Beamforming of sound from two-dimensional arrays using spatial matched filters

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(Received 24 October 2012; revised 30 April 2013; accepted 3 September 2013)

Fully-sampled two-dimensional (2D) arrays can have two-way focusing of the ultrasound beam in both lateral directions leading to high quality, real-time three-dimensional (3D) imaging. However, fully-sampled 2D arrays with very large element counts ($>16\,000$) are difficult to manufacture due to interconnect density and large element electrical impedance. As an alternative, row-column or crossed electrode arrays have been proposed to simplify transducer fabrication and system integration. These types of arrays consist of two one-dimensional arrays oriented perpendicular to each other. Using conventional delay-and-sum beamforming, each array performs one-way focusing in perpendicular lateral directions which yield higher sidelobe and acoustic clutter levels compared to fully-sampled 2D arrays with two-way focusing. In this paper, the use of spatial matched filters to improve focusing of row-column arrays is investigated. On receive, data from each element are first spatial match filtered in the elevation direction. After summation, the data are filtered again in the azimuth direction. Beam widths comparable to one-way focusing are seen in azimuth and beam widths comparable to two-way focusing are achieved in elevation. 3D beam patterns from computer simulation results using a 7.5 MHz 128×128 row-column array are shown with comparison to a fully sampled 2D array.

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PACS number(s): 43.60.Fg, 43.35.Yb [HCS]

Pages: 3697–3704

I. INTRODUCTION

Two-dimensional (2D) arrays offer high three-dimensional (3D) image quality since scanning is completely electronic and focusing in both elevation and azimuth is possible. Real-time volume rates and reliability is greater compared to mechanically moving one-dimensional (1D) arrays. However, dense interconnect requirements, high electrical impedance, and heating of integrated electronics are significant challenges that need to be overcome. A practical solution is a sparse array where transmit and receive elements are carefully chosen to yield a pulse-echo radiation pattern similar to the fully sampled 2D array. Mills cross arrays, vernier arrays, and random 2D arrays have been studied.^{1–4} Others have used various approaches such as simulated annealing⁵ or linear programming⁶ to arrive at an optimal solution given constraints of aperture size and channel counts. Still others have used the concept of the effective aperture or co-array to develop novel sparse 2D array configurations which provide a radiation pattern with low sidelobes and acoustic clutter.⁷ Philips developed the x6-1 matrix transducer, a fully-sampled 2D phased array with 9212 elements for abdominal, obstetric, and cardiac applications. Custom integrated circuits funnel data from these 2D array elements to 128 or 256 system channels.⁸ Active cooling mechanisms are likely required due to the large amount of heat generated by the dense electronics.⁹

As another possible solution to the sparse array problem, several researchers previously proposed a row-column addressing approach which minimizes the interconnect and electrical impedance challenges.^{10–12} In this approach, an $N \times N$ 2D array requiring N^2 separate elements can be

replaced by an array of comparable size but requires only $2N$ elements. This represents a substantial savings when N is on the order of 128 to 256. This approach has been investigated for piezoceramic-based arrays and more recently capacitive micromachined ultrasonic transducers.¹³

The disadvantage of this row-column or crossed electrode approach is that only one-way focusing in each direction is possible. For example, transmit focusing is only possible in the azimuth direction and receive focusing is only possible in the elevation direction. This leads to higher sidelobes and clutter compared to 2D array systems with two-way focusing capability in both lateral directions. For real-time 3D imaging, image quality of the row-column approach in the transmit lateral direction will suffer because of the single focus at one depth in that direction. The perpendicular receive direction will have better image quality throughout depth because of dynamic receive focusing. Alternative beamforming methods are needed to improve the image quality of row-column arrays throughout the entire volume.

3D beamforming with 2D arrays is another challenge due to the large amount of data produced by the elements and the need to interrogate a 3D volume. Reduction in computation complexity while maintaining real-time 3D imaging is desirable. Toward this goal, novel 3D beamforming methods have been previously proposed by using the frequency domain,¹⁴ chirp zeta transform,¹⁵ XY separable delays,¹⁶ and limited diffraction beams.¹⁷ In these studies, data from all elements of a fully sampled array is assumed. A noncoherent correlation-based beamforming technique was proposed for underwater imaging.¹⁸ This technique uses the envelope of the detected signals received by a 2D array of elements. With this technique, grating lobes from sparse 2D array geometries are absent. Development of novel 3D beamforming methods or adaptation of previously published 3D beamforming methods

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in combination with novel 2D array designs with reduced array complexity has not been widely explored.

Spatial filter methods have been previously proposed to improve focusing away from the transmit focus. Freeman *et al.* developed and evaluated a retrospective dynamic transmit focusing algorithm which deconvolves the out-of-focus transmit beam resulting in a new in-focus image.¹⁹ Their approach used a 1D array and assumed a single transmit focus and dynamic receive focusing for 2D imaging. Jensen and Gori also proposed a spatial filter approach for focusing ultrasound images.²⁰ Their approach required a synthetic aperture acquisition approach since spatial filtering was done for each transmit element and receive element combination.

Generally the purpose of matched filtering is to maximize the signal-to-noise ratio for the task of signal recognition. The transfer function of the matched filter is the phase-conjugated version of the signal spectrum. When a signal is passed match filtered, the output will have zero phase and all frequencies thereby add constructively to give the maximum signal-to-noise ratio.²¹ In the context of this paper, pulse-echo fields are considered to be the signal and its conjugate is the spatial matched filter. Out-of-focus pulse-echo fields have a distribution of phases such that the sum of the frequency components yields a broad beam. In-focus pulse-echo fields have nearly zero phase for all frequency components. In Kim *et al.*^{22,23} spatial matched filtering (SMF) methods are presented as an alternative to dynamic focusing and classic synthetic aperture methods. SMF methods convolve pulse-echo fields with spatial matched filters to achieve focusing. Analytically, the filters remove the quadratic phase factor associated with out-of-focus beams bringing the beam into focus. Two versions of SMF were investigated. One method is to spatial match filter pulse-echo fields from individual elements before summation of all of the receive elements. This approach yields results comparable to classic synthetic aperture methods, but can be computationally intensive. A more computationally efficient method is to spatial match filter pulse-echo fields after summation. This approach achieves focusing, but produces higher sidelobe levels compared to spatial match filtering before summing. For either approach, SMF methods require experimental measurement of the system's pulse-echo field to perform filtering.

In this paper, the application of both SMF methods to row-column arrays to improve beam focusing is investigated. SMF before summing is used in the receive or elevation direction, and SMF after summing is used in the transmit or azimuth direction. SMF is analytically evaluated using approximations of the Rayleigh-Sommerfeld diffraction formula. Field II (Ref. 24) simulations are performed to demonstrate the feasibility of this method. SMF results using a row-column array are compared to a gold standard fully sampled 2D array using delay-and-sum beamforming in terms of -6 , -20 , and -40 dB beam widths.

II. METHODS

A. Fully sampled 2D array

Our gold standard is a 128×128 fully sampled 2D array used for 3D rectilinear imaging. These 2D arrays are analogous to 1D linear arrays used for imaging of near-field targets such

as the breast, carotid, and musculoskeletal system. A 2D array sub-aperture is translated across the full array to scan a 3D rectilinear volume. Using the coordinate system shown in Fig. 1, the Rayleigh-Sommerfeld diffraction formula²⁵ for a 2D array aperture $a_t(x_0, y_0)$ located in the x_0y_0 plane is

$$\Phi_t(x, y, z) = \frac{1}{j\lambda} \int_{-\infty}^{\infty} dy_0 \int_{-\infty}^{\infty} dx_0 a_t(x_0, y_0) \frac{e^{jkR}}{R} \frac{z}{R}. \quad (1)$$

$\Phi_t(x, y, z)$ is the emitted pressure field. R is the distance from a point on the aperture to a point in the field given by $R = \sqrt{(x - x_0)^2 + (y - y_0)^2 + z^2}$. The variables λ and k are the wavelength and wave number, respectively, where $k = 2\pi/\lambda$. The coordinates x_0 and y_0 are the aperture coordinates, and x, y, z are coordinates used for the pressure or pulse-echo fields.

Applying the small angle and Fresnel approximations, Eq. (1) becomes

$$\begin{aligned} \Phi_t(x, y, z) = & \frac{e^{jkz}}{j\lambda z} \int_{-\infty}^{\infty} dy_0 \\ & \times \int_{-\infty}^{\infty} dx_0 a_t(x_0, y_0) e^{jk(x_0^2 + y_0^2)/2z} e^{-jkyy_0/z} e^{-jkxx_0/z}. \end{aligned} \quad (2)$$

Applying an on-axis focus at a depth z_f is equivalent to phase shifting the integrand in Eq. (2) by $-jk\Delta$ where $\Delta = \sqrt{x_0^2 + y_0^2 + z_f^2} - z_f$. Δ can be approximated as $\Delta \approx x_0^2/2z_f + y_0^2/2z_f$ by using a binomial expansion. Placing this approximation for Δ into Eq. (2) and assuming that the aperture function $a_t(x_0, y_0)$ is separable into $a_t(x_0)a_t(y_0)$ gives

$$\begin{aligned} \Phi_t(x, y, z) = & \frac{e^{jkz}}{j\lambda z} \int_{-\infty}^{\infty} dy_0 a_t(y_0) e^{jk y_0^2/2z} e^{-jk y_0^2/2z_f} e^{-jk y y_0/z} \\ & \times \int_{-\infty}^{\infty} dx_0 a_t(x_0) e^{jk x_0^2/2z} e^{-jk x_0^2/2z_f} e^{-jk x x_0/z}. \end{aligned} \quad (3)$$

At the focal depth $z = z_f$, Eq. (3) reduces down to the product of the two 1D Fourier transforms of $a_t(x_0)$ and $a_t(y_0)$ with spatial frequency $u_x = x/\lambda z$ and $u_y = y/\lambda z$. For two-way focusing, the pulse-echo field $\Psi(x, y, z)$ is the product of the transmit and receive fields

$$\Psi(x, y, z) = \Phi_t(x, y, z) \Phi_r(x, y, z), \quad (4)$$

due to the multiplicative process of transmit and receive. The term $\phi_r(x, y, z)$ is the receive field and can also be

FIG. 1. Coordinate system of a 2D array.

approximated using the product of two 1D Fourier transforms using Eqs. (1)–(3). When the transmit and receive apertures are equal and the focal depth for both apertures is set to $z = z_f$, the pulse-echo field becomes the square of the Fourier transform of the aperture.

1. 3D delay-and-sum beamforming using row-column arrays

Figure 2 shows a simplified schematic of the 3D scanning procedure using a row-column array design with only eight elements in each 1D array. The transmit array is a 1D linear array along the azimuth direction and performs beamforming in the azimuth direction using the gray sub-aperture elements [Fig. 2(A)]. The resulting transmit beam is focused in the azimuth direction but broad in the elevation as indicated by the beam profile curves. In receive, a second 1D linear array with elements oriented perpendicular with respect to the transmit array and performs receive beamforming with the elements shaded in gray [Fig. 2(B)]. The receive beam is focused in elevation but not in azimuth also indicated by the beam profile curves. Focusing in azimuth and elevation is achieved due to the multiplicative effect of pulse-echo. By moving the locations of the transmit and receive sub-aperture in azimuth and elevation, respectively, a 3D rectilinear volume can be scanned.

B. SMF methods using the row-column array

With SMF methods, the same scanning method described above is performed to acquire raw radio frequency (RF) data. The raw data is then filtered in a two step process. The first step is to spatial match filter each receive element along the elevation direction in the yz plane (SMF-EL), and is equivalent to spatial match filtering before summing in Ref. 22. The output of this filter is then filtered once more along the azimuth direction in the xz plane (SMF-AZ). This second filtering operation is equivalent to spatial match filtering after summing since the receive aperture appears as a single elongated element in azimuth. Both filters are 2D filters that can minimize computational burden compared to using 3D filters.

The 3D complex pulse-echo field for each receive element is first derived by multiplying transmit and receive echo fields for a row-column array. Beginning with the

Rayleigh Sommerfeld diffraction formula, assuming a separable 2D aperture $a_t(y_0)a_r(x_0)$, and applying the small angle and Fresnel approximations, the 3D complex pressure field $\phi_t(x, y, z)$ of the transmitted aperture is

$$\phi_t(x, y, z) = \frac{e^{jkz}}{j\lambda z} \int_{-\infty}^{\infty} dy_0 a_t(y_0) e^{iky_0^2/2z} e^{-jky_0 y/z} \times \int_{-\infty}^{\infty} dx_0 a_r(x_0) e^{jk\beta x_0^2} e^{-jkxx_0/z}, \quad (5)$$

where $\beta = 1/2z - 1/2z_f$ and z_f is the transmit focal depth. The β term only appears in the dx_0 integrand because transmit focusing is only possible in the x_0 direction with a row-column array. An element y_r in the receive aperture is approximated by a delta function in the elevation direction, and the 3D receive field for receive element y_r can be given by

$$\phi_r(x, y, z; y_r) = \frac{e^{jkz}}{j\lambda z} e^{iky_r^2/2z} e^{-jky_r y/z} \times \int_{-\infty}^{\infty} dx_0 a_r(x_0) e^{-jkxx_0/z} e^{jkx_0^2/2z}. \quad (6)$$

Multiplying Eqs. (5) and (6) together gives the transmit-receive field due to a focused transmit aperture and single receive element y_r ,

$$\psi(x, y, z; y_r) = \phi_t(x, y, z) \phi_r(x, y, z; y_r),$$

$$\psi(x, y, z; y_r) = C(x, z) \left[e^{iky_r^2/2z} e^{-jky_r y/z} \times \int_{-\infty}^{\infty} dy_0 a_t(y_0) e^{iky_0^2/2z} e^{-jky_0 y/z} \right],$$

where

$$C(x, z) = \frac{e^{jkz}}{j\lambda z} \frac{e^{jkz}}{j\lambda z} \left[\int_{-\infty}^{\infty} dx_0 a_t(x_0) e^{jk\beta x_0^2} e^{-jkxx_0/z} \times \int_{-\infty}^{\infty} dx_0 a_r(x_0) e^{-jkxx_0/z} e^{jkx_0^2/2z} \right]. \quad (7)$$

Convolving $\psi(x, y, z; y_r)$ along the y -direction with the transmit-receive field $\psi^*(0, -y, z; y_r)$ for each receive element y_r results in an output $\bar{\psi}(x, y, z; y_r)$ where $\otimes$ indicates convolution and $*$ indicates the complex conjugate:

$$\bar{\psi}(x, y, z; y_r) = \psi(x, y, z; y_r) \otimes \psi^*(0, -y, z; y_r),$$

$$\bar{\psi}(x, y, z; y_r) = C(x, z) C(0, z) \int_{-\infty}^{\infty} d\tau \left[e^{iky_r^2/2z} e^{-jk\tau y_r/z} \int_{-\infty}^{\infty} dy_0 a_t(y_0) e^{iky_0^2/2z} e^{-jk\tau y_0/z} \right] \times \left[e^{-jky_r^2/2z} e^{jk(\tau-y)y_r/z} \int_{-\infty}^{\infty} dy_1 a_t(y_1) e^{-jk\tau y_1^2/2z} e^{jk(\tau-y)y_1/z} \right],$$

$$\bar{\psi}(x, y, z; y_r) = C(x, z) C(0, z) \int_{-\infty}^{\infty} d\tau \left[\int_{-\infty}^{\infty} dy_0 a_t(y_0) e^{iky_0^2/2z} e^{-jk\tau y_0/z} \right] \left[e^{-jky_r y/z} \int_{-\infty}^{\infty} dy_1 a_t(y_1) e^{-jk\tau y_1^2/2z} e^{jk(\tau-y)y_1/z} \right],$$

$$\bar{\psi}(x, y, z; y_r) = C(x, z) C(0, z) \int_{-\infty}^{\infty} e^{-jk\tau y_0/z} e^{jk\tau y_1/z} d\tau \left[\int_{-\infty}^{\infty} dy_0 a_t(y_0) e^{iky_0^2/2z} \right] \left[e^{-jky_r y/z} \int_{-\infty}^{\infty} dy_1 a_t(y_1) e^{-jk\tau y_1^2/2z} e^{-jk\tau y_1/z} \right],$$

$$\bar{\psi}(x, y, z; y_r) = C(x, z) C(0, z) \frac{2\pi z}{k} e^{-jky_r y/z} \left[\int_{-\infty}^{\infty} dy_0 a_t^2(y_0) e^{-jk\tau y_0/z} \right]. \quad (8)$$

The term in the square brackets of Eq. (8) is the Fourier transform of the transmit aperture squared with spatial frequency $u_y = y/\lambda z$. Filtering with $\psi^*(0, -y, z; y_r)$ eliminated the quadratic phase factor $e^{iky_0^2/2z}$ which causes out-of-focus beams. Because of separability, the

function $C(x, z)$ in Eq. (8) has not been affected by filtering in the y -direction. Summing over all receive elements y_r and weighting by the receive aperture yields the 3D filtered pulse-echo field $\bar{\bar{\psi}}(x, y, z)$:

$$\begin{aligned}\bar{\bar{\psi}}(x, y, z) &= \int_{-\infty}^{\infty} dy_r a_r^2(y_0) \bar{\psi}(x, y, z; y_r), \\ \bar{\bar{\psi}}(x, y, z) &= C(x, y) C(0, z) \frac{2\pi z}{k} \int_{-\infty}^{\infty} dy_r a_r^2(y_0) e^{-jk y y_r / z} \left[\int_{-\infty}^{\infty} dy_0 a_t^2(y_0) e^{-jk y y_0 / z} \right], \\ \bar{\bar{\psi}}(x, y, z) &= C(x, z) C(0, z) \frac{2\pi z}{k} [\mathfrak{F}\{a_r^2(y_0)\}] [\mathfrak{F}\{a_t^2(y_0)\}].\end{aligned}\quad (9)$$

Equation (9) states that the pulse-echo field in the y -direction is the Fourier transform of the receive aperture squared times the Fourier transform of the transmit aperture squared where $\mathfrak{F}\{\}$ indicates the Fourier transform operation. Having a larger transmit or receive aperture will lead to a narrowing of the pulse-echo beam. The field in the x -direction is described by $C(x, z) C(0, z)$, shown in the last line of Eq. (7), which is the product of a transmit beam and an unfocused receive beam. The transmit beam is only focused at $z = z_f$.

A second filtering operation is performed in the x -direction. The resultant pulse echo-field $\equiv \bar{\psi}(x, y, z)$ in the x -direction is the convolution in the x -direction of $\bar{\bar{\psi}}(x, y, z)$ with $\bar{\psi}^*(-x, 0, z)$,

$$\begin{aligned}\bar{\bar{\psi}}(x, y, z) &= \bar{\bar{\psi}}(x, y, z) \otimes \bar{\bar{\psi}}^*(-x, 0, z) = \left(\frac{2\pi z}{k}\right)^2 \left(\frac{e^{jkz}}{j\lambda z}\right)^4 [\mathfrak{F}\{a_r^2(y_0)\}] [\mathfrak{F}\{a_t^2(y_0)\}] \int_{-\infty}^{\infty} dy_r a_r^2(y_0) \int_{-\infty}^{\infty} dy_0 a_t^2(y_0) \\ &\times \left[\int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} dx_0 a_t(x_0) e^{jk\beta x_0^2} e^{-jk\tau x_0/z} \int_{-\infty}^{\infty} dx_0 a_r(x_0) e^{jkx_0^2/2z} e^{-jk\tau x_0/z} \int_{-\infty}^{\infty} dx_1 a_t(x_1) e^{jk\beta x_1^2} e^{jk(\tau-x)x_1/z} \right. \\ &\times \left. \int_{-\infty}^{\infty} dx_0 a_r(x_0) e^{jkx_0^2/2z} e^{jk(\tau-x)x_1/z} \right].\end{aligned}\quad (10)$$

Equation (10) is slightly different from Eq. (13) in Ref. 22 where SMF after summing results in the Fourier transform of the aperture function raised to the fourth power. Their derivation was based on the geometry of a fixed identical f -number in transmit and receive. In this application, a plane wave transmit is used with an aperture size of 19.2 mm and a receive aperture size of 9.6 mm is focused. Equation (10) is not readily solved analytically when different transmit and receive aperture sizes are used. Improvement in focusing of the beam is evaluated with the aid of Field II simulation.

FIG. 2. 3D rectilinear scanning method for a row-column aperture in (A) transmit and (B) receive. Shaded elements indicate the active sub-apertures.

C. Field II simulations

The transducer center frequency is 7.5 MHz and has a 150 μm pitch. In this study, a 64×64 transmit sub-aperture is focused at a depth of 30 mm. A 19.2×19.2 mm 7.5 MHz fully sampled 2D array with 9.6×9.6 mm sub-aperture was simulated in Field II. The array element pitch was 0.15 mm in azimuth and elevation. A row-column array of equal total size was also simulated. Because of the design of the row-column array, the transmit (19.2×9.6 mm) and receive (9.6×19.2 mm) sub-apertures are rectangular in shape. The transmit focus is always set to 30 mm depth. On receive, the same sub-aperture is dynamically focused at depths of 20, 30, and 40 mm.

TABLE I. Fully-sampled 2D array and row-column array parameters.

Parameter	2D array	Row-column array
Frequency	7.5 MHz	7.5 MHz
Total aperture	19.2×19.2 mm	19.2×19.2 mm
Sub-aperture size	9.6×9.6 mm	19.2×9.6 mm in Tx 9.6×19.2 mm in Rx
Element size	$0.15 \text{ mm} \times 0.15 \text{ mm}$	19.2×0.15 mm in Tx 0.15×19.2 mm in Rx
Tx focus depth	30 mm	30 mm

FIG. 3. Simulated beam plots of a fully sampled array with transmit focus at $z = 30$ mm and receive focus at $z = 20$ mm. From left to right, beam plots for the fully-sampled, RC-DAS, SMF-EL, and SMF-AZ. 3D, elevational, and azimuthal views are provided in the (a) top, (b) middle, and (c) bottom rows, respectively.

Broadband discrete 2D spatial match filters in the elevation direction (SMF-EL), equivalent to $\psi^*(0, -y, z; y_r)$ shown in Eqs. (7) and (8), are created for each of the 64 receive elements y_r . To ensure minimal differences between the continuous aperture formulations shown in Sec. II B, a temporal sampling of 150 MHz and a lateral sampling of $75 \mu\text{m}$ was used. After filtering with the respective filter for each element, the RF data are summed together. A single 2D spatial match filter (SMF-AZ) equivalent to $\psi^*(-x, 0, z)$ in Eqs. (9) and (10) in the azimuth direction is also created for filtering every 2D azimuthal plane. Table I lists the parameters used for the fully-sampled 2D array and the row-column array used in this study.

III. RESULTS

3D beam plots are acquired for the 2D array and row-column array when the receive focus is set to 20, 30, and

40 mm. For the row-column array, beam plots with only delay-and-sum beamforming, after SMF-EL, and after SMF-AZ are also shown. Azimuthal and elevational views are shown in addition to a 3D view.

Figure 3 shows the magnitudes of the simulated beam plots at a depth of 20 mm. The first column shows the 3D view, elevational side view, and azimuthal side view of the fully sampled array in the top, middle, and bottom panel, respectively. The second column shows the same views of the row-column array with only delay-and-sum beamforming. The third column displays the beam of the row-column array with SMF before summing in the elevation direction, and the fourth column shows the final beam after SMF in the azimuth direction.

At $z = 20$ mm, the fully sampled array has a -6 dB beam width of 0.67 mm, a -20 dB beam width level of 2.20 mm, and a 5.46 mm width at -40 dB. The beam widths are always identical in azimuth and elevation due to symmetry of the 2D

TABLE II. Beam widths of the fully-sampled array and RC-DAS beamforming and SMF-EL and SMF-AZ. The depth is set to $z = 20$ mm.

Beam widths	Fully sampled		RC-DAS		SMF-EL		SMF-AZ	
	Azimuth	Elevation	Azimuth	Elevation	Azimuth	Elevation	Azimuth	Elevation
-6 dB	0.67	0.67	2.92	0.58	2.85	0.40	0.90	0.39
-20 dB	2.20	2.20	6.84	2.42	6.78	0.75	3.17	0.68
-40 dB	5.46	5.46	>15	>15	>15	4.10	>15	4.02

FIG. 4. Simulated beam plots of a fully sampled array with transmit focus at $z = 30$ mm and receive focus at $z = 30$ mm. 3D, elevational, and azimuthal views are provided in the (a) top, (b) middle, and (c) bottom rows, respectively. From left to right, beam plots for the fully-sampled, RC-DAS, SMF-EL, and SMF-AZ.

array. For the row-column array using only delay-and-sum beamforming, the -6 dB beam width is 2.92 mm in azimuth and 0.58 mm in elevation. This beam width is rather wide in azimuth because the transmit focus is located at 30 mm. The elevational beam width is narrower since a dynamic delay-and-sum receive focus is applied at this depth of 20 mm. The -20 dB beam width is 6.84 and 2.42 mm in azimuth and elevation, respectively. The sidelobe level in elevation is around -13 dB indicating that only one-way focusing is achieved. The -40 dB beam widths are greater than 15 mm in both directions. After spatial match filtering (SMF-EL) in the elevation direction, the -6 , -20 , and -40 dB elevational beam widths are 0.40, 0.75, and 4.10 mm. Meanwhile the azimuthal beam widths remain relatively unchanged at 2.92, 6.84, and >15 mm for -6 , -20 , and -40 dB levels, respectively. After SMF-AZ, azimuthal beam widths improve to 0.90 and 3.17 mm for -6 and -20 dB, respectively. The -40 dB beam width still remains greater than 15 mm. Table II summarizes

the azimuthal and elevational beam widths for the fully-sampled array, and the row-column array after delay-and-sum beamforming and subsequent spatial matched filtering.

Beam plots for the fully sampled and row-column arrays when focused at 30 mm depth are shown in Fig. 4. The beam widths for the fully-sampled 2D array at -6 , -20 , and -40 dB are 0.58, 1.09, and 3.56 mm, respectively. The beam widths are identical in azimuth and elevation due to symmetry of the array and identical focal depths in transmit and receive. These are the narrowest 2D array beam widths since the transmit and receive foci are both set to 30 mm depth. For the row-column array with delay-and-sum beamforming, the -6 , -20 , and -40 dB beam widths are 0.83, 3.14, and >15 mm in azimuth and are comparable in elevation. After spatial match filtering in elevation (SMF-EL), the elevational -6 , -20 , and -40 dB beam widths narrow to 0.53, 0.93, and 5.81 mm. Filtering in azimuth slightly increased the -6 and -20 dB beam widths to 0.89 and 3.27 mm. Table III

TABLE III. Beam widths of the fully-sampled array and RC-DAS beamforming and SMF-EL and SMF-AZ. The depth is set to $z = 30$ mm.

Beam widths	Fully sampled		RC-DAS		SMF-EL		SMF-AZ	
	Azimuth	Elevation	Azimuth	Elevation	Azimuth	Elevation	Azimuth	Elevation
-6 dB	0.58	0.58	0.83	0.83	0.82	0.53	0.89	0.55
-20 dB	1.09	1.09	3.14	3.28	2.67	0.93	3.27	0.96
-40 dB	3.56	3.56	>15	>15	>15	5.81	>15	5.93

FIG. 5. Simulated beam plots of a fully sampled array with transmit focus at $z = 30$ mm and receive focus at $z = 40$ mm. 3D, elevational, and azimuthal views are provided in the (a) top, (b) middle, and (c) bottom rows, respectively. From left to right, beam plots for the fully-sampled, RC-DAS, SMF-EL, and SMF-AZ.

summarizes the azimuthal and elevational beam widths for the fully-sampled array, and the row-column array after delay-and-sum beamforming and subsequent SMF for a depth of 30 mm.

Figure 5 shows the beam plots for the fully sampled array and the row-column array with a receive focus set to $z = 40$ mm. The beam widths for the fully-sampled 2D array are 0.92, 3.14, and 6.60 mm for -6 , -20 , and -40 dB levels in both azimuth and elevation. For the row-column array with only delay-and-sum beamforming, the azimuth beam widths are 2.58, 7.89, >15 mm for -6 , -20 , and -40 dB, respectively. With elevational filtering, the elevational beam widths are 0.71, 1.25, and 5.62 mm, which are all narrower than the fully-sampled 2D array. After azimuthal filtering, the -6 and -20 beam widths become narrower at 0.89 and 3.12 mm, but the -40 dB beam width remains >15 mm. Again, this is indicative of one-way focusing after azimuthal filtering. Table IV summarizes the azimuthal and elevational

beam widths for the fully-sampled array, and the row-column array after delay-and-sum beamforming and subsequent SMF for a depth of 40 mm.

IV. CONCLUSIONS

Using Field II simulations, spatial matched filter methods were demonstrated to be a potential method for improving focusing of row-column arrays after delay-and-sum (RC-DAS) beamforming. Delay-and-sum beamforming using row-column arrays lead to asymmetric beams which can lead to non-uniform image quality throughout the imaging volume because of a single transmit focal depth. Dynamically focusing in both transmit and receive would have symmetric beams but only offer one-way focused beams in azimuth and elevation throughout depth.

Elevational beam widths were much improved after spatial match filtering in the elevation direction. At the three

TABLE IV. Beam widths of the fully-sampled array and RC-DAS beamforming and SMF-EL and SMF-AZ. The depth is set to $z = 40$ mm.

Beam widths	Fully sampled		RC-DAS		SMF-EL		SMF-AZ	
	Azimuth	Elevation	Azimuth	Elevation	Azimuth	Elevation	Azimuth	Elevation
-6 dB	0.92	0.92	2.58	1.13	2.57	0.71	0.89	0.70
-20 dB	3.14	3.14	7.89	4.35	7.56	1.25	3.12	1.21
-40 dB	6.60	6.60	>15	>15	>15	5.62	>15	5.59

depths presented here, the -6 dB elevational beam widths for the row-column arrays after spatial match filtering were smaller than the elevational beam widths of the fully-sampled array. This result can be explained by noting that the transmit aperture in the elevation direction is larger in the row-column array than the fully-sampled array. Spatial match filtering effectively focuses this larger transmit aperture, as indicated in Eq. (9), yielding a narrower beam. In the azimuth direction, spatial match filtering improved the -6 dB beam widths compared to delay-and-sum beamforming of the row-column array at 20 and 40 mm depths. In all cases, the azimuthal beam plot showed first sidelobes around -13 dB indicating that equivalent one-way focusing was possible. This result is consistent with what is predicted in Ref. 22. As another alternative, it is possible to only filter in azimuth to achieve dynamic focusing in transmit while dynamic receive focusing with DAS beamforming is done in elevation. The resultant pulse-echo beam will have one-way focusing in both lateral directions.

Based on the results presented, the azimuthal and elevational beam widths for the row-column array with SMF are not identical which may lead to some image artifacts. Speckle size in elevation will appear smaller than in azimuth, and anechoic structures viewed from a C-scan perspective may see clutter primarily oriented in the azimuth direction. Further understanding of spatial match filtering after summing with different transmit and receive aperture sizes may yield additional strategies to improve beamforming in this context. Apodization may be another approach to achieve a more uniform speckle size. Achieving uniform image quality in both lateral directions is a focus of future work.

Additional future work will involve measurement of the impulse response of a prototype row-column or dual-layer array. Implementation of the SMF method is highly dependent on accurate measurements of the overall system impulse response to provide appropriate spatial filters. The SMF approach may be limited by unavoidable errors when measuring the system's pulse-echo field. Strategies to overcome or minimize these errors may involve novel methods for pulse-echo field measurement as well as additional signal processing strategies. For real-time 3D imaging, a bank of filters for different depths would be required for dynamic focusing.

ACKNOWLEDGMENTS

This work was funded by NIH Grant No. CA116379. The author would like to thank Dr. Michael F. Insana for helpful conversations on spatially matched filters.

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