Design optimization of MR-compatible rotating anode x-ray tubes for stable operation

Mihye Shin^{a)}

Department of Radiology, Stanford University, Stanford, California 94305 and Department of Mechanical Engineering, Stanford University, Stanford, California 94305

Prasheel Lillaney

Department of Radiology, Stanford University, Stanford, California 94305 and Department of Bioengineering, Stanford University, Stanford, California 94305

Waldo Hinshaw and Rebecca Fahrig

Department of Radiology, Stanford University, Stanford, California 94305

(Received 23 April 2013; revised 15 September 2013; accepted for publication 23 September 2013; published 11 October 2013)

Purpose: Hybrid x-ray/MR systems can enhance the diagnosis and treatment of endovascular, cardiac, and neurologic disorders by using the complementary advantages of both modalities for image guidance during interventional procedures. Conventional rotating anode x-ray tubes fail near an MR imaging system, since MR fringe fields create eddy currents in the metal rotor which cause a reduction in the rotation speed of the x-ray tube motor. A new x-ray tube motor prototype has been designed and built to be operated close to a magnet. To ensure the stability and safety of the motor operation, dynamic characteristics must be analyzed to identify possible modes of mechanical failure. In this study a 3D finite element method (FEM) model was developed in order to explore possible modifications, and to optimize the motor design. The FEM provides a valuable tool that permits testing and evaluation using numerical simulation instead of building multiple prototypes.

Methods: Two experimental approaches were used to measure resonance characteristics: the first obtained the angular speed curves of the x-ray tube motor employing an angle encoder; the second measured the power spectrum using a spectrum analyzer, in which the large amplitude of peaks indicates large vibrations. An estimate of the bearing stiffness is required to generate an accurate FEM model of motor operation. This stiffness depends on both the bearing geometry and adjacent structures (e.g., the number of balls, clearances, preload, etc.) in an assembly, and is therefore unknown. This parameter was set by matching the FEM results to measurements carried out with the anode attached to the motor, and verified by comparing FEM predictions and measurements with the anode removed. The validated FEM model was then used to sweep through design parameters [bearing stiffness (1×10^5 – 5×10^7 N/m), shaft diameter (0.372–0.625 in.), rotor diameter (2.4–2.9 in.), and total length of motor (5.66–7.36 in.)] to increase the fundamental frequency past the operating range at 50 Hz.

Results: The first large vibration during the prototype motor operation was obtained at 21.64 ± 0.68 Hz in the power spectrum. An abrupt decrease in acceleration occurred at 21.5 Hz due to struggling against the resonance vibrations. A bearing stiffness of 1.2×10^5 N/m in the FEM simulation was used to obtain a critical speed of 21.4 Hz providing 1.1% error. This bearing stiffness value and the 3D model were then confirmed by the experiments with the anode removed, demonstrating an agreement within 6.4% between simulation results and measurements. A calculated first critical frequency (fundamental frequency) of 68.5 Hz was obtained by increasing the bearing stiffness to 1×10^7 N/m and increasing the shaft diameter by 68.0%. Reducing the number of bearings in the design permits decreasing the total length of the motor by 1.7 in., and results in a fundamental frequency of 68.3 Hz in concert with additional changes (shaft diameter of 0.625 in., rotor diameter of 2.4 in., and bearing stiffness of 1×10^6 N/m).

Conclusions: An FEM model of the x-ray tube motor has been implemented and experimentally validated. A fundamental frequency above the operational rotation speed can be achieved through modification of multiple design parameters, which allows the motor to operate stably and safely in the MR environment during the repeated acceleration/deceleration cycles required for an interventional procedure. The validated 3D FEM model can now be used to investigate trade-offs between generated torque, maximum speed, and motor inertia to further optimize motor design. © 2013 American Association of Physicists in Medicine. [<http://dx.doi.org/10.1118/1.4824325>]

Key words: x-ray tube motor, hybrid x-ray/MR (XMR) system, mechanical resonance, resonant frequency, stability, magnetic fields, optimization

1. INTRODUCTION

1.A. Background

Hybrid x-ray/MR (XMR) systems have been established and two-room systems are now offered by several vendors.^{1–10} Combined XMR suites can enhance the diagnosis and treatment of endovascular, cardiac, and neurologic disorders using the complementary advantages of both modalities for image guidance during interventional procedures. The MR imaging system provides excellent soft tissue contrast in the three-dimensional images without radiation and permits monitoring of physiological parameters such as perfusion and flow, which are useful to evaluate the progression and success of an intervention. X-ray imaging provides high-resolution projection images of iodine-filled vessels and small, high contrast devices at 30 frames/s. By combining MR with x-ray, it is possible to obtain ideal image guidance during placement of catheters, guide wires, and prosthetic devices including real-time device monitoring and evaluation of outcomes.

A fully colocalized XMR system has been implemented in which the MR and x-ray projection image the same patient volume.¹¹ A static anode x-ray tube and digital flat panel detector were incorporated into the gap of a double-magnet 0.5 T interventional MRI (Signa SP, GE Milwaukee, WI). The hybrid system successfully demonstrated clinical applications such as hysterosalpingograms, chemoembolization of hepatic tumors, transjugular intrahepatic portosystemic shunts, loopograms, and brain biopsy.^{12–15} Using this system geometry, no patient movement is required when switching between the two systems. However, the instantaneous heat loading on the static anode restricts the maximum x-ray flux output, which hinders high flux angiographic procedures and abdominal interventions for larger patients. The low field strength of the MR system is also problematic because the maximum achievable signal to noise ratio (SNR) of the MR images is limited.

In order to overcome the limitations of the colocalized hybrid system, we proposed a close proximity hybrid closed bore XMR (CBXMR) that consists of a standard 1.5 T MR system and a C-arm fluoroscopy unit (Fig. 1). The C-arm unit, including a rotating anode x-ray tube and digital flat panel detector, is positioned no more than 1 m from the face of the MR shroud, and the x-ray and MR imaging fields of view are therefore separated by a minimum distance of 1.2 m. To minimize patient motion the x-ray tube should be able to operate in the presence of strong magnetic fields so the systems can be in close proximity. The C-arm gantry central axis and the cathode-anode axis will be parallel to the Z axis in cylindrical MRI coordinates, and orthogonal to the radial component of the MR fringe field. Existing rotating anode x-ray tubes fail near an MR imaging system, since MR fringe fields alter the electron trajectories in the x-ray tube cathode-anode gap and create eddy currents in the metal rotor which cause a reduction in the rotation speed of the motor.¹⁶ Several solutions for the electron ballistics problem have been proposed to correct for the displacement and change in shape of the focal spot,^{17–19} and a novel motor prototype has been developed to solve the problem of reduced rotation speed. The proposed x-ray tube motor design utilizes the MR fringe field to gen-

FIG. 1. (a) Schematic of the proposed hybrid x-ray/MR image guidance system illustrates the collimated x-ray flux output with the rectangular prism between the x-ray tube and the flat panel detector. The cathode-anode axis and the central axis of the x-ray tube motor are parallel to a dashed-dotted line aligned with the Z axis. (b) MR fringe field trajectories and the relative size comparison between the x-ray tube and the 1.5 T closed bore MR magnet.

erate torque, analogous to the permanent magnet stator field used in DC motors, but large vibrations indicating instability were detected during its operation.²⁰

1.B. Resonance

Mechanical resonance is a significant problem that occurs when a periodic external force is applied to a system having a natural frequency equal to the driving frequency; this often happens when the excitation force is derived from rotating parts. A harmonic driving force gives rise to large amplitude vibrations close to or at the resonant frequencies.²¹ If an assembly is operated continuously near resonance, it is possible to create repeated large deflections back and forth, which may repeatedly exceed the material strength limits and cause the structure to fail. One strategy for minimizing resonance effects is to quickly accelerate the assembly through the resonant frequency to reduce the length of time near the resonance region. However, momentary but large mechanical stresses will still be induced. When using the hybrid system for image-guided interventional procedures, the x-ray beam will likely be turned on and off at various times, which will require repeated acceleration and deceleration of the rotating anode x-ray tube motor. Due to the nature of these procedures, the possibly short but unwanted mechanical stresses and vibrations will be repeated, leading to fatigue which may damage the structure and shorten the structural life. Therefore, when developing a new x-ray tube motor it is important to consider the resonant frequencies of the design. One important design strategy is to eliminate any causes of the vibration so as to increase stability during operation. While the motor spins, it will undergo resonance with severe vibrations at critical angular speeds. In rotor dynamics, the critical angular speeds are defined as the rotation speeds where the speed-dependent natural frequencies intersect with an excitation line, the identity line in our case.²² If all critical speeds are located above the operational speed range, the motor will not encounter any resonances that could result in mechanical failure. To achieve this goal, the first critical frequency

FIG. 2. Cross sectional view of the commercial x-ray tube rotor. A rotating anode is supported by the center shaft, bearings, and support structure. The anode and shaft are fixed to the rotating part, and rotate together.

(fundamental frequency) must be above the operational range of the device. In this study the assembled new x-ray tube motor was evaluated for its rotor dynamics, with particular focus on resonance. A corresponding finite element model was developed in order to explore modification options and optimize motor design for mechanical stability.

1.C. Conventional rotating anode x-ray tube motor

A conventional x-ray tube uses an AC (alternating current) induction motor, which consists of a stator and a rotor. The stator includes a set of coils that surround the rotor, and current flow in the windings has phase differences so that the direction of the created magnetic field changes continuously; a net magnetic moment rotates on the center axis of the motor. The rotating net magnetic flux induces eddy currents in the metal rotor, and the induced eddy currents create their own magnetic fields. The magnetic fields in the stator and in the rotor interact with each other, and directional difference between the fields generates torque that causes the rotor to turn. This does not require direct electrical or mechanical contact between the stator and the rotor. Based on this operational mechanism, the rotation speed of the induction motor is determined by the frequency of the applied current in the stator. When the stator coils are operated using 60 Hz AC power, the rotation speed is approximately 50 Hz (3000 RPM). If a

180 Hz power supply is used, the motor can reach a speed of about 167 Hz (10 000 RPM).²³ The cases are commonly referred to as “normal (low) speed mode” and “high speed mode,” respectively.²⁴ Most rotating anode x-ray tubes show the first resonance in the range of 70–90 Hz, which is between the two operational modes. Our proposed MR-compatible rotating anode x-ray tube targets operation at the “normal speed mode.”

The inner structure of a commercial x-ray tube rotor is relatively compact and simple (Fig. 2). Bearing balls, bearing races, and a center shaft, to which the anode assembly is connected, are mounted in a stationary support. The bearings, center shaft, and anode form the rotor part of the tube assembly. The stationary support structure is connected to a housing through which thermal conduction occurs, which is one of the heat dissipation methods. A retaining clip and spring together exert force on the bearings and support, which in turn keeps all of the components tightly in place. This preload (spring force) on the bearing assembly improves structural stability, since increased contact points due to the preload generate a larger support area between the bearing balls and the bearing races based on removal of the internal clearance usually found in bearings.²⁵ In other words, the bearing stiffness is increased. Resultant compressive stress in the shaft also helps to compensate for the tensile stress that arises from mechanical loading.²⁶

2. NEW MOTOR DESIGN AND THEORY OF OPERATION

The proposed MR-compatible x-ray tube motor design satisfies the following requirements and has a performance that is similar to the existing x-ray tube motors, but can function in an MR environment (Fig. 3):

- (1) all components are nonmagnetic and vacuum compatible,
- (2) the anode has high heat capacity,
- (3) the rotor has low heat conductivity,
- (4) a desired minimum operational speed is 50 Hz (3000 RPM), and

FIG. 3. A new MR-compatible x-ray tube motor design consists of a nonmagnetic ceramic rotor, four support bearings, and insulating sleeves in between the bearings. The bearings also serve as electrical slip rings for the coil windings slotted into the rotor: one for each of the three sets of coil windings and one for the grounded anode. External magnetic fields, MR fringe fields in our case, are used to generate torque and make the motor (the anode and ceramic rotor) rotate on the dashed-dotted line, while the shaft is stationary.

- (5) the operation is not compromised by presence of the external magnetic field.

In order to avoid stalling due to magnetization, as well as to avoid eddy currents, the x-ray tube rotor is made out of non-metallic material. All components must be vacuum compatible for standard x-ray production using accelerated electron bombardment of the tungsten anode.²⁷ The vacuum environment inside the glass or metal envelope requires a bake-out process, which subjects the entire assembly to at least 300 °C for up to 48 h for outgassing.²⁰ The low thermal conductivity of the rotor material is also important in order to protect the x-ray tube components by preventing conductive heat loading from the anode through the rotor to the other parts of the assembly.²⁴ For the main components in the rotor, MACOR machinable glass ceramic (Corning Incorporated Lighting & Materials, NY) is suitable for ultrahigh vacuum environments and withstands high temperature up to 1000 °C (continuous operating temperature of 800 °C). The thermal conductivity of MACOR ceramic ranges between 1.35 and 1.5 W/(m · °C) from 0 to 600 °C, while that of normal tool steel is about 50 W/(m · °C) on average. Nonmagnetic stainless steel 316 is used for the support structures where high strength is required.

The thermal characteristics of the anode determine the minimum rotation speed of the x-ray tube motor. Most rotating anode x-ray tubes are affected by three distinct criteria: the heat load on the focal spot track, the heat storage of the anode body, and the heat dissipation by the tube housing. The heat storage and heat dissipation problems mainly rely on traditional heating and cooling mechanisms of the bulk material,

TABLE I. Input voltage change according to the motor state.

State	A	B	C
#1	+V	N/A	−V
#2	+V	−V	N/A
#3	N/A	−V	+V
#4	−V	N/A	+V
#5	−V	+V	N/A
#6	N/A	+V	−V

whereas the instantaneous heat load on the focal track is limited by the motor angular speed. The effective length of the focal spot track is determined by the rotation speed for a given exposure time. If the rotation speed is not sufficiently high, the electrons bombard a small area and concentrated energy immediately raises the focal spot track temperature before the instantaneous heat load can spread.^{23,28} The x-ray tube fails when the focal spot track temperature reaches the melting point of the anode material. For functionality, the standard minimum angular speed is about 3000 RPM (50 Hz).²⁹

The x-ray tube motor design is functionally analogous with an inverted BLDC (brushless direct current) motor. The design leverages the stationary external magnetic fringe field of the MR system (i.e., the field outside the MR scanner bore) using rotating coil windings slotted into the rotor, instead of the rotating magnetic fields of the conventional x-ray tube induction motor. The motor design incorporates three sets of coil windings that are spatially separated by 120°, and provide six possible states that can create torque [Figs. 4(a), 4(b), and Table I].

FIG. 4. (a) Three coil windings slotted into the rotor are connected to bearing outer races. The uppermost bearing is used to ground the anode through connection to the anode shaft which the anode has been welded on. The bearings are spaced and insulated by the inner sleeves. (b) Current is supplied from a power supply to each wire loop through inner wires and bearings. The anode, support mount, and other minor components have been omitted for clarity. (c) The coil windings are wired with each other in a wye configuration with a common point between all the windings.

FIG. 5. Cross sectional view diagram demonstrating how the net magnetic moment changes relative to the external magnetic field. (a), (b) The motor is in state #1 while rotating by 60° , so the wire loop of A and C are activated. The current flows following a path of power supply - wire loop A - common point - wire loop C - power supply. (c) The state changes immediately to state #2, so the wire loop of A and B are activated. Infinitesimal time of ΔT takes to switch from wire loop C off and to wire loop B on. (d) The motor is now in state #2 while rotating by 60° , and this routine is repeated according to the specific sequence in Table I.

The coil windings are connected together in a wye configuration, and each state has a specific pair of coil windings activated with the third coil winding inactive [Fig. 4(c)]. Hence, the direction of $\vec{\mu}$ for each state will be unique. Choosing the active state is determined by a feedback position system to maximize the generated torque, which can be accomplished by keeping $\vec{\mu}$ and $\vec{B}_R$ close to orthogonal (Fig. 5). By observing the angular position of the motor using an encoder, the feedback mechanism controls the electronic excitation through a power module. Thus, the rate at which the active state changes always follows the rotation speed of the motor, and is neither constant nor programmed. The amount of motor torque $\vec{\tau}$ is based on the Lorentz force and the interaction between magnetic moments [Eq. (1)]:

$$\vec{\tau} = \vec{\mu} \times \vec{B}_R. \quad (1)$$

In Eq. (1), $\vec{\mu}$ is the net magnetic moment of a certain state where two sets of wire loops are active in the rotor, and $\vec{B}_R$ is the magnetic field vector which is the MR fringe field in our case (Fig. 5).³⁰ Magnetic moment $\vec{\mu}$ is proportional to the number of turns in the wire loop, current flow through each loop, and the rectangular area of the wire loop. The bearings serve as electrical slip rings for the three coil windings and grounded anode in addition to supporting the structure between the rotor and the shaft. Therefore, the bearings are insulated from each other and the shaft is insulated by MACOR ceramic sleeves since the metal components are electrically conductive.

In a conventional x-ray tube motor, the shaft is rigidly mounted to the anode through the rotating body, and is in-

serted into a stationary hollow cylindrical support, so both the anode and shaft are rotating (Fig. 2). In our proposed motor design, the anode and outer rotor are rotating, while the center shaft on which the rotor is mounted is stationary (Fig. 3). The shaft plays an important role in the structural design and experiences high mechanical stress, because the weight of the rotor and anode apply a bending force to the shaft. We model the distributed masses as a lump mass at the free end of the shaft during the analysis. This is a conservative overestimate of the stresses placed on the shaft, but is simple to analyze. The cantilever configuration of the x-ray tube design increases instability since the anode, one of the heaviest components, is located at the free end of the stationary shaft. Because gravity is perpendicular to the shaft, the anode weight and the distributed rotor weight apply large moments and corresponding tensile bending stress on the shaft. The rotor is the next heaviest component due to its large volume; it supports the large wire loops that are needed to generate sufficient torque [see Eq. (1)]. The maximum value of the bending stress,

$$\sigma_{\max} = \frac{|M_{\max}| \times c}{I_{\text{area}}}, \quad (2)$$

is obtained using the maximum bending moment $M_{\max}$, the distance from the neutral axis to the outermost point of the cross section c (radius in our case), and the area moment of inertia of the cross-sectional area I_{area} .²⁶ The maximum bending stress must be lower than the strength of the material. The shaft design was conservatively chosen based on this static stress analysis including a safety factor (the material yield strength divided by the working stress) of 10.

TABLE II. Hardware comparison between rotating anode x-ray tubes.

	MR compatible x-ray tube motor	Conventional x-ray tube motor
Rotor material	MACOR (Ceramic)	Copper/Iron
Shaft material	Stainless steel 316	Tool steel
The number of bearings	Four	Two
Rotor diameter	2.9 in	1.47 in
The length of rotating part	6.5 in	3.3 in
Shaft diameter	0.372 in	0.36 in
Shaft length	7.2 in	2.94 in

Wave springs are placed between the inner sleeves and the bearing inner races to reduce the gap in the bearings which, in turn, reduces the electrical resistance across the bearing and increases the resulting torque achieved per unit applied power. As the internal clearance decreases, the support area increases and thus the preload from the spring force makes the structure sturdy.^{31,32}

The MR-compatible x-ray tube motor structure is compared with a conventional x-ray tube motor in Table II. A conventional tube rotor diameter and length are about half the size of the new concept x-ray tube rotor. This is because the stator of a commercial x-ray tube generates magnetic fields to turn the rotor, so the rotor part does not have to contain space to accommodate the wiring of the motor. Given the differences in both composition and geometry, the dynamic characteristics of the rotating object must be analyzed to identify possible modes of mechanical failure in addition to the static analysis.

3. METHODS AND MATERIALS

3.A. Finite element method model of rotor system

In order to investigate the stability and safety of the spinning MR-compatible x-ray tube motor, a finite element method model of the motor was developed including the effects of rotational inertia, gyroscopic moments, and applied load (ANSYS v 13.0, ANSYS, Inc., PA) (Fig. 6). Discrete natural frequency values for a rotation speed are computed every 20 Hz from 0 to 120 Hz by ANSYS, and a cubic curve fit is applied to obtain a natural frequency curve as a function of angular speed.^{33,34} For all simulation datasets, an R-Square correlation coefficient was 1.00 between the discrete values and the output of a fitted curve based on the cubic curve fit. The lowest critical frequency (fundamental frequency) indicates the first resonance point that the motor meets while accelerating from a stationary state, so that the fundamental frequency can be found by the intersection between the natural frequency curve and the identity line.

3.A.1. Solid motor component

Rotor dynamics are related to several physical properties such as mass, stiffness, damping ratio, and moment of inertia.

Combining the shaft and bearing models, the governing equation is given by³⁵

$$[\mathbf{M}] \{\ddot{\mathbf{Q}}\} + [\mathbf{G} + \mathbf{C}] \{\dot{\mathbf{Q}}\} + [\mathbf{K}] \{\mathbf{Q}\} = \{\mathbf{F}\}_b + \{\mathbf{F}\}_e. \quad (3)$$

Parameters $[\mathbf{M}]$, $[\mathbf{G}]$, $[\mathbf{C}]$, and $[\mathbf{K}]$ are mass, gyroscopic, damping, and stiffness matrices for the rotor system, respectively. The mass matrix, $[\mathbf{M}]$, consists of the mass and the radial moment of inertia; the polar moment of inertia is coupled by motion in the gyroscopic matrix, $[\mathbf{G}]$. The matrix $\{\mathbf{Q}\}$ represents the displacement vector including the bending and axial displacements of the elements.³⁶ On the right hand side of Eq. (3), the force vector inducing the motion is considered; $\{\mathbf{F}\}_b$ is associated with the bearing reaction loads, and $\{\mathbf{F}\}_e$ refers to gravity and other unbalanced forces and moments.

The FEM model must use masses and moments of inertia as equivalent as possible to the actual machined components (Fig. 7) to maximize accuracy in addition to using the same mechanical coefficients as the prototype, for instance, the modulus of elasticity, Poisson's ratio, and the material density, although only the shape of components and mechanical coefficients can be assigned in ANSYS. Table III summarizes the actual values as compared to the values used in the FEM model, where the moment of inertia has been computed on the axis of rotation (Fig. 6). The density of each material was first inspected using the actual mass of the components and the volume of SolidWorks 3D models (SolidWorks 2012, Dassault Systèmes SolidWorks Corporation, Waltham, MA) that are exactly the same as the prototype components, and the moment of inertia of the prototype was calculated based on the SolidWorks models. When developing

FIG. 6. Finite element model to simulate rotor dynamics of the novel x-ray tube motor. Predicted and measured fundamental frequencies are used to verify the model.

FIG. 7. Machined components of the MR-compatible x-ray tube motor prototype.

the FEM model of the motor, we leave out details of some of the parts such as threaded holes and grooves to increase use of regular shaped meshes for the convergence of the simulations, and use a hollow dumbbell-shaped rotor to represent space for coil windings; these approximations could induce small errors between the actual values for the prototype and those in the FEM model. Nevertheless, the errors for the moment of inertia are under 1% for the rotating components, and this should provide accurate estimates of resonance properties.

3.A.2. Bearing part

As described in Eq. (3), the bearing properties are also essential in solving the equation of motion. In the direct stiffness approach, the linearized bearing is represented as

$$\begin{bmatrix} c_{yy}^b & c_{yz}^b \\ c_{zy}^b & c_{zz}^b \end{bmatrix} \begin{Bmatrix} \dot{y}^b \\ \dot{z}^b \end{Bmatrix} + \begin{bmatrix} k_{yy}^b & k_{yz}^b \\ k_{zy}^b & k_{zz}^b \end{bmatrix} \begin{Bmatrix} y^b \\ z^b \end{Bmatrix} = \begin{Bmatrix} f_y^b \\ f_z^b \end{Bmatrix} \quad (4)$$

with the coordinate system as shown in Fig. 6.³⁷ In this equation, c_{ij}^b and k_{ij}^b describe bearing damping and stiffness coefficients in the i and j directions, respectively, and $\{f_y^b, f_z^b\}^T$ is the bearing force vector.

Four identical bearings (6004-RS, VXB Ball Bearings, Anaheim, CA), idealized as undamped and isotropic, are evenly distributed inside the MR-compatible x-ray tube rotor (Fig. 3). In practice, the finished prototype included a gap (at least 0.001 in. in diameter) required for slip or loose fit condition in each bearing assembly within the brittle MACOR material in addition to the internal clearance that a bearing itself has. This gap is needed to compensate for thermal expansion of the bearings due to heat load produced by electron bombardment on the anode. During operation the x-ray tube motor assembly will be heated from the electron bombardment on the anode, and the thermal expansion of the bearings will make tighter assembly against the inner wall of the rotor. For the current bearing characteristics, the interfaces between the ceramic inner sleeve and bearing inner race, as well as between the bearing outer race and ceramic rotor must be considered to verify the FEM simulations.³¹ The effect of these gaps is difficult to estimate theoretically, so experimental data was used as described below.

3.B. Data acquisition and analysis

The 3D FEM model was verified experimentally both by monitoring changes in angular acceleration and by using the

TABLE III. Comparison between the FEM model and the actual prototype about mass (top) and the moment of inertia (bottom).

Mass (g)	Assembled prototype	Finite element model	Error (%)
Anode	653.7	659.8	0.93
Rotor	738.3	778.4	5.43
Bearing	47.5	43.3	8.84
Shaft	98.4	103.3	4.99
Moment of inertia (g · cm ²)	Assembled prototype	Finite element model	Error (%)
Anode	5121.1	5155.6	0.67
Rotor	6319.5	6281.9	0.59
Bearing	160.0	159.1	0.56
Shaft	11.0	11.8	7.27

FIG. 8. Experimental setup with the accelerometer measuring the amplitude of vibrations. The angular position of the motor is being monitored by the quadrature sensor and magnet wheel.

power spectrum of rotor vibration response according to rotation speed (Fig. 8). Succinctly, the experimental protocol is as follows:

- (1) supply current to the coil windings of the motor (Table I) while gathering rotation speed data,
- (2) reach the maximum angular speed and steady state,
- (3) gather and average power spectrum data while the motor coasts down, and
- (4) obtain rotation speed and power spectrum datasets until the motor stops turning.

In the experimental setup, Helmholtz coils (Stangenes Industries, Palo Alto, CA) were used to generate the magnetic field which mimics the MR fringe field. The magnitude of the magnetic fields is determined by the amount of current flowing in the coils. The Helmholtz coils produce a magnetic field up to approximately 45 mT, covering an appropriate range of conditions that would be seen in the fringe field of an MR magnet. During deceleration, an accelerometer (353B34, PCB Piezotronics, Inc., Depew, NY) and a corresponding sensor signal conditioner (482A21, PCB Piezotronics, Inc.) send the time domain vibration response to a spectrum analyzer (3562A, Hewlett Packard, Palo Alto, CA), which generates the power spectrum. In the frequency domain, peaks of the power spectrum signify large amplitude vibrations at a certain frequency, for example, a resonant frequency. The rotation speed curves showing changes in angular acceleration are used to support the experimental results from the power spectrum. The angular speed curve is obtained from a position-reading encoder including a Hall effect quadrature sensor and ferrous nylon magnet wheel (both components purchased from Electro-Sensors, Minnetonka, MN) that consists of 120 alternating north and south poles, from which the feedback system acquires the angular position of the motor to prop-

erly excite the coil windings (Table I). The magnet wheel is mounted at the base of the x-ray tube rotor thus the stationary quadrature sensor outside of the rotor can detect the magnetic field changes. As the ferrous magnet wheel spins with the rotor, two square wave outputs (channels A and B) are generated by the quadrature sensor and recorded by a data acquisition card (SCB-68 and PXI-8176, National Instruments, Austin, TX). Based on the square wave signals, the angular speed is calculated by using the time interval between rising and falling edges of signals on channels A and B (LabVIEW 8.6, National Instruments). After the motor starts to spin, the increasing rotation speed (ω) asymptotically converges to the steady-state speed because angular acceleration (α) decreases. Consistent electrical input power ($V \cdot i$) is converted to mechanical output power ($\tau \cdot \omega = I \cdot \alpha \cdot \omega$), so the torque for acceleration decreases continuously as the angular speed increases. The steady state eventually occurs when the generated torque equals the resistive torque.

During the acceleration of the x-ray tube an abrupt decrease in the slope of the angular speed curve occurs when large vibrations from resonance are present. The x-ray tube motor spends more time at the critical rotation speed struggling against the vibrations, since the large wobble increases frictional torque and decreases the angular acceleration. Two kinds of measurements, the first peak location in the power spectrum and the angular acceleration change, can both be obtained and should match, providing an estimate of the first critical frequency.

3.C. Finite element method model validation

The development of the finite element model is needed to interpret the physics of the new motor design in depth and to evaluate modified designs using FEM models rather than constructing multiple prototypes. As stated in Sec. 3.A.2, the bearing stiffness cannot be determined from bearing geometry and materials alone when the bearing is used as a part in an assembly. The rotation speed curve and power spectrum were first measured using the fully assembled prototype. After verifying that the two measurement techniques provided the same fundamental frequency, we fixed the unknown bearing parameter by comparing the measured first critical frequency value to the FEM simulation results while varying bearing stiffness from 1×10^4 to 5×10^7 N/m. In order to confirm the adjusted finite element rotor model, measurements and simulations were repeated with the anode detached from the motor assembly. The experimentally determined critical frequency and theoretically predicted critical frequency from the FEM model will match if the determined bearing stiffness is correct.

3.D. Modification options for resonance effects

Natural frequencies, which are inherent to a structure, depend on many design parameters, and a mode is defined by the natural frequency and its mode shape. The mode shape of a rotor forms a pattern which describes the shape of vibration if it were to vibrate only at the corresponding resonant

FIG. 9. Mode shapes of the proposed x-ray tube motor at (a) the lowest and (b) one of the higher critical frequencies. The bearing stiffness and the center shaft mainly determine each mode. The axial symmetric views are exaggerated to demonstrate that the motor is about to fail due to putting excessive energy in the resonance modes.

frequency.²¹ Mode shapes can be used to determine which parameter has a major impact on the resonance phenomenon because relatively large physical displacement occurs in the component corresponding to the parameter at a certain resonance mode. The crucial parameter differs for each mode shape; a major property that controls the lower order mode may be distinct from a property that dominates the higher order mode. The bearing components deform more than the other components at the lowest critical frequency, and the deflection of the center shaft is more obvious in the higher mode (Fig. 9). As expected from the large displacement in each mode shape, the bearing stiffness dominantly determines the lowest mode.³⁸ The number of bearings and the length of the motor are also significant design factors. Based on the actuation principle of the new x-ray tube motor, a floating common point exists between wire loops, where the voltage is close to 0 V (Fig. 4).²⁰ The voltage at the common point can be regarded as an electrical ground compared to the magnitude of the voltage at the cathode that would be rectified and relatively constant in an x-ray tube.^{24,39} Thus, the common point could be used to ground the anode, and only three bearings are needed to run the proposed motor, offering increased freedom to vary the length and the inner structure. Considering the above discussion of mode shapes, motor operation, and theoretical analyses [Eqs. (2) and (4)], we investigated following design parameter space:

- (i) bearing stiffness varied between 1×10^5 and 5×10^7 N/m,
- (ii) shaft diameter varied between 0.372 and 0.625 in.,
- (iii) rotor diameter varied between 2.4 and 2.9 in., and
- (iv) the total length of the motor decreased by up to 1.7 in. (1.2 in. from rotor length reduction region and 0.5 in. from additional shaft length reduction region) (Fig. 10).

The above variable ranges were determined by design constraints. The stiffness of a commercial sturdy ball bearing is roughly at least 1×10^7 N/m, and cost-effective off-the-shelf bearings have standard sizes.⁴⁰ The inner diameter of the currently used bearing determines the outer size of the MACOR ceramic inner sleeve due to the fixed size of the wave spring

which lies on top of the inner race of the bearing, and the inner sleeve size affects the center shaft diameter. The weakness and brittleness of the ceramic material used for the sleeves also restrict the maximum size of the shaft diameter. The maximum diameter and length of the MACOR ceramic rotor were limited by sizes available from the manufacturer (Corning Incorporated Lighting & Materials). The rotor must have the space internally for ceramic bearing spacers to insulate each bearing and externally for coil windings to generate adequate torque, which determines the maximum decrease in motor length and diameter. In order to examine design parameters (iii) and (iv) regarding the rotor geometry, a hollow cylindrically shaped rotor was used instead of the hollow dumbbell-shaped rotor for FEM simulations, but with the same outermost diameter (Figs. 6 and 10).

4. RESULTS AND DISCUSSION

4.A. Finite element method model validation

The MR-compatible x-ray tube motor with anode attached demonstrated a first resonance at 21.64 ± 0.68 Hz from 12 measured sets of the power spectrum. The peak location

FIG. 10. Cross sectional outline of the three-bearing motor design for FEM simulations showing regions that were changed to reduce the length of the rotor (colored) as well as for additional shaft length reduction (hatched). The darker portions indicate the maximum decrement of each region to shorten the motor length: 1.2 in. in the rotor region and 0.5 in. in the additional center shaft length reduction region. Along with the main components such as the rotor and shaft, inner sleeves and bearing spacers are also modified.

FIG. 11. The first critical frequency for the new x-ray tube motor (a) with the anode, as well as (b) without the anode based on the experiments measuring the rotation speed change and frequency response.

indicating large vibrations was measured at the critical frequency in the frequency response. The abrupt decrease in acceleration (slope) occurred at 21.5 Hz in the rotation speed curve [Fig. 11(a)]. These measurements were used to fix the bearing stiffness to 1.2×10^5 N/m in the FEM model, providing 1.1% error between expected resonance at 21.4 Hz and the measured value (Fig. 12). The model was validated by performing the same experiments with the anode removed, and the corresponding physical prototype was modeled. For this setup, the FEM predicted that the first mode resonance would occur at 36.8 Hz. This result is close to the average measured resonance peak of 34.59 ± 1.39 Hz from 6 sets of the power spectrum, and the reduction of the angular acceleration was detected at 34.4 Hz in the angular speed curve [Fig. 11(b)].

4.B. Parameter space investigation

Figure 13(a) demonstrates that the fundamental frequency increases asymptotically to a maximum as shaft diameter increases, with the maximum value of 24 Hz for the bearing stiffness of 1×10^5 N/m. On the other hand, the fundamen-

tal frequency rises asymptotically to 32.1 Hz with increased bearing stiffness for the initial shaft diameter of 0.372 in. [Fig. 13(b)]. The two factors are not completely independent as is seen in the changing fundamental frequency associated with the shaft diameter between 0.372 and 0.625 in. and the bearing stiffness between 1×10^5 and 1×10^7 N/m [Fig. 13(a)]. For stiffer bearings, the asymptotic behavior was less pronounced as shaft diameter increased, and thus higher resonant frequencies could be attained. The rate of change in the first critical frequency becomes larger as both the diameter and bearing stiffness are increased. A fundamental frequency as high as 68.5 Hz can be achieved for reasonable shaft diameter of 0.625 in. and bearing stiffness of 1×10^7 N/m. To achieve the higher stiffness required, a compact ball bearing may be appropriate with stiffness of 3.3×10^7 N/m and internal clearance of 0.001 in. or a roller bearing would be another option.^{41,42} Higher preload on bearings can also have a positive influence on increasing bearing stiffness.

The effects of the rotor moment of inertia have been examined by changing the outer diameter of the rotor. The outer diameter also changed the fundamental frequency, although the overall impact of this parameter is much smaller than that of the other two parameters [Fig. 13(c)]. Decreasing the outer diameter of the ceramic rotor from 2.9 to 2.4 in. increased the fundamental frequency from 20.1 to 21.4 Hz. This variation corresponds to the change in the rotor moment of inertia from 9263.5 to 3763.4 $\text{g} \cdot \text{cm}^2$ based on the hollow cylindrically shaped rotor. The reduction of the outer diameter of the rotor component restricts the space for wire windings, and in turn degrades the motor performance in terms of the total amount of torque that can be produced due to reduced moment area and decreased number of turns of the windings [see Eq. (1)].

Although four bearings provide more support than three bearings, moving to a three-bearing design permits decreasing the motor length, which also increases the critical frequency (Fig. 14). Losing one support spot can be sufficiently compensated by using a pair of narrow super-precision bearings (718 (SEA) bearing series, SKF Group, Gothenburg, Sweden) at each remaining location. The well-honed and compact-structured super-precision bearings have higher stiffness (i.e., over 7×10^7 N/m) than regular bearings, and using the

FIG. 12. Resonant frequency plot based on the FEM simulation results, regarding different conditions of the MR-compatible x-ray tube motor: with and without the anode. The predicted fundamental frequencies, respectively, are 21.4 and 36.8 Hz.

FIG. 13. The fundamental frequencies as a function of changing (a) the center shaft diameter, (b) the bearing stiffness, and (c) the outer diameter of the cylindrically shaped rotor. The other parameters are fixed at the initial values while changing each specific design parameter. Resonant frequencies increase with increasing bearing stiffness and shaft diameter, and decreasing the rotor diameter.

bearings in pairs makes them even stiffer similar to the effect that is seen when connecting springs side by side. These face-to-face or back-to-back bearings in a pair also apply force on each other, thus this is as effective as preload with increasing contact points between their balls and races. In addition to this

FIG. 14. The first critical frequencies increase as motor length is decreased. The FEM simulation is based on the three-bearing model with shaft diameter of 0.625 in., and bearing stiffness of 1×10^6 N/m. When the motor model uses a 2.4 in. diameter rotor, the fundamental frequencies are higher than the results from the simulation model using a 2.9 in. rotor. Triangle-spotted lines include 0.2–0.5 in. more reduction in the center shaft length in addition to the decrement of 1.2 in. in the rotor length.

setup, thermal expansion of the bearing material reduces internal clearances including the gap between the bearings and the outer rotor, and will increase the stiffness of bearing assembly in real operation.^{43,44} The bearing stiffness should be increased to more than 1×10^7 N/m through this series of refinements, but to be safe we will assume that the bearing stiffness reaches only 1×10^6 N/m. Even with this constraint we can increase the critical frequency using other parameters. In 3D FEM simulations based on the three-bearing motor design, the model is composed of a 0.625 in. diameter shaft and stiff bearings of 1×10^6 N/m. Both 2.9 in. diameter rotor and 2.4 in. diameter rotor have been evaluated as the length of the rotor decreases from 5.6 in. (the initial length of the rotor) to 4.4 in., which corresponds to the lines with circle symbols in Fig. 14. Lines with triangle symbols show the effect of a length reduction in the shaft region in addition to the rotor length reduction, thus the FEM model for the triangular points has an additional 0.2, 0.4, and 0.5 in. shaft length reduction, respectively, compared to solid filled circular points with the length of the rotor fixed at 4.4 in. (Fig. 10). The small amount of length reduction in the center shaft causes a relatively large increase in the first critical frequency, because both the entire massive weight load and the locations of the support provided by each bearing were moved closer to the clamped end by shortening the bottom section of the shaft. A conservative design of the motor for stable operation can be finally achieved by shaft diameter greater than 0.625 in., rotor diameter smaller than 2.4 in., total length reduction greater than 1.7 in., and bearing stiffness higher than 1×10^6 N/m. The resonant frequency properties can be further enhanced if we alleviate currently existing waviness on the inner surface of the outer rotor which the bearing outer races border based on precise machining in place of hand-grinding.⁴⁵ Additionally,

the narrow and thin super-precision bearing can provide more space for the ceramic inner sleeve and center shaft.

5. CONCLUSIONS

We have designed and built a new rotating anode x-ray tube motor assembly which can operate in strong magnetic fields. The finite element model describing x-ray tube rotor dynamics has been developed and accurately predicted resonant frequencies using *a priori* parameters. Both the frequency response and angular speed measurements of the rotating anode x-ray tube motor were used to validate the FEM model. After adjusting the value for the unknown bearing stiffness, the FEM simulation result of 21.4 Hz is closely matched with the measured critical frequency values of 21.64 Hz from the power spectrum and 21.5 Hz from the rotation speed curve for the motor. The resulting resonance occurred within the targeted operational range of the tube, and therefore design improvements are needed. Based on simulations, the contributions of different constituent components to the resonance phenomena were determined; the effects of bearing stiffness and shaft diameter dominate the lowest mode and one of the higher modes, respectively. Modification of those two significant parameters increased the fundamental frequency up to 68.5 Hz which is higher than the operational speed of 50 Hz. The moment of inertia of the ceramic rotor has relatively little influence on the resonant frequency. Redesigning the motor to have only three bearings, a larger shaft diameter (0.625 in.), a smaller rotor diameter (2.4 in.), and an overall length reduction (by 1.7 in.) provides a conservative estimate of fundamental frequency of 68.3 Hz for an assumed bearing stiffness of 1×10^6 N/m, which could be achieved relatively easily with commercially available bearings and improved manufacturing processes. Iterative simulations based on the verified FEM model can now be used to optimize dynamic performance of the motor.

ACKNOWLEDGMENTS

This work is supported by NIH R01 EB007626 and the Richard M. Lucas Foundation. The authors would like to acknowledge Ricky Smith and Christopher Artig at Varian Medical Systems for their advice on the experiment and on conventional x-ray tube design. We would also like to give a special thanks to Kyung-Taek Kim for advice related to the finite element method modeling of the rotor.

^{a)} Author to whom correspondence should be addressed. Electronic mail: mhshin@stanford.edu

¹ T. J. Vogl, J. O. Balzer, M. G. Mack, G. Bett, and A. Oppelt, "Hybrid MR interventional imaging system: combined MR and angiography suites with single interactive table: Feasibility study in vascular liver tumor procedures," *Eur. Radiol.* **12**(6), 1394–1400 (2002).

² K. S. Rhode, D. L. G. Hill, P. J. Edwards, J. Hipwell, D. Rueckert, G. Sanchez-Ortiz, S. Hegde, V. Rahunathan, and R. Razavi, "Registration and tracking to integrate X-ray and MR images in an XMR facility," *IEEE Trans. Med. Imaging* **22**(11), 1369–1378 (2003).

³ M. W. Wilson, N. Fidelman, O. M. Weber, A. J. Martin, R. L. Gordon, J. M. LaBerge, R. K. Kerlan, K. A. Wolanske, and M. Saeed, "Experimental

renal artery embolization in a combined MR imaging/angiographic unit," *J. Vasc. Interv. Radiol.* **14**(9), 1169–1175 (2003).

⁴ R. M. Chu, R. P. Tummala, J. Kucharczyk, C. L. Truwit, and R. E. Maxwell, "Minimally invasive procedures - Interventional MR image-guided functional neurosurgery," *Neuroimaging Clin. N. Am.* **11**(4), 715–725 (2001).

⁵ J. Kucharczyk, W. A. Hall, W. C. Broaddus, G. T. Gillies, and C. L. Truwit, "Cost-efficacy of MR-guided neurointerventions," *Neuroimaging Clin. N. Am.* **11**(4), 767–772 (2001).

⁶ P. Acher, S. Puttagunta, K. Rhode, S. Morris, J. Kinsella, A. Gaya, P. Dasgupta, C. Deehan, R. Beaney, R. Popert, and S. Keevil, "An analysis of intraoperative versus post-operative dosimetry with CT, CT-MRI fusion and XMR for the evaluation of permanent prostate brachytherapy implants," *Radiother. Oncol.* **96**(2), 166–171 (2010).

⁷ M. Carlsson, A. J. Martin, P. C. Ursell, D. Saloner, and M. Saeed, "Magnetic resonance imaging quantification of left ventricular dysfunction following coronary microembolization," *Magn. Reson. Med.* **61**(3), 595–602 (2009).

⁸ P. P. Chinchapatnam, K. S. Rhode, A. King, G. Gao, Y. Mat, T. Schaeffter, D. Hawkes, R. S. Razavi, D. L. G. Hill, S. Arridge, and M. Sermesant, "Anisotropic wave propagation and apparent conductivity estimation in a fast electrophysiological model: Application to XMR interventional imaging," *Lect. Notes. Comput. Sci.* **4791**, 575–583 (2007).

⁹ M. Matsumae, J. Koizumi, H. Fukuyama, H. Ishizaka, Y. Mizokami, T. Baba, H. Atsumi, A. Tsugu, S. Oda, Y. Tanaka, T. Osada, M. Imai, T. Ishiguro, M. Yamamoto, J. Tominaga, M. Shtmoda, and Y. Imai, "World's first magnetic resonance imaging/x-ray/operating room suite: a significant milestone in the improvement of neurosurgical diagnosis and treatment," *J. Neurosurg.* **107**(2), 266–273 (2007).

¹⁰ M. Sermesant, Y. Coudiere, V. Moreau-Villegier, K. S. Rhode, D. L. G. Hill, and R. S. Razavi, "A fast-marching approach to cardiac electrophysiology simulation for XMR interventional imaging," *Lect. Notes. Comput. Sci.* **3750**, 607–615 (2005).

¹¹ R. Fahrig, K. Butts, J. A. Rowlands, R. Saunders, J. Stanton, G. M. Stevens, B. L. Daniel, Z. Wen, D. L. Ergun, and N. J. Pelc, "A truly hybrid interventional MR/X-ray system: Feasibility demonstration," *J. Magn. Reson. Imaging* **13**(2), 294–300 (2001).

¹² R. Fahrig, K. Butts, Z. Wen, R. Saunders, S. T. Kee, D. Y. Sze, B. L. Daniel, F. Laerum, and N. J. Pelc, "Truly hybrid interventional MR/X-ray system: Investigation of in vivo applications," *Acad. Radiol.* **8**(12), 1200–1207 (2001).

¹³ S. T. Kee, A. Ganguly, B. L. Daniel, Z. Wen, K. Butts, A. Shimikawa, N. J. Pelc, R. Fahrig, and M. D. Dake, "MR-guided transjugular intrahepatic portosystemic shunt creation with use of a hybrid radiography/MR system," *J. Vasc. Interv. Radiol.* **16**(2 Pt 1), 227–234 (2005).

¹⁴ A. Ganguly, Z. F. Wen, B. L. Daniel, K. Butts, S. T. Kee, V. Rieke, H. M. Do, N. J. Pelc, M. T. Alley, and R. Fahrig, "Truly hybrid x-ray/MR imaging: Toward a streamlined clinical system," *Acad. Radiol.* **12**(9), 1167–1177 (2005).

¹⁵ R. Fahrig, G. Heit, Z. Wen, B. L. Daniel, K. Butts, and N. J. Pelc, "First use of a truly-hybrid X-ray/MR imaging system for guidance of brain biopsy," *Acta Neurochir.* **145**(11), 995–997 (2003).

¹⁶ L. Brzozowski, A. Ganguly, M. Pop, Z. Wen, R. Bennett, R. Fahrig, and J. A. Rowlands, "Compatibility of interventional x-ray and magnetic resonance imaging: Feasibility of a closed bore XMR (CBXMR) system," *Med. Phys.* **33**(8), 3033–3045 (2006).

¹⁷ J. A. Bracken, G. DeCrescenzo, P. Komljenovic, P. V. Lillaney, R. Fahrig, and J. A. Rowlands, "Closed bore XMR (CBXMR) systems for aortic valve replacement: Active magnetic shielding of x-ray tubes," *Med. Phys.* **36**(5), 1717–1726 (2009).

¹⁸ Z. F. Wen, R. Fahrig, and N. J. Pelc, "Robust x-ray tubes for use within magnetic fields of MR scanners," *Med. Phys.* **32**(7), 2327–2336 (2005).

¹⁹ P. Lillaney, M. Shin, S. M. Conolly, and R. Fahrig, "Magnetostatic focal spot correction for x-ray tubes operating in strong magnetic fields using iterative optimization," *Med. Phys.* **39**(9), 5567–5583 (2012).

²⁰ P. Lillaney, M. Shin, W. Hinshaw, N. R. Bennett, N. Pelc, and R. Fahrig, "Novel motor design for rotating anode x-ray tubes operating in the fringe field of a magnetic resonance imaging system," *Med. Phys.* **40**(2), 022302 (14pp.) (2013).

²¹ D. J. Inman, *Engineering Vibration*, 2nd ed. (Prentice Hall, Upper Saddle River, 2001).

²² C. W. Lee and Y. H. Seo, "Enhanced campbell diagram with the concept of H-infinity in rotating machinery: Lee diagram," *ASME J. Appl. Mech.* **77**, 021012 (12pp.) (2010).

- ²³P. Sprawls, *The Physical Principles of Diagnostic Radiology* (University Park, Baltimore, 1977).
- ²⁴J. T. Bushberg, *The Essential Physics of Medical Imaging*, 3rd ed. (Wolters Kluwer Health/Lippincott Williams & Wilkins, Philadelphia, 2012).
- ²⁵J. E. Shigley, C. R. Mischke, and R. G. Budynas, *Mechanical Engineering Design*, 7th ed. (McGraw-Hill, New York, 2004).
- ²⁶A. Pytel and J. Kiusalaas, *Mechanics of Materials* (Thomson, Australia; Pacific Grove, CA, 2003).
- ²⁷J. Beutel, *Handbook of Medical Imaging* (SPIE, Bellingham, 2000).
- ²⁸T. Koller, "Thermal characteristics of rotating anode X-ray tubes," *Proc. SPIE* **70**, 44–51 (1975).
- ²⁹J. T. Bushberg, *The Essential Physics of Medical Imaging*, 2nd ed. (Lippincott Williams & Wilkins, Philadelphia, 2002).
- ³⁰J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (Wiley, New York, 1999).
- ³¹J. Kraus, J. J. Blech, and S. G. Braun, "In situ determination of rolling bearing stiffness and damping by modal analysis," *ASME J. Vib., Acoust., Stress, Reliab. Des.* **109**, 235–240 (1987).
- ³²T. C. Lim and R. Singh, "Vibration transmission through rolling element bearings, part I: Bearing stiffness formulation," *J. Sound. Vib.* **139**(2), 179–199 (1990).
- ³³H. F. Bauer, "Vibration of a rotating uniform beam. 1. Orientation in the axis of rotation," *J. Sound. Vib.* **72**(2), 177–189 (1980).
- ³⁴G. Genta, *Dynamics of Rotating Systems* (Springer, New York, 2005).
- ³⁵Y. S. Lee and C. W. Lee, "Modelling and vibration analysis of misaligned rotor-ball bearing systems," *J. Sound. Vib.* **224**(1), 17–32 (1999).
- ³⁶Y. D. Kim and C. W. Lee, "Finite-element analysis of rotor bearing systems using a modal transformation matrix," *J. Sound. Vib.* **111**(3), 441–456 (1986).
- ³⁷H. D. Nelson and J. M. McVaugh, "The dynamics of rotor-bearing systems using finite elements," *J. Eng. Ind.* **98**, 593–600 (1976).
- ³⁸M. F. While, "Rolling element bearing vibration transfer characteristics: Effect of stiffness," *J. Appl. Mech.* **46**, 677–684 (1979).
- ³⁹J. Hsieh, *Computed Tomography: Principles, Design, Artifacts, and Recent Advances*, 2nd ed. (Wiley Interscience/SPIE, Bellingham, Hoboken, 2009).
- ⁴⁰Y. Kang, P. C. Shen, C. H. Chen, C. C. Huang, and L. K. Yang, "A computation strategy based on neural network for stiffness determination of deep-groove ball bearings," *Ind. Lubr. Tribol.* **56**(2-3), 147–157 (2004).
- ⁴¹F. P. Wardle, "Vibration forces produced by waviness of the rolling surfaces of thrust loaded ball-bearings. 2. Experimental validation," *Proc. Inst. Mech. Eng., Part C: J. Mech. Eng. Sci.* **202**(5), 313–319 (1988).
- ⁴²H. Prashad, "Determination of stiffness of roller bearings - An alternative approach," *J. Inst. Eng.* **84**, 186–192 (2004).
- ⁴³M. Tiwari, K. Gupta, and O. Prakash, "Effect of radial internal clearance of a ball bearing on the dynamics of a balanced horizontal rotor," *J. Sound. Vib.* **238**(5), 723–756 (2000).
- ⁴⁴B. Stone, "The state of the art in the measurement of the stiffness and damping of rolling element bearings," *CIRP Ann. Manuf. Technol.* **31**(2), 529–538 (1982).
- ⁴⁵B. Changqing and X. Qingyu, "Dynamic model of ball bearings with internal clearance and waviness," *J. Sound. Vib.* **294**(1), 23–48 (2006).