocenter; see Fig. 3). The effect of no phase correction can be seen in Fig. 6b. The interpretation of the phase error leading to a shift along the slice encoding direction is demonstrated by observing that contributions from both slices are present in the resulting image.

The 3D FLASH results for four simultaneously excited slices are displayed in Figure 7. In row a, the sum-of-squares combination of an inverse 3D FFT reconstruction from a fully encoded SMS 3D k-space is shown. Coil sensitivities were then derived from this fully encoded scan by dividing individual coil images by the sum-of-squares combination of all coils. The phase correction for off-center slices was applied prior to reconstruction. The left columns display results for the 12-channel coil and the right columns displays the results from the 32-channel coil. The 12-channel coil has an arrangement of the elements in a single axial ring, hence providing less sensitivity variation along z compared to the 32-channel coil. Row b shows the reconstruction of an $L = 4$ blipped-CAIPI k-space that was synthesized from the fully sampled data. Row c shows the reconstruction from a zig-zag blipped-CAIPI under sampling pattern (see Figure 2c), also synthesized from the fully sampled data. Virtually no differences can be observed between the fully encoded SMS reconstruction (a) and the four-fold undersampled reconstruction (b) for both the 12- and the 32-channel coils. Although the four-fold undersampled zig-zag pattern (c) shows some image quality loss compared to the full sampled reconstruction (a) for the 12-channel coil, the 32-channel images appear comparable.

Rows d and e display the noise amplification in b and c, respectively. The noise amplification maps are relative to the fully encoded reconstruction and were derived from a pseudo-replica-method (27). Although no differences can be observed between the fully encoded SMS reconstruction (a) and the four-fold undersampled reconstruction (b) for both the 12- and the 32-channel coils, the four-fold accelerated reconstructions have twice as much background noise as the non-accelerated scans. Furthermore, due to the coil geometry the accelerated images (b) have considerably less g-factor penalty for the 32-channel coil than for the 12-channel coil. A different behavior for both coils is observed for the alternative zig-zag blip pattern. No residual artifacts are visible for the 32-channel coil (c) and the corresponding noise amplification map (e) shows reduced and more evenly distributed g-factor noise compared to the standard blip pattern (d). For the 12-channel coil the reconstruction clearly shows some residual artifacts in the center of the image (c), corresponding to the stronger noise enhancement shown to the left of row e.

Blipped-CAIPIRINHA EPI data

In vivo human brain data with five simultaneous slices were acquired using the $L = 3$ blipped-CAIPI EPI sequence using the 32-channel coil. The generalized SENSE reconstruction is shown in Figure 8a. Individually acquired and reconstructed fully encoded 2D EPI reference slices are shown in Figure 8b. Panel (a) shows that the slices are well separated and the contrast is preserved. Some degradation of image quality due to the five-fold acceleration can be observed.

Blipped spiral data

The multi-shot stack-of-spirals trajectory with 16 segments for full SMS 3D k-space sampling was used to acquire two simultaneous slices with a slice separation of 40 mm. The sum-of-squares combination of individually reconstructed coil elements is displayed in Figure 9a. The reconstruction of two simultaneous slices using the fully sampled single-shot blipped spiral trajectory can be seen in Figure 9b. The total sampling time was around 40ms, leading to pronounced off-resonance blurring in the sum-of-squares combination of individually reconstructed coil elements. The total readout time was then reduced to 20 ms using two-fold undersampling resulting in reduced off-resonance blurring for the parallel imaging reconstruction in Figure 9c. By visual inspection, the slices are nicely separated and no slice cross talk is visible for any of the three spiral acquisitions.

Discussion

We demonstrated that the generalization of the common 3D Fourier encoding framework to the acquisition and reconstruction problem of simultaneously excited slices provides a comprehensive understanding of SMS encoding, including using RF phase modulation, blipped-CAIPI with z-gradient encoding, and non-Cartesian sampling such as spiral. We demonstrated that if the SMS 3D k-space is fully sampled, simultaneously excited slices could be reconstructed without relying on coil sensitivities to disentangle the slices. Although this offers no acceleration advantage, once the fully sampled SMS 3D k-space is conceptualized, undersampling patterns, for instance that for blipped-CAIPI acquisitions, can be easily derived. We showed that a slice and k-space position dependent phase correction prior to reconstruction is also necessary (see Eq. [7]). The phase correction essentially shifts off-center slice groups back to the isocenter as discussed in the original blipped-CAIPI work and is analogous to a phase shift in the slice direction for off-center slab excitation in volumetric 3D imaging. We also demonstrated that there is no difference between Fourier encoding using RF modulation or magnetic field gradients, which then enables implementation of hybrid methods using both types of encoding. Finally we showed that the SMS 3D k-space picture allows for the application of SENSE to blipped-CAIPI EPI and for utilizing blipped spiral trajectories.

Importantly, the previously published blipped-CAIPI approach can be fully and consistently explained in the 3D Fourier formalism. We have shown that the blipped-CAIPI gradients are identical to Cartesian phase encoding along the slice direction in the SMS 3D encoding framework. It also follows that the blipped-CAIPI approach represents an undersampled k-space, where undersampling occurs in the k_y - k_z -plane and the patterns depend on the actual blip scheme employed (16). From a sampling perspective, ignoring T2* decay and off-resonance effects, the information content of a conventional $L = 2$ blipped-CAIPI scheme, for example, is identical to that of a single-blip (i.e. two k_z -planes) Echo Volumar Imaging (EVI) trajectory with two-fold in-plane undersampling and a relative shift of k in the phase direction. Both would provide SMS results with the same g-factor. However, an EVI approach is likely not desirable because of the well-known voxel shift artifact along the slow phase encoding direction for Cartesian trajectories. The combination of a small number of relatively large voxels with a very pronounced slow phase encoding along k_z results in

strong overlap of the slices. The fact that neighboring voxels in the reconstruction no longer correspond to neighboring regions in the brain makes this effect more dramatic for SMS acquisitions compared to volumetric 3D acquisitions.

The SMS 3D framework also allows for further optimization of undersampling patterns. We applied an alternative blip pattern for four slices (see Figure 2c) which samples the k_z direction in a zig-zag pathway instead of diagonal lines. For the 32-channel coil with sensitivity variations along the z-direction this pattern results in improved acceleration properties whereas the noise properties are clearly worse when the 12-channel coil without sensitivity variation along z is used. This is because the 12-channel coil can only restore missing k-space samples in the y-direction and not along z, and the gap along y for the two outer lines corresponding to six-fold acceleration is too large. In contrast, the 32-channel coil is able to restore missing samples in the y and z-directions and therefore benefits from the shift of the locations of the fold-over artifacts due to the modified sampling pattern, a fact that was previously shown for 2D CAIPIRINHA applications in Refs. (16) and (28).

Extending the 3D Fourier encoding framework to non-Cartesian sampling patterns has no inherent limitations. Equations [3] span a 3D k-space ($k_{\max}$) and dictate a minimum k-space increment k that has to be sampled to fulfill the Nyquist criterion. Every k-space trajectory that fulfills these conditions is thus feasible for SMS acquisitions. We used a fully encoded multi-shot stack-of-spirals (16 ms readout duration) to demonstrate the ability to separate two slices without the need for coil sensitivity information in Figure 9a. Furthermore, if the trajectory under-samples k-space, the missing points can be restored by means of parallel imaging. The same considerations as for the blipped-CAIPI EPI trajectory with phase encoding along z apply to a single-shot blipped spiral as well. A blipped spiral trajectory (Figure 5b) with alternating gradient blips after each full revolution avoids the slow phase encoding direction along z present in a stack-of-spirals trajectory. A factor two undersampled single-shot blipped spiral 20 ms acquisition results in significantly less distorted images shown in Figure 9c and are comparable to standard single-slice spirals (images not shown). This acquisition is comparable to a factor two blipped-CAIPI acquisition. The use of the blipped spiral-in trajectory for high temporal resolution fMRI applications will be investigated in the future.

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Figure 1.

This figure depicts the relationship between standard volumetric 3D and SMS3D sampling geometries. The gray boxes in **a** represent four simultaneous slices and the horizontal arrows represent the position of the δ -functions, which compose the total sampling function $U(z)$ for either volumetric or SMS sampling. For connected slices (no gaps in **a**) the imaging geometry can either be expressed in terms of the slice thickness or the slice separation. For separated slices the imaging geometry is expressed by the slice separation. The volumetric reconstruction for four simultaneously excited slices in a phantom projected onto the yz - and xy -planes is shown in **b** and **c**. The corresponding volumetric k-space is shown in **d** by the red and blue circles that represent lines in the read direction. The SMS 3D k-space is shown in **e** and was generated by taking only a subset of the volumetric k-space (the red circles) in **d**. The 3D FFT reconstruction of the SMS 3D k-space is displayed in **f**, which is virtually identical to the four corresponding planes taken from the volumetric reconstruction in **c**.

Figure 2.

This figure shows the SMS 3D k-space representation of blipped-CAIPI acquisitions with gray circles indicating full sampling. A three slice and relative FOV shift of $1/3$ k-space is shown in **a**. For five slices the k-space representation for FOV shifts of $1/5$ between neighboring slices is shown in **b** as blue dots. Relative FOV shifts of $1/3$ are displayed as red dots for comparison. An alternative blip scheme resulting in a zig-zag k-space traversal is shown in **c** for four simultaneously excited slices.

Figure 3.

This figure shows the image space representation of k-space encoding for two simultaneous slices. The blue line represents the phase modulation imposed by the z-blips on the spins prior to signal acquisition. Any offset of the simultaneous slices relative to the isocenter adds an additional phase to the acquired signal that needs to be removed during reconstruction.

Figure 4.

Encoding of $N=4$ simultaneously excited slices ($z = 25\text{ mm}$) using phase modulated multiband RF pulses. Each multiband RF excitation with a phase modulation at the slice locations according to Eq. [8] yields, after conventional gradient encoding in the x- and y-directions, a plane in the SMS 3D k-space at location k_z^m . Full sampling of the SMS 3D k-space requires N excitations, which can then be reconstructed using an inverse 3D Fourier transform.

Figure 5.

A multi-shot "stack-of-spirals" trajectory with 16 segments is plotted in **a**. A single-shot $L = 2$ blipped-CAIPI-like spiral is plotted in **b**. The gradients consist of alternating z -blips played out every full rotation of a planar spiral, as can be seen in **c**. As in Cartesian blipped-CAIPI, the trajectory can be described as two two-fold undersampled spiral planes.

Figure 6.

In **a** two simultaneous slices are reconstructed from a fully sampled SMS 3D k-space with the proper slice offset phase correction. The effect of no phase correction can be seen in **b**, where contributions from both slices are visible in each image.

Figure 7.

This figure shows a phantom reconstruction of four simultaneous slices acquired with the 12- and 32-channel head coils. The data were synthesized from a volumetric 3D FLASH scan using a multiband excitation. A fully sampled SMS 3D reconstruction from four k_z planes is shown in **a** and parallel imaging reconstruction with a factor four blipped-CAIPIRHNA pattern is shown in **b**. Parallel imaging reconstruction with the alternative zig-zag blip pattern from Figure 2c is shown in row **c**. Noise amplification maps relative to a fully sampled reconstruction for **b** and **c** are shown in **d** and **e**, respectively.

Figure 8.

This figure shows *in vivo* blipped-CAIPI EPI reconstructions for five simultaneous slices. Coil sensitivities were derived from a fully encoded pre-scan. The generalized SENSE reconstruction is shown in **a** and the fully encoded reference slices are shown in **b**.

Figure 9.

SMS spiral acquisitions for two 2.5 mm slices (separation = 40mm) are shown in this figure. In **a**, the SMS k-space was fully sampled with the segmented (16 shot) stack-of-spirals from Figure 5a with a duration of 8ms for each spiral segment. Blipped-CAIPI likespiral acquisitions using full sampling and two-fold undersampling are shown in **b** and **c**, respectively. Figure 5b shows the trajectory used for image **c**. Due to the prolonged acquisition of 40ms for the fully sampled spiral in **b**, off-resonance blurring can be observed. Halving the acquisition time by means of parallel imaging results in less blurring for the two-fold undersampled SMS spiral in **c**.