Erratum: On the eigensolutions of one-dimensional Duffin-Kemmer-Petiau oscillator [J. Math. Phys. 49, 022302 (2008)]

Abdelmalek Boumali^{a)}

Laboratoire de Physique Théorique et Appliquée (LPTA), Université de Tébessa,
12000 W. Tebessa, Algeria

(Received 28 August 2013; accepted 30 August 2013; published online 13 September 2013)

[<http://dx.doi.org/10.1063/1.4821200>]

In the paper of Ref. 1, and according the eigensolutions of the case of spin-1 particles, we note that (i) first, when we put $\omega = 0$ in Eq. (25) of Ref. 1, the well-known relation of a free particles $E^2 = c^2 p^2 + m^2 c^4$, is not well established, and (ii) second, the form of the spectrum of energy (Eqs. (37) and (41) in Ref. 1) led to the possibility of the existence of a complex eigenvalues.

The problem is due mainly in the incorrect choice of the β matrices in Ref. 1. These matrices do not completely fulfilled the rules of commutations of the DKP equation described by the following equation:

$$\beta^\mu \beta^\nu \beta^\rho + \beta^\rho \beta^\nu \beta^\mu = \beta^\mu g^{\nu\rho} + \beta^\rho g^{\nu\mu}, \quad (1)$$

where $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ is the metric of the Minkowski space. (Following the β matrices of Ref. 2, we can see, for example, that $(\beta^1)^3 = \beta^1$ instead $(\beta^1)^3 = -\beta^1$ according the algebra defined by Eq. (1).) In order to overcome this problem, and also to obtain the correct eigensolutions, we have used the β matrices given in Ref. 3. Their matrices verified completely the algebra gives by Eq. (1).

Then, when we start with the modified β matrices, given in Ref. 3, and which are by

$$\beta^0 = \begin{pmatrix} 0_{3 \times 3} & 0_{3 \times 3} & -I_{3 \times 3} & 0_{3 \times 1}^\dagger \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 1}^\dagger \\ -I_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 1}^\dagger \\ 0_{1 \times 3} & 0_{1 \times 3} & 0_{1 \times 3} & 0 \end{pmatrix}, \quad \beta^k = \begin{pmatrix} 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & iK^{k\dagger} \\ 0_{3 \times 3} & 0_{3 \times 3} & s_{3 \times 3}^k & 0_{3 \times 1}^\dagger \\ 0_{3 \times 3} & -s_{3 \times 3}^k & 0_{3 \times 3} & 0_{3 \times 1}^\dagger \\ iK^k & 0_{1 \times 3} & 0_{1 \times 3} & 0 \end{pmatrix}, \quad (2)$$

with

$$0_{3 \times 3} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (3)$$

$$s^1 = i \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad s^2 = i \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad s^3 = i \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (4)$$

$$K^1 = (1 \ 0 \ 0), \quad K^2 = (0 \ 1 \ 0), \quad K^3 = (0 \ 0 \ 1), \quad (5)$$

the equation (Eq. (3) in Ref. 1)

$$[\beta^0 E - c\beta^1 (p_x - i m \omega \eta^0 x) - mc^2] \psi(x) = 0, \quad (6)$$

^{a)}Electronic mail: boumali.abdelmalek@gmail.com

gives, instead the system of equations from (9) to (18) in Ref. 1, the following algebraic equations:

$$mc^2\psi_1 + E\psi_7 + ic(p_x + im\omega x)\psi_{10} = 0, \quad (7)$$

$$mc^2\psi_2 + E\psi_8 = 0, \quad (8)$$

$$mc^2\psi_3 + E\psi_9 = 0, \quad (9)$$

$$\psi_4 = 0, \quad (10)$$

$$mc^2\psi_5 - ic(p_x - im\omega x)\psi_9 = 0, \quad (11)$$

$$ic(p_x - im\omega x)\psi_8 + mc^2\psi_6 = 0, \quad (12)$$

$$mc^2\psi_7 + E\psi_1 = 0, \quad (13)$$

$$mc^2\psi_8 + E\psi_2 + ic(p_x + im\omega x)\psi_6 = 0, \quad (14)$$

$$ic(p_x + im\omega x)\psi_5 - E\psi_3 - mc^2\psi_9 = 0, \quad (15)$$

$$ic(p_x - im\omega x)\psi_1 + mc^2\psi_{10} = 0. \quad (16)$$

From these equations, we get, instead Eqs. (19)–(20) in Ref. 1, the following results:

$$\psi_2 = -\frac{E}{mc^2}\psi_8, \psi_3 = -\frac{E}{mc^2}\psi_9, \psi_4 = 0, \psi_5 = \frac{ic(p_x - im\omega x)}{mc^2}\psi_9, \quad (17)$$

$$\psi_6 = -\frac{ic(p_x - im\omega x)}{mc^2}\psi_8, \psi_7 = -\frac{E}{mc^2}\psi_1, \psi_{10} = -\frac{ic(p_x - im\omega x)}{mc^2}\psi_1. \quad (18)$$

As in Ref. 1, we choose to solve this equation with ψ_1 . The wave function ψ_1 verifies the following differential equation:

$$\left[c^2(p_x^2 + m^2\omega^2 x^2 - m\omega\hbar) - (E^2 - (mc^2)^2) \right] \psi_1 = 0, \quad (19)$$

instead Eq. (25) in Ref. 1. From Eq. (19), we recover the well-known relation $E^2 = c^2 p^2 + m^2 c^4$ for free particles.

Now, when we follow the same procedure used in Ref. 1 for solving Eq. (19), the solutions are

$$\psi_1(x) = Ae^{-\frac{m\omega}{2\hbar}x^2} {}_1F_1\left(a; \frac{1}{2}; \frac{m\omega}{\hbar}x^2\right) + Be^{-\frac{m\omega}{2\hbar}x^2} \sqrt{\frac{m\omega}{\hbar}} x {}_1F_1\left(a + \frac{1}{2}; \frac{3}{2}; \frac{m\omega}{\hbar}x^2\right), \quad (20)$$

where A and B are normalizing factors, and the parameter a is given by

$$a = -\left(\frac{\kappa}{2} - \frac{1}{4}\right), \quad \text{with } \kappa = \frac{k^2}{2\lambda} \quad (21)$$

with $\lambda = \frac{m\omega}{\hbar}$ and $k^2 = \frac{E^2 - (mc^2)^2}{\hbar^2 c^2} + \frac{m\omega}{\hbar}$ instead Eq. (27) in Ref. 1. These solutions are decoupled according to the parameter a in two possible cases:

- $a = -n$, and $B = 0$ we have

$$\psi_1(x) = Ae^{-\frac{m\omega}{2\hbar}x^2} {}_1F_1\left(-n; \frac{1}{2}; \frac{m\omega}{\hbar}x^2\right), \quad (22)$$

$$E_n = \pm mc^2 \sqrt{1 + 4 \frac{\hbar\omega}{mc^2} n}, \quad (23)$$

instead Eq. (37) in Ref. 1. The non-relativistic limit is obtained by setting $E_n = mc^2 + \epsilon_{nr}$ and considering $\epsilon_{nr} \ll mc^2$. Taylor expansion of Eq. (23) would give us

$$E_n = \pm mc^2 \left(1 + 2 \frac{\hbar\omega}{mc^2} n + 2 \left(\frac{\hbar\omega}{mc^2} \right)^2 n^2 \right), \quad (24)$$

instead Eq. (38) in Ref. 1.

- $a + \frac{1}{2} = -n$, and $A = 0$ we obtain

$$\psi_1(x) = B e^{-\frac{m\omega}{2\hbar}x^2} \sqrt{\frac{m\omega}{\hbar}} x {}_1F_1 \left(-n; \frac{3}{2}; \frac{m\omega}{\hbar} x^2 \right), \quad (25)$$

$$E_n = \pm mc^2 \sqrt{1 + (2 + 4n) \frac{\hbar\omega}{mc^2}}, \quad (26)$$

instead Eq. (41) in Ref. 1. The non-relativistic limit is obtained by setting $E_n = mc^2 + \epsilon_{nr}$ and considering $\epsilon_{nr} \ll mc^2$. Taylor expansion of Eq. (26) would give us

$$E_n = \pm mc^2 \left(1 + (1 + 2n) \frac{\hbar\omega}{mc^2} + \frac{1}{2} \left(\frac{\hbar\omega}{mc^2} \right)^2 (1 + 2n)^2 \right), \quad (27)$$

instead Eq. (42) in Ref. 1.

Finally, Eq. (30) in Ref. 1 should be read as

$$y \frac{d^2 \varphi(y)}{dy^2} + \left(\frac{1}{2} - y \right) \frac{d\varphi(y)}{dy} + \left(\frac{\kappa}{2} - \frac{1}{4} \right) \varphi(y).$$

In the original paper of Ref. 1, the use of the β matrices of Ref. 2 led to incorrect eigensolutions of the case of the spin-1 particles.

¹ A. Boumali, *J. Math. Phys.* **49**, 022302 (2008).

² W. Greiner, *Relativistic Quantum Mechanics: Wave Equations*, 3rd ed. (Springer-Verlag, Berlin, 2000).

³ A. Boumali, L. Chetouani, and H. hassanabadi, *Can. J. Phys.* **91**, 1–11 (2013).