A Heuristic Approach to Automated Nipple Detection in Digital Mammograms

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Abstract In this paper, a heuristic approach to automated nipple detection in digital mammograms is presented. A multithresholding algorithm is first applied to segment the mammogram and separate the breast region from the background region. Next, the problem is considered separately for craniocaudal (CC) and mediolateral-oblique (MLO) views. In the simplified algorithm, a search is performed on the segmented image along a band around the centroid and in a direction perpendicular to the pectoral muscle edge in the MLO view image. The direction defaults to the horizontal (perpendicular to the thoracic wall) in case of CC view images. The farthest pixel from the base found in this direction can be approximated as the nipple point. Further, an improved version of the simplified algorithm is proposed which can be considered as a subclass of the Branch and Bound algorithms. The mean Euclidean distance between the ground truth and calculated nipple position for 500 mammograms from the Digital Database for Screening Mammography (DDSM) database was found to be 11.03 mm and the average total time taken by the

algorithm was 0.79 s. Results of the proposed algorithm demonstrate that even simple heuristics can achieve the desired result in nipple detection thus reducing the time and computational complexity.

Keywords Breast cancer · Nipple detection · Mammography · DDSM database · Multithresholding

Introduction

The nipple serves as an important landmark in many Computer Aided Diagnosis (CAD) techniques as well as a useful reference for radiologists. Nipple detection is a precursor in many automated CAD algorithms such as alignment of left and right breast (via registration), automatic segmentation, and establishment of an anatomical coordinate system [1].

Several methods have been suggested to address this problem each with varying degrees of complexity. R. Chandrasekhar and Y. Attikouzel [2] suggested that the nipple could be detected by characterizing the variation of a gradient vector along the skin air interface. Kinoshita et al. [3] found that the nipple point lies at the intersection of breast tissue components and this was implemented using a Radon transform. Although these are excellent algorithms, they are limited by the requirement of accurate detection of skin–air interface. This is tackled in the algorithm of Kinoshita et al. [3] by selecting the best segmentation boundary among several algorithms output by a radiologist. Thus, such methods cannot be used for a completely automated CAD system.

Nipple detection algorithm based on Genetic Algorithms has been suggested by Karnan and Thangavel [4]. In this article, first the rough breast boundary is found by a histogram based method and then fine-tuned using Genetic Algorithms. Once the final breast boundary has been found, the nipple position is found on the boundary using Genetic Algorithms taking the intensity value of the border pixels as

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the fitness function. Zhou et al. [5] have developed a very rigorous method for nipple detection. They developed a two stage process—in the first, a rule-based method was designed to detect nipple position based on significant changes in intensity values along the breast border. In the second stage, the nipple position was detected based on convergence of texture patterns around the nipple. The final nipple point was found based on a confidence measure and a set of rules. However, dense mammograms are likely to have structured noise and are therefore likely to give false positives when this method is used. Yin et al. [6] proposed a simplified version of the different intensity based method for nipple detection and subsequent alignment of left and right breast mammograms.

Petroudi and Brady [7] have proposed a method based on segmentation of a “fat band” and its curvature. On the other hand, Tzikopoulos et al. [8] have tried to model the nipple as an ellipse in a 10-mm search band located just outside the breast boundary. All of these methods are intensity based and this is likely to pose as a limitation as image acquisition technology varies over time and this will change optical density, contrast and several other parameters which affect intensity. Moreover, methods that rely solely on intensity have the disadvantage that radiopaque artifacts with an intensity profile similar to the nipple can be detected as the nipple.

One way to circumvent this issue can be considered by applying the algorithm developed by Iglesias and Karssemeijer [9] where nipple positions from multiple atlas images were fused to give the location. This method tries to improve the accuracy by introducing redundancy (more atlas images than required for the problem at hand) in the system and using a priori information from the atlas images. Unfortunately, such a method ceases to be practical in most situations. Every new dataset introduced would require manually annotated nipple positions on a training dataset to determine the best subset of atlases to be used for the fusion.

Mustra et al. [10] have attempted to use a combination of geometric properties of the breast region as well as the cumulative intensity profile for two different cases—nipple in profile and nipple outside profile. Interestingly, the authors select an angular band within which the nipple position is presumed to lie and then apply an intensity based method. Unfortunately, the method was not tested on medio-lateral oblique (MLO) view images and relied on the intensity profile for the final detection, which can be a problem for the reasons discussed earlier.

In the present work, our aim is to overcome the challenges posed by the intensity based methods and come up with a procedure that is both simple and accurate. The proposed method is scalable as it searches the image space in recursive order similar to the Branch and Bound algorithm [11]. Moreover, it can estimate the nipple location accurately even when there is architectural distortion such

as inversion. Since it requires only the binary mask of the image, it does not depend on the intensity or quality of the acquired image. Thus, this method can be adopted easily across different applications without any difficulty.

While the reader peruses through the methods section, it is useful to keep in mind the key proposals of the algorithm which are very simple and could be taken into account while creating more sophisticated algorithms in the future. First, the nipple is considered to lie within a search band, i.e., it cannot be located at unreasonable locations such as near the axilla or the rib. While Mustra et al. [10] have considered an angular band, we shall consider a rectangular band but whose angular direction depends on the direction of the pectoral muscle. Choosing a band has an added advantage of narrowing down the search space making the method computationally more efficient. At the same time, it allows a certain degree of uncertainty rather than being fully deterministic based on certain geometrical features. Second, the method does not make any implicit assumptions regarding the intensity profile of the nipple. This is important since there is no evidence to believe that radiopaque artifacts cannot show the same fluctuations in intensity as the nipple. Third, our method makes use of the location of the centroid which can be a useful indicator of how the nipple position varies as the global shape and size of the breast varies.

Proposed Method

Our procedure involves three steps—multithresholding-based segmentation of breast region, pectoral muscle edge detection, and nipple search (Fig. 1). Pectoral muscle edge detection is only required for MLO view and not for craniocaudal (CC) view images.

Multithresholding Based Segmentation of the Breast Region

In the first step, the rough image boundary is identified. This would require a method that is computationally inexpensive while yielding a smooth breast boundary at the same time.

The method of multithresholding based breast segmentation [12] (MTBS) serves our purpose in satisfying these criteria. In this paper, a method implemented by Seth et al. [12] is used. It is an extension of clustering based histogram thresholding [13]. In this method, the ratio of the within class to the between class variance is optimized. A recursive relation is used at each step for the variances to achieve the results faster [13]. For our purposes, the threshold is set at a level between the fourth and fifth clusters. Furthermore, we refine the segmentation mask obtained by this thresholding using morphological opening operator (radius 3 pixel) followed by removal of horizontal artifacts and finally applying connected component analysis.

Fig. 1 **a** Flowchart demonstrating the different sub-blocks of the proposed nipple detection algorithm. **b** Nipple detection block expanded in more detail. MoveStrip represents a function that changes the position of the search strip based on the current position, step size, and direction of search

It may be noted that the choice of segmentation algorithm can be made based on the dataset. While the current dataset gives good segmentation results for a multithresholding algorithm, multithresholding can be a disadvantage in certain situations where the mammogram edge is fuzzy. However, the nipple detection algorithm is not affected by this choice and gives good results if the images are segmented optimally.

Pectoral Muscle Edge Detection

In the next step, the pectoral muscle edge is detected for MLO view mammograms. We use the method proposed by Chakraborty et al. [14]. In this procedure, the boundary is first approximated by a straight line. A weighted average gradient is applied to detect points of discontinuity along horizontal line. A shape-based method is used to determine bands which divide these discontinuity points. The band with maximum number of points is considered as the most probable band containing pectoral muscle edge. Finally, a straight line is estimated based on the discontinuity points lying in this band. This gives us an accurate estimate of pectoral muscle edge direction.

Search for the Nipple Location

Finally, to identify the nipple position, a search algorithm is applied on the binary mask corresponding to the segmented breast region in a direction perpendicular to the pectoral muscle edge for MLO view images. This is done by first identifying all the pixels lying on the line passing through

the centroid ($C(x_c, y_c)$) of the segmented breast and perpendicular to the pectoral muscle edge, which can be called as the median line. (The careful reader may note that the median line is different from the posterior central line PCL) [15]. While PCL passes through the nipple point, the median line passes through the centroid. In the best case scenario, when PCL is parallel to the median line and within the distance defined by the search band, our algorithm would work perfectly. However, in most practical situations, we can get a close result even when this assumption is not perfectly valid.) These pixels then serve as the center points of narrow search strips parallel to the pectoral muscle edge. The search is progressively moved away from the centroid and along the median line looking for foreground pixels (Fig. 2a). The average x and y coordinates of the white pixels in the last strip containing foreground pixels gives us the nipple position $N(x_n, y_n)$.

Let α be the angle of the pectoral muscle edge with the vertical, and $P(x_p, y_p)$ be a point on the median line, then the equation of a line passing through P and parallel to the pectoral muscle edge can be written as:

$$(y - y_p) = (\cot \alpha)(x - x_p) \quad (1)$$

It can be shown that the distance d of a point $A(x_a, y_a)$ from this line is:

$$d = |\sin \alpha \{ \cot \alpha * (x_p - x_a) + (y_a - y_p) \}| \quad (2)$$

By choosing the criterion $d < 1$, the set of pixels belonging to the search strip can be determined. The search is limited in space by reducing the length of the strips. Two

Fig. 2 Pictorial representation of the algorithm. In the simplified version, **a** the search strip (shown in *light gray*) moves within the dark gray region (in a direction perpendicular to the pectoral muscle edge) from the centroid *C* towards the edge of the segmented region. **b** The search process in the modified algorithm. The step size is decreased by

a factor of two in successive passes till it decreases to one. The final position of the search strip containing the nipple is indicated using the arrow mark. **c** The bounds of the possible location of the nipple column as determined by the algorithm in successive passes

factors are taken into account while deciding the length. First, the breast is asymmetric (due to its mass) about the median line. Second, this reduction excludes the possibility of radiopaque artifacts outside the breast region which might otherwise be included in the strip. Hence, the strip length is taken to be 10 % of the pectoral muscle edge length above the center point and 25 % of pectoral muscle edge length below the center point (Fig. 2a). This means that the ratio of length of strip above the median line and below it is 5:2. This ratio is retained for CC view images as well.

Since the CC view mammograms are geometrically symmetric with respect to the *x* axis, the search is conducted along the horizontal direction. Thus, for CC view images, the pectoral muscle edge detection is not required while rest of the algorithm remains unchanged.

Modified Search Algorithm for Nipple Detection

The modified algorithm can be loosely considered to be a subclass of the popularly known heuristics Branch and Bound [16]. Heuristics of Branch and Bound first require a splitting procedure that returns two or more smaller sets from a larger set. Next, a bounding procedure is required that helps in narrowing down the search space by specifying a lower bound and an upper bound. This is carried out recursively till the set is reduced to a single element.

In the present problem, the splitting procedure can be thought to divide the available columns (Fig. 2c) into two or more parts (decided by the step size). Then, the bounding procedure moves the search strip along the upper and lower bound of these parts till the strip falls outside the foreground. The last strip which was inside the foreground

region becomes the new lower bound of the available columns. With this new lower bound and the same upper bound as before, the splitting procedure is carried out again and then the bounding procedure. This continues in a recursive manner till the set size decreases to one.

In practice, the nipple search process is divided into multiple passes (Fig. 2b). In the first pass, the starting step size *s* is calculated from the maximum possible step size θ so that it remains a power of two (Eq. (3)). For example, if θ is 33, *s* can be determined to be 16, which would then be the initial step size (Fig. 2b)

$$s = 2^{\lfloor \log_2 \theta \rfloor - 1} \quad (3)$$

Once the search strip moves outside the segmented region, the current pass is terminated and the next pass is started from the point where the search strip was last found inside the segmented region. In each successive pass, we conduct the search with a reduced (by a factor of 2) step size than the previous one and finally terminate the method with a step size of one pixel.

Results

Dataset

Five hundred images (252 CC view and 248 MLO view) selected randomly from the Digital Database for Screening Mammography (DDSM) [17] were used in this study. All images had a uniform sampling rate of 50 μm in both the *x* and *y* directions of the image. For comparison purposes, the

algorithm of Kinoshita et al. is taken as the benchmark. Since the image sizes in the DDSM database were not uniform, they were down sampled and padded on both sides to $1,024 \times 1,024$. This created a different sampling rate for each image, but the resolution is taken into account while reporting the final results.

Qualitative Results

First, a visual verification of the results was done by marking the ground truth and the calculated nipple position on the original mammogram (Fig. 3). This helps prevent and diagnose any large scale systematic errors in the calculation of the nipple position.

It can be observed that the algorithm performs accurately (Fig. 3) in most of the cases.

Fig. 3 Result of the proposed nipple detection algorithm on four different categories of the images: **a** left CC view, **b** left MLO view, **c** right CC view, and **d** right MLO view. The calculated position is indicated with a small white circle while the ground truth is indicated with a white cross

Quantitative Results

The calculated nipple position is compared against the ground truth which was verified by a radiologist (A. S.). The Euclidean distance between the ground truth and the calculated nipple position is found for every image. Then, the percentage of images lying in 5 mm bins was calculated (Table 1). This gives a quantitative measure of the accuracy of the proposed method and demonstrates that this method clearly outperforms the method suggested by Kinoshita et al. To implement the nipple detection algorithm proposed by Kinoshita et al., we use the same breast boundary which was used for the proposed method.

As expected, it is found that the accuracy of the algorithm improves if higher resolution images are used. Next, the overall accuracy of the proposed algorithm is computed with Euclidean distance as the criterion. The average accuracy in the proposed method is found to be 11.03 ± 12.80 mm. Thus, this method outperforms the method of Kinoshita et al. by around 3.5 mm whose overall accuracy is found to be 14.48 ± 15.82 mm.

The proposed method does significantly better than existing geometry based nipple detection algorithms. Our results indicate that 64.8 % of mammograms lie within the 1 mm mark as compared to 56.03 % of images in the method proposed by Mustra et al. [10]. The reader may additionally note that the proposed method was tested on a much larger database of 500 images consisting of both CC and MLO view images as compared to 144 images consisting of only CC images in the competing method.

The algorithm is tested on an Intel Core 2 Duo CPU at 2 GHz and 3 GB of RAM running 32-bit Windows Vista. All functions for the proposed algorithm (and competing algorithms) were written in MATLAB (R2010b). The down sampling factor for the method of Kinoshita et al. and the proposed method is taken

Table 1 Euclidean distance of proposed method vs. the method of Kinoshita et al.

Euclidean distance in bins	Proposed method (%)		Kinoshita et al. (%) (Down sampled)
	Down sampled	Original size	
0 to 5 mm	30.4	33.2	21.8
5 to 10 mm	32.0	31.6	26.8
10 to 15 mm	17.8	15.4	19.6
15 to 20 mm	8.2	8.4	11.2
20 to 25 mm	2.4	3.4	6.2
25 mm and above	9.2	8.0	14.4

A total of 500 images from the DDSM database were tested and the percentage of images falling in different bins was compared based on the Euclidean distance between actual nipple position (ground truth) and the calculated nipple position

Table 2 Average time taken by the different blocks in the algorithm

Block	Proposed method (s)		Kinoshita et al. (s)	
	MLO view	CC view	MLO view	CC view
Multithresholding	0.4097	0.4024	0.4097	0.4024
Pectoral muscle edge detection	0.2065	–	0.2065	–
Nipple detection algorithm	0.3558	0.2077	4.9690	5.499
Total	0.9720	0.6101	5.5852	5.9104

to be the same for comparison purposes. The average time taken by the algorithm was found to be 0.79 s, while in case of the method by Kinoshita et al., it was found to be 5.75 s. This means that the present algorithm outperforms the method proposed by Kinoshita et al. by a factor of around 7. It may be noted that the pre-processing steps (MTBS and pectoral muscle edge detection) consumes around 63 % of the total time in case of MLO view images and 66 % of the total time in case of CC view images (Table 2). Thus, the proposed method may become faster in the future if faster pre-processing blocks are utilized.

The results indicate that the proposed method outperforms existing methods even with simple heuristics based on anatomical shape.

Discussion

The proposed nipple detection algorithm shows that simple shape based heuristics can outperform advanced algorithms for a sufficiently large dataset (Tables 1 and 2). Moreover, it has been demonstrated that it is possible to find the nipple position using a priori knowledge about the geometry of a breast. Third, the proposed algorithm does not require any

human intervention thus automating the entire workflow for the nipple detection algorithm. Finally, it is possible to improve the algorithm further by considering the fact that the weight of the breast can be used to roughly determine the deflection of the nipple position from the position determined geometrically.

Furthermore, the proposed algorithm is different than that suggested by Méndez et al. [18] who have taken the maximum height of the breast border as the nipple position. Even though the proposed method ultimately finds a point of inflection on the breast border, the detected position may or may not be the point of maximum height in the breast border. This is because the search region is localized and only a segment of the foreground lying around the center line is considered for the nipple search. Moreover, this algorithm can be successfully applied to MLO view images as well where the maximum height does not always give a good approximation of the nipple position.

Next, different cases of how the proposed algorithm performs in terms of accuracy and time complexity will be considered along with suggested improvements for the future.

Detection of Nipple Position in Case of Inversion

The proposed method is robust to mammographic images where the nipple position has undergone inversion. In Fig. 4a, it can be seen that the nipple position (indicated by G) has been inverted. In this case, the search strip does not stop moving once it reaches G but continues till $N1$ and $N2$ is reached. Since either $N1$ or $N2$ are not good approximations of the nipple position, the average x and y coordinates of these points are found to give the point N . This point is a much better approximation of the actual nipple position G .

As is evident from Fig. 4b, extraneous points such as $N1$ or $N2$ will not be detected as the nipple position if the length of search strip is reduced. However, reduction of the length of strip must be adaptive depending on the particular image.

Fig. 4 Robustness to retraction.

a Result with a longer search strip but taking the mean for multiple detections. Calculated nipple position N (circle with white foreground and black border) and actual nipple position G (circle with light gray foreground and dark gray border) are shown for comparison. The centroid is marked with C and the points where the search strip exits the segmentation mask are denoted by $N1$ and $N2$. **b** Result with shorter search strip gives a more accurate result

This can be implemented in future versions of the algorithm to achieve even better results.

Accuracy in CC View vs MLO View

The 2D histogram of estimated position relative to actual position (Fig. 5) demonstrates that the CC view gives more accurate results than the MLO view. We observe that the results for the MLO view are not as accurate as the results for the CC view in the horizontal direction (Fig. 5a).

Moreover, we find that the accuracy is better in the x direction as compared to the y direction in the CC view.

However, in MLO view, we find that the accuracy along the top-right to bottom-left diagonal is better than the accuracy along the top-left to bottom-right diagonal. This difference can be attributed to the fact that the search direction in CC view is along the x direction while in the MLO view, it is along a line perpendicular to the pectoral muscle edge. This perpendicular line approximately lies in the direction of the top-right to bottom-left diagonal.

Thus, we can conclude that in both the views, the accuracy is lower parallel to the length of the search strip than in the direction in which the search strip moves.

Fig. 5 **a** A scatter plot is made showing the distribution of the detected nipple position relative to the actual nipple position. **b** A 2D histogram is constructed by taking bins of 1 mm in both the x and y direction and the number of occurrences of the nipple position in a 1×1 -mm area are counted

Effect of Gravity on the Nipple Position

As can be observed from the scatter-plots (Fig. 5a), the accuracy along the y direction is lower than in the x direction. One particular reason for this might be that the weight of the breast lowers the nipple position from the geometrically predicted nipple position. To take this into account, the search strip is longer below median line. Specifically, we determine by experimentation, that a ratio of 5:2 can detect nipple positions which stoop below the geometrically predicted position due to weight of the breast.

Effect of Density on the Nipple Position

The density of the breast at a point may be considered roughly correlated to the optical density and thus the intensity of the pixel under consideration. In general, a normal or malignant breast does not have uniform density. Thus, the algorithm can be improved further if local variations in the density were taken into account while calculating the nipple position. This may be roughly achieved by first smoothing the segmented mammogram using a Gaussian filter, normalizing for the contrast in the image and thereby finding the center of mass.

Alternatively, these local density variations could be used to find a more accurate estimate of the total weight and thus to vary the ratio of search strip above and below the median line. We shall consider such modifications for the future and outside the scope of our present work.

Scalability of the Proposed Algorithm

The algorithm has been optimized for time by using an adaptive step size and an iterative search process. This means that the algorithm is likely to be much faster than existing methods. On investigation, it was found that the time taken by the nipple search block in the algorithm scales linearly with the number of pixels in the image (Fig. 6).

Regression lines were plotted to find the approximate rate at which the algorithm scales. It can be observed that this rate is higher for MLO view images as compared to CC view images. This can be partly attributed to the varying directions of search in MLO view which poses as an additional computational burden. Thus, we can conclude that the algorithm can be easily extended in the future when higher resolution images are available to detect the nipple position even more reliably but still keeping the time complexity at a minimum.

Limitations

It must be pointed out that the accuracy of the algorithm depends on the accuracy of segmentation and detection of

Fig. 6 Scalability of the algorithm with increase in number of pixels. Original image sizes have been used for testing scalability

pectoral muscle edge. Although accurate segmentation of the entire breast region is not necessary, correct segmentation near the nipple position is critical to the success of the method. Since the pectoral muscle edge is curvilinear, a straight line approximation may not be always accurate if the curvature is higher. However, we expect the accuracy of our method to only improve as the accuracy of these steps increases.

Secondly, we do not consider nipples inside breast region separately. Our method is likely to produce erroneous results if the nipple position is considerably inside the breast region. For those cases, an intensity based method could be implemented as a second stage of the algorithm to fine-tune the position within a small neighbourhood of the detected position. In any case, it is found that the advantages of a geometry based nipple detection algorithm greatly outweigh the possible shortcomings.

Conclusions

Thus, in summary, the major advantages of the proposed algorithm are as follows. The proposed method achieves accurate results for both CC view and MLO view images. Moreover, the algorithm is different from intensity based methods and relies on the geometry of the breast to find the nipple position, thus overcoming the limitations posed due to quality of the image and the technology used in the scanner. Finally, this approach is scalable to larger and higher resolution images without the need for greater computational complexity.

This method has tremendous scope for future work. As discussed, taking center of mass instead of geometrical centroid could improve the accuracy. Furthermore, adaptively changing the length of search strip can have its advantages, in particular, for special cases like nipple inversion. Eventually, this method can be expanded by adding more features and hence making it more comprehensive.

References

1. Bandyopadhyay SK: An Approach for registration method to find corresponding mass lesions in temporal mammogram pairs. *Int J Comput Appl Technol* 1(1):16–23, 2010
2. Chandrasekhar R, Attikiouzel Y: A simple method for automatically locating the nipple on mammograms. *IEEE Trans Med Imaging* 16(5):483–494, 1997
3. Kinoshita S, Azevedo-Marques P, Pereira R, Rodrigues JAH, Rangayyan R: Radon-domain detection of the nipple and the pectoral muscle in mammograms. *J Digit Imaging* 21(1):37–49, 2008
4. Karnan M, Thangavel K: Automatic detection of the breast border and nipple position on digital mammograms using genetic algorithm for asymmetry approach to detection of microcalcifications. *Comput Methods Prog Biomed* 87(1):12–20, 2007
5. Zhou C, Chan HP, Paramagul C, Roubidoux MA, Sahiner B, Hadjiiski LM, Petrick N: Computerized nipple identification for multiple image analysis in computer-aided diagnosis. *Med Phys* 31(10):2871–2882, 2004
6. Yin FF, Giger ML, Doi K, Vyborny CJ, Schmidt RA: Computerized detection of masses in digital mammograms: Automated alignment of breast images and its effect on bilateral-subtraction technique. *Med Phys* 21(3):445–452, 1994
7. Petroudi S, Brady M: Automatic nipple detection on mammograms. *Med Image Comput Comput-Assist Interv—MICCAI* 2003:971–972, 2003
8. Tzikopoulos SD, Mavroforakis ME, Georgiou HV, Dimitropoulos N, Theodoridis S: A fully automated scheme for mammographic segmentation and classification based on breast density and asymmetry. *Comput Methods Prog Biomed* 102(1):47–63, 2011
9. Iglesias JE, Karssemeijer N: Robust initial detection of landmarks in film-screen mammograms using multiple FFDM atlases. *IEEE Trans Med Imaging* 28(11):1815–1824, 2009
10. Mustra M, Bozek J, Grgic M: Nipple detection in craniocaudal digital mammograms. In: *ELMAR, 2009. ELMAR'09. International Symposium*, 2009
11. Marti R, Reinelt G: Branch-and-Bound. In: *The Linear Ordering Problem*, Berlin: Springer Berlin Heidelberg, 2011, 85–94
12. Seth S, Mukhopadhyay S: Multi-level Thresholding-based Breast Segmentation in Mammograms. In: *International Conference on Communication, Computers and Devices*, Kharagpur, India, 2010
13. Goparaju S, Acharya J, Ray AK, Goswami JC: A Fast Hierarchical \Multilevel Image Segmentation Method using Unbiased Estimators. *CoRR abs/0712.4015* 2007. Available at <http://arxiv.org/abs/0712.4015>. Accessed July 29, 2012
14. Chakraborty J, Mukhopadhyay S, Singla V, Khandelwal N, Bhattacharyya P: Automatic detection of pectoral muscle using average gradient and shape based feature. *J Digit Imaging* 25(3):387–399, 2012
15. Kwok SM, Chandrasekhar RA, Attikiouzel Y: Automatic assessment of mammographic positioning on the mediolateral oblique view. In: *2004 International Conference on Image Processing*, 2004. *ICIP '04*, 2004
16. Land AH, Doig AG: An automatic method of solving discrete programming problems. *Econometrica: J Econ Soc* 497–520, 1960
17. Heath M, Bowyer K, Kopans D, Moore R, Kegelmeyer Jr. P: The Digital Database For Screening Mammography. In: *5th International Workshop on Digital Mammography*, Toronto, Canada, 2000
18. Méndez AJ, Tahoces PG, Lado MJ, Souto M, Correa J, Vidal JJ: Automatic detection of breast border and nipple in digital mammograms. *Comput Methods Prog Biomed* 49(3):253–262, 1996