Prospects for *in vivo* estimation of photon linear attenuation coefficients using postprocessing dual-energy CT imaging on a commercial scanner: Comparison of analytic and polyenergetic statistical reconstruction algorithms

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Purpose: Accurate patient-specific photon cross-section information is needed to support more accurate model-based dose calculation for low energy photon-emitting modalities in medicine such as brachytherapy and kilovoltage x-ray imaging procedures. A postprocessing dual-energy CT (pDECT) technique for noninvasive *in vivo* estimation of photon linear attenuation coefficients has been experimentally implemented on a commercial CT scanner and its accuracy assessed in idealized phantom geometries.

Methods: Eight test materials of known composition and density were used to compare pDECT-estimated linear attenuation coefficients to NIST reference values over an energy range from 10 keV to 1 MeV. As statistical image reconstruction (SIR) has been shown to reconstruct images with less random and systematic error than conventional filtered backprojection (FBP), the pDECT technique was implemented with both an in-house polyenergetic SIR algorithm, alternating minimization (AM), as well as a conventional FBP reconstruction algorithm. Improvement from increased spectral separation was also investigated by filtering the high-energy beam with an additional 0.5 mm of tin. The law of propagated uncertainty was employed to assess the sensitivity of the pDECT process to errors in reconstructed images.

Results: Mean pDECT-estimated linear attenuation coefficients for the eight test materials agreed within 1% of NIST reference values for energies from 1 MeV down to 30 keV, with mean errors rising to between 3% and 6% at 10 keV, indicating that the method is unbiased when measurement and calibration phantom geometries are matched. Reconstruction with FBP and AM algorithms conferred similar mean pDECT accuracy. However, single-voxel pDECT estimates reconstructed on a $1 \times 1 \times 3 \text{ mm}^3$ grid are shown to be highly sensitive to reconstructed image uncertainty; in some cases pDECT attenuation coefficient estimates exhibited standard deviations on the order of 20% around the mean. Reconstruction with the statistical AM algorithm led to standard deviations roughly 40% to 60% less than FBP reconstruction. Additional tin filtration of the high energy beam exhibits similar pDECT estimation accuracy as the unfiltered beam, even when scanning with only 25% of the dose. Using the law of propagated uncertainty, low Z materials are found to be more sensitive to image reconstruction errors than high Z materials. Furthermore, it is estimated that reconstructed CT image uncertainty must be limited to less than 0.25% to achieve a target linear-attenuation coefficient estimation uncertainty of 3% at 28 keV.

Conclusions: That pDECT supports mean linear attenuation coefficient measurement accuracies of 1% of reference values for energies greater than 30 keV is encouraging. However, the sensitivity of the pDECT measurements to noise and systematic errors in reconstructed CT images warrants further investigation in more complex phantom geometries. The investigated statistical reconstruction algorithm, AM, reduced random measurement uncertainty relative to FBP owing to improved noise

performance. These early results also support efforts to increase DE spectral separation, which can further reduce the pDECT sensitivity to measurement uncertainty. © 2013 American Association of Physicists in Medicine. [<http://dx.doi.org/10.1118/1.4828787>]

Key words: dual-energy computed tomography, alternating minimization, filtered backprojection, photon attenuation coefficient estimation

1. INTRODUCTION

Accurate *in vivo* mapping of patient-specific photon cross-section information has been identified as one of the crucial tasks needed to improve dose-calculation accuracy for low energy photon-emitting radiation modalities such as brachytherapy, mammography, and x-ray computed tomography (CT).^{1,2} Large dose calculation errors can occur in these modalities if tissue composition inhomogeneities are ignored. Absorbed dose calculations for photon energies below 50 keV are exquisitely sensitive to tissue composition since energy deposition in this range is dominated by photoelectric absorption, which is strongly dependent on the absorbing material's effective atomic number. For example, neglecting tissue composition differences in low energy Pd-103 and I-125 permanent seed implants of the breast and prostate have been shown to lead to errors in dose-volume histogram (DVH) metrics used for prescription and plan assessment ranging from 8% to 40%.^{3–6} Even higher energy brachytherapy sources, such as Ir-192 and Yb-169, have been shown to suffer from dose calculation errors on the order of 5%–30% when neglecting patient-specific tissue composition and geometry.^{7,8}

Recommended bulk tissue compositions⁹ are derived from a handful of measurements that exhibit large sample-to-sample variability.^{10,11} In addition, large patient-to-patient variations in bulk tissue composition have been observed in some anatomical sites, for example the fraction of adipose tissue in the female breast has been shown to vary from 32% to 84%.^{12,13} Using recommended bulk tissue compositions is thus not a satisfactory substitute for *in vivo* measurement of patient-specific tissue composition. For example, Sechopoulos *et al.* utilized a Monte Carlo algorithm to calculate breast dose from mammography and demonstrated that ignoring patient-specific heterogeneity information and instead using an average breast tissue composition approximation lead to average and maximum glandular tissue dose overestimates of 27% and 100%, respectively, in a sample of 20 patients.¹⁴ A method to noninvasively measure patient-specific, low-energy photon cross-section information would thus be of great value for improving kilovoltage dose-calculation accuracy for radiation therapy modalities, such as permanent seed and electronic brachytherapy, as well as kV imaging procedures. In addition, accurate voxel-by-voxel cross-section information as a function of energy enables the implementation of energy-selective reconstruction,¹⁵ which has the potential to alleviate streaking and cupping artifacts arising from the polychromatic x-ray spectrum in support of a broad range of quantitative CT applications.¹⁶

Single-energy CT (SECT) information has been shown to support sufficiently accurate tissue heterogeneity corrections

for megavoltage (MV) radiotherapy dose calculations.^{17,18} However, for low kilovoltage (kV) photon modalities, SECT methods have been shown to introduce linear-attenuation coefficient measurement errors in excess of 20%,¹⁹ due to the dependence on two independent material parameters; electron density and effective atomic number. CT scans at multiple energies can be used to decouple the dependence of photon attenuation on two or more independent material parameters. Much effort has been focused on the use of dual-energy CT (DECT) for material characterization, often in terms of electron density and effective atomic number.^{20–22} Electron density (ρ_e) and effective atomic number (Z_{eff}) have been shown to be estimated quite effectively with errors up to approximately 5% and 12%, respectively, from experimentally acquired DECT data of known test materials.^{23,24} Bazalova *et al.* further reported dose calculated using the DECT estimated Z_{eff} and ρ_e values to be within 1% of dose calculated with exactly assigned material parameters for 18 MV, 6 MV, and 250 kVp photon beams.²³ The effect of the Z_{eff} and ρ_e errors on dose for the low energy regime of brachytherapy seeds (10–30 keV) where the PE mechanism dominates is less clear. Landry *et al.* showed Monte Carlo dose calculations using DECT based material segmentation to agree with exactly assigned composition dose calculations to within $\pm 4\%$ for low energy Pd-103 seeds in an idealized noiseless phantom simulation study, though further work is warranted to investigate the effect of image noise on DECT-based material estimates and subsequent dose calculations in patient geometries.²⁵

Multi-energy CT information can also be used to directly estimate a material's radiological cross-section information to be used as input to more accurate model-based dose calculation algorithms, however the accuracy of this approach has not been extensively studied. Midgley developed a non-separable four-parameter model capable of fitting low energy photon linear attenuation coefficients to within 1.5% for energies greater than 30 keV in an idealized simulation.²⁶ Midgley further demonstrated cross-section estimation accuracy on the order of 1.5% (for energies between 32 and 66 keV) in an experimental study using near-monochromatic characteristic x-ray beam scanning.²⁷ However, clinical implementation of this approach may not be feasible, as it requires scanning at four energies using an x-ray source that cannot be easily extrapolated to clinical practice. In a simulation study, Williamson *et al.* demonstrated that a simple two-parameter basis-vector model could fit low-energy linear-attenuation coefficients from idealized DECT image information with around 1% accuracy,²⁸ though their error propagation analysis illustrated that the pDECT estimates are extremely sensitive to uncertainties in the reconstructed CT images; for example a 1% reconstructed image error propagates to approximately

a 3%–4% linear attenuation coefficient error at 30 keV via the pDECT system of equations.²⁸ Similarly, Goodsitt *et al.* recently reported difficulties obtaining accurate low energy attenuation coefficient estimates, in the form of synthesized monochromatic images, for known tissue-equivalent phantom materials from data acquired on a commercially available DECT scanner,²⁴ though details of the commercial DECT algorithm that was utilized are not available.

In this work, we evaluate the accuracy of photon linear attenuation coefficients in the 10 keV to 1 MeV energy range estimated using dual-energy CT images acquired on a commercial fan-beam CT scanner and the two-parameter basis-vector model (BVM) described by Williamson *et al.*²⁸ The BVM approach investigated here is classified as a postprocessing method since the CT data at two scanning energies are reconstructed before being utilized for DECT cross-section estimation.

Since statistical iterative reconstruction (SIR) algorithms exploit the inherently statistical nature of x-ray transmission data, they are able to reconstruct images with less noise, for similar resolution, than conventional filtered backprojection (FBP) reconstruction.^{29,30} SIR algorithms can also incorporate more accurate models of the CT signal formation process, such as the polychromatic x-ray spectrum and scatter, which has been shown to mitigate systematic artifacts such as cupping^{31–33} and streaking.³⁴ In this work, we reconstruct images from the dual-energy sinogram data with both FBP and a polyenergetic SIR algorithm, alternating minimization (AM).^{33,35} By reducing random and systematic uncertainties of the reconstructed CT images, we hypothesize that the polyenergetic statistical AM algorithm can improve the accuracy of pDECT-estimated photon cross-sections relative to conventional FBP image reconstruction.

Aqueous solutions and industrial plastics of known composition simulating a range of biological tissues are used to compare pDECT-estimated linear attenuation coefficients to NIST reference coefficients in idealized phantom geometries scanned on a commercial Philips Brilliance Big Bore CT scanner. Increased DE spectral separation is also investigated for potential pDECT performance improvement via additional filtration of the high kVp beam. Most quantitative DECT reports in the literature utilize DECT models to estimate a material's effective atomic number and electron density or use the DECT information to generate synthesized mono-energetic images to increase image contrast, often at 40 keV or above. To the best of the authors' knowledge, this is the first work to systematically assess the achievable accuracy of a postprocessing DECT method using data acquired on a commercially available CT scanner to estimate linear attenuation coefficients throughout the low energy range for a set of precisely benchmarked test materials.

2. MATERIALS AND METHODS

2.A. The two-parameter basis vector model and postprocessing dual-energy CT

Two-parameter models are the core of pDECT methods, as they relate the reconstructed CT image intensity to the

underlying material properties to be estimated. Williamson *et al.*²⁸ found that the two-parameter BVM, in which photon cross sections of an arbitrary material are approximated as a weighted mixture of two dissimilar reference materials (basis materials), more accurately represents low-energy photon cross sections than the more widely used parametric fit model, which represents cross sections as power functions of a material's effective atomic number (Z_{eff}), energy, and electron density (ρ_e).²¹ In this paper, we use the term cross section to refer collectively to all the radiological quantities of a material, e.g. linear attenuation coefficients, interaction frequencies, differential cross-sections, regardless of their associated units.

The BVM (Ref. 36) model utilized in this work assumes that the linear attenuation coefficient of an unknown material in voxel x can be accurately represented as a linear combination of two basis substances α and β :

$$\mu(x, E) = w_\alpha(x) \cdot \mu_\alpha(E) + w_\beta(x) \cdot \mu_\beta(E), \quad (1)$$

where $w_\alpha(x)$ and $w_\beta(x)$ are the weighting coefficients for each basis substance. Assuming that reconstructed CT image intensities are proportional to the linear attenuation coefficient of the substance occupying the location x , at the effective scanning energies E_1 and E_2 , the BVM results in a pair of linear equations with two unknowns, $w_\alpha(x)$, $w_\beta(x)$:

$$\begin{aligned} \mu(x, E_1) &= w_\alpha(x)\mu_\alpha(E_1) + w_\beta(x)\mu_\beta(E_1), \\ \mu(x, E_2) &= w_\alpha(x)\mu_\alpha(E_2) + w_\beta(x)\mu_\beta(E_2), \end{aligned} \quad (2)$$

which can be solved on a voxel-by-voxel basis, yielding a basis-vector image ($w_\alpha(\mathbf{x})$, $w_\beta(\mathbf{x})$).

This postprocessing dual-energy CT (pDECT) technique does not require explicit knowledge of E_1 and E_2 , as the basis material expansion vectors ($\mu_\alpha(E_1)$, $\mu_\alpha(E_2)$, $\mu_\beta(E_1)$, $\mu_\beta(E_2)$) are obtained from calibration images of the basis materials acquired using the same CT scanning technique as the unknown substance occupying the voxel at location x . Given that the composition and density of the basis materials are known, the basis coefficient images $w_\alpha(\mathbf{x})$ and $w_\beta(\mathbf{x})$ derived from the pair of DECT images can be used to estimate the entire photon linear attenuation curve as a function of energy for the substance at location x from Eq. (1). Williamson *et al.* showed that the basis material weights can be further used to accurately estimate other radiological quantities, such as partial cross-sections, mass-energy absorption coefficients, and differential cross-sections²⁸ required by model-based dose calculation algorithms such as Monte Carlo. In this work, the focus is on the accuracy with which the BVM model can estimate the total linear attenuation coefficient of the test substances.

2.B. Test substances and phantom geometries

The three basis materials and eight test substances used in the theoretical study of postprocessing DECT photon cross-section estimation accuracy of Williamson *et al.* were physically realized in this work, as reported in Table I. The meaning of the basis pair (α , β) column reported in Table I is further explained near the end of Sec. 2.D. The solutions of

TABLE I. Basis materials and test substances investigated in this work. Composition percentages denote fraction by mass. “Basis pair” refers to the two materials used to represent the test substance, as described in Secs. 2.B and 2.C.

Substance	Composition	State	ρ (g/cm ³)	Z_{eff}^a	Basis pair (α , β)
Basis materials					
Water	H ₂ O	Liquid	1.000	2.3	–
Polystyrene	[C ₈ H ₈] _n	Solid	1.044	2.0	–
23% CaCl ₂	23% CaCl ₂ , 77% H ₂ O	Liquid	1.214	3.5	–
Test substances					
18% CaCl ₂	18% CaCl ₂ , 82% H ₂ O	Liquid	1.166	3.3	(Water, 23% CaCl ₂)
7% CaCl ₂	7% CaCl ₂ , 93% H ₂ O	Liquid	1.066	2.7	(Water, 23% CaCl ₂)
29% NaClO ₃	29% NaClO ₃ , 71% H ₂ O	Liquid	1.237	3.0	(Water, 23% CaCl ₂)
Teflon	[C ₂ F ₄] _n	Solid	2.155	3.9	(Water, 23% CaCl ₂)
ETOH	100% C ₂ H ₅ OH	Liquid	0.787	1.9	(Water, polystyrene)
50% ETOH	50% C ₂ H ₅ OH, 50% H ₂ O	Liquid	0.909	2.1	(Water, polystyrene)
Methyl-ethyl ketone (MEK)	CH ₃ CH ₂ COCH ₃	Liquid	0.804	1.9	(Water, polystyrene)
PMMA (Lucite)	[C ₅ H ₈ O ₂] _n	Solid	1.186	2.2	(Water, polystyrene)

^aEffective atomic numbers, Z_{eff} , shown here for comparison purposes were calculated using the freely available “Auto Z_{eff} ” software described in Taylor *et al.* (Ref. 38) and were energy-weighted for an I-125 radioactive source.

precise percent-by-mass composition were fabricated by mixing high purity dehydrated salts or ethanol (ETOH), with high purity water using a calibrated analytical balance with an estimated uncertainty of 0.1 mg. An elemental analysis (Elemental Analysis Inc., Lexington, KY) was performed on the solid plastic rods (Teflon, polystyrene, and PMMA) to account for high and medium Z impurities, which, if neglected in calculating the reference cross-section curves, were found to induce mass-attenuation coefficient differences of up to 1.5% in the low energy range of interest. The mass density of each material was estimated to within 0.5% uncertainty by measuring the mass of a known volume, determined by machining samples of known dimensions for the solid plastic materials and measured using an adjustable-volume pipette for the aqueous solutions. The composition, measured impurities, and mass density were used to derive the total linear attenuation coefficient benchmark, $\mu_{\text{NIST}}(E)$, as a function of photon energy for each material in Table I, using the NIST XCOM program (NIST Standard Reference Database 8).³⁷ The total attenuation coefficient, including coherent scattering, is used for all basis and test materials in this work.

All solutions were contained in 30 ml polyethylene Nalgene bottles having outer diameters of 34 mm, with the exception of methyl-ethyl ketone (MEK), which, due to its corrosiveness, was stored in a 30 ml Teflon bottle with dimensions of 31 mm in diameter and 1 mm thick walls. The solid plastic rods all had a diameter of 26 mm. The solutions and solid rods were immobilized in the center of a cylindrical water phantom using an out-of-field acrylic plate. The water phantom (Victoreen CT performance phantom; Nuclear Associates, Carle Place, NY) consists of a 20.3 cm diameter water cylinder enclosed by a 6 mm thick acrylic shell, for a total diameter of 21.5 cm. The 21.5 cm water cylinder is referred to as the *head phantom* geometry. To test accuracy within a phantom more representative of a pelvic patient, a 26 cm × 35 cm elliptical acrylic shell was placed around the water cylinder. This setup

is referred to as the *body phantom* geometry. Images of the head and body phantoms with a pure ethanol (ETOH) sample positioned in the center are displayed in Fig. 1.

2.C. Dual-energy data acquisition

All dual-energy CT data were acquired on the Philips Brilliance 16 detector-row Big Bore CT scanner (Philips Medical Systems, Cleveland, OH) located in Virginia Commonwealth University’s (VCU) radiation oncology clinic, utilizing clinically available protocols and tube potentials. Axial scan protocols were used (denoted in the vendor’s software as *Axial Pelvis*) for scanning both head and body phantom geometries. The data from the central 4 rows of detectors, each 0.75 mm wide in the z -direction, were averaged together to give an axial slice thickness of 3 mm. This is the narrowest beam collimation available and was chosen to minimize the amount of scattered radiation in the measured data.

FIG. 1. FBP images of the (a) *head phantom* [diameter = 21.5 cm] and (b) the *body phantom* [26 cm × 35 cm], with 100% ethanol (ETOH) samples in the center. These images were scanned at 90 kVp, 220 mAs, and 3 mm beam collimation. Images are windowed to [−40%: +20%] of the mean water intensity.

FIG. 2. Equivalent x-ray energy spectra (normalized to unit area for comparison) for DECT imaging in this work. It is seen that additional tin filtration of the 140 kVp beam increases the spectral separation with the standard 90 kVp beam. Equivalent spectra were estimated by fitting a Birch-Marshall spectrum model to measured attenuation data as described in Sec. 2.F.

Maximum allowable tube currents for the chosen scan protocols of 220 mAs and 175 mAs were used to acquire dual-energy data at tube potentials of 90 kVp (90 *Std*) and 140 kVp (140 *Std*), respectively. Here the term “*Std*” denotes the use of the standard x-ray beams on the commercial system. These acquisition parameters lead to CTDI_{vol} dose values, measured in a standard CTDI pelvis phantom of 9.4 mGy for 90 kVp and 25.2 mGy for 140 kVp.

As previous literature has shown that additional filtration of the high kVp beam improved the performance of dual-energy material decomposition³⁹ by reducing overlap of the high- and low-energy scanning spectra, a high purity tin filter of 0.5 mm thickness was machined to be retrofitted to the Brilliance CT’s collimation system. Figure 2 shows equivalent x-ray energy spectrum (refer to Sec. 2.F for more details) for the three tube potentials used here and illustrates how the additional 0.5 mm of tin filtration increases the spectral separation between the low- and high-energy scans. The term “*Tin*” is used to denote data acquired with the additional 0.5 mm of tin filtration. However, the 0.5 mm tin filter also reduces the 140 kVp beam particle fluence incident on the scan subject. Since maximum allowable tube currents are already being used for the *Std* unfiltered scans, 140 *Tin* scanning doses will not be exactly matched. The tin filtered 140 kVp (140 *Tin*) CTDI_{vol} is 6.1 mGy, approximately 25% of the unfiltered 140 *Std* dose.

The raw sinogram data, corrected only for dark current, were exported from the scanner for offline processing and reconstruction. Proprietary software provided by Philips enabled any of the standard data preprocessing corrections to

be applied or omitted. System corrections, such as detector gain, reference detector normalization, slice normalization and crosstalk, were applied to all data in this work. The Brilliance scanner uses a 1D anti-scatter grid (ASG) for physical scatter rejection, and does not apply any additional scatter correction to the data. Two sets of processed data were generated from each raw dataset; one with the vendor’s proprietary water-equivalent beam hardening (BH) correction turned on for conventional FBP reconstruction, and one with the BH correction omitted for reconstruction with the polyenergetic AM algorithm, which requires energy uncompensated data as discussed in Sec. 2.F. Also note that the 140 *Tin* data were only able to be reconstructed with polyenergetic AM, as there is not a vendor supported beam-hardening correction for this x-ray energy spectrum.

2.D. pDECT basis material calibration procedure

For calibrating the basis materials, repeat scans were acquired and averaged to reduce statistical noise. However, all of the test substances were acquired with the standard scanning protocol described above in Sec. 2.C to mimic scanning doses a patient could receive, unless otherwise noted in Sec. 3. Basis material scans were obtained within both the head and body phantom geometries, using the same system corrections and reconstruction algorithm as used for the test substance scans. In this situation where basis and test material intensities were measured under nearly identical phantom conditions, the effect of residual artifacts arising from nonlinearities such as beam-hardening and scatter will be minimized on the pDECT process.

From the reconstructed basis material images, the calibration basis material image intensities ($\mu_\alpha(E_1)$, $\mu_\alpha(E_2)$, $\mu_\beta(E_1)$, $\mu_\beta(E_2)$) required by the pDECT system of equations were averaged within 7 or 9 mm diameter circular regions-of-interest (ROIs) centered on each plastic rod or solution, respectively. These circular ROIs were also used within the test substance images to quantify pDECT-estimated attenuation coefficient accuracy as described in Sec. 2.E.

Previous work found that BVM model fitting accuracy was improved by using two pairs of basis materials for the range of test substances investigated here.²⁸ Test substances with Z_{eff} lower than water were modeled by a water-polystyrene basis pair, while higher Z materials ($Z_{\text{eff}} > \text{water}$) were represented with a basis pair of water and a 23% CaCl_2 aqueous solution as denoted in Table I. In theory, each image pixel x can be classified as low-Z or high-Z, and assigned the corresponding basis pair, by comparing the relative CT image intensities at low and high-energy:

$$\begin{cases} \left(\frac{\mu_x}{\mu_{\text{water}}} \right) (E_1) \leq \left(\frac{\mu_x}{\mu_{\text{water}}} \right) (E_2); (\alpha, \beta) = (\text{Water, polystyrene}) \\ \left(\frac{\mu_x}{\mu_{\text{water}}} \right) (E_1) > \left(\frac{\mu_x}{\mu_{\text{water}}} \right) (E_2); (\alpha, \beta) = (\text{Water, 23\%CaCl}_2). \end{cases} \quad (3)$$

Once the basis pair is assigned for each pixel, the basis material weighting coefficients, $(w_\alpha(\mathbf{x}), w_\beta(\mathbf{x}))$, are then calculated for each pixel x from the pDECT system of Eq. (2), using the reconstructed DE image intensities, $(\mu(\mathbf{x}, E_1), \mu(\mathbf{x}, E_2))$, and the averaged basis material intensities calibrated in the appropriate phantom geometry and at the appropriate scan energies, $(\overline{\mu}_\alpha(E_1), \overline{\mu}_\alpha(E_2), \overline{\mu}_\beta(E_1), \overline{\mu}_\beta(E_2))$. From the basis material weighting coefficient images, the linear attenuation coefficient can be estimated for every pixel location x at any energy E using the BVM Eq. (1) and the NIST reference attenuation coefficient for the basis materials, $(\mu_\alpha(E), \mu_\beta(E))$.

2.E. Dual-energy attenuation coefficient accuracy endpoints and propagated uncertainty analysis

For pixels within the test sample ROI, pDECT accuracy was quantified by the ratio of the pDECT-estimated attenuation coefficient to the NIST reference attenuation coefficient as a function of energy in the 10–1000 keV range. The mean pDECT bias (i.e., systematic uncertainty) was quantified by averaging the attenuation coefficient ratio of the pixels within the test substance ROI, $(\overline{\mu}_{DE}(E)_{\mathbf{x} \in \text{ROI}} / \mu_{\text{NIST}}(E))$. The standard deviation of the distribution of linear attenuation coefficient ratios within each test material ROI was used to quantify the statistical uncertainty (i.e., random uncertainty) of the pDECT linear attenuation coefficient estimates.

In addition to directly comparing pDECT-estimated linear attenuation coefficients to NIST reference values, we also employed the law of propagation of uncertainty⁴⁰ to investigate the sensitivity of the pDECT-estimated coefficients to uncertainties in the reconstructed image intensities in a well-controlled environment. As described in detail by Williamson *et al.*,²⁸ the mean image intensity and the estimated uncertainty for each of the six reconstructed CT images required to solve the pDECT system of Eq. (2) are used as input to compute the uncertainty of the resulting pDECT-estimated attenuation coefficient, $\mu_{DE}(x, E)$, for each of the test materials. Note that the term “uncertainty” here refers to the total uncertainty arising from both systematic and random components.

First, we utilize the propagated uncertainty analysis to investigate the material-dependent pDECT sensitivity. Each test substance’s mean reconstructed intensity is used in the propagated uncertainty system of equations, but with matched reconstructed uncertainty to investigate if some test materials are more sensitive to input errors than others. We also use the propagated uncertainty analysis to further investigate the pDECT improvement from additional tin filtration of the 140 kVp beam. Recall that 140 *Tin* measurements were not able to be exactly dose matched as the maximum allowable tube currents were already being used for the 140 *Std* scans. Using the uncertainty propagation method with the mean reconstructed intensities from scans using the 140 *Std* and 140 *Tin* x-ray spectra, while assuming the same reconstructed image uncertainty simulates a dose-matched comparison. And finally, we utilize the propagated uncertainty analysis to work backwards and estimate what image uncertainty would be required to meet a target estimated-coefficient uncertainty. This allows us to assess whether either of the two CT recon-

struction algorithms presented in Secs. 2.F and 2.G have the potential to meet a target pDECT-estimated linear attenuation coefficient uncertainty, chosen here to be 3% at 28 keV.

2.F. Polyenergetic alternating minimization image reconstruction

The alternating minimization algorithm is employed in this work to investigate the hypothesis that a polyenergetic statistical reconstruction algorithm can improve pDECT linear attenuation coefficient estimation performance by reducing random and systematic errors in the reconstructed CT images. Here, the sinogram data exported from the scanner is denoted by $d(\mathbf{y})$, where the data space, $\mathbf{y} = (\gamma, \phi)$, is defined by the angle of each source-detector pair ray, γ , and each gantry angle, ϕ . The 2D image space is composed of a 512×512 rectangular array of 1 mm square pixels (1.0 mm length on a side), where $\mathbf{x}$ denotes the indices of each pixel. For the AM algorithm, an object is represented in image space as a map of linear attenuation coefficients that depend on spatial location x and energy E . The object is represented as a weighted sum of N basis materials:

$$\mu(\mathbf{x}, E) = \sum_{i=1}^N \mu_i(E) c_i(\mathbf{x}), \quad (4)$$

where $\mu_i(E)$ denotes the linear attenuation coefficient as a function of energy of the i th basis material. The AM algorithm will estimate N images that represent the partial density of each basis material in each voxel; $c_i(\mathbf{x})$. In this work, we reconstruct images with a single basis substance ($N = 1$) of water. Given this restriction, AM models the sinogram data, by computing the expected data means g from an image estimate $\mathbf{c}'$ according to the forward model

$$g(\mathbf{y} : \mathbf{c}') = \sigma(\mathbf{y}) + \sum_E I_0(\mathbf{y}, E) \cdot \exp \left(- \sum_{x \in X} h(\mathbf{y}|\mathbf{x}) \cdot \mu_{\text{water}}(E) \cdot \mathbf{c}'(\mathbf{x}) \right). \quad (5)$$

The system matrix, $h(\mathbf{y}|\mathbf{x})$, is the average distance traveled by photons crossing pixel x that are incident on the face of detector γ for gantry angle ϕ and is precomputed to increase the speed of the iterative algorithm. An estimate of scattered radiation, $\sigma(\mathbf{y})$, can be included in the forward model. $I_0(\mathbf{y}, E)$ denotes the x-ray particle fluence spectrum incident on the scan subject. Incorporating the x-ray spectrum directly in the AM algorithm’s forward model represents an implicit beam-hardening correction. The polyenergetic AM algorithm thus operates on energy-uncompensated data; that is data with all system preprocessing corrections performed excluding the vendor’s BH correction.

The AM algorithm derivation is based upon minimizing the discrepancy between the noisy measured sinogram, $d(\mathbf{y})$, and the noiseless mean computed sinogram, $g(\mathbf{y})$, as quantified by Csiszar’s I-divergence,⁴¹ $I(d||g)$; a scalar-valued information-theoretic measure of the discrepancy between two functions. The I-divergence is proportional to the negative of the Poisson log-likelihood, which means that the image

which minimizes the I-divergence also maximizes the log-likelihood of observing the data $d(\mathbf{y})$. The reformulation of the optimization problem in terms of the I-divergence and a novel application of the convex decomposition lemma allow for a closed form image update step. The reader is referred to O'Sullivan and Benac³³ for a more complete treatment of the AM algorithm.

The penalized objective function includes a penalty term to enforce the *a priori* assumption of image smoothness:

$$\Phi(\mathbf{c}') = I(d||g) + \lambda \cdot R(\mathbf{c}'), \quad (6)$$

where $\mathbf{c}'$ is the current image estimate and λ controls the relative weighting of the penalty function. A value of $\lambda = 5.0 \times 10^{-4}$ is used for all AM reconstructions in this work and was chosen from preliminary reconstructions that were found to well balance noise and resolution. The roughness penalty $R(\mathbf{c}')$ computes a penalty for all pixels $\mathbf{x}$ as a function of the neighboring pixel intensities $\mathbf{x}'$. The edge-preserving log-cosh penalty function^{32,42} used in this work is defined as

$$R(\mathbf{c}') = \sum_{\mathbf{x}} \sum_{\mathbf{x}' \in N(\mathbf{x})} w(\mathbf{x}') \cdot \left(\frac{1}{\delta} \right) \log[\cosh(\delta(c'(\mathbf{x}) - c'(\mathbf{x}')))], \quad (7)$$

where $N(\mathbf{x})$ is the set of neighboring pixels weighted as $w(\mathbf{x}') = 1$ for the four directly adjacent pixels and as $w(\mathbf{x}') = 0$ for all other pixels. The parameter δ controls the penalty transition from quadratic to linear growth. Here $\delta = 15$ is used, which corresponds to a transition at intensity differences of 10% from background. For a more detailed treatment of the parameters λ and δ in the log-cosh penalized AM algorithm, the reader is referred to Evans *et al.*⁴³ Ordered subsets are utilized to increase the convergence rate.⁴⁴ All AM images in this work are reconstructed using 1024 iterations and 33 ordered subsets. The choice to stop the AM algorithm at 1024 iterations was made in prior work in which phantoms of similar size and composition as the ones in this paper were judged to be well-converged in terms of incremental I-divergence decreases and stable mean reconstructed image intensities.³⁵

The x-ray spectrum, $I_0(\mathbf{y}, E)$, required by the AM forward projection model was obtained by fitting the semiempirical Birch-Marshall spectrum model, including tungsten characteristic x-rays, to narrow-beam attenuation curves through high purity aluminum and copper filters of varying thickness for the central-axis ($\gamma = 0$) source-detector ray. The resultant equivalent spectrum model (shown in Fig. 2) fit the measured attenuation data to within 1.5% RMS error for all tube potentials investigated in this work, including the 140 *Tin* beam. The off-axis hardening of the spectrum due to the bowtie filter was modeled by computationally hardening the central axis equivalent spectrum using the known composition and geometry of the filter. More details concerning the x-ray spectrum measurement methodology are presented in Evans *et al.*³⁵ No scatter estimate, $\sigma(\mathbf{y})$ in Eq. (5), is used in the AM algorithm's forward model in this work since using the same phantom geometry for test substances and basis material calibrations

is expected to mitigate the effect of scatter in the pDECT results.

2.G. Filtered backprojection image reconstruction

An in-house weighted filtered backprojection algorithm⁴⁵ was used in this work to ensure that FBP and AM used the same system matrix $h(\mathbf{y}|\mathbf{x})$, and to allow management of the FBP reconstruction kernel. For FBP reconstruction, the sinogram data were preprocessed with all standard Philips corrections, including the BH correction. FBP reconstruction of the 140 *Tin* sinogram data was not possible since no vendor-supported beam-hardening correction was available for the tin-filtered beam. Thus, pDECT linear attenuation coefficient estimation was not performed for FBP reconstruction of the 90 and 140 *Tin* DE dataset. The in-house FBP filter, $H(f)$, is a modified ramp filter defined in frequency space as

$$H(f) = s \cdot |f| \cdot W(f) \cdot G(f), \quad (8)$$

where s is a constant scale factor that ensures the image intensities represent units of linear attenuation, mm^{-1} , and $|f|$ is the ramp function. $W(f)$ is a rectangular window function up to frequencies of 90% of Nyquist and then rolls off with a raised cosine function to zero at Nyquist frequency. More detailed information regarding this modified ramp filter can be found in Evans *et al.*⁴³ $G(f)$ is the Fourier transform of a Gaussian smoothing kernel, which controls reconstructed image noise and resolution by further reducing the amplitude of high spatial frequencies. In this work, FBP image resolution was approximately matched to the reconstructed AM images by varying the FWHM of the Gaussian smoothing kernel and observing the sharpness of the reconstructed test material edges. Gaussian smoothing kernels with FWHM of 1.0 mm and 2.0 mm, for the head and body phantoms, respectively, were found to give very good edge profile agreement for the datasets used in this work. Figure 3 illustrates the edge matching for the PMMA rod in the body phantom for which a FBP kernel with FWHM = 2.0 mm was used for the pDECT estimation results.

3. RESULTS

3.A. Systematic estimated attenuation coefficient uncertainty

The pDECT process is illustrated in Fig. 4 for the 50% ETOH solution that is ideally modeled by the $(\alpha, \beta) = (\text{water}, \text{polystyrene})$ basis pair, in the head phantom scanned with the 90 + 140 *Std* energy pair. The basis pair coefficient images show that the 50% ETOH solution is modeled as predominantly polystyrene (high intensity in the w_β image). In the water background, the water basis, w_α , is near unity as expected. The total linear attenuation coefficient in each pixel can be estimated at any energy using the basis coefficient images, w_α and w_β , and Eq. (1). Figure 4 shows simulated monoenergetic images calculated at 28 and 100 keV. It is seen that the contrast between the polystyrene test substance and the water background is increased in the 28 keV image, however,

FIG. 3. Illustration of the first-order resolution matching performed in this work. (Inset) Zoomed 140- kVp image of the PMMA rod in the body phantom reconstructed with FBP using a smoothing kernel FWHM of 2.0 mm. The horizontal line represents the location of the profile used for resolution comparison. The square ROI near the top represents the pixels used for noise comparison (coefficient of variation around the mean) for the varying smoothing levels. Profiles through the PMMA rod edge compare the AM reconstruction and three FBP reconstructions with varying smoothing strengths. The legend specifies the FWHM of the Gaussian smoothing kernel in millimeters. The noise assessed in the water ROI displayed on the inset is also reported for each image illustrating that FBP requires more aggressive smoothing to achieve comparable noise levels as AM.

the image noise is greater due to increased uncertainty propagation at lower energies.

The mean ratio of pDECT-estimated attenuation coefficient to the NIST reference value for the pixels within each test substance ROI, $(\overline{\mu_{DE}}(E)_{x \in \text{ROI}} / \mu_{\text{NIST}}(E))$, is plotted as a function of energy in Fig. 5. In both the head and body phantom geometries, FBP and polyenergetic AM estimate total linear attenuation with a mean error (or bias) of 0.5%–1.0% for energies above 30–40 keV. However, for lower energies, mean error exceeds 1% rising to 2%–3% at 10 keV for most materials and 3%–6% for Teflon.

Data with inadequate signal statistics, i.e., acquired without enough dose, can suffer from systematic streaking artifacts due to photon starvation.⁴⁶ Using the standard scanning protocol with 3 mm slices as described in Sec. 2.C, mean linear attenuation coefficient estimates for MEK deviated by as much as 60% from NIST reference at 10 keV in the body phantom. Recall from Sec. 2.B that due to its corrosiveness, the MEK test substance required a Teflon bottle with 1 mm thick walls, which in conjunction with the additional attenuation of the 35 cm body phantom, is thought to be responsible for the poor signal statistics and the resultant streaking artifact. The level of mean accuracy for MEK displayed in Fig. 5 required a higher effective scanning dose to avoid systematic streaking artifacts, achieved here by averaging 4 adjacent sinogram slices (total slice thickness = 12 mm versus the original 3 mm) over 7 repeat scans. While signal averaging will not exactly match a higher dose scan due to nonlinear

FIG. 4. Illustration of the pDECT method for 50% ETOH centered in the head phantom reconstructed with FBP (left) and AM (right). (Top row) Reconstructed images of 90 kVp and 140 kVp Std sinogram data. (Middle row) In conjunction with the corresponding basis material calibration values, the DE images are used to calculate the basis material weighting coefficient images. (Bottom row) From the basis coefficient images, monoenergetic images can be calculated at any energy. Displayed here are mono-energetic linear attenuation images computed at 28 and 100 keV. Note that all images are windowed to $[-50\%:+20\%]$ of the mean water intensity, except the w_α , w_β images, which are windowed to weights of $[-0.1:1.1]$.

FIG. 5. Ratio of mean pDECT estimated coefficients (averaged over all pixels within the material ROI) to NIST reference values as a function of energy for all test substances. Columns compare test substances centered in the (left) head phantom compared to the (right) body phantom. Rows compare scanning energies and reconstruction algorithms: (top) FBP with 90 + 140 *Std* data, (middle) AM with 90 + 140 *Std* data and (bottom) AM with 90 + 140 *Tin* data. All test material data was acquired with the standard scanning doses and 3 mm slice thicknesses, except for the MEK test substance in the body phantom in which a higher effective dose was achieved by averaging seven repeat acquisitions of a 12 mm slice thickness.

FIG. 6. Distribution of attenuation coefficient ratios for pixels within the PMMA ROI at 28 keV. Images were all acquired with the standard scanning protocol and reconstructed with $1 \times 1 \times 3 \text{ mm}^3$ voxel dimensions. Left column is for the head phantom and right column is for the body phantom. Rows compare the 90 + 140 Std DE images reconstructed with (top) FBP and (middle) AM, and (bottom) AM reconstruction of the 90 + 140 Tin image pair with increased spectral separation. For each distribution of ratios, the mean and standard deviation is reported.

effects such as electronic noise, the averaging employed here to give an effective scanning dose around 28 times the standard scanning protocol is a practical method for mitigating the source of the observed systematic streaking artifact.

From Fig. 5, it appears that increasing the spectral separation by using the 90 + 140 Tin spectral pair does not improve the pDECT-AM mean accuracy, and is in fact slightly worse for certain materials. However, it is difficult to draw conclusions on the efficacy of increasing spectral separation at this point since the 140 Std imaging dose is 4 times that of the 140 Tin scans.

3.B. Random uncertainty of estimated attenuation coefficients

While the mean pDECT estimation accuracy within each material is important, the accuracy will vary from pixel-to-

pixel due to image noise. Figure 6 displays the distribution of $\mu_{DE}(\mathbf{x}, E)/\mu_{NIST}(E)$ ratios for pixels within the PMMA ROI at 28 keV for the various pDECT scenarios. The mean of each distribution in Fig. 6 corresponds to the mean bias at 28 keV, as displayed in Fig. 5. The standard deviation of the distribution provides a measure of how widely the pDECT-estimated attenuation coefficient varies for pixels within the test substance ROI. For example, Fig. 6 shows that while both AM and FBP images give a mean bias of less than 1.4% for PMMA at 28 keV in the head phantom, the pDECT estimates have a wider spread using FBP reconstruction ($\sigma = 12.2\%$) than when using AM reconstruction ($\sigma = 7.5\%$). Similar results are observed for the body phantom case.

The width of the attenuation coefficient ratio distribution for each test substance provides important information regarding the impact of reconstructed image noise on pDECT performance. To summarize these results, Table II reports the

TABLE II. Mean percent error of measured pDECT-estimated attenuation coefficients relative to NIST reference values within the test substance ROIs at 28 and 200 keV. Values in parentheses denote the corresponding standard deviation of the pDECT error distribution.

Test substance	Energy (keV)	Phantom	FBP (90 + 140)		AM (90 + 140)		AM (90+140 Tin)	
18% CaCl ₂	28	Head	0.5%	(4.7%)	0.6%	(2.6%)	0.6%	(2.3%)
		Body	0.9%	(8.5%)	0.5%	(3.9%)	−0.6%	(2.6%)
	200	Head	−0.2%	(1.5%)	−0.2%	(0.8%)	−0.2%	(1.0%)
		Body	−0.5%	(2.4%)	−0.4%	(1.2%)	0.1%	(1.0%)
7% CaCl ₂	28	Head	1.2%	(7.4%)	1.1%	(4.2%)	0.6%	(4.2%)
		Body	0.6%	(9.7%)	0.7%	(3.7%)	−0.3%	(4.3%)
	200	Head	−0.5%	(1.6%)	−0.4%	(0.9%)	−0.3%	(1.1%)
		Body	−0.3%	(1.6%)	−0.4%	(0.6%)	−0.1%	(1.1%)
29% NaClO ₃	28	Head	−0.5%	(6.0%)	−0.2%	(3.1%)	0.4%	(3.8%)
		Body	−1.0%	(10.2%)	−1.1%	(4.0%)	−2.4%	(4.3%)
	200	Head	−0.4%	(1.4%)	−0.4%	(0.7%)	−0.7%	(1.4%)
		Body	−0.5%	(2.0%)	−0.4%	(0.8%)	0.0%	(1.3%)
Teflon	28	Head	−1.7%	(6.3%)	−0.6%	(2.8%)	−1.0%	(2.7%)
		Body	−1.2%	(8.8%)	−0.6%	(2.4%)	−2.3%	(3.2%)
	200	Head	0.0%	(1.1%)	0.2%	(0.5%)	0.3%	(0.7%)
		Body	−0.2%	(1.2%)	0.1%	(0.4%)	0.5%	(0.7%)
ETOH	28	Head	0.0%	(17.1%)	−0.6%	(10.8%)	−0.9%	(10.7%)
		Body	1.0%	(24.9%)	0.9%	(12.2%)	0.5%	(8.8%)
	200	Head	0.0%	(1.8%)	0.0%	(1.1%)	0.0%	(1.5%)
		Body	0.1%	(2.2%)	−0.2%	(1.2%)	−0.1%	(1.3%)
50% ETOH	28	Head	−0.6%	(12.9%)	−0.7%	(7.9%)	−0.4%	(8.8%)
		Body	0.6%	(22.3%)	1.3%	(9.4%)	−2.2%	(5.9%)
	200	Head	0.4%	(1.5%)	0.4%	(0.9%)	0.4%	(1.5%)
		Body	0.4%	(2.5%)	0.1%	(1.1%)	0.7%	(0.9%)
MEK ^a	28	Head	−0.8%	(20.0%)	−0.8%	(12.7%)	1.1%	(9.4%)
		Body ^a	−0.2%	(10.3%)	−0.1%	(14.6%)	−1.9%	(8.3%)
	200	Head	0.1%	(2.0%)	−0.1%	(1.3%)	−0.4%	(1.3%)
		Body ^a	0.6%	(0.7%)	0.4%	(1.1%)	−0.4%	(1.0%)
PMMA	28	Head	−1.4%	(12.2%)	−1.0%	(7.5%)	−0.9%	(6.2%)
		Body	−1.7%	(19.6%)	−0.9%	(7.4%)	0.0%	(5.2%)
	200	Head	0.4%	(1.2%)	0.5%	(0.7%)	0.5%	(0.8%)
		Body	0.4%	(1.9%)	0.5%	(0.8%)	0.3%	(0.7%)

^aScanning dose CTDI_{vol} measured in a pelvis phantom were 9.4 mGy, 25.2 mGy, and 6.1 mGy for 90 *Std*, 140 *Std*, and 140 *Tin*, respectively, for all test materials except for MEK in the body phantom in which data averaging was used to achieve an effective scanning dose around 28 times higher to mitigate photon starvation streaking artifacts since MEK required a Teflon-walled container.

mean and standard deviation of the pDECT estimated attenuation coefficient distribution within each test substance for all pDECT scenarios. 200 keV is chosen as representative of energies between 100 keV and 1 MeV, where mean bias of less than 1% is consistently achieved and 28 keV is chosen as representative of low-energy applications, where mean attenuation coefficient estimation errors often exceed 1%. Comparing values at 28 and 200 keV shows that both the mean bias (systematic uncertainty) and the standard deviation of the bias distribution (random uncertainty) of estimated attenuation coefficients are larger at low energies.

3.C. Uncertainty propagation analysis

The law of propagation of uncertainty was employed to investigate the sensitivity of pDECT-estimated linear attenuation coefficients to errors in the reconstructed CT images. Figure 7 plots the percent unexpanded uncertainty (coverage factor of $k = 1$) of the pDECT-estimated linear attenuation

coefficient as a function of energy for each test material using mean image intensities of the unfiltered 90 + 140 *Std* spectral pair. For this propagated uncertainty analysis, the total input image uncertainty for all eight test materials is 0.50% and 0.25% at 90 kVp and 140 kVp, respectively. These levels of input image-intensity uncertainty were chosen as they are similar to the image noises reconstructed using the standard scanning protocol employed in this work. Basis material image uncertainty was chosen to be 0.07% (90 kVp) and 0.03% (140 kVp), since much higher doses can be used for basis calibration scans.

Figure 7 shows that this level of reconstructed image uncertainty leads to pDECT attenuation-coefficient uncertainties of around 1% for energies greater than 50 keV, with uncertainty increasing as energy decreases, ranging from 4% to 20% at 10 keV. The propagated uncertainty reaches a minimum around 80 keV, which falls roughly between the effective energies of the 90 and 140 kVp scanning spectra. In the review paper of Yu *et al.* concerning monochromatic

FIG. 7. Percent unexpanded ($k = 1$) uncertainty of the DE estimated linear attenuation coefficient as a function of energy. For this comparison all eight test materials were assumed to have the same input image uncertainty of 0.50%, 0.25% (90, 140 kVp) and basis material uncertainty of 0.07%, 0.03%, which was chosen as it is similar to the reconstructed image noises using the standard scanning protocol.

imaging from DECT information⁴⁷ plots of monochromatic image noise as a function of energy exhibit a similar shape to the propagated uncertainty curves displayed here, which is expected as noise transfer through the BVM system of equations is one source of the total estimated attenuation coefficient uncertainty. The uncertainty propagation curves shown here also appear closely related to the shape of plots of monochromatic image CNR as a function of energy. Yu *et al.* show monochromatic image CNR to be maximized around 70 keV,⁴⁷ which is very close to the energy of minimum uncertainty propagation shown here in Fig. 7. Monochromatic image CNR is also shown to decrease at lower energies, which can be explained by the rapidly increasing noise propagation through the BVM system of equations for decreasing energy exhibited here.

Figure 7 also illustrates the point that given the same reconstructed CT image uncertainty, the resultant pDECT-estimated attenuation coefficient is more sensitive to propagated uncertainties for some test materials than others. Broadly speaking, the low-energy coefficient estimates for test substances that use the (water, polystyrene) basis pair are more sensitive to input image error than the higher Z substances that use the (water, 23% CaCl_2) basis pair.

Figure 7 compared the propagated uncertainty for all test materials using the standard 90 kVp and unfiltered 140 kVp spectral pair (140 *Std*). Figure 8 illustrates the effect of increased spectral separation by an additional 0.5 mm of tin filtration for the 140 kVp beam (140 *Tin*). Here, the same reconstructed image uncertainty (90 kVp = 0.5%, 140 kVp = 0.25%) is assumed for two test substances, MEK and 7% CaCl_2 . Clearly, the use of the 140 *Tin* spectrum results in less propagated uncertainty than the 140 *Std* spec-

FIG. 8. Percent unexpanded uncertainty ($k = 1$) of the DE estimated linear attenuation coefficient as a function of energy. All four cases shown here were assumed to have the same input image uncertainty of 0.50%, 0.25%, and basis uncertainty of 0.07%, 0.03%. This figure illustrates that the increased spectral separation of the 140 *Tin* spectrum reduces the uncertainty propagated into the estimated attenuation coefficient from the reconstructed images.

trum, reinforcing the use of dual energy scanning spectrum with less overlap to better condition the pDECT system of equations.

We can further use the error propagation model to answer the question, “what reconstructed image uncertainty is required to achieve a target attenuation coefficient uncertainty at a given energy?” Table III reports the 140 kVp image uncertainty needed to achieve DE-estimated attenuation coefficient uncertainty of 3% at 28 keV. Our target estimated linear attenuation coefficient uncertainty of 3% at 28 keV seems reasonable as previous Monte Carlo investigations into I-125 dose calculations have shown that a 3% uncertainty in the linear attenuation coefficient of water at 30 keV leads to a 3.5% uncertainty in the I-125 seed’s dose rate constant.⁴⁸ For this analysis, the scanning doses are assumed to scale such that the 90 kVp image uncertainty is twice that of the 140 kVp uncertainty and basis material intensities are assumed to have an uncertainty of 0.07% and 0.03% at 90 kVp and 140 kVp, respectively.

When scanning with the standard spectral pair, it is seen that for the high-Z materials modeled by the (water, 23% CaCl_2) basis pair, 140 kVp image uncertainty of less than 0.15% to 0.25%, is required to meet our target DECT uncertainty. For the low-Z materials modeled by the (water, polystyrene) basis pair, even smaller image uncertainties of less than 0.10% are required. When scanning with the tin-filtered spectrum, larger reconstructed image uncertainty can be tolerated to meet the same target pDECT-estimated attenuation coefficient uncertainty. Clearly, these reconstructed image uncertainty targets cannot be achieved at clinically acceptable patient doses and image resolutions using currently available scanning and image reconstruction technologies.

TABLE III. Percent reconstructed image uncertainty (σ_X) required to achieve a target linear attenuation coefficient uncertainty of 3% at 28 keV. The 90 kVp uncertainty is assumed to be twice as large as the 140 kVp uncertainty. From measured image noises reconstructed using $1.0 \times 1.0 \times 3.0 \text{ mm}^3$ voxels, the in-plane pixel size (for the same slice thickness of 3.0 mm) necessary to achieve the target image uncertainty can be estimated for each algorithm.

Basis pair (α, β)	Test substance (X)	Target image noise σ_X @ 140 kVp (%)		In-plane pixel size (length of a side in mm) to meet target image noise			
		140 Std	140 Tin	FBP 140 Std	AM 140 Std	AM 140 Tin	AM 140 Tin ^a
Water, 23% CaCl_2	18% CaCl_2	0.25	0.36	1.7	1.0	1.9	0.9
	7% CaCl_2	0.18	0.27	2.7	1.6	2.8	1.4
	29% NaClO_3	0.19	0.29	2.4	1.3	3.5	1.7
	Teflon	0.15	0.22	2.4	1.3	2.3	1.2
Water, polystyrene	ETOH	0.09	0.16	6.7	4.2	8.0	4.0
	50% ETOH	0.10	0.18	5.3	3.3	6.8	3.4
	MEK	0.08	0.16	8.4	5.5	7.3	3.6
	PMMA	0.09	0.17	4.8	2.9	4.1	2.1

^aMeasured AM 140 Tin image noises are scaled in this column to simulate dose-matching with the 140 Std spectrum.

We can further estimate the reconstructed pixel size that would be necessary to achieve the target uncertainty for each material using the scanning dose of the standard protocol in this work. This analysis assumes that the image uncertainty (σ_X) is dominated by random image noise which can be reduced by averaging neighboring pixels according to the familiar $1/\sqrt{N}$ relationship, where N is the number of pixels averaged. For example, if 4 adjacent pixels are averaged, meaning $4\times$ the area is averaged, then the random image noise is assumed to be $\frac{1}{2}$ of the original value. The AM and FBP noise values used in this analysis were measured from 140 kVp head phantom images ($1 \times 1 \times 3 \text{ mm}^3$ voxel dimensions) acquired with the standard scanning dose of 25.2 mGy, which would be a clinically acceptable patient dose. Table III reports the estimated in-plane pixel size necessary to achieve the target image uncertainty, for a constant cross-plane pixel dimension of 3 mm.

Table III shows that the increased sensitivity of the low-Z materials to input image uncertainty requires larger pixel dimensions than for the high-Z materials. For the low-Z materials, in-plane pixel dimensions on the order of $4 \times 4 \text{ mm}^2$ to $8 \times 8 \text{ mm}^2$ pixels are needed to achieve 3% attenuation coefficient uncertainty at 28 keV at clinically acceptable doses. The increased noise-resolution performance of the AM algorithm has the potential to achieve the target image uncertainty with in-plane pixel dimensions around 40% smaller than FBP. Even with less propagated uncertainty (as shown in Fig. 8), the measured 140 Tin image noise values give required pixel sizes that are larger than the 140 Std AM images. Recall, however, that the 140 Tin images were acquired with around 25% of the dose as the 140 Std images. In theory, increasing the scanning dose by a factor of four will decrease the image noise by one half. The last column in Table III (AM 140 Tin with footnote “a”) simulates dose-matching by repeating the analysis when dividing the measured AM 140 Tin image noises in half. For most materials, it is seen that when simulating dose-matching, the 140 Tin spectral pair allows a target attenuation coefficient uncertainty to be achieved with pixels smaller than the AM 140 Std spectral pair.

4. DISCUSSION

The two-parameter models used to directly represent photon linear attenuation coefficients often have model-fitting accuracy around the 1% level.^{26,28,49} The mean accuracy of pDECT total linear attenuation coefficient estimation from real data demonstrated here, which includes both modeling and measurement error, is thus encouraging. In the case where a large number of voxels can be averaged to reduce statistical uncertainty, mean linear attenuation coefficient errors are less than 1% for energies between 30 keV and 1 MeV with errors rising to between 3% and 6% at 10 keV.

The propagated uncertainty analysis presented here highlights the sensitivity of the postprocessing DECT method to reconstructed image errors, with low atomic-number materials being particularly sensitive. Our experimental results are qualitatively consistent with the predictions from the error propagation analysis. Systematic streaking artifacts known to arise from excessively noisy data⁴⁶ were found to compromise mean pDECT accuracy for the MEK test substance in the larger body phantom. For MEK in the body phantom, data averaging to increase the effective scanning dose was needed to achieve acceptable accuracy, suggesting that random error was the dominant uncertainty in our idealized measurement geometry.

In this work, the increased spectral separation by using the 140 Tin spectrum is found to have similar mean attenuation coefficient estimation accuracy as the unfiltered 140 Std spectrum, despite delivering 25% of the scanning dose. The uncertainty propagation analysis confirms that increased spectral separation should result in less uncertainty propagated into the DE-estimated attenuation coefficients. This, in turn, relaxes the stringent reconstructed image uncertainty requirements to meet a target DE attenuation coefficient uncertainty. The data acquisition protocols used in this work utilized the maximum tube current available on the commercial scanner. Work is ongoing to explore data acquisition protocols with which the mAs per rotation can be increased allowing future studies to compare pDECT performance between the Std and

Tin spectral pairs using measured data with matched scanning dose.

Very few experiences of directly estimating attenuation coefficients from experimentally acquired dual-energy data are reported in the literature. Goodsitt *et al.* recently published on their experience using the GE dual-energy system to estimate the effective atomic number of and synthesized monochromatic images for a range of phantom materials.²⁴ The GE system utilizes fast-kVp switching for data acquisition and a proprietary prereconstruction method for decomposing the sinogram data into basis components. In the range of 40–120 keV, they observed synthesized monochromatic CT # errors on the order of 1% up to 20%. It is difficult to directly compare results of Goodsitt *et al.*²⁴ to those reported here. While both results show a dependence on test material, different test materials were chosen. In addition, no information regarding GE's proprietary basis materials or calibration procedure is available. However, Goodsitt *et al.*²⁴ provide a good discussion regarding the sensitivity of their results to phantom size and assumed test material density, which are similar to the results presented here.

In this work, both FBP and polyenergetic AM image reconstruction are shown to confer similar mean pDECT attenuation coefficient estimation accuracy, likely due to the idealized calibration procedure used here in which basis materials and test substances were scanned in identical phantom geometries. The ideal situation of nearly matched basis and test substance phantom geometries was intentional in this work to form a baseline for which to compare work with more complicated phantom geometries in the future. Future work will test DECT accuracy in more complex phantom arrangements, including varying basis and test material geometries and multiple test materials. We hypothesize the AM algorithm's more accurate forward model may confer more advantage over FBP in terms of mean DECT accuracy for these more complex arrangements, since the AM algorithm has been shown to minimize reconstructed image errors due to data nonlinearities such as beam-hardening and scatter.³⁵

The statistically motivated AM algorithm employed in this work demonstrates the ability to reconstruct attenuation coefficient estimates with less pixel-to-pixel variation around the mean value than FBP, as shown in Fig. 6 and Table II. Reconstructed image uncertainties targets on the order of 0.1%–0.3%, as shown in Table III, are unlikely to be practically achievable using existing CT hardware and image reconstruction algorithms. The noise advantage of the statistically motivated AM algorithm implies that to achieve a target level of estimated coefficient uncertainty either fewer pixels will need to be averaged, less smoothing applied to the images, or DECT data can be acquired at lower imaging dose than for conventional FBP reconstruction.

This raises the important question of what noise and image resolution is required in our target application of Monte Carlo dose calculation for low energy brachytherapy sources. The minimum spatial resolution required to achieve a specific dose calculation accuracy is difficult to estimate as there is little experience with model-based brachytherapy dose calculation at this point, especially in terms of voxel-based cross-

section assignment.² We view Table III as a first-order attempt to assess the lower bound on spatial resolution, in terms of basic pixel size, to achieve a target estimated attenuation coefficient uncertainty. The noise advantage of the AM algorithm shows the potential to achieve a target level of pDECT uncertainty for reconstructed pixels with in-plane dimensions approximately 40% smaller than FBP.

More aggressive image smoothing may prove to be a useful strategy for limiting pixel-to-pixel variability in the pDECT-estimated cross-section information. However, some early work suggests that the required DECT-estimated cross-section resolution will depend on the spatial gradient of the underlying tissue compositions, not on the gradients of the resultant dose distribution.⁵⁰ Patient tissue composition can certainly exhibit high frequency content, for example, adipose-glandular tissue boundaries in breast and calcifications in prostate, suggesting high spatial resolution for estimated cross-section information may be necessary. Future work will be needed to address the noise-resolution tradeoff issue in the specific context of using DECT-derived information to support Monte Carlo dose calculation for low energy brachytherapy sources.

We are also investigating different algorithms to utilize the DECT scan data for cross-section estimation. For example, use of the integrated dual-energy AM algorithm, which simultaneously operates on the DE sinogram data to estimate the basis component fractions,⁵¹ may be a more promising approach than the postprocessing method investigated here that uses a pair of fully reconstructed single-energy CT images. Another possibility is to constrain the DECT solution using *a priori* information, perhaps based upon anatomic appearance, to segment ROIs within the image that are likely to have similar composition but varying absolute densities.

5. CONCLUSIONS

This work has experimentally assessed the accuracy of a postprocessing DECT technique for estimating the total linear attenuation coefficient in the 10 keV to 1 MeV energy range from data acquired on a commercially available fan beam CT scanner. Inhomogeneous phantom inserts with compositions and densities spanning the range of biological tissues were investigated in idealized phantom geometries. Estimated photon attenuation coefficients were found to have mean bias on the order of 1%–3%, for both small headlike phantoms and larger bodylike phantoms for energies greater than 30 keV. For lower energies down to 10 keV, mean bias was found to increase to around 5%–6%. The larger body phantom necessitated scanning the basis materials and one test substance at a higher dose to avoid systematic streaking artifacts. Additional tin filtering of the high kVp beam to increase the spectral separation of the DE scans was found to confer similar mean accuracy to the standard x-ray spectra. Low-Z materials were found to be more sensitive to uncertainties in reconstructed images than high-Z materials.

In this scenario, with matched basis material calibration and test substance geometries, the main advantage of polyenergetic statistical image reconstruction is the improved noise

performance with a corresponding decrease in pixel-to-pixel estimated coefficient variability for a given scanning dose and spatial resolution. This implies that SIR can support pDECT cross-section estimation to a specified target uncertainty with less pixel averaging, less image smoothing, or less imaging dose than conventional FBP reconstruction. Polyenergetic SIR, which incorporates a more physically accurate forward model, is expected to demonstrate an advantage over FBP in future work that plans to utilize more complex phantom geometries. The fact that a simple postprocessing DECT method can estimate photon attenuation coefficients from experimental data acquired on a commercially available scanner on the order of 1% mean accuracy is encouraging for kV dose calculation applications, which are sensitive to material composition. However, the highly sensitive nature of the postprocessing DECT method may limit its clinical utility in more complex patient geometries where image uncertainty can easily exceed 0.5% and thus encourages research in other DECT methods for cross-section estimation.

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