Attenuation correction in SPECT without attenuation map

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Abstract

A method of attenuation correction in Single Photon Emission Computed Tomography (SPECT), based on emission data only is presented. The algorithm uses the well known Helgason-Ludwig consistency conditions. However, it does not attempt to find the attenuation map, but rather the correction factors for the projection data, which makes the problem simpler and does not need to assume any particular template of the attenuation map. Although the method alone gives only approximate correction, it can be combined with other approaches to provide an effective improvement for scanning systems without the transmission scan functionality.

I. Introduction

Attenuation correction is a necessary step not only for physically and quantitatively accurate image reconstruction, but also for reliable clinical diagnosis. Modern SPECT scanners often come with the CT, providing the accurate anatomical image, which can be easily converted to attenuation map and used for attenuation correction. However, a large number of devices without the capability of acquiring transmission scans is still in use. In procedures where only the functional image is used for diagnosis, e.g. in cardiac imaging, the attenuation correction is often not a sufficient advantage to justify the increase the complexity and cost of the scanner by fitting a transmission source. Moreover, since the transmission scan is performed separately from the emission measurement, there is always the possibility of artifacts due to e.g. misalignment of the two datasets.

Such problems are avoided in emission-data-only based attenuation correction. Several methods of this kind have been studied. They usually rely on the Helgason-Ludwig consistency conditions [1]. The idea is to assume an attenuation map in a simplified form, described by a small number of parameters, e.g. a uniformly attenuating ellipse. These parameters are fitted to the measured data using the consistency equations [2-4].

In this paper we propose a method of attenuation correction using the emission data and based on the consistency conditions, but without referring explicitly to the attenuation map.

II. Materials and Methods

The consistency conditions for the attenuated projections of a 2D activity distribution can be written as

$$\int_0^{2\pi} \int_{-\infty}^{\infty} s^m e^{ik\varphi} \exp([T(s, \varphi) + i\mathcal{H} T(s, \varphi)]/2) E(s, \varphi) ds d\varphi = 0 \quad (1)$$

where $T(s, \varphi) = \int_{-\infty}^{\infty} \exp(-\int_l^{\infty} \mu(l', \varphi) dl')$ - is the transmission projection, $\mathcal{H}$ - Hilbert transform, $m \geq 0$ and $k > m$ are integer parameters, and $E(s, \varphi)$ denotes the measured projections in standard sinogram variables. In practical applications the integers m and k should be rather small in order to prevent numerical and statistical errors from expanding.

This limits the number of “useful” equations (1) to the order of 10. The usual way to apply the consistency conditions for attenuation correction is to assume an attenuation map μ in some constrained form, dependent on a few free parameters which can be solved for after plugging the parameterised μ into (1). This typically leads to a nonlinear minimisation problem. The effective attenuation map obtained in this way is then used for attenuation correction.

In the method presented in this paper we do not try to find the attenuation map μ , but start from the obvious fact that the measured projections are related to the ideal attenuation free projections $E_0(s, \varphi)$ through the equation

$$E_0(s, \varphi) = C(s, \varphi) E(s, \varphi) \quad (2)$$

where $C(s, \varphi)$ is a projection correction function, depending on both the attenuation map and activity distribution; eq. (2) can be regarded as the definition of $C(s, \varphi)$. To perform attenuation correction it is sufficient to find the function $C(s, \varphi)$.

We use the fact that the attenuation free projections have to fulfill the consistency conditions (1) in the limit $\mu \rightarrow 0$:

$$\int_0^{2\pi} \int_{-\infty}^{\infty} s^m e^{ik\varphi} C(s, \varphi) E(s, \varphi) ds d\varphi = 0. \quad (3)$$

Similarly as for the Eq. (1), the consistency conditions (3) are not sufficient to fully determine the function $C(s, \varphi)$. However, to reduce the number of unknowns it can be constrained by expanding it in an orthogonal function basis and restricting to a few most significant terms. We use the Fourier expansion:

$$C(s, \varphi) = \sum_{\nu_\varphi=-N_\varphi}^{N_\varphi} \sum_{\nu_s=-N_s}^{N_s} C_{\nu_\varphi \nu_s} e^{i\nu_\varphi \varphi} e^{i2\pi \nu_s s/L} \quad (4)$$

where L is the width of the detector. Inserting (4) into (3) we obtain

$$\sum_{\nu_\varphi=-N_\varphi}^{N_\varphi} \sum_{\nu_s=-N_s}^{N_s} W_{\nu_\varphi \nu_s}^{(m,k)} C_{\nu_\varphi \nu_s} = 0, \quad (5)$$

where the coefficients $W_{\nu_\varphi \nu_s}^{(m,k)} = \int_0^{2\pi} \int_{-\infty}^{\infty} s^m e^{i(k+\nu_\varphi)\varphi} e^{i2\pi \nu_s s/L} E(s, \varphi) ds d\varphi$ can be computed directly from the measured projections. The set of equations (5), indexed by m and k , is a linear homogenous problem. It is equivalent to finding the null space of the W matrix, and can be solved efficiently using e.g. singular value decomposition (SVD). Since the correction function is real, the complex coefficients fulfil $C_{\nu_\varphi \nu_s} = C_{-\nu_\varphi -\nu_s}^*$. As the number of meaningful equations, indexed by m and k , is small, the system of equations (5) is typically underdetermined. Then SVD yields a subspace of “base” solutions, $c_n(s, \varphi)$, and any linear combination

$$C(s, \varphi) = \sum_n a_n c_n(s, \varphi) \quad (6)$$

is also a solution. Thus, the consistency conditions (3) define a subspace of functions in which the true correction function should be embedded. In order to obtain the correction function for a particular set of measured projections we need to pick up one solution from the subspace defined in Eq. (6), by determining the coefficients a_n . This can be done in

many ways; here we simply project a “prototype” correction function $C_0(s, \varphi)$ onto the subset of consistent solutions, i.e. we assume

$$a_n = \int \int C_0(s, \varphi) c_n(s, \varphi) \quad (7)$$

In the simplest case when no a priori knowledge of the correction function is assumed the function $C_0(s, \varphi)$ can be defined as uniform, equal to 1 for all nonzero projection pixels; this corresponds to the “no attenuation correction” case. One can also use a more sophisticated function which reflects some known properties of the correction function.

III. Results

The algorithm has been applied to simulated projections of the NCAT phantom [5] with activity distribution typical for the myocardial perfusion imaging and the attenuation map for ^{201}Tl principal energies of 70–80 keV. 360° scan has been simulated; no scatter or collimator-detector response has been modeled.

Fig. 2 shows the reconstructed standard views of the myocardium for the perfect attenuation correction and no attenuation correction as the reference for the presented method.

Consecutively, the presented algorithm has been applied to the simulated projections. The consistency conditions (3) were used with the maximal parameter values $m=1$ and $k=3$. For higher values the LHS of the equations were equal to zero within the numerical noise even without any correction. We used the Fourier expansion (3) with $N = N_s = 3$. The SVD resulted in 19 linearly independent vectors c_n spanning the null space of (5). Two different prototype functions C_0 were used to obtain the correction function by means of (7) and (6): I. uniform: $C_0(s, \varphi) = 1$ for all nonzero projection pixels II. parabolic: $C_0(s, \varphi) = p(s-s_l)(s-s_r) + 1$, where s_l and s_r are the left- and rightmost nonzero pixels, respectively, in each projection, and p is a parameter determining the maximum of the parabola, which is linearly dependent on the width of the projection perpendicular to the one considered, i.e.

$p(\varphi) \sim |s_l(\varphi + \pi/2) - s_r(\varphi + \pi/2)|$. The maximal values of the parabolas range from about 8 to 12 for the phantom considered. This simple model gives a rough approximation to the true correction function, which is nevertheless better than in the case I. Examples of images obtained from the attenuation corrected projections in the noise-free case are shown in Fig. 3.

The apparent perfusion defects due to attenuation, visible in the uncorrected image (Fig. 2, right), are at least partly removed. The parabolic prototype function gives considerably better results than the uniform one.

The method has also been applied to noisy data. Fig. 4 shows an example of the reconstruction from data corrected with the presented method using the parabolic prototype correction function, compared to the image obtained from the same noisy dataset without correction.

It can be noticed that the method is quite robust with respect to noise. This can be attributed to the fact that the consistency conditions (3), and consequently the coefficients W in Eq. (5) contain weighted sums (integrals) of the data and do not rely on single pixel counts.

IV. Discussion and Conclusions

Consistency conditions for the SPECT projection data can be used to find the attenuation correction factors for the projection data, and recover the unattenuated projections, at least approximately, without referring to the attenuation map. The method consists of computing Fourier-like coefficients based on the measured data and solving a linear homogenous problem to obtain the subspace of consistent solutions in which the correction function should be embedded. The actual projection correction function can be obtained e.g. by projecting a prototype function, which may contain some a priori information, on the computed subspace. The corrected data can be reconstructed using the standard FBP algorithm, or alternatively the correction function can be included in an iterative reconstruction. Recovering the projection correction factors rather than the attenuation map makes the method more general, independent of a particular form of the map. The computation problem is also simpler (linear instead of nonlinear set of equations). It can be implemented very efficiently.

The consistency conditions are in general too weak to provide unique, exact correction function, nevertheless, they can provide a significant improvement with respect to no correction at all. In particular, since Eq. (5) is invariant with respect to multiplication by a constant factor, the constant term c_{00} in (4), corresponding to the average correction factor over all projection pixels, needs to be fixed by other means.

The method has been implemented using the expansion into Fourier series. One could try to find a more appropriate basis, e.g. the wavelets, in which the higher order expansion coefficients decay even more rapidly. To increase the accuracy, the method could be combined with other projection-data-only attenuation correction methods, e.g. the differential attenuation method for radionuclides emitting multiple photon energies, like ^{201}Tl .

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References

- [1]. Natterer F. Determination of tissue attenuation in emission tomography of optically dense media. *Inverse Problems*. 1993; 9:731.
- [2]. Welch A, Clack R, Natterer F, Gullberg GT. Toward accurate attenuation correction in SPECT without transmission measurements. *IEEE Trans Med Im*. 1997; 165:532.
- [3]. Welch A, et al. Attenuation correction in PET using consistency information. *IEEE Trans. Nucl. Sci*. 1998; 45:3134.
- [4]. Bronnikov A. Reconstruction of attenuation map using discrete consistency conditions. *IEEE Trans. Med. Im*. 2000; 19:451.
- [5]. Segars WP, Lalush DS, Tsui BMW. A realistic spline-based dynamic heart phantom. *IEEE Trans. Nucl. Sci*. 1999; 46:503–6.

Fig. 1.
Example slices of activity distribution (left) and attenuation map (right) of the NCAT phantom used to generate SPECT projections.

Fig. 2. Standard views of the myocardium reconstructed with the FBP from projections with perfect attenuation correction (left) and no attenuation correction (right).

Fig. 3. Standard views of the myocardium reconstructed with the FBP from noise-free projections with attenuation correction based on projection consistency conditions, using a uniform (left) parabolic (right) prototype correction C_0 .

Fig. 4. Standard views of the myocardium reconstructed with the FBP from noisy projections corresponding to typical myocardial perfusion scan with about 200k counts from the myocardium; left: no attenuation correction, right: with attenuation correction based on projection consistency conditions using parabolic prototype correction.