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Magnetoacoustic Tomography with Magnetic Induction: Bioimpedance reconstruction through vector source imaging

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Abstract

Magneto acoustic tomography with magnetic induction (MAT-MI) is a technique proposed to reconstruct the conductivity distribution in biological tissue at ultrasound imaging resolution. A magnetic pulse is used to generate eddy currents in the object, which in the presence of a static magnetic field induces Lorentz force based acoustic waves in the medium. This time resolved acoustic waves are collected with ultrasound transducers and, in the present work, these are used to reconstruct the current source which gives rise to the MAT-MI acoustic signal using vector imaging point spread functions. The reconstructed source is then used to estimate the conductivity distribution of the object. Computer simulations and phantom experiments are performed to demonstrate conductivity reconstruction through vector source imaging in a circular scanning geometry with a limited bandwidth finite size piston transducer. The results demonstrate that the MAT-MI approach is capable of conductivity reconstruction in a physical setting.

Index Terms

Electrical impedance imaging; Helmholtz decomposition; Magneto acoustic tomography (MAT-MI); reconstruction

I. Introduction

Electrical conductivity imaging of biological tissue has attracted considerable interest in recent years owing to research indicating that electrical properties especially electrical conductivity and permittivity are indicators of underlying physiological and pathological conditions in biological tissue [1][2]. Also, the knowledge of electrical conductivity of biological tissue is of interest to researchers conducting electromagnetic source imaging [3] and in design of high field MRI to estimate the specific absorption rate (SAR) [4]. Several techniques have been proposed to image electrical properties of tissue. Among these methods Electrical Impedance Tomography (EIT) was first developed [5][6][7] with the use of surface electrodes to inject currents and measure the resultant surface voltage for image reconstruction. The EIT technique is a low cost, real time and safe method. However the ill-posed inverse problem and use of surface electrodes leads to low spatial resolution. Magnetic resonance Electrical Impedance Tomography (MREIT) [8] overcomes this limitation by combining Magnetic resonance Current Density Imaging (MRCDI) [9] with EIT to measure the internal magnetic flux distribution induced by the injected currents from surface electrodes using an MRI machine. High spatial resolution conductivity images were obtained in both in-vitro and in-vivo experiments [8]. However, MREIT is currently limited

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by its requirement of high level current injection to obtain an acceptable signal-to-noise (SNR) level. In addition as with EIT the use of surface electrodes to inject currents leads to shielding effect [10] caused by insulating or low conductive region in the object, such as fat tissue. This problem of “shielding effect” is avoided in magnetic induction tomography (MIT) [11], which uses oscillating magnetic fields to induce eddy currents in the tissue and measure the resultant secondary magnetic field using sensing coils placed around the object. The limited number of sensors of MIT and the need to solve an ill-posed inverse problem, however, leads to poor spatial resolution with this technique. Besides these electromagnetic techniques other hybrid imaging techniques such as combining electromagnetic fields and acoustic fields, as reported in magneto acoustic tomography (MAT) [12][13] and Hall effect imaging (HEI) [10][14], have been explored. In MAT and HEI, spontaneous or injected currents in a static magnetic field are used to induce Lorentz-force-based acoustic vibrations in the object and acoustic measurements from around the object are used to reconstruct the image. These imaging methods also use surface electrodes for current injection/voltage measurements, which leads to shielding effects. Magnetoacoustic tomography with magnetic induction (MAT-MI) [15]–[28] is a recently proposed approach to image electrical impedance distribution with high spatial resolution. In MAT-MI, magnetic fields are used to induce current, and hence, the method does not suffer from the shielding effect. Ultrasound is generated in the object to be imaged by placing it in a dynamic and a static magnetic field. The dynamic field induces eddy currents in the object and the static field leads to generation of acoustic vibrations from Lorentz force on the induced currents. The acoustic vibrations are at the same frequency as the dynamic magnetic field, which is chosen to match the ultrasound frequency range. This allows us to reconstruct the acoustic source distribution in the object using possible ultrasound imaging approaches. The conductivity distribution of the object can then be reconstructed from the obtained acoustic source map [16][18][22].

Previously with MAT-MI few image reconstruction methods have been implemented [16][21][22]. As in scalar reconstruction to image the acoustic source distribution [16][22] and vector acoustic tomography [21]. In the scalar reconstruction method after obtaining the acoustic source distribution from the ultrasound measurements the conductivity can be estimated by making a piece wise homogeneous approximation [18]. However this method involves neglecting the boundary acoustic source while estimating the conductivity and as seen in previous simulation and experimental studies [17]–[19] this is the dominant source in MAT-MI. To obtain a reliable conductivity reconstruction with this method a complete acoustic source reconstruction is necessary which requires an unlimited bandwidth ultrasound measurement, which is not feasible in practice. To take into account the strong boundary acoustic signal while reconstructing conductivity a multi-excitation based method was proposed [22][23]. This method uses multiple magnetic excitations with different coil configurations to generate MAT-MI acoustic signal in the object and the reconstructed acoustic source distribution from the various excitations is then combined to solve for the conductivity distribution. It is shown in MAT-MI studies that if unlimited bandwidth acoustic data are available, the internal conductivity contrast of an object can be accurately reconstructed [22]. However with limited bandwidth data, a modified reconstruction algorithm is used and only the relative conductivity contrast can be obtained. The other reconstruction method developed for MAT-MI reconstructs the vector source which is the Lorentz force inducing the ultrasound signal [21]. The proposed vector acoustic tomography method for image reconstruction uses a spherical or cylindrical measurement geometry satisfying the far field assumption relative to the dimension of the object and requires ultrasound detectors which have unlimited bandwidth. The reconstructed vector field is then combined with the knowledge of the induced electric field to obtain the conductivity distribution in the object. However the induced electric field is dependent on the conductivity distribution and is not known a priori especially for objects with complex geometries. As such, the vector reconstruction method [21] due to its rigid requirements on

the ultrasound detection system and the a priori knowledge of the induced electric field to estimate conductivity distribution is not applicable for all objects.

In the present study we propose a conductivity reconstruction method based on vector acoustic source imaging as the intermediate step. In this study we have obtained a relationship between the conductivity distribution of the object and the MAT-MI source component of the induced current using Helmholtz decomposition under the assumption of piece wise homogeneous conductivity. A beamformer algorithm is applied to the ultrasound scanning setup to design an imaging method with a directionally dependent point spread function (psf), which gives the vector source distribution on image reconstruction. The vector source field which is a component of the induced eddy current, unlike the boundary signal dominant scalar acoustic field is, uniformly present throughout the conductive object and such a field requires lesser frequency bandwidth to be reliably estimated. The developed reconstruction algorithm is applied to MAT-MI simulation and phantom experiments with limited bandwidth acoustic measurement setup to test its performance. It is shown in these experiments that reliable conductivity estimation is possible with the proposed reconstruction method under limited bandwidth ultrasound measurement and using a single magnetic excitation. This allows for practical implementation of the MAT-MI approach for human imaging applications for differentiation of tissue types based on conductivity distribution, which has potential application for cancer imaging.

II. Theory

A. Forward Problem

The forward problem for MAT-MI describes the physical process of obtaining the induced ultrasound pressure signal from the stimulating magnetic fields. In MAT-MI a conductive object with an isotropic conductivity of $\sigma(\mathbf{r})$ is placed in the presence of a static magnetic field with flux density $\mathbf{B}_0(\mathbf{r})$ and a time varying magnetic pulse $\mathbf{B}_1(\mathbf{r}, t)$ is applied to the object. This magnetic field induces an electric field $\mathbf{E}(\mathbf{r}, t)$ in the object which for a conductive sample gives rise to eddy currents $\mathbf{J}(\mathbf{r}, t) = \sigma(\mathbf{r})\mathbf{E}(\mathbf{r}, t)$ in the object.

The magnetic pulse used in MAT-MI is generated using approximately microsecond long currents through coils which lead to magnetic fields with slow time variation such that the propagation time of the field through the object is negligible. This allows the magnetic induction problem in MAT-MI to be considered as quasi-static. The skin depth of the corresponding MHz field in general biological tissue is at the level of meters which is much larger than the characteristic dimension of the object, so the magnetic diffusion can be ignored [29]. This allows the applied magnetic field to be uniquely determined by the coil setup and the separation of the spatial, and temporal functions in the electromagnetic field analysis of MAT-MI as in $\mathbf{B}_1(\mathbf{r}, t) = \mathbf{B}_1(\mathbf{r})f(t)$, $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r})f'(t)$ where the prime indicates first derivative. In addition to this the microsecond long magnetic stimulation also leads to the displacement current being much weaker than the ohmic current due to the fact that in biological tissue in the 1 MHz range [16], where ω is the angular frequency of the stimulating field and ϵ is the permittivity of the object. After applying the above approximations the induced current in MAT-MI can be described by the following equations:

$$\begin{cases} \nabla \cdot (-\sigma \frac{\partial \mathbf{A}}{\partial t} - \sigma \nabla \phi) = 0 \text{ in } \Omega \\ \mathbf{J} \cdot \mathbf{n} = 0 \text{ on } \partial\Omega \end{cases} \quad (1)$$

Where Ω denotes the domain in which the object is present, $\mathbf{A}$ is the magnetic vector potential corresponding to magnetic stimulation $\mathbf{B}_1 = \nabla \times \mathbf{A}$, $\nabla \cdot \mathbf{A} = 0$ and ϕ is the

electrical scalar potential; $\mathbf{n}$ is the unit vector normal to the boundary surface. The quasi-static condition in MAT-MI allows estimating the magnetic vector potential $\mathbf{A}$ directly from the configuration of the coils and the currents applied to them. For a known conductivity distribution and vector potential $\mathbf{A}$ the above magnetic induction problem can be solved in the domain for the electric potential ϕ using the finite element method (FEM) simulations [29]. The electric field can then be obtained as

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \phi \quad (2)$$

Now in the presence of a static magnetic field $\mathbf{B}_0$ the induced eddy current is subject to Lorentz force $\mathbf{J} \times \mathbf{B}_0$ and the time-varying Lorentz force gives rise to a traveling acoustic wave. In MAT-MI the divergence of the Lorentz force $\nabla \cdot (\mathbf{J} \times \mathbf{B}_0)$ acts as the source of the acoustic wave. Now the static magnetic field $\mathbf{B}_0$ in MAT-MI is generated from external sources outside the conductive object, so its spatial distribution can be determined by the setup generating the magnetic field. The field from the permanent magnets used in the current study can be approximated to $\mathbf{B}_0 = B \mathbf{k}$, where B is a constant and $\mathbf{k}$ is the unit vector in the Z direction. This leads to $\nabla \times \mathbf{B}_0$ being negligible inside the object space and the acoustic source can be further simplified as $\nabla \cdot (\mathbf{J} \times \mathbf{B}_0)$. Now, the induced eddy current $\mathbf{J}$ is a smooth function that vanishes outside the object space with its first derivative, so it can be decomposed into a rotational part $\mathbf{J}_c$ and an irrotational part [30]. The current $\mathbf{J}$ can be written as:

$$\mathbf{J} = \mathbf{J}_c + \nabla \theta \quad (3)$$

Of these two components the term $\mathbf{J}_c$ acts as the source for the acoustic wave, as $\nabla \times \mathbf{J} = \nabla \times \mathbf{J}_c$ and the MAT-MI acoustic source can then be written as $\nabla \cdot (\mathbf{J}_c \times \mathbf{B}_0)$. The generated acoustic waves can be described by the following equation [13]:

$$\nabla^2 p - \frac{1}{c_s^2} \frac{\partial^2 p}{\partial t^2} = \nabla \cdot (\mathbf{J}_c \times \mathbf{B}_0) \quad (4)$$

where P is the pressure and c_s is the acoustic speed in the medium.

Suppose the induced eddy currents are confined to the object space, the acoustic source is then zero outside the conductive object. Now the detected acoustic pressure $p(\mathbf{r}, t)$ on the scanning aperture can be obtained using Green's function solution to the wave equation as follows [16]

$$p(\mathbf{r}, t) = -\frac{1}{4\pi} \iiint d^3r' \nabla_{r'} \cdot (\mathbf{J}_c(\mathbf{r}') \times \mathbf{B}_0(\mathbf{r}')) G(\mathbf{r}, \mathbf{r}', t) \quad (5)$$

Where $\mathbf{r}'$ is the location of the acoustic source, $\mathbf{r}$ is the location detector, $\nabla_{r'}$ implies derivative with respect to source space $\mathbf{r}'$, the Green's function for this solution

$G(\mathbf{r}, \mathbf{r}', t) = \frac{\delta(t - \frac{R}{c_s})}{R}$ where $R = |\mathbf{r} - \mathbf{r}'|$. The volume integration is carried out over the acoustic source distribution in the object space. This equation gives the observed pressure for an impulse source i.e. a source with its time function as $\delta(t)$. An induced acoustic source field with time dependence $h(t)$ gives an observed pressure $p(\mathbf{r}, t) = h(t)$, where $\mathbf{r}$ is the

convolution operator. The Green's function for this time dependence can be written as

$$G(\mathbf{r}, \mathbf{r}', t) = \frac{h\left(t - \frac{R}{c_s}\right)}{R} [24].$$

B. Image Reconstruction

The image reconstruction algorithm for MAT-MI can be broken into two steps. In the first step the acoustic source distribution is obtained and in the second step this acoustic source is used to estimate the conductivity of the object.

In the present study the rotational component of the induced current density is reconstructed through the design of vector point spread functions for the ultrasound measurement system. For an unfocussed small aperture detector acting as a point receiver the pressure signal at the receiver can be described by equation (5). The current source is zero outside the conductive object region, so this equation can be rewritten as follows:

$$p(\mathbf{r}, t) = \frac{1}{4\pi} \iiint d^3\mathbf{r}' (\mathbf{J}_c(\mathbf{r}') \times \mathbf{B}_0(\mathbf{r}')) \cdot \nabla_{\mathbf{r}'} G(\mathbf{r}, \mathbf{r}', t) \quad (6)$$

As the ultrasound measurement aperture used for MAT-MI imaging comprises of a finite number of receivers the formulation of the image reconstruction can readily be discretized. Now applying a beamformer algorithm [31] with the signal received at the sensor locations we can design the imaging psf to extract the orthogonal components of the vector source. The beamformer algorithm comprises of summing up the magnitude weighted and time delayed signal at all the receiver locations. This can be described by the following equations

$$V(\mathbf{r}_s) = \sum_n W_a p(\mathbf{r}, t - t_a) \quad (7)$$

Where W_a is the weight applied to the ultrasound signal, t_a is the time delay applied to the signal, n is the number of ultrasound receiver signal used in the beamformation process and V is the resultant signal obtained at source location $\mathbf{r}_s$ from the beamformation process. Now substituting for the Pressure $p(\mathbf{r}, t)$ using equation (6) in (7) we get

$$V(\mathbf{r}_s) = \sum_n W_a \iiint d^3\mathbf{r}' (\mathbf{J}_c(\mathbf{r}') \times \mathbf{B}_0(\mathbf{r}')) \cdot \nabla_{\mathbf{r}'} G(\mathbf{r}, \mathbf{r}', t - t_a) \quad (8)$$

Interchanging the integration over the source space and the summation over the sensor space, (8) can be written as

$$V(\mathbf{r}_s) = \iiint d^3\mathbf{r}' (\mathbf{J}_c(\mathbf{r}') \times \mathbf{B}_0(\mathbf{r}')) \cdot \sum_n W_a \nabla_{\mathbf{r}'} G(\mathbf{r}, \mathbf{r}', t - t_a) = \iiint d^3\mathbf{r}' \mathbf{J}_c(\mathbf{r}') \cdot (\mathbf{B}_0(\mathbf{r}') \times \sum_n W_a \nabla_{\mathbf{r}'} G(\mathbf{r}, \mathbf{r}', t - t_a)) \quad (9)$$

The vector term $\mathbf{S}(\mathbf{r}) = \sum_n W_a \nabla_{\mathbf{r}'} G(\mathbf{r}, \mathbf{r}', t - t_a)$ is the point spread function (psf) of the ultrasound imaging system. The beamforming algorithm is designed such that the vector $\mathbf{S}(\mathbf{r})$ is a spatial impulse pointing in one of the orthogonal spatial directions, for example $\mathbf{S}(\mathbf{r}) \approx (\mathbf{r}) \hat{\mathbf{n}} + 0\hat{\mathbf{j}}$, where $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ are the orthonormal vectors describing X and Y directions. Another design with beamformer algorithm is implemented using a new set of weights and time delay to obtain a vector psf pointing in the other orthogonal direction i.e. $\mathbf{S}(\mathbf{r}) \approx 0\hat{\mathbf{i}} + \mathbf{S}(\mathbf{r})\hat{\mathbf{j}}$. Such psf vectors combined with the knowledge of $\mathbf{B}_0(\mathbf{r})$ can be used to extract the individual components of the current density $\mathbf{J}_c$ which induces the MAT-MI acoustic waves.

After reconstructing the vector source we can then derive the conductivity distribution of the object, which is of more interest for clinical applications.

The induced current density $\mathbf{J}$ can be expanded in terms of equation (2) using Ohm's Law as

$$\mathbf{J} = -\sigma \mathbf{E} = -\sigma \frac{\partial \mathbf{A}}{\partial t} - \sigma \nabla \phi \quad (10)$$

This can be rewritten as follows

$$\mathbf{J} = -\sigma \frac{\partial \mathbf{A}}{\partial t} + \phi \nabla \sigma - \nabla(\sigma \phi) \quad (11)$$

Now in an object with piece-wise homogeneous conductivity distribution $\sigma \approx 0$ at points other than at the conductivity boundaries. So in regions away from the boundary the induced current can be approximated as follows

$$\mathbf{J} \approx -\sigma \frac{\partial \mathbf{A}}{\partial t} - \nabla(\sigma \phi) \quad (12)$$

Comparing equation (12) with (3), in the region away from the boundary, the rotational component of the induced current, which is the source of MAT-MI acoustic wave, is then

given by the term $-\sigma \frac{\partial \mathbf{A}}{\partial t}$. This term is a purely rotational vector, due to the design of the magnetic vector potential $\mathbf{A}$. So the reconstructed vector source $\mathbf{J}_c$, which is the rotational part of the induced current, under a piece wise homogeneous conductivity distribution can be represented as follows

$$\mathbf{J}_c \approx -\sigma \frac{\partial \mathbf{A}}{\partial t} \quad (13)$$

The conductivity can then be estimated from the above vector equation.

Due to the limited bandwidth of the ultrasound transducer only a portion of information about the acoustic source can be restored in MAT-MI. To obtain a millimeter level spatial resolution in the reconstruction, ultrasound transducers in the megahertz central frequency range are desired. With these transducers the dc to low frequency component of the signal is not available for the reconstruction. This leads to the point spread function of the image reconstruction to be a non ideal spatial pulse which acts as a band pass filter in the reconstructed image. This psf is determined by the ultrasound imaging setup and the reconstruction algorithm. The estimated psf is then used to filter the reconstructed image to obtain a more complete acoustic source distribution [32]. In the presence of noise there is a tradeoff between the completeness of the inverse filtering process and noise amplification. To better handle the inaccuracies in the acoustic source reconstruction the conductivity estimation can be done with modifications of equation (13) as follows

$$\sigma \approx S_1 + S_2 \quad (14)$$

Where

$$S_1 = (\mathbf{J}_c(r'_1, r'_2 \dots r'_n) \cdot \hat{i} - \mathbf{J}_c(r'_m) \cdot \hat{i}) \setminus (\mathbf{A}'(r'_1, r'_2 \dots r'_n) \cdot \hat{i} - \mathbf{A}'(r'_m) \cdot \hat{i})$$

$$S_2 = (\mathbf{J}_c(r'_1, r'_2 \dots r'_n) \cdot \hat{j} - \mathbf{J}_c(r'_m) \cdot \hat{j}) \setminus (\mathbf{A}'(r'_1, r'_2 \dots r'_n) \cdot \hat{j} - \mathbf{A}'(r'_m) \cdot \hat{j})$$

Where, $A' = \frac{\partial A}{\partial t}$, \set operator is the least square fit, $r_1, r_2 \dots r_n, r_m$ are points within a region of uniform conductivity in the object. The boundaries of such a region can be determined from the MAT-MI acoustic source distribution, which is the strongest at the location of conductivity change in the object [17][22][24].

III. Methods

The proposed image reconstruction technique has been verified with computer simulation and MAT-MI experiment studies. In these studies, the conductive samples and the magnetic fields were assumed to be homogeneous in the z direction. With these approximations and an appropriate setup for acoustic measurements, we can simplify the corresponding MAT-MI imaging problems to 2-D problem.

A. Computer Simulation

Figure 1 shows the schematic of the MAT-MI imaging setup implemented in the current simulation study. The pulsed magnetic field is applied with two coils of 14 cm diameter in a Helmholtz pair arrangement with a separation of 5 cm in the z direction. This setup ensures a uniform magnetic field over the object pointing in z direction. The pulsed magnetic field is designed to generate eddy currents with the same bandwidth as the ultrasound measurement system to ensure the maximum coupling of the acoustic signals with the ultrasound transducer. The magnetic field strength $\mathbf{B}_1$ is assumed to be 1 Tesla in simulation. The static magnetic field $\mathbf{B}_0$ is assumed to be uniform in the imaging area and pointing in z direction with the flux density set to 1 Tesla.

The magnetic induction problem for MAT-MI as described in (1) is solved through FEM simulations using COMSOL. The object space is set to be a $9 \times 9 \times 3$ cm volume in the simulation. The object and the coil setup are centered at the origin. The uniform conductivity distribution of the object along the z direction and the implemented setup of the $\mathbf{B}_1$ field ensure that the induced current is also uniform in the z direction. The FEM simulation for MAT-MI can then be approximated as a 2D problem and the FEM solution is implemented at the object slice with a thickness of 1 mm at $z=0$. The COMSOL FEM simulation first computes the electric field potential ϕ which is then used to compute the electric field E and the induced current density $\mathbf{J}$. This current density $\mathbf{J}$ is then interpolated to a square mesh of size 9×9 cm with a spacing of 0.15 mm to compute the acoustic source distribution $\nabla \cdot (\mathbf{J} \times \mathbf{B}_0)$.

Ultrasonic imaging simulations were performed with Field II software [33]. The software allows defining transducers of various geometries. The spatial impulse response of the transducer is calculated in the software by breaking the transducer surface into smaller mathematical elements. The program also allows simulating the transducer's electromechanical impulse response. In the current simulation a flat 2 cm diameter piston transducer is used. The transducer has a center frequency of 500 kHz and a bandwidth of 60 % of the center frequency. A pulse with this frequency distribution is set as the transducers electromechanical response. The transducers are placed on a circular scanning geometry 20

cm away from the center of the object on $z=0$ plane. The 20 cm scanning radius leads to an approximately uniform sensitivity over the object space. Thus the piston transducer can be assumed to be acting as a point receiver. The large scanning radius also ensures that the spherical acoustic waves from the MAT-MI source can be approximated as plane waves at the transducer, which allows the ultrasound imaging setup to be also approximated as 2D. The received pressure signal is computed according to equation (5). At a given transducer location the spatial impulse response of the transducer is computed and the received signal at this location is then obtained by adding up the spatial impulse response multiplied with the acoustic source amplitude. This process is then repeated for each transducer location employed in the scanning scheme. Using this transducer signals vector source is reconstructed and the conductivity is estimated according to section II.b. The maximum error in the reconstruction is around the conductivity boundaries in the object and it is extended over one ultrasound wavelength around the boundaries due to the smoothing effect of limited bandwidth reconstruction. This error is reduced by obtaining the conductivity in this region by extrapolating the conductivity values from the neighboring regions.

B. Experiment Method

We have evaluated the proposed conductivity reconstruction method in MAT-MI experiments. The experiment system [17] has a similar design to the schematic setup shown in Fig 1. The static magnetic field $\mathbf{B}_0$ is applied with two permanent magnets used like a Helmholtz pair to get an approximately uniform magnetic field of 0.2 T (Gaussmeter, Alpha Laboratory) between them. Two coils of three turn each and arranged as a Helmholtz pair are used to generate the pulsed magnetic field $\mathbf{B}_1$. A stimulator based on a transistor switch was developed to drive these coils. The current flowing in the coils has a waveform approximating a bipolar single cycle sinusoid, which lasts $2\mu\text{s}$ and gives a center frequency of 500 kHz which matches with the bandwidth of the ultrasound imaging transducer. The dynamic magnetic excitation generated by the stimulating coils was measured by a sensing coil with radius of 1.5 cm connected to an oscilloscope. The estimated maximum current changing rate in the coils is $1.5\text{e}8\text{ A/s}$, which corresponds to a magnetic field changing rate of $6\text{e}3\text{ T/s}$ at the coordinate center. The maximum dynamic magnetic field strength $\mathbf{B}_1$ is estimated to be around 0.006 T at the coordinate center. The MAT-MI acoustic signal measurement was conducted using a circular scanning scheme. During experiments, both the sample and the transducer were submerged in distilled water media for acoustic coupling. A 500 KHz flat transducer (Panametrics V301) with 29 mm diameter and around 60% bandwidth was employed in this study. The transducer was mounted to a scanning frame driven by a stepper motor that can scan around the sample over a 320° view angle and 2.5° scanning step. The scanning radius was about 20 cm. While scanning, the object and the magnetic field setup are fixed in position to reliably acquire the acoustic measurements. The acoustic data collection is synchronized with the magnetic excitation. Again as in simulation the uniformity along Z direction of conductivity distribution of the object, the uniformity of the magnetic fields and the implemented scanning setup allows the corresponding MAT-MI problem to be simplified as 2D problem. The acoustic signals collected using this transducer was fed into preamplifiers with 90 dB gain and digitized by a 5 MHz data acquisition card. Signal averaging was used to increase SNR. After each experiment the conductivity of the sample was measured using a four electrode probe [25]. The conductivity distribution reconstructed in experiments was normalized to the expected range of values using a scaling factor to account for the various gains in the system. The scaling factor was determined by estimating the conductivity of a 50 mm diameter test sample of 0.6 % salinity whose conductivity was measured to be 0.98 S/m using the four electrode probe.

In the experiments the time varying magnetic field leads to electromagnetic interference (EMI) at the ultrasound transducer. This continues for certain time duration due to the turn

off transients associated with the stimulator and the impulse response of the transducer. The observed EMI signal in the experiment data is seen to have significant low frequency variations. This noise on backprojection leads to slow varying spatial signals, which interferes with the reconstruction of the desired low spatial frequency component of the vector source distribution. This leads to errors in conductivity estimation so the received acoustic signal is bandpass filtered to eliminate most of the interfering low frequency and high frequency noise. A simple filter implemented [34][35] for this purpose in frequency domain is

$$F(f) = \begin{cases} 1, & 100kHz < f < 900kHz \\ 0, & otherwise \end{cases}$$

Further a hanning window based filtering is applied to the estimated conductivity image to improve the SNR of the reconstruction. Such a filter can be designed as follows

$$H(n, n) = h \times h'$$

Where

$$h(n, 1) = 0.5 * (1 - \cos(2 * \pi * \frac{0:n-1}{n-1})), n=6$$

Where n is the width of the hanning window filter H.

IV. Results and Discussion

A. Computer Simulation

We have conducted computer simulation to verify the proposed conductivity reconstruction method. The result of the vector source reconstruction under noise free condition is shown in Fig 2. Fig. 2(a) shows the conductivity distribution of the simulated object and Fig. 2(b) shows the induced MAT-MI acoustic source distribution for the object. To verify the proposed Helmholtz decomposition to obtain the rotational component of the induced current, the acoustic source distribution was obtained with two methods, one using the induced current $\mathbf{J}$ and the other with the rotational component of the current $\mathbf{J}_c$ as described in the theory section.

The two source distributions were compared and were found to approximately equal. A portion of the induced current density is seen in Fig. 2(c) and the corresponding reconstructed current density is seen in Fig. 2(d). The induced eddy currents are cyclic, as seen in this figure, due to the magnetic vector potential generated by the coils being cyclic. The individual orthogonal components of the reconstructed vector source. $\mathbf{J}_x$ and $\mathbf{J}_y$ have phase inversion about the x and y axis, respectively, due to the cyclic nature of the current. Fig. 2(e) shows a line profile of the reconstructed current source with the points of conductivity boundaries marked by arrows. Due to the limited bandwidth of the imaging system, spatial smoothing is observed in the reconstructed current density especially at the conductivity boundaries where a sharp change in the current density is expected.

The conductivity reconstruction from vector source imaging is seen in Fig. 3 for various noise levels. Fig. 3(a) shows the conductivity distribution of the simulated object. Figs. 3(b), (c), (d) show the conductivity reconstruction for SNR level of 50, 10 and 1 respectively. The reconstructed conductivity values when compared to the target have correlation coefficients (CC) of 99.6%, 99.2%, and 98.3% respectively. The corresponding relative error (RE) is 6.3%, 10.87%, and 24.68%, respectively. Due to the smoothing effect of limited bandwidth reconstruction on current density the estimated conductivity at the boundaries of the object have the maximum error. To reduce this error the conductivity in this region around the boundary is estimated by extrapolating the conductivity at the neighboring points.

B. Experimental Results

We have also evaluated the proposed technique in phantom experiments. Phantoms made of pork skin gel were used in these experiments. Fig. 4(a) shows a tested phantom, which was photographed after the experiment. The phantom was made with a 0.4 % salinity gel in the background with a diameter is 5cm. A 2.5 cm diameter cylindrical column is embedded in the gel. This region is filled with 1.2% salinity gel after the background gel is set. The conductivity values 0.4% and 1.2% saline solutions are 0.67 S/m, 2.01 S/m respectively. A photograph of the phantom was taken immediately after the experiments with a black dye applied to the inner layer for photo contrast enhancement. The reconstructed acoustic source distribution is shown in Fig. 4(b) and as expected with limited bandwidth reconstruction the acoustic source is mainly distributed around the conductivity boundaries in the object. Using the vector imaging method the MAT-MI current source components were reconstructed as shown in Figs. 4(c) and 4(d). The cyclic nature of the induced current can be seen in this figure and also the current density is stronger in the higher conductivity inner region as compared to its neighboring lower conductivity region. Fig. 4(e) shows the estimated conductivity distribution using the reconstructed vector source. A conductivity profile at $x=-1$ mm is shown in Fig. 4(f). The reconstructed conductivity distribution seen in these results represents well the expected conductivity of the tested phantom. The extrapolation of conductivity values at the boundary gives a good fit with the expected conductivity profile for the piece wise homogeneous objects.

We also tested, in experiments, the imaging contrast of the reconstruction technique, as this is an important index for biological imaging. Fig. 5(a) shows the photograph of the phantom imaged in this experiment; the phantom consists of a conductive 25 mm square block placed in non-conductive background. Three different samples with salinity 0.3 %, 0.6 % and 0.9 % were prepared for this experiment. The conductivity of the samples were measured to be 0.54, 0.97, and 1.52 S/m, respectively. The reconstructed conductivity images are shown in Figs. 5(b), (c) and (d) corresponding to the phantoms with salinity 0.3 %, 0.6 % and 0.9 %, respectively. Fig. 5(e) shows the plot of the estimated and measured conductivity against the salinity of the phantom. The mean estimated conductivity of the three samples was 0.57, 0.92 and 1.48 S/m respectively. It can be seen from the error bar in Fig. 5(e) that approximately 10% variation is seen in the reconstructed conductivity.

Spatial resolution is another important index for biological imaging. Fig. 6(a) shows the photograph of the phantom imaged. The phantom consists of two square gels of 0.9 % salinity embedded in a background gel of 0.4 % salinity. The embedded blocks have a dimension of 9 and 13 mm, respectively. The reconstructed image is shown in Fig. 6(b); it agrees with the original sample well. Specifically the relative location and size of the square gel are clearly resolved and matches well with the phantom. A line profile at $x=3$ mm from the reconstructed image is shown in Fig. 6(c) and it includes the two of square inclusions. The reconstructed profile as seen here is in good agreement with the original profile, which was a grayscale profile from the photo in Fig. 6(a). Some distortion of the object occurred

around the boundary over one ultrasound wavelength, which is 3mm due to the errors in estimating the conductivity around this region.

V. Conclusion

MAT-MI is an emerging imaging approach, which combines the high resolution of ultrasound imaging with the good contrast provided by tissue electrical properties for biological imaging. The previously developed image reconstruction algorithms, for MAT-MI, using the acoustic source distribution as an intermediate quantity and a single magnetic excitation, are only able to reconstruct the conductivity boundaries when the ultrasound setup has limited bandwidth [25]. With the use of multiple magnetic excitations and a modified reconstruction algorithm the relative conductivity contrast can be obtained with the limited bandwidth ultrasound setup [22][23]. In this study we demonstrate that, using the proposed vector source reconstruction algorithm combined with the relationship obtained using Helmholtz decomposition of the induced eddy current, the conductivity distribution can be estimated reliably even under limited bandwidth measurement scheme. As seen in the computer simulations and phantom experiments, the estimated conductivity value is fairly accurate in regions of uniform conductivity in the object. In regions around the boundaries with the current implementation of image reconstruction the conductivity distribution is estimated by extrapolating the values from the surrounding uniform conductivity regions. To improve the accuracy of the reconstruction in regions of variable conductivity the dependence of the vector source on conductivity gradient and the electric potential would have to be taken into account. The electric potential is a term that depends on the magnetic stimulation and the conductivity distribution of the object and a priori knowledge of this term is not possible for all objects, so an iterative reconstruction algorithm as proposed in [22][23] would have to be implemented. Conductivity anisotropy of biological tissue is another factor that would have to be taken into account for accurate imaging [20]. In this case a multi-excitation method as proposed in [22][36] can be used to estimate the directionally dependent components of conductivity.

In this preliminary verification study of the proposed method the computer simulations and phantom experiments were conducted on a simplified 2D space. This assumption is satisfied when the conductivity of the phantom, the stimulating magnetic fields and the spatial response of the ultrasound receiver is uniform along the Z direction. However the proposed reconstruction algorithm can also be applied for 3D MAT-MI imaging using a 3D acoustic receiver aperture [21]. This setup can be used to obtain the 3D distribution of MAT-MI vector source and the conductivity can then be estimated using the proposed method.

To generate the eddy currents in the object, MAT-MI uses a strong pulsed magnetic field which could cause heating of tissue and nerve stimulation. In the current setup, the induced electrical field applied to the sample is under 550 V/m which is comparable to the stimulating strength in transcranial magnetic stimulation (TMS) [25]. However, the total energy applied to the samples in MAT-MI is much lower than TMS as the MAT-MI pulse duration is at the microsecond level instead of hundreds of microsecond in TMS.

In MAT-MI to obtain a reliable reconstruction a good measurement of the acoustic signal is necessary. In the experimental implementation of the MAT-MI method, this is ensured by using a stable scanning frame and a good synchronization between the magnetic excitation and acoustic data collection. The magnetic excitation generated by the coil also induces significant electromagnetic interference (EMI) at the transducer. To reduce the effect of the EMI a stable coil driver with a short turn-off transient is desired. The transducer also has a good EM shielding to suppress this interference and it is placed at a certain distance away from the imaging region of interest such that by the time the acoustic signal from this region

arrives at the transducer the EMI fades away. To avoid the effect of EMI in MAT-MI an optical ultrasound transducer as proposed in [37] can be used. Since in the proposed reconstruction method the slow spatially varying conductivity distribution is being estimated the low frequency component of the noise would also get amplified through the imaging process. Since the MAT-MI acoustic signal is bandwidth limited and distributed over a certain percentage of the central frequency, the low frequency part of the signal is dominated by noise. This frequency component of the data is carefully processed [34][35] while reconstruction to remove the noise prior to conductivity estimation for reliable imaging.

The acoustic homogeneity assumption used in the theoretical derivation restricts the proposed imaging method to primarily soft tissue imaging. The acoustic heterogeneity of soft tissue is less than 10 % and its effect can be considered to be negligible for MAT-MI [16][21].

In conclusion we have developed a current density based conductivity reconstruction method for MAT-MI imaging. Computer simulation and phantom experimental studies have been conducted to demonstrate the ability and evaluate the performance of the proposed method in reconstructing the conductivity contrast using ultrasound measurements.

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Fig. 1.
Schematic diagram of MAT-MI setup used in this study.

Fig. 2. (a) Simulated conductivity distribution. (b) Induced acoustic source in the object. (c) Induced current density. (d) Reconstructed current density. (e) Current density profile through $y=3.25$ mm of J_y , the arrows mark the conductivity boundaries.

Fig. 3. (a) Target conductivity image. (b), (c) and (d) Reconstructed conductivity images with SNR of 50, 10 and 1, respectively.

Fig. 4. (a) Photograph of the gel phantom tested. (b) Reconstructed acoustic source distribution. (c) Reconstructed x component of the current: J_x . (d) Reconstructed y component of the current: J_y . (e) Reconstructed conductivity distribution. (f) Conductivity profile at $x=-1$ mm showing the comparison between the target and reconstructed conductivity value.

Fig. 5. (a) Photograph of gel phantom used in the study. (b),(c) and (d) The reconstructed conductivity distributions corresponding to phantoms with salinity 0.3%,0.6% and 0.9% respectively.

Fig. 6. (a) Photograph of the gel phantom imaged. (b) Reconstructed conductivity distribution. (c) Conductivity profile through $x=3$ mm comparing the reconstructed conductivity distribution with the target conductivity distribution from the photograph.