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Statistical analysis of the pulse-coupled synchronization strategy for wireless sensor networks

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Abstract

Pulse-coupled synchronization is attracting increased attention in the sensor network community. Yet its properties have not been fully investigated. Using statistical analysis, we prove analytically that by controlling the number of connections at each node, synchronization can be guaranteed for generally pulse-coupled oscillators even in the presence of a refractory period. The approach does not require the initial phases to reside in half an oscillation cycle, which improves existing results. We also find that a refractory period can be strategically included to reduce idle listening at nearly no sacrifice to the synchronization probability. Given that reduced idle listening leads to higher energy efficiency in the synchronization process, the strategically added refractory period makes the synchronization scheme appealing to cheap sensor nodes, where energy is a precious system resource. We also analyzed the pulse-coupled synchronization in the presence of unreliable communication links and obtained similar results. QualNet experimental results are given to confirm the effectiveness of the theoretical predictions.

Index Terms

Pulse-coupled synchronization; refractory period; energy efficiency; wireless sensor networks; idle listening; unreliable links

I. Introduction

Pulse-coupled synchronization is a new decentralized synchronization strategy that is receiving increased attention in the communication community [1], [2], [3], [4]. It is inspired by the flashing of fireflies, the contraction of cardiac cells, and the firing of neurons [5], [6]. In the pulse-coupled synchronization strategy, synchronization is spontaneously established by exchanging simple identical pulses. Due to its inherent scalability and simplicity [2], [4], [7], the pulse-coupled synchronization strategy naturally applies to wireless sensor networks, which have shown a clear developmental trend to large network size, small node size, and low energy cost.

However, compared with the mature packet-based synchronization strategies such as RBS (Reference-Broadcast Synchronization) [8], TPSN (Timing-sync Protocol for Sensor

Networks) [9], FTSP (Flooding Time Synchronization Protocol) [10], GTSP (Gradient Time Synchronization Protocol) [11], PulseSync [12], and Glossy [13], the pulse-coupled synchronization strategy is relatively novel and is still under active development. One topic of particular interest is the refractory period in pulse based interaction, which naturally arises if antennas cannot transmit and receive signals simultaneously. The refractory period is the interval during which a node ignores all incoming pulses. In the pulse-coupled synchronization strategy, the refractory period starts immediately after firing (the start of the phase) since firing means signal transmission, during which the antenna is occupied and hence incoming signals have to be ignored. Due to its prevalence, the refractory period is receiving increased attention in the study of pulse-coupled synchronization [2], [14], [15], [16]. The authors in [2] showed that a refractory period is necessary to guarantee stable synchronization. The authors in [15] and [16] discussed the maximal allowable refractory period in an all-to-all connected oscillator network. Recently we proposed a new pulse-coupled synchronization protocol for arbitrarily-connected networks by strategically including a large refractory period to reduce idle listening [14]. Since the time to synchronization is analytically proven to be independent of the included refractory period, the new synchronization protocol can significantly improve energy efficiency, which is of great appeal to wireless sensor networks where nodes are typically battery driven. However, the protocol requires the initial phases to be distributed within half of an oscillation cycle. Although the requirement is fulfilled in [14] by using an initial flood, this extra flood increases the complexity of the synchronization protocol.

Another important topic under active investigation in pulse-coupled synchronization is sensor networks with unreliable communication links [3], [17]. Multi-hop wireless sensor networks have widely been known to be subject to message loss [18]. However, most existing results addressing unreliable pulse-based communication links are based on numerical methods [3], [17], and to our knowledge, the effect of unreliable message delivery on pulse-coupled synchronization has not been analytically studied.

This paper discusses pulse-coupled synchronization on a general-topology network in the presence of a refractory period. Since it does not restrict the maximal phase difference, the result extends the energy-efficient pulse-coupled synchronization algorithm in [14], which is only applicable when the initial phases are distributed within half of an oscillation cycle. The key idea of the paper is to increase the synchronizable region by controlling the minimal number of connections at local nodes. Given the inherent stochasticity in wireless communication and initial phase distribution, the analysis is conducted in a statistical framework. The probability of synchronization is also analyzed for pulse-coupled networks in the presence of unreliable communication links on which messages can only be delivered with a certain successful rate (probability). In this case, synchronization is also proven to be guaranteed by controlling the minimal number of connections at local nodes. The results are verified using QualNet experiments.

II. Related work

Although for spatially remote clocks, the problem of network synchronization has been widely investigated and various methods have been proposed [19], these methods cannot be used to synchronize wireless sensor networks, which require more precise time synchronization than in traditional applications (e.g., the Internet)—sometimes on the order of $1 \mu\text{sec}$ —due to their close coupling with the physical world and their energy constraints [8]. In recent years, numerous synchronization algorithms have been proposed for wireless sensor networks. Examples include RBS [8], TPSN [9], FTSP [10], PulseSync [12], and Glossy [13]. However, most of these algorithms have limited scalability because they either divide the network into connected single-hop clusters or organize the whole network into a

spanning tree, which is computationally challenging to construct for large-scale networks. To overcome the limitation, decentralized synchronization algorithms are receiving increased attention recently. Typical examples include Reachback Firefly Algorithm (RFA) [3], Scalable synch [20], GTSP (Gradient Time Synchronization Protocol) [11], Average TimeSynch [21] and Energy-efficient Sync [14]. These decentralized synchronization algorithms contain no special nodes (e.g., root) and all nodes run exactly identical algorithms, which make them scalable and robust to node failure as well as topology changes. Among these decentralized synchronization algorithms, pulse-coupled synchronization based algorithms (e.g., Scalable Sync [20], and Energy-efficient Sync [14]) stand out due to their inherent simplicity and efficiency. Pulse-coupled synchronization was first studied in mathematical biology to describe the synchronous flashing of fireflies, and firing neurons [5], [22], [23]. Recently, it has been shown to be extremely valuable for wireless sensor networks [4], [24], [25]. For example, the pulse-coupled synchronization strategy can operate exclusively at the physical layer [4], by transmitting simple identical pulses instead of packet messages. This eliminates the imprecision in traditional packet-based synchronization strategies due to Media Access Control (MAC) layer delays, packet processing, or software implementation. Furthermore, the pulse-coupled synchronization strategy treats each received pulse identically, since exchanged pulses are independent of their origin [2], [20], [26], thus avoiding requirements for memory storage of node information. Recently, the pulse-coupled synchronization strategy has also been shown to have a comparable, or superior, accuracy to the state of the art packet-based synchronization strategies [14], such as FTSP [10]. Experimental implementation of the pulse-coupled synchronization strategy have also confirmed its advantages over conventional sensor network synchronization protocols (e.g., RBS) [20], [27].

The main obstacle in the application of pulse-coupled synchronization to wireless sensor network synchronization lies in analysis and design. Pulse-coupled synchronization involves both nonlinear and impulsive dynamics, which make it very difficult to analyze analytically under a general interaction topology. Existing analytical treatments of pulse-coupled synchronization are conducted either under a special network topology (e.g., the all-to-all structure [2]) or under a special initial phase distribution (e.g., within half an oscillation cycle [14]), and do not give any rigorous mathematical guarantee for synchronization when topology is general and initial phases are unrestricted. This greatly restricted the application of pulse-coupled synchronization strategy to wireless sensor networks. In this paper, we prove analytically that for pulse-coupled oscillators with a general topology, synchronization can be guaranteed by controlling the number of connections at each node even in the presence of unrestricted initial phases. Moreover, by adding a refractory period wisely, we can reduce idle listening and hence energy consumption without sacrificing the synchronization probability. The results are applicable even in the presence of unreliable communication links, on which messages can only be delivered with a certain successful rate (probability).

III. Pulse-coupled synchronization and Phase response functions

In the pulse-coupled synchronization strategy, each sensor node behaves as an oscillator, whose phase rises from 0 to 2π with a constant speed. When the phase reaches 2π , a node will fire (emit a pulse), and simultaneously reset its phase to 0, after which the cycle repeats. When a node receives a pulse from a neighboring node, it shifts its phase by a certain amount. A schematic of pulse-coupled oscillators is given in Fig. 1, where (a) shows the phase evolution of an isolated sensor node, and (b) shows the phase evolution when a node can interact with neighbors and hence its phase can be modified during interaction (upon receiving a pulse from a neighbor). The magnitude of the modification (phase shift) ε depends on the exact timing of the incoming pulse relative to the phase of the node's own

oscillation. It is usually described by a phase response function, which is defined as the phase shift induced by a pulse as a function of the phase at which the pulse is received [6], [28]. In this paper, we consider the delay-advance phase response function shown in Fig. 2. It can be seen that under such a phase response function, the phase of a node can be either delayed (if a pulse is received when a node's phase is within $[D, \pi)$) or advanced (if a pulse is received when a node's phase is within $[\pi, 2\pi)$). D is the length of the refractory period and it is assumed to be within $(0, \pi)$ in this paper. If a pulse is received when a node's phase φ is within the refractory period $[0, D)$, the pulse will have no influence on φ , i.e., φ will evolve freely towards 2π . In other words, a node will not respond to incoming pulses during the refractory period. When a node's phase is outside the refractory period, the node will receive external pulses and make appropriate phase adjustments according to the phase response function. If multiple pulses are received simultaneously when a node's phase is outside the refractory period, then they can be regarded as one pulse since there is neither source nor destination information in each pulse. Since a node can ignore external pulses when its phase is within the refractory period $[0, D)$, it can shut down its transceiver and stays in the sleep mode during this time interval. So a refractory period is closely related to energy consumption in a synchronization process. Moreover, a refractory period has also been shown to be necessary to guarantee the stability of network synchronization [2]. In the following, we will discuss the synchronization of arbitrarily-connected pulse-coupled oscillator networks in the presence of a refractory period. We will consider both the reliable communication link case where every message can be successfully delivered and the unreliable communication link case where messages may be lost during delivery

IV. Problem formulation and preliminary results

We consider a network of N pulse-coupled oscillators with oscillation phases given by $\varphi_1, \varphi_2, \dots, \varphi_N$. The oscillators interact in a unidirectional manner, i.e., the fact that oscillator i can receive pulses from oscillator j does not necessarily mean that oscillator j can receive pulses from oscillator i . The network can be formulated as a directed graph $G = (V, E)$, where $V = \{v_1, v_2, \dots, v_N\}$ is a set of vertices corresponding to nodes in the network and E is a set of directed edges corresponding to the interaction between a pair of oscillators [29]. An edge directed from v_i to v_j is represented as (v_i, v_j) . It means that v_j can receive pulses from v_i but not necessarily vice versa. v_i and v_j are called the tail and head of the edge, respectively. A directed path in graph G is a sequence of edges $(v_{k_1}, v_{k_2}), (v_{k_2}, v_{k_3}), (v_{k_3}, v_{k_4}), \dots$. For a node i , the number of edges entering node i (using node i as edge head) is called the indegree of node i and is represented as δ_i^- ; the number of edges leaving node i (using node i as edge tail) is called the outdegree of node i and is represented as δ_i^+ . The value $\delta^- \triangleq \min_{1 \leq i \leq N} \delta_i^-$ is called the indegree of graph G , and $\delta^+ \triangleq \min_{1 \leq i \leq N} \delta_i^+$ is called the outdegree of graph G .

To guarantee the connectedness of the network, we make the following assumption:

Assumption 1: There is a directed multi-hop path from each vertex in the graph to every other vertex.

In order to get a rigorous mathematical treatment of pulse-coupled synchronization, we also make the following two assumptions:

Assumption 2: All nodes have identical oscillation frequency.

Assumption 3: Time delay in pulse exchange is negligible.

It is worth noting that since the radio coverage of a sensor is usually short, the delay caused by signal propagation is negligible in wireless sensor networks [9]. Moreover, the pulse-

coupled synchronization strategy can be implemented at the MAC layer or even lower, the physical layer, which prevents high-layer processing delays existing in conventional packet-based synchronization algorithms. Hence the pulse-coupled synchronization strategy has a much smaller delay in message communication than the conventional packet-based synchronization algorithms.

Next, we show some preliminary results which are necessary for the statistical analysis of pulse-coupled synchronization.

Lemma 1—[14] For pulse-coupled oscillators with a phase response function given in Fig. 2 (the length of the refractory period is $D < \pi$), if the phases of all nodes can fall within half of an oscillation cycle at some time instant, then pulse-coupled synchronization can be ensured for a network satisfying Assumptions 1–3. Moreover, the time to synchronization is independent of the length of the refractory period D .

In wireless communication, due to propagation impairment such as multi-path fading and shadowing, the links are usually subject to random failure [18]. In these cases, the pulsatile communication between a pair of nodes is associated with a certain probability of success. We represent the probability as p . Namely, when there is an edge between two nodes such that the tail node can send pulses to the head node, every pulse sent by the tail node may be successfully received by the head node (with probability p , $0 < p < 1$), or corrupted/lost during transmission (with probability $1 - p$). Lemma 2 gives a deterministic synchronization condition when the phases are restricted.

Lemma 2—For pulse-coupled oscillators with a phase response function given in Fig. 2 (the length of the refractory period is $D < \pi$), suppose the oscillators' initial phases are restricted to half of an oscillation cycle and Assumptions 1–3 are satisfied, then the oscillators can be perfectly synchronized for any D less than π even when communication is unreliable such that pulses are delivered with a successful rate (probability) of $0 < p < 1$. Moreover, the time to synchronization is independent of the length of the refractory period D .

Proof: Similar to the reasoning in Theorem 1 in [14], we can prove that after one oscillation period, the phase difference will be reduced (with probability p) or unchanged (with probability $1 - p$). Since p is larger than 0, the phase difference will finally be reduced to 0. The time to synchronization is determined by the initial phase difference and average reduction in phase difference in each iteration, which is independent of the length of the refractory period D according to the reasoning in Theorem 1 in [14], so the time to synchronization is independent of the length of the refractory period D .

V. Statistical analysis of pulse-coupled synchronization under reliable communication

Next, we show that for an arbitrary pulse-coupled network satisfying Assumptions 1–3, its probability of synchronization can be obtained analytically:

Theorem 1—For a size- N pulse-coupled network satisfying Assumptions 1–3 and having a phase response function given in Fig. 2, if the initial phases of component nodes are independent uniformly distributed over the phase space (the oscillation cycle), then the probability that the network will synchronize is no less than

$$P_{\text{lower}}\{\text{sync}\} = \prod_{i=1}^N \left\{ 1 - \frac{1}{2} \left(\frac{\pi+D}{2\pi} \right)^{\delta_i^-} + \frac{1}{2} \left[\left(\frac{\pi+D}{2\pi} \right)^{\delta_i^-} - \left(\frac{1}{2} \right)^{\delta_i^-} \right] \left[1 + \left(\frac{\pi+D}{2\pi} \right)^{\delta_i^-} \right] \right\} \quad (1)$$

where δ_{i*}^- denotes the minimal indegree among the nodes that have edges entering node i .

Proof: According to Lemma 1, if the phases can fall within half of an oscillation cycle at some time instant, then the network will be ensured to synchronize. So the probability of synchronization is no less than the probability of the event that the phases can fall within half of an oscillation cycle at some time instant.

Without loss of generality, we assume that the period of oscillation is 2π and the initial time instant is t_0 . If at time instant t_0 , for some node i , any of the following three events holds:

- event A_i : among all nodes that have an edge entering node i , at least one of them has phase residing in the interval $[\pi + D, 2\pi)$;
- event B_i : among all nodes that have an edge entering node i , all have phase residing in the interval $[0, \pi + D)$, and at least one of them satisfies the following two conditions: (1) having phase residing in the interval $[\pi, \pi + D)$; (2) having an entering edge whose tail node has phase residing in the interval $[\pi + D, 2\pi)$;
- event C_i : the phase of node i resides in the interval $[0, D)$ or $[\pi + D, 2\pi)$

then the phase of node i is ensured to be within $[0, \pi)$ at time instant $t_0 + \pi - D$. This is because 1) events A_i and B_i mean that node i will receive a pulse before time instant $t_0 + \pi - D$, and hence its phase will be less than π at $t_0 + \pi - D$; 2) event C_i guarantees that the phase of node i will be residing in the interval $[0, \pi)$ at time instant $t_0 + \pi - D$ (Note that the phase of node i cannot be advanced when its phase resides in $[0, \pi)$). So with probability $P\{A_i \cup B_i \cup C_i\}$, the phase of node i will be residing in the interval $[0, \pi)$ at time instant $t_0 + \pi - D$. Here ‘ $\cup$ ’ denotes the union operation of events.

Since both events A_i and B_i ensure that node i will receive a pulse within time $\pi - D$, and according to the phase response function, upon receiving a pulse, node i will either emit a pulse or stay silent, the fact that node i receives a pulse will make other nodes more likely to receive pulses. Therefore, we have

$$\begin{aligned} P\{(A_i \cup B_i) \cap (A_j \cup B_j)\} &= P\{(A_i \cup B_i)\} P\{(A_j \cup B_j) | (A_i \cup B_i)\} \\ &\geq P\{(A_i \cup B_i)\} P\{(A_j \cup B_j)\} \end{aligned} \quad (2)$$

for any $1 \leq i, j \leq N$. Here ‘ $\cap$ ’ denotes the intersection operation of events and $P\{\bullet | \bullet\}$ denotes conditional probability.

Furthermore, the independence between all initial phases leads to the independence between C_i ($1 \leq i \leq N$), which in combination with (2) yields

$$P\{\Delta_i \cap \Delta_j\} \geq P\{\Delta_i\} P\{\Delta_j\}, \quad \Delta_i \triangleq A_i \cup B_i \cup C_i \quad (3)$$

for any $1 \leq i, j \leq N$. Note that in (3), the independence between C_i and $\{A_i, B_i\}$ is employed.

Therefore, the probability that the phases of all N nodes reside in the interval $[0, \pi)$ at time instant $t_0 + \pi - D$ satisfies the following inequality:

$$P\{\Delta_1 \cap \Delta_2 \cap \dots \cap \Delta_N\} \geq P\{\Delta_1\}P\{\Delta_2\} \dots P\{\Delta_N\} \quad (4)$$

From probability theory, for each node, the probability $P\{\Delta_i\}$ can be obtained as follows:

$$\begin{aligned} P\{\Delta_j\} &= P\{A_i \cup B_i \cup C_i\} = P\{A_i \cup C_i\} + P\{B_i\} - P\{(A_i \cup C_i) \cap B_i\} \\ &= P\{A_i \cup C_i\} + P\{B_i\} - P\{(A_i \cap B_i) \cup (C_i \cap B_i)\} \end{aligned} \quad (5)$$

Taking into account the fact that A_i and B_i are pairwise disjoint, (5) can be rewritten as

$$P\{\Delta_i\} = P\{A_i \cup C_i\} + P\{B_i\} - P\{C_i \cap B_i\} \quad (6)$$

Since events B_i and C_i are independent of each other, (6) can be further rewritten as

$$P\{\Delta_i\} = P\{A_i \cup C_i\} + P\{B_i\} - P\{C_i\}P\{B_i\} \quad (7)$$

According to simple probability theory, we have the following relationship:

$$P\{A_i \cup C_i\} = 1 - P\{\overline{A_i \cup C_i}\} = 1 - P\{\overline{A_i} \cap \overline{C_i}\} = 1 - \frac{1}{2} \left(\frac{\pi + D}{2\pi} \right)^{\delta_i^-} \quad (8)$$

The probabilities of events B_i and C_i are given by

$$P\{B_i\} = \left[\left(\frac{\pi + D}{2\pi} \right)^{\delta_i^-} - \left(\frac{1}{2} \right)^{\delta_i^-} \right] \left[1 - \left(\frac{\pi + D}{2\pi} \right)^{\delta_{i^*}^-} \right] \quad (9)$$

and

$$P\{C_i\} = \frac{1}{2} \quad (10)$$

respectively, which, in combination with (8), (6), and (4) give (1).

Remark 1—Different from [14], the result in Theorem 1 is applicable even when the initial phases are not restricted to half of an oscillation cycle.

Remark 2—From the reasoning in the proof of Theorem 1, we can see that after one oscillation period, the phase difference will be reduced to within half an oscillation cycle. Then the phase difference will decrease to zero, as guaranteed by Lemma 1. Moreover, [14] has proven analytically that the time taken to reduce the phase difference from within half an oscillation cycle to zero is independent of the length of the refractory period D . So the length of the refractory period can affect time to synchronization only before the phase difference is reduced to within half an oscillation cycle. Given that the phase difference will be reduced to within half an oscillation cycle in one cycle's time (i.e., one oscillation period), the difference in the time to synchronization under different lengths of the refractory period is at most one oscillation period.

Theorem 1 shows that the probability of pulse-coupled synchronization depends on the number of connections at each node, i.e., δ_i^- and δ_{i*}^- . In the following, we will show that the probability of synchronization is an increasing function of both δ_i^- and δ_{i*}^- .

Theorem 2—The probability of synchronization in (1) increases with δ_i^- and δ_{i*}^- for all $i = 1, 2, \dots, N$.

Proof: Since $\left[\left(\frac{\pi+D}{2\pi} \right)^{\delta_i^-} - \left(\frac{1}{2} \right)^{\delta_i^-} \right]$ is always positive and D is within the interval $(0, \pi)$, it can be easily obtained that $P_{\text{lower}}\{\text{sync}\}$ in (1) is an increasing function of δ_i^- .

To prove that $P_{\text{lower}}\{\text{sync}\}$ in (1) is an increasing function of δ_i^- , we rewrite (1) as follows:

$$\begin{aligned} P_{\text{lower}}\{\text{sync}\} &= \prod_{i=1}^N \left\{ 1 - \frac{1}{2} \left(\frac{\pi+D}{2\pi} \right)^{\delta_i^-} + \frac{1}{2} \left(\frac{\pi+D}{2\pi} \right)^{\delta_i^-} - \frac{1}{2} \left(\frac{1}{2} \right)^{\delta_i^-} - \frac{1}{2} \left[\left(\frac{\pi+D}{2\pi} \right)^{\delta_i^-} - \left(\frac{1}{2} \right)^{\delta_i^-} \right] \left(\frac{\pi+D}{2\pi} \right)^{\delta_{i*}^-} \right\} \\ &= \prod_{i=1}^N \left\{ 1 - \frac{1}{2} \left(\frac{1}{2} \right)^{\delta_i^-} - \frac{1}{2} \left[\left(\frac{\pi+D}{2\pi} \right)^{\delta_i^-} - \left(\frac{1}{2} \right)^{\delta_i^-} \right] \left(\frac{\pi+D}{2\pi} \right)^{\delta_{i*}^-} \right\} \\ &= \prod_{i=1}^N \left\{ 1 - \frac{1}{2} \left[1 - \left(\frac{\pi+D}{2\pi} \right)^{\delta_{i*}^-} \right] \left(\frac{1}{2} \right)^{\delta_i^-} - \frac{1}{2} \left(\frac{\pi+D}{2\pi} \right)^{\delta_{i*}^-} \left(\frac{\pi+D}{2\pi} \right)^{\delta_i^-} \right\} \end{aligned} \quad (11)$$

Given that $\left[1 - \left(\frac{\pi+D}{2\pi} \right)^{\delta_{i*}^-} \right]$ and $\left(\frac{\pi+D}{2\pi} \right)^{\delta_{i*}^-}$ are positive, it can be easily verified that $P_{\text{lower}}\{\text{sync}\}$ is an increasing function of δ_i^- .

In fact, it can also be easily inferred that when δ_i^- and δ_{i*}^- tend to infinity, the lower-bound probability of synchronization $P_{\text{lower}}\{\text{sync}\}$ in (1) will tend to 1. So in combination with the property in Theorem 2 that the probability of synchronization is an increasing function of δ_i^- and δ_{i*}^- , we can conclude that any given probability of synchronization can be obtained by choosing appropriate δ_i^- and δ_{i*}^- .

Theorem 3—For a pulse-coupled oscillator network satisfying Assumptions 1–3 and having a phase response function given in Fig. 2, if the initial phases of nodes are independent uniformly distributed on the oscillation cycle, then any given probability of synchronization $P_0 \in (0, 1)$ can be ensured if δ^- (the indegree of graph G , which is defined as $\delta^- \triangleq \min_{1 \leq i \leq N} \delta_i^-$) is no less than δ determined by the following equation:

$$P_0 = \left\{ 1 - \frac{1}{2} \left(\frac{\pi+D}{2\pi} \right)^{\delta} + \frac{1}{2} \left[\left(\frac{\pi+D}{2\pi} \right)^{\delta} - \left(\frac{1}{2} \right)^{\delta} \right] \left[1 - \left(\frac{\pi+D}{2\pi} \right)^{\delta} \right] \right\}^N \quad (12)$$

Proof: Since δ^- is defined as $\delta^- \triangleq \min_{1 \leq i \leq N} \delta_i^-$, it satisfies $\delta^- \leq \delta_i^-$ and $\delta^- \leq \delta_{i*}^-$ for all $1 \leq i \leq N$. Then making use of the facts that $P_{\text{lower}}\{\text{sync}\}$ in (1) is an increasing function of δ_i^- and δ_{i*}^- , and $P_{\text{lower}}\{\text{sync}\}$ tends to 1 when δ_i^- and δ_{i*}^- tend to infinity, the theory can be easily obtained by replacing δ_i^- and δ_{i*}^- with δ^- .

Remark 3—Similar with Theorem 2, it can be easily verified that the right hand side of (12) is an increasing function of δ . Therefore, δ in (12) can be easily obtained using a numerical search method such as bisection.

An interesting property of δ in Theorem 3 is that it increases only logarithmically with the network size N . As shown in Table I, to guarantee a synchronization probability of 0.99, the required value of δ increases less than twice when the network size is increased by 10^3 fold (from 1×10^2 to 1×10^5) for any refractory period D . This makes the approach especially appealing for applications in wireless sensor networks, where the scale is getting larger [7].

Remark 4—Another interesting property of δ in Theorem 3 is that it is not sensitive to the length of the refractory period D when D is less than 0.5π (cf. Fig. 3). In fact, for a network with size $N = 1000$ and number of connection $\delta = 20$, compared with the refractory period free case, the probability of synchronization is only reduced by 0.0029 if a refractory period of 0.5π is strategically included in the phase response function. Given that the refractory period reduces idling listening and hence reduces energy consumption [14], it can increase energy efficiency. In other words, a refractory period can be strategically added in the phase response function to reduce energy consumption, with almost no sacrifice in synchronization performance.

Remark 5—Under commonly studied deployment scenarios, such as grid deployment [30], [31], random uniform deployment [30], [31], and Poisson deployment [30], the required number of indegree δ^- to guarantee a given probability of synchronization can be achieved by controlling node transmission range locally, and the needed transmission range can be easily calculated [30]. In scenarios where the indegree δ^- cannot be controlled using node transmission range due to, e.g., existence of walls and external interference, we conducted simulations and found that the pulse-coupled synchronization strategy may still be able to synchronize the network (but the probability of synchronization is lower).

VI. Statistical analysis of pulse-coupled synchronization under unreliable communication

Next, we give analytically the probability of synchronization under unreliable communication links and show that synchronization can still be achieved in this case. Due to unreliable communication links, each message sent can either be lost (with probability $1 - p$) or be successfully delivered (with probability p).

According to Lemma 2, when $0 < p < 1$ and the phases of all nodes are restricted within half an oscillation cycle, the network can always be synchronized. So when the phases are not restricted, the probability of synchronization is no less than the probability that the phases can reside within half an oscillation cycle at some time instant. Following the line of reasoning of Theorem 1, we can get a lower bound on the probability of synchronization even when the links are not reliable:

Theorem 4—For a size- N pulse-coupled network satisfying Assumptions 1–3 and having a phase response function given in Fig. 2, if the initial phases of component nodes are independent uniformly distributed over the phase space (the oscillation cycle), and every transmission is associated with a successful delivery rate $p \in (0, 1]$, then the probability that the network will synchronize is no less than

$$P'_{\text{lower}}\{\text{sync}\} = \prod_{i=1}^N \left\{ 1 - \frac{1}{2} \left(\frac{\pi+D}{2\pi} + (1-p) \frac{\pi-D}{2\pi} \right)^{\delta_i^-} + \frac{1}{2} \left[\left(\frac{\pi+D}{2\pi} \right)^{\delta_i^-} - \left(\frac{1}{2} + (1-p) \frac{D}{2\pi} \right)^{\delta_i^-} \right] \left[1 - \left(\frac{\pi+D}{2\pi} + (1-p) \frac{\pi-D}{2\pi} \right)^{\delta_{i^*}^-} \right] \right\} \quad (13)$$

Proof: Making use of Lemma 2 and a derivation similar to Theorem 1, one gets that the probability of synchronization under unreliable communication is no less than the probability of the event that all phases can fall within half of an oscillation cycle at some time instant.

Without loss of generality, we still assume that the period of oscillation is 2π and the initial time instant is t_0 . If at time instant t_0 , for some node i , any of the following three events holds:

- event A'_i : among all nodes that have an edge entering node i , at least one of them has phase residing in the interval $[\pi + D, 2\pi)$ and, its pulse sent at the end of this oscillation period can successfully reach node i ;
- event B'_i : among all nodes that have an edge entering node i , all have phase residing in the interval $[0, \pi + D)$, and at least one of them satisfies the following two conditions: (1) having phase residing in the interval $[\pi, \pi + D)$ and, its pulse sent at the end of this oscillation period can successfully reach node i ; (2) having an entering edge whose tail node ① has phase residing in the interval $[\pi + D, 2\pi)$ and ② when fires at the end of this oscillation period, successfully sends the pulse through the edge;
- event C'_i : the phase of node i resides in the interval $[0, D)$ or $[\pi + D, 2\pi)$

then the phase of node i is ensured to be within $[0, \pi)$ at time instant $t_0 + \pi - D$, even under unreliable communication. This is because 1) events A'_i and B'_i mean that node i will receive a pulse before time instant $t_0 + \pi - D$, and hence its phase will be less than π at time instant $t_0 + \pi - D$; 2) event C'_i guarantees that the phase of node i will be residing in the interval $[0, \pi)$ at time instant $t_0 + \pi - D$ (Note that the phase of node i cannot be advanced when its phase resides in $[0, \pi)$). So with probability $P\{A'_i \cup B'_i \cup C'_i\}$, the phase of node i will be residing in the interval $[0, \pi)$ at time instant $t_0 + \pi - D$.

Define $\Delta'_i \triangleq A'_i \cup B'_i \cup C'_i$. Then similar to the reliable communication link case, we can derive that

$$P\{\Delta'_1 \cap \Delta'_2 \cap \dots \cap \Delta'_N\} \geq P\{\Delta'_1\}P\{\Delta'_2\} \dots P\{\Delta'_N\}, \quad (14)$$

$$P\{\Delta'_i\} = P\{A'_i \cup C'_i\} + P\{B'_i\} - P\{C'_i\}P\{B'_i\} \quad (15)$$

and

$$P\{A'_i \cup C'_i\} = 1 - P\{\overline{A'_i \cup C'_i}\} = 1 - P\{\overline{A'_i} \cap \overline{C'_i}\} = 1 - P\{\overline{A'_i}\}P\{\overline{C'_i}\} \quad (16)$$

The probability $P\{\overline{A'_i}\}$ is determined by the probability of the union of the following events:

- event $\overline{A}_{i,0}$: all the nodes having an edge entering node i have phase residing in the interval $[0, \pi + D)$;

- event $\bar{A}'_{i,1}$: among all the nodes having an edge entering node i , only one node has phase residing in the interval $[\pi + D, 2\pi)$, but when it fires at the end of this oscillation period (which is within time $\pi - D$), its pulse fails to reach node i ;
- event $\bar{A}'_{i,2}$: among all the nodes having an edge entering node i , only two nodes have phase residing in the interval $[\pi + D, 2\pi)$, but when they fire at the end of their current respective oscillation periods (which are within time $\pi - D$), both of their pulses fail to reach node i ;
- ...;
- event $\bar{A}'_{i,j}$: among all the nodes having an edge entering node i , only j nodes have phase residing in the interval $[\pi + D, 2\pi)$, but when they fire at the end of their current respective oscillation periods (which are within time $\pi - D$), all of their pulses fail to reach node i ;
- ...;
- event $\bar{A}'_{i,\delta_i^-}$: among all δ_i^- nodes having an edge entering node i , all have phase residing in the interval $[\pi + D, 2\pi)$, but when they fire at the end of their current respective oscillation periods (which are within time $\pi - D$), all of their pulses fail to reach node i .

Since all events $\bar{A}'_{i,0}, \bar{A}'_{i,1}, \bar{A}'_{i,2}, \dots, \bar{A}'_{i,\delta_i^-}$ are disjoint with each other, the probability of event $\bar{A}'_i$ can be obtained as follows:

$$\begin{aligned} P\{\bar{A}'_i\} &= P\{\bar{A}'_{i,0} \cup \bar{A}'_{i,1} \cup \dots \cup \bar{A}'_{i,\delta_i^-}\} = P\{\bar{A}'_{i,0}\} + P\{\bar{A}'_{i,1}\} + \dots + P\{\bar{A}'_{i,\delta_i^-}\} \\ &= \left(\frac{\pi+D}{2\pi}\right)^{\delta_i^-} + (1-p)C_{\delta_i^-}^1 \left(\frac{\pi-D}{2\pi}\right)^1 \left(\frac{\pi+D}{2\pi}\right)^{\delta_i^- - 1} + (1-p)^2 C_{\delta_i^-}^2 \left(\frac{\pi-D}{2\pi}\right)^2 \left(\frac{\pi+D}{2\pi}\right)^{\delta_i^- - 2} + \dots + (1-p)^{\delta_i^- - 1} C_{\delta_i^-}^{\delta_i^- - 1} \left(\frac{\pi-D}{2\pi}\right)^{\delta_i^- - 1} \left(\frac{\pi+D}{2\pi}\right)^1 + (1-p)^{\delta_i^-} \left(\frac{\pi-D}{2\pi}\right)^{\delta_i^-} \\ &= \left(\frac{\pi+D}{2\pi} + (1-p)\frac{\pi-D}{2\pi}\right)^{\delta_i^-} \end{aligned}$$

where C_n^k denotes the k -combination of a set of n identical elements.

Event B'_i is in fact the intersection of the following events:

- event $B'_{i,1}$: among all δ_i^- nodes that have an edge directed to node i , all have phase in the interval $[0, D + \pi)$, and at least one of them has phase residing in the interval $[\pi, \pi + D)$ and furthermore, can successfully deliver its pulse to i when sending a pulse at the end of this oscillation period;
- event $B'_{i,2}$: the node in event $B'_{i,1}$ (owning an edge directed to node i and having phase residing in the interval $[\pi, \pi + D)$) has an entering edge whose tail has phase residing in the interval $[\pi + D, 2\pi)$ and can successfully deliver its pulse through the edge at the end of this oscillation period.

Following the reasoning in the calculation of $P\{A'_i\}$, we can get the probability of $P\{B'_{i,1}\}$ and $P\{B'_{i,2}\}$, respectively:

$$P\{B'_{i,1}\} = \left(\frac{\pi+D}{2\pi}\right)^{\delta_i^-} - \left[\left(\frac{1}{2}\right)^{\delta_i^-} + (1-p)C_{\delta_i^-}^1 \frac{D}{2\pi} \left(\frac{1}{2}\right)^{\delta_i^- - 1} + (1-p)^2 C_{\delta_i^-}^2 \left(\frac{D}{2\pi}\right)^2 \left(\frac{1}{2}\right)^{\delta_i^- - 2} + \dots + (1-p)^{\delta_i^-} C_{\delta_i^-}^{\delta_i^-} \left(\frac{D}{2\pi}\right)^{\delta_i^-} \right] \\ = \left(\frac{\pi+D}{2\pi}\right)^{\delta_i^-} - \left(\frac{1}{2} + (1-p)\frac{D}{2\pi}\right)^{\delta_i^-} \quad (18)$$

$$P\{B'_{i,2}\} = 1 - \left[\frac{\pi+D}{2\pi} + (1-p)\frac{\pi-D}{2\pi} \right]^{\delta_i^-} \quad (19)$$

Note that events $B'_{i,1}$ and $B'_{i,2}$ are mutually independent, so the probability of event B'_i can be obtained as follows:

$$P\{B'_i\} = P\{B'_{i,1} \cap B'_{i,2}\} = P\{B'_{i,1}\}P\{B'_{i,2}\} \quad (20)$$

The probability of event C'_i is given by

$$P\{C'_i\} = \frac{1}{2} \quad (21)$$

Combining (14), (15), (16), (17), (20), and (21) gives (13).

Remark 6—If each link has a different reliability, we can still analyze the probability of synchronization following the line of reasoning in the proof of Theorem 4 (although the complex derivation prevents an explicit general description of the probability). Moreover, if we denote the minimal reliability among all links as p , then we can prove that (13) can still give a lower bound on the probability of synchronization in the heterogeneous reliability case. The reason is as follows. We can prove that, in the heterogeneous reliability case,

$P\{\bar{A}'_{i,j}\}$ does not increase with the reliability of any link for all $j=1, 2, \dots, \delta_i^-$, hence $P\{\bar{A}'_i\}$ does not increase with the reliability of any link. We can also prove that $P\{B'_i\}$ does not decrease with the reliability of any link and $P\{C'_i\} = \frac{1}{2}$. Given that (15) can be rewritten as

$$P\{\Delta'_i\} = P\{A'_i \cup C'_i\} + (1 - P\{C'_i\})P\{B'_i\} \quad (22)$$

we know $P\{\Delta'_i\}$ does not decrease with an increase in the reliability of any link and hence $P\{\Delta'_1 \cap \Delta'_2 \cap \dots \cap \Delta'_N\}$ does not decrease with an increase in the reliability of any link. Therefore, we know if the minimal reliability among all links is p , then the probability that the network will synchronize is no less than (13), which is obtained under the assumption that all links have the minimal reliability p .

Remark 7—When the probability of successful delivery tends to $p = 1$, it can be easily verified that the probability of synchronization in (13) tends to the probability of synchronization in (1).

Remark 8—When the transmission range of every node covers the whole network, that is, every node has an edge entering every other node, then under unreliable communication the

considered network becomes an Erdős-Rényi network [32], where all nodes are connected randomly with independent probability p .

Remark 9—From the same reasoning in Remark 2, we can get that the length of the refractory period can affect the time to synchronization by at most one oscillation period.

Similar with Theorem 2, we can also prove that the probability of synchronization $P'_{\text{lower}}(\text{sync})$ always increases with δ_i^- and δ_{i*}^- , and it will also tend to 1 for any $0 < p < 1$ when δ_i^- and δ_{i*}^- are sufficiently large. This leads to Theorem 5:

Theorem 5—For a pulse-coupled oscillator network with phase response function given in Fig. 2 and probability of successful transmission $p \in (0, 1)$, if Assumptions 1–3 are satisfied and the initial phases of nodes are independent uniformly distributed on the oscillation cycle, then any given probability of synchronization $P'_0 \in (0, 1)$ can be ensured if δ^- (the indegree of graph G) is no less than δ' determined by the following equation:

$$P'_0 = \left\{ 1 - \frac{1}{2} \left(\frac{\pi + D}{2\pi} + (1-p) \frac{\pi - D}{2\pi} \right)^{\delta'} + \frac{1}{2} \left[\left(\frac{\pi + D}{2\pi} \right)^{\delta'} - \left(\frac{1}{2} + (1-p) \frac{D}{2\pi} \right)^{\delta'} \right] \left[1 - \left(\frac{\pi + D}{2\pi} + (1-p) \frac{\pi - D}{2\pi} \right)^{\delta'} \right] \right\}^N \quad (23)$$

Fig. 4 and Fig. 5 show the probability of synchronization under different link reliabilities p . It is clear from Fig. 4 that for a network of 1000 nodes, when the network indegree δ^- is no less than 50, synchronization can still be guaranteed even when the successful message delivery probability p is as low as 0.4. Moreover, Fig. 5 confirms that for the same network, even when the link is not reliable, a moderate refractory period (e.g., less than 0.3π) can still be added to reduce idle listening and hence energy consumption at almost no sacrifice to the probability of synchronization.

VII. QualNet experiments

A. Verification of the theoretical predictions

In this section, we use a network evaluation tool, QualNet, to verify the theoretical results. QualNet was released by Scalable Network Technologies in 2000 and has been widely used to predict the performance of sensor networks, MANETs, satellite networks, among others [33].

We considered a network of 200 nodes distributed on a $100m \times 100m$ square. Of all the nodes, 121 are fixed in the square with position represented by the blue dots in Fig. 6. They divide the square into 100 equal subsquares. The rest of the nodes ($200 - 121 = 79$) are initially uniformly distributed in the square and then move according to the random waypoint model [34]. That is, each of these nodes chooses at random a destination in the square and moves towards it along a straight line with a velocity chosen at random in the interval $(0m/s, 10m/s)$. When it reaches the destination, it remains stationary for a pause time of 5 seconds, and then it starts moving again according to the same rule. Every node is assumed to have an omnidirectional antenna with the same transmission range of $40m$.

From Theorem 3, it can be derived that if every node i has an indegree δ_i^- that is no less than 17, then the probability of synchronization is guaranteed to be above 0.99 for any refractory period $D \leq 0.5\pi$. Under the current node deployment and node transmission range, it can be derived that every node is guaranteed to access at least 16 fixed nodes and at the same time with probability over 0.999 accesses at least one mobile node. This means that with

probability over 0.999, every node has an indegree no less than 17, which can guarantee a lower bound of probability 0.99 of synchronization for any D not larger than 0.5π according to Theorem 3. To verify the prediction, we simulated the network under different lengths of the refractory period D . For each D , we ran the simulation for 10,000 times with initial phases in each run randomly chosen from a uniform distribution on $[0, 2\pi)$. The probabilities of synchronization under different refractory periods are summarized in Table II, all of which are indeed higher than 0.99, hence confirming the theoretical prediction. Moreover, we also compared the energy consumption, which is equal to the product of antenna power (can be obtained from the QualNet specifications), number of nodes, and time to synchronization [35]. The results are given in Table II, it can be seen that compared with the small refractory period case ($D = 0.1\pi$), using a refractory period of $D = 0.5\pi$ indeed reduced energy consumption by about a half while at the same time did not affect the probability of synchronization.

We also simulated the network in the presence of unreliable communications links. According to (23), we can calculate the required number of connections for each node to guarantee a probability of synchronization 0.99 under different message delivery reliabilities. The results are shown in Table III. In the implementation, we used a link reliability $p = 0.9$, i.e., any sent pulse can only reach destination with probability 0.9, and with probability 0.1 it will be lost. We also used a refractory period (of length $D = 0.3\pi$) to increase energy efficiency. According to Table III, if $\mathcal{J}$ is no less than 24, we can still guarantee that the network will synchronize with a probability no less than 0.99 even in the presence of unreliable communication. To verify the prediction, we simulated the network under different refractory periods $D = 0.1\pi, 0.15\pi, 0.2\pi, 0.25\pi, 0.3\pi$. For each D , we simulated the network for 10,000 times with initial phases in each run randomly chosen from a uniform distribution on $[0, 2\pi)$. The transmission range is set to 50 m and hence each node can access at least 26 nodes. The results are given in Table IV, which show that the network indeed synchronized with a probability larger than 0.99, and moreover, the refractory period indeed reduced energy consumption at almost no sacrifice to synchronization probability.

B. Performance evaluation of the proposed pulse-coupled synchronization approach

In this subsection, we evaluate the proposed pulse-coupled synchronization approach using QualNet experiments. More specifically, we evaluate the pulse-coupled synchronization strategy under different link reliabilities, refractory periods, network indegrees, and frequency drifts. To guarantee a fair evaluation, we only kept the 121 fixed nodes in Fig. 6 and remove the 79 mobile nodes to reduce uncertainties from random mobility.

1) Influence of link reliability on time to synchronization—When 121 nodes are distributed in a square according to Fig. 6, for any link reliability, we can calculate the required indegree that guarantees a probability of synchronization 0.99. To evaluate how the synchronization time is affected by link reliability, we set the transmission range to 58.4 m, which can guarantee an indegree 31, and hence, according to Theorem 5, can guarantee a synchronization probability no less than 0.99 even when the link reliability is 0.7. The times to synchronization under different link reliabilities are given in Table V. It can be seen that the time to synchronization increases with a decrease in link reliability.

2) Influence of refractory period on time to synchronization—We also evaluated the influence of refractory period on the time to synchronization using QualNet experiments. In the implementation, we set link reliability p to 1 and transmission range to 36.1 m. For the deployment in Fig. 6, the transmission range can ensure that the network indegree is no less than 13, and hence, according to Theorem 3, can synchronize the network with probability

no less than 0.99. We ran the network under different refractory periods $D = 0.2\pi, 0.3\pi, 0.4\pi, 0.5\pi$ to study the influence of the refractory period. The results are summarized in Table VI, which confirms the analysis in Remark 2 that the refractory period affects time to synchronization by at most one oscillation period (which is 1 s in the implementation).

3) Influence of network indegree on time to synchronization—We also studied the influence of network indegree on time to synchronization. In the implementation, we increased the transmission range to increase the network indegree. The transmission ranges used in the implementation are $d, 1.25d, 1.5d, 1.75d$, and $2d$, respectively, where $d = 36.1 m$. The corresponding network indegrees and times to synchronization are given in Table VII. It can be seen that the time to synchronization decreases with an increase in the network indegree.

4) Influence of network size on time to synchronization—We also evaluated the influence of network size on time to synchronization using QualNet experiments. The network sizes considered in the implementation are summarized in Table VIII. For all the network sizes considered, the transmission range is fixed to $36.1m$, which can guarantee a network indegree 13, and hence, according to Theorem 3, can guarantee a no less than 0.99 probability of synchronization. The time to synchronization under different network sizes are given in Table VIII. It can be seen that with an increase in network size, the time to synchronization increases.

5) Evaluation of the synchronization strategy in the presence of heterogeneous frequencies—Using the network deployment in Fig. 6 and transmission range $36.1 m$, which give an indegree 13 (and hence according to Theorem 3, can guarantee 0.99 probability of synchronization), we also evaluated the performance of the synchronization strategy in the presence of heterogeneous frequencies. Given that the clocks in typical sensor nodes can have clock drifts up to 100 ppm (part per million) [11], we set the frequency of each node to $1 + \Delta$ with Δ randomly chosen from the uniform distribution on $[-10^{-4}, 10^4]$. This corresponds to up to 100 ppm clock drift. Implementation results confirm that the network is still synchronized with a probability 1.0000. The time to synchronization is 3.86 s, which is comparable to the homogeneous frequency case. This shows that the proposed synchronization strategy is still applicable even in the presence of heterogeneous frequencies.

VIII. Conclusions

Pulse-coupled synchronization is a newly proposed decentralized synchronization strategy for wireless sensor networks. Based on statistical analysis, we show that the pulse-coupled synchronization strategy can guarantee synchronization in general coupling topologies with relaxed initial conditions, even in the presence of a large refractory period. The key point is to control the number of connections at each node, which can be realized by controlling the transmission range of each node locally. Since the required number of connections is shown to increase only logarithmically with the network size, the proposed strategy is applicable to large-scale networks. When the size is moderate, a refractory period is proven to have negligible influence on the probability of synchronization. Hence it can be strategically added in node cooperation to improve energy efficiency given that idle listening and energy consumption decrease with the length of the refractory period. The proposed strategy is proven to be able to guarantee synchronization even when the communication is unreliable and thus messages are delivered at a certain successful rate (probability). The theoretical predictions are confirmed by QualNet experiments.

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Fig. 1. Phase evolution of pulse-coupled oscillators. (a) Isolated oscillators (b) Coupled oscillators (ε : phase shift)

Fig. 2. Delay-advance phase response function used in the paper. D is the length of the refractory period.

Fig. 3.
The influence of refractory period on synchronization probability under different δ . The size of network is 1000.

Fig. 4.

The effects of link reliability on the probability of synchronization. The total number of nodes is $N = 1000$ and the network indegree is $\delta^- = 50$. The length of the refractory period is set to $D = 0$.

Fig. 5.

The effects of link reliability and refractory period on the probability of synchronization. The total number of nodes is $N = 1000$ and the network indegree is $\delta^- = 50$.

Fig. 6. The positions of the 121 fixed nodes in the QualNet experiments. The rest of the nodes are initially uniformly distributed in the square and then move according to the random waypoint model. The total number of nodes is 200.

The required value of δ for a probability of synchronization 0.99 under different network sizes and refractory periods

TABLE I

D	0.0π	0.1π	0.2π	0.3π	0.4π	0.5π	0.6π	0.7π	0.8π
	N								
1×10^2	13	13	13	13	14	16	20	27	41
1×10^3	16	16	16	16	17	19	25	34	52
1×10^4	19	19	19	19	20	23	30	41	63
1×10^5	23	23	23	23	23	27	35	48	74

Probability of synchronization and energy consumption under different lengths of the refractory period

TABLE II

	$D = 0.1\pi$	$D = 0.2\pi$	$D = 0.3\pi$	$D = 0.4\pi$	$D = 0.5\pi$
Fraction of synchronized runs	1.0000	1.0000	1.0000	1.0000	1.0000
Total energy consumption [J]	6.90	5.30	4.42	4.18	3.71

TABLE III

The required value of δ for a probability of synchronization 0.99 under different link reliabilities and refractory periods ($N = 200$)

p	$p = 1.0$	$p = 0.95$	$p = 0.9$	$p = 0.85$	$p = 0.8$	$p = 0.75$	$p = 0.7$	$p = 0.65$	$p = 0.6$
D									
0.0π	14	15	16	17	18	20	22	24	26
0.1π	14	16	18	19	21	23	25	27	30
0.2π	14	18	20	22	24	26	28	31	34
0.3π	14	21	24	26	28	31	33	36	40

Probability of synchronization and energy consumption under different lengths of the refractory period when the communication link is unreliable ($p = 0.9$)

TABLE IV

	$D = 0.1\pi$	$D = 0.15\pi$	$D = 0.2\pi$	$D = 0.25\pi$	$D = 0.3\pi$
Fraction of synchronized runs	0.9987	0.9987	0.9981	0.9982	0.9985
Total energy consumption [J]	25.03	19.32	17.60	16.35	14.50

TABLE V

Time to synchronization under different link reliabilities

Performance	$p = 0.90$	$p = 0.85$	$p = 0.80$	$p = 0.75$	$p = 0.70$
Fraction of synchronized runs	1.0000	1.0000	1.0000	1.0000	1.0000
Time to synchronization [s]	7.75	12.96	18.57	23.97	44.74

TABLE VI

Time to synchronization under different refractory period lengths

Performance	$D = 0.2\pi$	$D = 0.3\pi$	$D = 0.4\pi$	$D = 0.5\pi$
Fraction of synchronized runs	1.0000	1.0000	1.0000	1.0000
Time to synchronization [s]	2.66	2.30	2.54	3.27

TABLE VII
Time to synchronization under different transmission ranges and network indegrees

Transmission range	$d = 36.1$ m	$1.25d = 45.1$ m	$1.5d = 54.1$ m	$1.75d = 63.1$ m	$2d = 72.2$ m
Indegree	13	22	30	37	49
Fraction of synchronized runs	1.0000	1.0000	1.0000	1.0000	1.0000
Time to synchronization [s]	3.27	3.08	2.34	2.15	1.82

TABLE VIII

Time to synchronization under different network sizes

Network size	11 × 11	10 × 10	9 × 9	8 × 8	7 × 7	6 × 6
Fraction of synchronized runs	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Time to synchronization [s]	3.27	2.48	1.67	1.21	1.06	0.96