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A Statistical Analysis of the Bloch-Siebert B_1 Mapping Technique

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Abstract

A number of B_1 mapping methods have been introduced. A model to facilitate choice among these methods is valuable, as the performance of each technique is affected by a variety of factors, including acquisition SNR. The Bloch-Siebert Shift B_1 mapping method has recently garnered significant interest. In this manuscript, we present a statistical model suitable for analysis of the Bloch-Siebert Shift method. Unlike previously presented models, the analysis is valid in both low SNR and high SNR regimes. We present a detailed analysis of the performance of the Bloch-Siebert Shift B_1 mapping method across a broad range of acquisition scenarios, and compare it to two other B_1 mapping techniques (the Dual Angle method and the Phase Sensitive Method). Further validation of the model is presented through both Monte Carlo simulations and experimental results. The simulations and experimental results match the model well, lending confidence to its accuracy. Each technique is found to perform well with high acquisition SNR. However, our results suggest that the Dual Angle method is not reliable in low SNR environments. Furthermore, the Phase Sensitive method appears to outperform the Bloch-Siebert Shift method in these low-SNR cases, although variations of the Bloch-Siebert method may be possible that improve its performance at low SNR.

Keywords

Magnetic Resonance Imaging (MRI); B_1 Mapping; Dual-Angle; Bloch-Siebert Shift; Phase Sensitive; Statistical Analysis; Flip-Angle Mapping

1. Introduction

Variations in the radio frequency (RF) transmit field, or B_1 inhomogeneity, are a confounding factor for many different magnetic resonance imaging pulse sequences and applications. B_1 inhomogeneity results in flip angle variations across the imaging volume, leading to image intensity variations, errors in T_1 and other quantitative measurements (Deoni et al. 2003, Schabel & Morrell 2009, Schabel & Parker 2008, Warntjes et al. 2008), errors in actual RF waveforms resulting in degraded excitation profiles (e.g., in parallel transmission (Katscher et al. 2003, Zhu 2004, Xu et al. 2008)), and other undesired

consequences. Rapidly producing accurate maps of the non-uniform B_1 field is an active area of research. Flip angle maps can be used to correct errors in T_1 measurement or other quantitative MRI methods and are also necessary for the design of RF pulses for parallel transmission.

Several B_1 mapping techniques have been proposed (Morrell 2008, Cunningham et al. 2006, Insko & Bolinger 1993, Kerr et al. 2007, Stollberger & Wach 1996, Yarnykh 2007), but the relative performance of each technique across a range of conditions is not well understood. A thorough analysis of each method would facilitate choice among them. Such an analysis was recently conducted by Morrell & Schabel (2010) for five promising B_1 mapping techniques, including comparisons of mean bias and standard deviation of flip angle estimate across a range of basic measurement signal-to-noise ratios (SNRs). More recent work compared the efficacy of dual-angle B_1 mapping with Morrell's Phase Sensitive method in the low SNR environment of sodium imaging, validating some of Morrell & Schabel's previous analysis (Allen et al. 2011). However, since that analysis Sacolick et al. (2010) have introduced the Bloch-Siegert shift (BSS) method of B_1 mapping, which has garnered significant interest and yielded promising results. An analysis including the BSS method was done by Pohmann & Scheffler (2013) with a focus on the challenges of imaging at 7T. However, the choice of parameters used in their implementation severely limited the signal-to-noise ratio (SNR) efficiency of the methods that usually require longer repetition time (i.e., the Phase Sensitive method and the dual-angle method), limiting the applicability of the analysis. Therefore, a thorough analysis with optimal implementation of both the BSS and other methods has not yet been conducted.

The work of Morrell & Schabel (2010) compared five different B_1 mapping methods: the Phase Sensitive (PS) method (Morrell 2008); the gradient recalled echo dual-angle (GRE-DA) method (Cunningham et al. 2006, Insko & Bolinger 1993, Kerr et al. 2007, Stollberger & Wach 1996); an extended DA method (Insko & Bolinger 1993); the actual flip angle imaging (AFI) method (Yarnykh 2007); and an extended AFI method (Morrell & Schabel 2010).

In this study, we adapt the statistical framework used in Morrell & Schabel (2010) to analyze the performance of the BSS method, and compare it to the performance of both the PS and GRE-DA methods. We first present a theoretical derivation and statistical analysis of each technique, followed by a Monte Carlo simulation to validate theory. Finally, we provide quantitative experimental measurement data to validate the model and analysis. We then summarize the performance of each technique (as measured by mean bias and standard deviation of flip angle estimate) across a range of basic measurement SNRs. Our statistical model is shown to be consistent with both the Monte Carlo simulations and experimental results, which gives confidence in the model's accuracy.

2. Theory

B_1 mapping methods can be categorized into two groups: phase-based and magnitude-based. Phase-based B_1 mapping methods cause variations in the phase of an image that are dependent on the strength of the B_1 field, and use those variations to map flip angle. Magnitude-based B_1 mapping methods exploit variations in signal magnitude that are dependent on the B_1 field strength. Most methods require acquisition of two separate images. Magnitude-based methods typically use a ratio of signal intensities between two acquired images. Phase-based methods use phase differences between two acquired images to remove sources of phase other than flip angle. The PS and GRE-DA methods are included in this analysis as a phase-based and a magnitude-based reference point to the previous analysis of Morrell & Schabel (2010). While the GRE-DA method is magnitude-based, the

PS and BSS methods treated in this study are phase-based. Other methods could be included. However, the focus of this research is to explore the Bloch-Siebert B_1 mapping technique.

The PS and GRE-DA B_1 mapping techniques use the excitation pulse of the sequence to map the B_1 field as a flip angle. This leads to a flip angle which is directly proportional to the maximum B_1 field strength, $B_{1,\text{peak}}$. However, the BSS B_1 mapping technique estimates B_1 directly with a separate RF pulse.

The B_1 field may be related to the flip angle, α , by solving the Bloch equation, yielding

$$\alpha = \int_0^T \gamma B_1(t) dt, \quad (1)$$

where T is the duration of the applied B_1 pulse. Normalization of $B_1(t)$ by the maximum field strength, $B_{1,\text{peak}}$, yields $B_{1,\text{norm}}(t)$ which has a maximum of unity. $B_{1,\text{peak}}$ and α are then related by

$$\alpha = K_{\text{ex}} B_{1,\text{peak}}, \quad (2)$$

where

$$K_{\text{ex}} = \int_0^T \gamma B_{1,\text{norm}}(t) dt. \quad (3)$$

Therefore, when the excitation pulse is used for B_1 mapping and α is directly estimated, the excitation pulse may be used to calculate $B_{1,\text{peak}}$ from the flip angle estimates. Likewise, flip angle may also be calculated from an estimated $B_{1,\text{peak}}$ for any excitation pulse. Given this relationship, the terms B_1 and flip angle may be used interchangeably.

Below, we briefly review the theory for each treated B_1 mapping technique. While these are developed in the provided references, a summary of the relevant equations is provided here for clarity and consistency of notation.

2.1. Bloch-Siebert Shift B_1 Mapping Method

The BSS B_1 mapping method exploits the fact that the application of an off-resonance RF pulse slightly changes the effective frequency of spins that are on resonance (Bloch & Siebert 1940). This results in accrual of phase by previously excited magnetization in the transverse plane, with total phase accrual proportional to the RF pulse energy and inversely proportional to the off-resonance frequency. Specifically, the phase ϕ_{BSS} imparted to the transverse magnetization by the Bloch-Siebert RF pulse is described by

$$\phi_{\text{BSS}} = \int_0^T \omega_{\text{BSS}}(t) dt, \quad (4)$$

where

$$\omega_{\text{BSS}}(t) = -\Delta\omega_{\text{RF}}(t) + \sqrt{\Delta\omega_{\text{RF}}^2(t) + (\gamma B_1(t))^2} \approx \frac{(\gamma B_1(t))^2}{2\Delta\omega_{\text{RF}}(t)}, \text{ for } \Delta\omega_{\text{RF}}(t) \gg \gamma B_1(t). \quad (5)$$

The off-resonance frequency $\Delta\omega_{\text{RF}}(t)$ of the BSS pulse is defined relative to the Larmor frequency, and $B_1(t)$ represents the magnetic field strength of the BSS pulse. This expression can be simplified, as noted above, by defining the time-varying RF field strength $B_1(t)$ as the

product of the maximum B_1 amplitude, $B_{1,\text{peak}}$, and the normalized RF waveform $B_{1,\text{norm}}(t)$. Then the BSS phase can be defined in terms of $B_{1,\text{peak}}$ and a constant, K_{BSS} , which fully describe the RF waveform's potential for creating BSS phase shift:

$$\phi_{\text{BSS}} \approx \int_0^T \frac{(\gamma B_1(t))^2}{2\Delta\omega_{\text{RF}}(t)} dt = B_{1,\text{peak}}^2 \int_0^T \frac{(\gamma B_{1,\text{norm}}(t))^2}{2\Delta\omega_{\text{RF}}(t)} dt = B_{1,\text{peak}}^2 \cdot K_{\text{BSS}}, \quad (6)$$

where

$$K_{\text{BSS}} = \int_0^T \frac{(\gamma B_{1,\text{norm}}(t))^2}{2\Delta\omega_{\text{RF}}(t)} dt \quad (7)$$

and T is the period of time the BSS pulse is applied. Note that the rotation ϕ_{BSS} occurs approximately about the axis of the B_0 field when $\Delta\omega_{\text{RF}} \gg B_{1,\text{peak}}$. However, the actual rotation axis may be modeled with methods detailed by Rabi et al. (1954). Rabi et al. models the off-resonance frequency, $\Delta\omega_{\text{RF}}$, as a vector coaxial with the B_0 field and the B_1 field strength as a vector in the transverse plane with magnitude γB_1 . The sum of these two vectors is the axis of rotation. This vector sum may vary greatly from the axis of the B_0 field when the condition $\Delta\omega_{\text{RF}} \gg B_{1,\text{peak}}$ is not satisfied, in which case the approximation given in (5) no longer holds.

The $B_{1,\text{peak}}$ estimate is formed from the phase difference of two acquisitions, with the off-resonance frequency of the BSS pulse set to $\pm\Delta\omega_{\text{RF}}$ to eliminate other sources of phase in the image (Sacolick et al. 2010).

The BSS method uses two RF pulses. The first is a conventional RF excitation which creates the transverse magnetization. The second is the off-resonance BSS pulse which creates the incremental phase in the transverse magnetization on which B_1 estimation is based. These pulses can be chosen independent of each other. Once they are both chosen, and assuming the same coil is used for excitation and the BSS pulse, they will be proportional and scale with B_1 .

2.1.1. Optimal choice of RF waveform for Bloch-Siegert Shift B_1 Mapping

Method—As shown in (4) and (5), the phase difference created by the BSS RF pulse is proportional to the integral of the pulse amplitude squared and inversely proportional to the off-resonance frequency of the BSS pulse. B_1 estimation will have improved accuracy when the spread of measured phase with respect to variation in $B_{1,\text{peak}}$ is maximized; i.e., by using a BSS pulse with the largest possible energy at the lowest possible off-resonance frequency. BSS pulse energy is limited in practice by specific absorption rate (SAR), and BSS pulse off-resonance frequency is constrained by the frequency profile of the BSS pulse.

2.1.2. SAR constraints on BSS pulse—Accuracy of B_1 estimation improves with the energy of the BSS pulse, but in practice this is limited by SAR considerations. SAR constraints vary widely depending on main field strength, patient weight, and the RF coil being used. In general, larger pulse energy requires longer sequence repetition time (TR) in order to keep SAR within safe limits.

2.1.3. Constraints on the BSS pulse off-resonance frequency—Minimizing the off-resonance frequency of the BSS pulse maximizes ϕ_{BSS} , thereby maximizing the accuracy of B_1 estimation. However, the minimum off-resonance possible is constrained by the requirement that the BSS pulse must not cause excitation at the on-resonance frequency. To allow the smallest possible off-resonance frequency without creating excitation on-

resonance, the BSS pulse should have a very narrow transition band between the stopband and passband of its frequency spectrum. However, no matter how small the transition band, off-resonance is limited by the range of B_0 homogeneity expected in the experiment. In general, the BSS off-resonance frequency must be at least equal to the maximum expected frequency deviation from the Larmor frequency due to chemical shift and B_0 inhomogeneity plus the width of the pulse's transition band and half passband. Small residual excitation from the BSS pulse may be mitigated without affecting the prepared magnetization by surrounding the BSS pulse with reverse polarized crusher gradients to dephase any excited spins (Khalighi et al. 2012a).

The approximation in (5) and (6) is based on the assumption that $\Delta\omega_{\text{RF}} \gg \gamma B_1(t)$. When the $\Delta\omega_{\text{RF}}$ is decreased to the point where this assumption is invalid, a more accurate way to estimate the expected BSS phase must be used, such as Bloch simulations.

2.2. Phase Sensitive B_1 Mapping Method

The Phase Sensitive (PS) B_1 mapping method uses a composite excitation pulse of $2\alpha_y - \alpha_x$ to generate phase in the image which varies with flip angle. Measurement of this phase allows estimation of flip angle. Other sources of image phase are removed by taking the phase difference of two acquisitions, one with the $2\alpha_y$ pulse sign reversed. Equations for the magnetization after the composite excitation pulse of the PS method are

$$M_x = \pm \frac{M_0 \alpha \Delta\omega \tau}{\beta^2} [4\sin^2(\beta)\cos(\beta)] - \frac{M_0 \alpha \sin\beta}{\beta^3} [\alpha^2 \cos(2\beta) + \Delta\omega^2 \tau^2] \quad (8)$$

$$M_y = \pm \frac{M_0 2\alpha \sin(\beta)}{\beta^3} [\Delta\omega^2 \tau^2 \cos(2\beta) + \alpha^2 \cos(\beta)] + \frac{M_0 \alpha \Delta\omega \tau}{\beta^4} [1 - \cos(\beta)] [\alpha^2 \cos(2\beta) + \Delta\omega^2 \tau^2] \quad (9)$$

$$M_z = \pm \frac{2M_0 \alpha^2 \Delta\omega \tau \sin(\beta)}{\beta^3} [\cos(\beta) - \cos(2\beta)] + \frac{M_0 \alpha^2 \cos(2\beta) + \Delta\omega^2 \tau^2}{\beta^4} [\alpha^2 \cos(\beta) + \Delta\omega^2 \tau^2], \quad (10)$$

where $\beta = \sqrt{\alpha^2 + \Delta\omega^2 \tau^2}$, M_0 is the thermal equilibrium magnetization magnitude, $\Delta\omega$ is the off-resonance frequency, and τ is the duration of the α_x hard-pulse. The $2\alpha_y$ flip is achieved with a hard-pulse which is double the length of the α_x pulse while maintaining equal magnitude, and $\pm$ represents the sign of the 2α flip (Morrell 2008).

As seen in (8)–(10), flip angle estimation with the PS method is affected by off-resonance ($\Delta\omega$). For modest B_0 inhomogeneity, this effect contributes only minimally to the flip angle estimate and can be ignored (e.g. reference Allen et al. (2011)). In situations with higher B_0 inhomogeneity, the effect of off-resonance can be corrected with a B_0 map, which somewhat increases scan time. With severe B_0 inhomogeneity, the PS method can fail altogether for some ranges of flip angles.

2.3. GRE-Dual Angle B_1 Mapping Method

The GRE-DA B_1 mapping method uses the magnitudes of two images: one acquired at a nominal flip angle of α and another acquired at a nominal flip angle of 2α with signal magnitudes of

$$m_1 = M_0 \sin(\alpha) \quad (11)$$

and

$$m_2 = M_0 \sin(2\alpha) \quad (12)$$

respectively. M_0 denotes the magnitude of the thermal equilibrium magnetization. Other sources of variations in image magnitude are removed by forming the ratio of these two images,

$$r = \frac{m_1}{m_2} = \frac{1}{2\cos\alpha}. \quad (13)$$

This ratio of image magnitudes is used to estimate α (Insko & Bolinger 1993).

The GRE-DA B_1 mapping method is inherently an image magnitude-based method. Image magnitude has a dependence on T_1 that is not modeled accurately with (13) unless complete T_1 recovery is achieved with a long TR or saturation recovery (e.g. reference Cunningham et al. (2006)). If T_1 effects are not eliminated by long TR or saturation recovery, an additional mean bias will be introduced into the B_1 estimate.

3. Methods

In this study, we derived the theoretical probability density function (PDF) of the flip angle estimate provided by each of the three methods for a range of SNRs. We then verified the calculated PDFs with Monte Carlo simulation. Finally, we performed actual repeated phantom measurements at 3 T to verify both theory and simulation.

All experiments were performed on a Siemens (Erlangen, Germany) Tim Trio 3 T whole body scanner.

3.1. Derivation of PDF of flip angle estimate

The PDF of the flip angle estimate for a given true flip angle provides complete information about the accuracy of the method. Once the PDF has been determined, simpler useful measures of accuracy such as mean bias and variance of the flip angle estimate are also readily determined. To derive the PDF of the flip angle estimate for each B_1 mapping method, we begin with the assumption of Gaussian white noise corrupting the real and imaginary part of each image pixel value, and then derive the PDF of the flip angle estimate based on the functional relationships unique to each method between the measured image pixel values and the flip angle.

3.2. Definition of System SNR and SNR efficiency

We assume for each B_1 mapping method the same equilibrium longitudinal magnetization, M_0 , and the same standard deviation of noise, σ , defined as the standard deviation of the real or imaginary part of each complex image pixel value (for a given voxel size, receive bandwidth, and total readout time). System SNR is defined as the ratio of these values, M_0/σ .

Different B_1 mapping methods are optimally implemented with different sequence repetition times (TR), and therefore some methods are potentially faster than others. Methods with short repetition times might be conveniently implemented with a gradient recalled echo (GRE) acquisition with acquisition of one k-space line with each TR. Methods requiring longer repetition times may be more conveniently implemented with readout methods requiring fewer repetitions, such as echo-planar or spiral readout. An example of this approach would be the recent implementation of the dual angle method with spiral readout by Cunningham et al. (2006) for rapid cardiac B_1 mapping. We are interested in comparing

the accuracy of methods without regard to details of readout. To accomplish this while allowing each method to function at its optimal TR, we incorporate the concept of signal-to-noise efficiency in our analysis. The signal-to-noise efficiency is proportional to the SNR and to the inverse of the square root of the imaging time. If one imaging method runs optimally with half the TR of another method, the faster method can acquire two signal acquisitions for every one acquisition of the slower method, thereby achieving a signal-to-noise efficiency advantage of a factor of $\sqrt{2}$. This effect is accounted for in our analysis by scaling the SNR of the faster methods (i.e., the BSS method) by a factor equal to the square root of the number of signal averages they can perform for each single acquisition of the slower methods (e.g., the PS method). This approach correctly quantifies and compares the accuracy of each method operating at its optimal TR. This approach ignores the potential drawbacks of specific acquisition schemes (such as image distortion in an echo planar readout) and focuses instead only on the inherent accuracy of each method independent of specific readout methods. However, the optimal TR for the BSS method cannot be generally specified. Under time constraints, the BSS method performs best by maximizing the max B_1 strength of the BSS pulse subject to SAR constraints (Sacolick et al. 2010). Without time constraints, we determine the maximum B_1 our scanner allows. Using that B_1 value, we minimize the TR under SAR constraints and use the idea of SNR efficiency to compare methods. We present results from the BSS method with various parameters determined empirically to present as fair a comparison as possible.

3.3. Practical SAR Limitations Experiment

As previously mentioned, the performance of the BSS method improves with increasing BSS pulse power, but this is limited by SAR constraints. Work has been done to improve the performance of the method by investigating the tradeoffs between pulse length and SAR for various values of T_2^* (Khalighi et al. 2012b). In our analysis, we used a typical Fermi pulse envelope as published by Sacolick et al. (2010). Some improvement in performance may be possible with other RF pulse shapes.

To quantify the effect of SAR limits on achievable BSS pulse energy in a realistic scenario, we performed phantom experiments in which the BSS pulse energy was varied to explore the SAR limits built in to our scanner software (Park et al. 2012). For a given BSS pulse energy, we determined the minimum sequence TR allowed by the scanner. This experiment was performed twice with different RF coils: the built-in body coil, and a commercially available transmit-receive knee coil. Note that SAR constraints are dictated by many factors, including coil, anatomy being imaged, mass of volume being imaged, and method of calculation. Further work needs to be done in evaluating the SAR constraints of the BSS method, but is beyond the scope of this paper; we simply wanted to determine a set of reasonable parameters to allow us to validate our analytical framework. Safe B_1 field strengths for the BSS pulse may be very different than those in our experiments, depending on implementation and application.

3.4. Parameter Choice

The parameters chosen for the theoretical PDF analysis were based on experimentally determined BSS SAR limitations and human tissue-like parameters at 3 T. Tissue parameters chosen were $T_1 = 1$ s and $T_2 = 40$ ms. Total scan time was assumed constant, and parameters were then selected for each method to yield optimal accuracy and minimize noise in the allotted scan time. The PS and GRE-DA methods require a long TR for complete relaxation. The BSS method does not have the same requirement and is typically used in time constrained scenarios. However, as mentioned above, the B_1 strength will reach a peak value without time constraints, so multiple B_1 maps may be acquired and averaged in the same amount of time as a single B_1 map for PS or DA, resulting in an effectively higher

SNR. Four BSS pulse strength/TR combinations were considered. The first combination corresponds to the parameters used in the phantom experiment for direct comparison to the experimental data. The remaining three combinations were based on the experiment to find practical SAR limitations described above: the second and third are based on data from the extremity coil, and the last on data from the body coil. An 8 ms Fermi pulse was used for the BSS pulse with a $K_{\text{BSS}} = 7401 \text{ rad/mT}^2$, similar to that published by Sacolick et al. (2010). The echo times were chosen to correspond to the phantom experiment, which employed the minimum TE allowed by the sequence for the given parameters. BSS parameters were: TR = 3 s, 91 ms, 651 ms, and 260 ms; TE = 13 ms; $B_{1,\text{nom}} = 10 \mu\text{T}$, $4 \mu\text{T}$, $11.5 \mu\text{T}$, and $24 \mu\text{T}$;

$K_{\text{BSS}} = 7401 \frac{\text{rad}}{\text{mT}^2}$. A longer echo time for the BSS method is a result of the 8 ms Fermi pulse and 1 ms gradient crushers before and after the BSS pulse. The Ernst angle was calculated and used as the nominal excitation angle. DA Parameters were: TR = 3 s, TE = 3.84 ms, $\alpha_{\text{nom}} = 60^\circ$. PS parameters were: TR = 3 s, TE = 4.24 ms, $\alpha_{\text{nom}} = 90^\circ$.

3.5. Theoretical Model of Statistical Accuracy

We present below the derivation for the BSS method PDF. Following that derivation we present highlights of the PS and DA method PDF derivations (further details may be found in Morrell & Schabel (2010)).

3.5.1. Derivation of the PDF for the BSS B_1 Mapping Method—We modeled the noise in each pixel of an MR image as a bivariate Gaussian distribution. Assuming a noise standard deviation σ and a pixel magnitude ρ , the PDF of the phase Φ of that pixel is given by Blachman (1981):

$$f_{\Phi}(\phi) = \frac{1}{2} \exp\left(-\frac{\rho^2}{2\sigma^2}\right) + \frac{\rho \cos(\phi - \phi_{\text{BSS}})}{2\sqrt{2\pi}\sigma^2} \exp\left(-\frac{\rho^2}{2\sigma^2} \sin^2(\phi - \phi_{\text{BSS}})\right) \times \text{erfc}\left(-\frac{\rho}{\sqrt{2}\sigma^2} \cos(\phi - \phi_{\text{BSS}})\right), \quad (14)$$

where ϕ_{BSS} is the flip angle dependent phase resulting from the BSS pulse and $\text{erfc}(x)$ is the complementary error function. The function $\text{erfc}(x)$ is given by

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-u^2) du. \quad (15)$$

The pixel magnitude ρ depends on factors such as sequence timing and relaxation times T_1 and T_2 . Our analysis assumes the BSS method is used in a GRE sequence, resulting in a pixel magnitude given by the GRE signal equation

$$\rho = \frac{M_0(1 - \exp(-\frac{TR}{T_1}))\sin\alpha}{1 - \cos(\alpha)\exp(-\frac{TR}{T_1})}. \quad (16)$$

The phase difference of the two acquisitions for a given pixel is the random variable

$$\Theta = \Phi^+ - \Phi^-, \quad (17)$$

where Φ^+ and Φ^- are the random variables representing the phase of the signal acquired after application of a BSS pulse with off-resonance frequency $+\Delta\omega_{\text{RF}}$ and $-\Delta\omega_{\text{RF}}$ respectively. The PDF of Θ , $f_{\Theta}(\theta)$, is given by the convolution of the PDFs of the phase of each acquisition:

$$f_{\Theta}(\theta) = \int_{-\pi}^{\pi} f_{\Phi+}(-\tau) \cdot f_{\Phi-}(\theta - \tau) d\tau = f_{\Phi+}(-\phi) * f_{\Phi-}(\phi). \quad (18)$$

Solving (6) for $B_{1,\text{peak}}$ results in

$$B_{1,\text{peak}} \approx \left(\frac{\Theta}{2K_{\text{BSS}}} \right)^{\frac{1}{2}} = g(\Theta). \quad (19)$$

The PDF of $B_{1,\text{peak}}$ can then be calculated by

$$f_{B_{1,\text{peak}}}(x) = \frac{f_{\Theta}(g^{-1}(x))}{|g'(g^{-1}(x))|} \quad (20)$$

as described by Papoulis (1984), where $g^{-1}(x)$ is the inverse of $g(\Theta)$ and $g'(\Theta) = dg(\Theta)/d\Theta$. The functions $g^{-1}(a)$ and $g'(\Theta)$ are then given by

$$g^{-1}(a) \approx a^2 \cdot 2K_{\text{BSS}} \quad (21)$$

and

$$g'(\Theta) \approx \left(\frac{1}{8\Theta K_{\text{BSS}}} \right)^{\frac{1}{2}}, \quad (22)$$

which leads to

$$f_{B_{1,\text{peak}}}(x) = f_{\Theta}(x^2 \cdot 2K_{\text{BSS}}) 4K_{\text{BSS}} \cdot x. \quad (23)$$

We used numerical methods to solve (23) (e.g. differences to approximate derivatives and interpolation to estimate function values at unavailable sample points as needed). Since the approximations used in (5) and (19) do not hold over the complete range simulated, we used Bloch simulations, which do not require those assumptions, to solve for the BSS signal level as well (Park et al. 2011).

3.5.2. Derivation of the PDF for the PS B₁ Mapping Method—Analysis for the PS method parallels the BSS method. Equations (17), (18), and (20) are equivalent where ϕ_{PS} is substituted for ϕ_{BSS} in (18). The difference from the BSS method is in the signal level (ρ and ϕ_{PS}) which also results in a change of $g(x)$ in (20). Equations (8) – (10) are the basis of the signal magnitude and phase with

$$\rho = \sqrt{M_x^2 + M_y^2} \quad (24)$$

and

$$\phi_{PS} = \tan^{-1} \left(\frac{M_y}{M_x} \right). \quad (25)$$

The function, $g(x)$, that maps Φ to the estimated flip angle is calculated numerically using the same methods mentioned with the BSS PDF derivation. Further details are given by Morrell & Schabel (2010).

3.5.3. Derivation of the PDF for the DA B_1 Mapping Method— B_1 mapping with the DA method has a similar derivation to the BSS and PS methods. However, it is a magnitude-based method so (18) becomes

$$f_M(m) = \frac{m}{\sigma^2} I_0\left(\frac{m\rho}{\sigma^2}\right) \exp\left(-\frac{m^2 + \rho^2}{2\sigma^2}\right) \quad (26)$$

where $I_0(x)$ is a modified Bessel Function of the first kind expressed by

$$I_0(x) = \frac{1}{2\pi} \int_0^{2\pi} \exp(x \cos \theta) d\theta. \quad (27)$$

The PDF of (13) is

$$f_r(r) = \int_{-\infty}^{\infty} |x| f_{m_1}(rx) f_{m_2}(x) dx \quad (28)$$

according to Papoulis (1984). Equation (20) is used again with

$$g(x) = \frac{1}{2 \cos \alpha} \quad (29)$$

to calculate the final PDF for the B_1 estimate of the DA method.

3.6. Monte Carlo Verification

Bloch equations were used for Monte Carlo verification of theoretical results. Signal was estimated using Bloch equations for each method. Complex Gaussian noise was added to the calculated signal levels. Noise magnitude was determined by the desired system SNR. Following the addition of noise, a B_1 /flip angle estimate was calculated. The process was repeated 10,000 times per flip angle across a range of true flip angles, and PDFs, mean bias, and standard deviation of estimates were then calculated.

3.7. Experimental Verification

We constructed a 2% agarose phantom (by weight) doped with 0.61 ± 0.01 mM CuSO_4 with a T_1 of 890 ± 40 ms and a T_2^* of 49 ± 3 ms. T_1 and T_2^* measurements were verified immediately before all B_1 acquisitions. A planar phantom was used which measured $15 \text{ cm} \times 15 \text{ cm} \times 0.6 \text{ cm}$. The phantom size was chosen to produce a low system SNR, allowing the statistical model to be validated in a regime with more mean bias and larger standard deviation in a reasonable scan time. For all methods we used a TR of 5 s to eliminate any T_1 effects, matrix size of 512×64 , 2D acquisition with a non-selective excitation, a $17 \text{ cm} \times 17 \text{ cm}$ in plane field of view, and bandwidth of 400 Hz/pixel. We acquired 17 B_1 maps at 3 T with each method. Echo times for each technique were as follows: BSS TE = 13.9 ms, DA TE = 3.84 ms, and PS TE = 4.24 ms. The phantom was imaged in the periphery of a commercial transmit-receive bird cage coil, where significant B_1 variation is present.

A truth dataset was constructed from the Dual Angle method by averaging the 17 constituent images (increasing the effective SNR by a factor of $\sqrt{17}$) and creating a B_1 map from the averaged images. We then low-pass filtered the B_1 map from the averaged images for further reduction of noise. Data was used from the most uniform sections of the phantom, both in physical thickness and B_0 uniformity. The standard deviation and mean bias were then plotted as functions of the truth dataset.

The system SNR in the phantom experiment was calculated in order to correlate the experimental data with the theoretical results. The thermal equilibrium signal M_0 for each pixel was estimated from the mean of the signal magnitude from the 17 images for each flip angle mapping method, based on the signal equation for each method and the transmit and receive coil sensitivities. Coil sensitivity at each pixel was based on the flip angle “truth” from a B_1 map formed with the DA method from the average of the 17 images. Measured T_1 and T_2 values of the phantom also figured into the calculation of M_0 according to the signal equation for each method. The standard deviation of noise was estimated from the real channel in the background of each method across the 17 images and over multiple pixel values. The standard deviation of the image noise for all methods was found to be identical within the expected error.

Both the analytical treatment and Monte Carlo simulations assumed a single system SNR across a range of flip angles. Our experiment used a coil in an area of non-uniform receive sensitivity, so that system SNR varied with location. Therefore, we assumed the receive field is proportional to the transmit field by the principle of reciprocity for direct comparison with experimental results.

The major focus of this experimental work is to validate the analytical framework for studying the performance of the various B_1 mapping methods. Such a validation does not require optimal choice of parameters, and we chose for the phantom experiment to normalize TR across each of the methods. Once the analytical framework is experimentally validated, it can be used to predict the performance of each method with any choice of parameters. New excitation schemes for the BSS method can be evaluated in this analytical framework as they become available.

4. Results

4.1. Practical SAR Limitations Experiment

Experimental results of minimum allowed TR for a given BSS pulse strength are shown in Figure 1 for three different cases. First, the blue trace uses an extremity bird cage coil with an 8 ms Fermi pulse used for the BSS pulse. Second, the green trace uses the same setup used in the first case except the BSS pulse is lengthened to 16 ms with unchanged amplitude, resulting in twice the BSS phase and twice the SAR. Finally, the red trace uses the body coil with an 8 ms Fermi pulse.

4.2. Theoretical Model of Statistical Accuracy

Figure 2 shows the theoretical PDFs for the Bloch-Siegert Shift method, GRE-Dual Angle method, and the Phase Sensitive method for different values of flip angle. For concise visualization of results, we have displayed many one dimensional PDFs in a two dimensional picture: the horizontal axis represents the flip angle actually achieved; the vertical axis represents the PDF of the flip angle estimate; and the intensity is the measure of probability for estimation. Thus, each vertical line represents a PDF for a given true flip angle corresponding to the horizontal position on the line. Figures 3 and 4 show the mean bias and standard deviation, respectively, of the flip angle estimates at many actual flip angles.

A PDF with zero mean bias and zero standard deviation is a delta function at the theoretical flip angle. That PDF would appear as a diagonal line from the bottom left to the top right in figure 2. As standard deviation increases, the maximum value of the PDF decreases, which leads to a more broad and less distinct line (as is seen in the DA method). The BSS and PS methods have crisp lines except in areas of low SNR. This indicates a low standard

deviation of noise. Any curve in the PDF is a deviation from the ideal and represents a mean bias in the estimate. All methods show this deviation from the ideal at low flip angles in both standard deviation and mean bias.

Figures 3, 4, and 5 are simplified ways to display some of the information found in the PDFs. The axes on those figures represent $B_{1,nom}$ for the BSS method and α_{nom} for the PS and GRE-DA methods. $B_{1,nom}$ is related to the flip angle by a constant, K_{EX} , which depends on the excitation pulse being used. For example, a 1 ms hard pulse has a K_{EX} of 268 rad/mT which translates into an α_{nom} of 122.8° for a $B_{1,nom}$ of 8 μT and an α_{nom} of 153.6° for a $B_{1,nom}$ of 10 μT . The greatest variation among methods is at low flip angle and low SNR.

4.3. Monte Carlo Verification

Figure 6 shows the mean bias, standard deviation, and out of bounds measurements of the flip angle estimates at many actual flip angles comparing theory to Monte Carlo simulation. Note there is excellent agreement for all methods, giving us confidence in our theoretical model. The only obvious deviation is in the DA method. As the actual flip angle of the DA method approaches 90° it becomes undefined.

4.4. Experimental Verification

Figure 7 shows the mean bias and the standard deviation results of the phantom experiment compared to theory. There is excellent agreement between the experimental results and theory. In the GRE-DA method the experimental data appears to diverge in mean bias from the analytical results. This is likely due to fewer data points being available at lower flip angles which causes the increase in mean bias to appear more steep as the density of data points in the horizontal direction decreases toward the left of the figure. Note that a larger standard deviation of noise leads to a more broad clustering of experimental points around the theoretical line.

5. Discussion

Our theoretical derivation of the PDF of the flip angle estimate for the BSS, DA, and PS methods of flip angle mapping is verified by both the Monte Carlo simulations and the experimental data from a phantom study. This analysis shows that the performance of the BSS method depends heavily on the power of the BSS pulse that is used, which is constrained by SAR. SAR in turn depends heavily on the geometry of individual RF coils. When used at the SAR limits of the whole-body RF coil, the BSS method performs more accurately than either the DA or PS methods. For the smaller extremity coil, the PS method has the best performance not accounting for off-resonance effects. Including off-resonance in the analysis would potentially skew the mean bias or require additional time for a B_0 map, effectively lowering the SNR efficiency of the PS method. Sacolick et al. (2010) stated that an optimal pulse shape maximizes the BSS without directly exciting spins in a sample. Interestingly, an optimal pulse translates into SNR efficiency of BSS B_1 mapping when TR is constrained by SAR. At times, image SNR efficiency increases with lowered flip angle and TR. However, when operating at the practical SAR limits of our scanner, a higher power BSS pulse and corresponding longer sequence TR give better SNR efficiency than a lower power BSS pulse and shorter TR. Our simulations suggest that this trend continues even to large values of TR; the accuracy of the BSS method is helped more by increased BSS pulse power at the cost of longer TR than by the possibility of signal averaging with shorter TR.

Use of the body coil for RF transmission allowed higher BSS pulse strength for a given TR than use of a less homogeneous extremity coil, giving better accuracy of flip angle measurement. However, measurement of flip angle is typically of most importance when

coil sensitivity is inhomogeneous. Thus the performance of the BSS method with the body RF coil may not be a realistic measure of the usefulness of this method in real world applications, where coil sensitivity inhomogeneity causes tighter SAR constraints which hurt the performance of the BSS method. In these situations the PS method is likely to give better accuracy, as was the case with the transmit-receive extremity volume coil.

A recent analysis of the PS and BSS methods (Pohmann & Scheffler 2013) has suggested that the BSS method gives better accuracy than the PS method. As mentioned above in the Introduction, this analysis is limited by a choice of parameters which cripples the performance of the methods which typically require longer TR. For ease of implementation, the authors of that study chose the same TR (60 ms) for every B_1 mapping method they evaluated. This short TR resulted in SAR constraints which limited the PS method to a nominal flip angle of 35° at 9.4 T rather than the typical nominal flip angle of 90° that would be used with this method. Cunningham et al. (2006) published results of DA B_1 maps *in vivo* during a breath hold with longer TR. The TR of the PS method could be lengthened in a similar way. At this short TR and low flip angle, the performance of the PS method is severely compromised. Our simulations have shown that for the T_1 of 1750 ms used in (Pohmann & Scheffler 2013) the PS method achieves much higher SNR efficiency with a TR of 3000 ms, flip angle of 90° , and a single signal acquisition than with the parameters used by Pohmann & Scheffler (2013) at 9.4T (TR of 60 ms and flip angle of 35°) and fifty signal averages. This underscores the need when comparing flip angle mapping methods to operate each in the parameter regime where its SNR efficiency is optimized. Our simulations confirm that the PS method gives best SNR efficiency when used with relatively long TR.

Interestingly, when using a constant off-resonance frequency the length or shape of the BSS RF waveform does not affect the tradeoff of SAR with B_1 estimation accuracy. The phase created by the BSS pulse depends on the pulse energy, or the time integral of the squared RF pulse magnitude. SAR is represented by the same quantity. Therefore, any pulse with a given SAR has the same efficacy in creating BSS phase. When designing the constant off-resonance frequency BSS pulse for a given B_1 mapping experiment, it is reasonable to determine what the maximum allowable SAR is for the desired TR, and then implement the shortest and highest amplitude BSS pulse possible which creates this SAR. The shortest possible BSS pulse minimizes SNR loss due to T_2^* signal decay. However, there is a tradeoff between the shortness of the BSS pulse and the sharpness of the transition band of the pulse's frequency response, which is important for achieving the smallest off-resonance possible.

Monte Carlo simulations and experiment results shown in Figures 6 and 7 demonstrate excellent agreement with the model which leads to a high level of confidence that the model is accurate. Having been validated by experiment, the model may be used to facilitate choice among B_1 mapping methods for specific cases.

Our analysis has focused on the SNR efficiency of B_1 mapping methods without regard to other specific details of pulse sequence design such as readout strategies. In practical implementation of a B_1 mapping sequence, other factors may be important. For instance, the BSS method lends itself to slice-selective B_1 mapping, while the PS and DA methods require non-selective excitation to avoid slice profile effects. This might make the BSS method advantageous for rapid single-slice B_1 mapping. The PS method has some sensitivity to B_0 inhomogeneity which may limit its performance in some environments. Implementation of echo-planar or spiral readout for rapid B_1 mapping with the PS or DA methods incurs increased complexity of pulse sequence design and image reconstruction, and may introduce image artifacts. In high SNR environments, the noise performance of a

method may be less important than its ease of implementation. It is likely for this reason that the DA method remains in widespread use despite its inferior noise performance. In any B_1 mapping application, the need for accuracy in the presence of noise must be balanced with practical considerations of implementation.

6. Conclusion

In this work, we have presented a theoretical derivation of the statistical performance (probability distribution functions and derived mean bias, standard deviation, and probability of out of bounds measurements) of the BSS, GRE-DA, and PS methods of B_1 mapping. We validated the theoretical framework with both Monte Carlo simulations and phantom experiments. All three showed excellent agreement. Our statistical framework may be used to analyze the mean bias and standard deviation of flip angle estimates for any choice of parameters for these methods. Such analysis will inform choice among B_1 mapping methods for specific applications.

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Figure 1. Empirical measurement of TR constraints on BSS method due to SAR

Minimum allowed TR as a function of BSS pulse amplitude determined empirically on our 3T scanner with different RF coils. Note that the extremity coil has more stringent constraints because its field is localized and likely less uniform than the body coil. The 16 ms BSS pulse used with the extremity coil has the same $B_{1,peak}$ as the 8 ms pulse, but twice the power.

Figure 2. Theoretical probability distribution functions

Bloch–Siegert Shift method (left column), Phase Sensitive method (center column), and Dual Angle method (right column). System SNR of 10, 25, 50, and 100 from top to the bottom rows. Note that the PS and BSS method PDFs show up more clearly than the DA method because all PDFs are scaled the same and the DA method has a larger standard deviation which leads to a broader PDF. Each column in the image represents a probability distribution function at a specific flip angle or B_1 strength. The image intensity represents the value of the PDF for each possible flip angle (row). The nominal flip angles for the DA

and PS method are 60° and 90° . The nominal field strength (which is proportional to flip angle if the pulse shape is known) for the BSS method is $10 \mu T$.

Figure 3. Analytical mean bias of flip angle estimate

Calculated at system SNRs of 10, 25, 50, and 100 (from top to bottom). Both axes are normalized by α_0 , the nominal flip angle for each method. Note that the Phase Sensitive method performs consistently well and the Bloch-Siebert Shift method can perform well over a range of flip angles. However, the performance of the BSS method varies greatly with parameter choice. The choice of parameters for the BSS method which show less mean bias at lower flip angles than the PS method are determined from SAR limitations caused by a homogenous body coil and likely not practical for cases with an inhomogenous B_1 field where a B_1 map is required.

Figure 4. Analytical standard deviation of flip angle estimate

Calculated at system SNRs of 10, 25, 50, and 100 (from top to bottom). Both axes are normalized by α_0 , the nominal flip angle for each method. Notice that all methods have increasing standard deviation of noise with decreasing flip angle and decreasing SNR. However, the phase-based methods (Bloch-Siebert Shift and Phase Sensitive) can degrade slowly with careful parameter choice. This will lead to more accurate estimates of flip angle. The Bloch-Siebert Shift method has the best theoretical performance in low SNR or low flip angle areas, but may be limited by SAR constraints.

Figure 5. Analytical out of bound measurements of flip angle estimates

Calculated at system SNRs of 10, 25, 50, 100 (from top to bottom). Both axes are normalized by α_0 , the nominal flip angle for each method. The Dual Angle method flip angle estimates are undefined above actual flip angles of 90° ($1.5 \cdot \alpha_0$) or above. The probability of out of bounds measurements for the phase-based methods is significantly lower over a larger range of flip angles than for the DA method. The Phase Sensitive method demonstrates a lower probability of out of bounds measurements than all other methods simulated with practical parameters at lower flip angles. The Bloch-Siebert shift method has consistently low probability of out of bounds measurements as the flip angles become greater.

Figure 6. Comparison between Monte Carlo simulations and analytical results
Plots of the mean bias (top), standard deviation (middle), and probability of out of bounds measurements (bottom). Note the excellent agreement between theory and Monte Carlo simulations. The comparisons were calculated with a system SNR of 50.

Figure 7. Experimental results compared to analytic model

Plot of mean bias (top row) and standard deviation (bottom row) for the Dual Angle method (first column), Phase Sensitive method (second column), and Bloch-Siebert Shift method (third column) for the experimental results (blue dots) compared to the analytic model (black line). The system SNR, T_1 , and T_2 were estimated to be 93, 900 ms, and 49 ms respectively. The experimental results and model are in very good agreement.