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Viscous damping and spring force calculation of regularly perforated MEMS microstructures in the Stokes' approximation

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Abstract

There are a number of applications for microstructure devices consisting of a regular pattern of perforations, and many of these utilize fluid damping. For the analysis of viscous damping and for calculating the spring force in some cases, it is possible to take advantage of the regular hole pattern by assuming periodicity. Here a model is developed to determine these quantities based on the solution of the Stokes' equations for the air flow. Viscous damping is directly related to thermal-mechanical noise. As a result, the design of perforated microstructures with minimal viscous damping is of real practical importance. A method is developed to calculate the damping coefficient in microstructures with periodic perforations. The result can be used to minimize squeeze film damping. Since micromachined devices have finite dimensions, the periodic model for the perforated microstructure has to be associated with the calculation of some frame (edge) corrections. Analysis of the edge corrections has also been performed. Results from analytical formulas and numerical simulations match very well with published measured data.

Keywords

Perforated Microstructures; Viscous Damping; Spring Force; Stokes' equations

1 Introduction

Many microelectromechanical systems (MEMS) devices such as microphones, accelerometers, microswitches, and micromirrors are comprised of a moving plate and a fixed backplate. The two parallel plates typically serve as electrodes for capacitance sensing or electrostatic actuation mechanisms. In order to increase the sensitivity for sensing or the force for actuation, designers try to increase the electrode area and decrease the gap between plates. The thin air layer squeezed in the small gap between the transversely moving planar structure and the fixed substrate gives rise to damping and spring forces that can adversely affect the dynamic response of the device. The backplate often contains perforations in order to reduce the time required to remove sacrificial materials between the moving structure and the fixed substrate during the wet etching process. These perforations can be used to control the viscous damping. The squeeze-film damping is reduced by the presence of holes (in one or both plates). However, the longitudinal motion of the air within the holes gives rise to a viscous resistance which adds to the squeeze-film damping.

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Quantitative analysis of the viscous damping in perforated microstructures is critical in designing these types of MEMS transducers, and there is an extensive literature dedicated to the study of squeeze-film damping in MEMS (Sadd and Stiffler 1975; Starr 1990; Yang and Senturia 1990; Nayfeh and Younis 2004). As was recognized by Bleach 1983, the spring force is equally important in describing the dynamics of the structure: its connection with the damping force determines the cut-off frequency of the microstructure. Below the cutoff frequency, viscous damping is the dominant effect, while above that frequency the spring force is more important. The performance and stability of these types of micromachined capacitive devices can be greatly improved with proper design of the micromachined perforations that are typically arranged spatially in a regular pattern.

A comprehensive analysis of the damping and spring force would require the numerical solution of the Navier-Stokes equations for the air flow in the entire 3-D domain composed of the space between the plates and the holes. Numerically simulating the air flow for perforated microstructures containing a large number of holes using finite element or other approximation solution methods of the Navier-Stokes equations in the entire 3-D domain are generally impractical owing to the computational requirements. When the distance between the bounding plane surfaces is small, the lubrication approximation is appropriate to model the airflow in the gap. In this case, it is possible to integrate the Navier-Stokes equations across the gap and obtain a 2-D equation for the pressure that is known as the Reynolds equation. Most of the work concerning squeeze-film damping is based on analytical or numerical solutions of the Reynolds equation.

In a series of papers, Homentcovschi and Miles (2004; 2005; 2007; 2008) took advantage of the periodic spatial arrangement of the holes, present in many applications, to develop an integrated method for analyzing the viscous damping in perforated microstructures based on the Reynolds equation for the flow in the gap and Poiseuille flow type solutions in the backplate holes. However, in the case of perforated microstructures, the hypothesis that the flow domain is thin is not always valid in the region of holes. To address this problem, Bao et al. (2003a,b) developed a modified Reynolds equation by adding a term related to the flow through the holes to the classical equation. The modified equation includes the flow resistance in the gap, the circular perforation resistance and the constant elongation of the plate. The resulting 2-D Partial Differential Equation (PDE) was integrated analytically for the case of a rectangular damper that is much longer than its width by reducing it to an ordinary differential equation. Some authors also introduced several secondary losses such as those due to the open ends of the holes, the turning of the flow from horizontal to vertical, wall elongations, rarefied gas effects, etc. Solutions of these types of 2-D compact models can be found in the work of Mohite et al. (2005; 2008).

Homentcovschi et al. (2010) developed an analytical method, based on the Stokes equations, for determining the damping and spring force coefficients of a basic cell. The cell was initially approximated by an axi-symmetrical domain composed of two circular cylinders. The analysis shows that the theory based on the Reynolds equation in the gap combined with the Poiseuille solution in the holes works well only when the pitch of holes is large compared with the diameters of the holes (like 5 to 10 times larger). In another study based on the Stokes approximation, compressibility effects were studied by Homentcovschi and Miles (2010). A recent paper by Nigro et al. (2012) reviewed a number of analytical and numerical models for squeeze-film damping problems in addition to performing their own finite element simulations. They compared the results from several models with available experimental data. They concluded that their numerical simulations and the results presented in Homentcovschi et al. (2010) both agree well with the available experimental data.

For the basic (axi-symmetric) cell geometry, the analytical formulas for the damping coefficient given in Homentcovschi et al. (2010) and Homentcovschi and Miles (2010) give good results for perforated microstructures with small and medium area ratio A values (defined as the ratio of the hole area over total area) ($A < 0.4$), but cannot be applied in the case of high area ratio microstructures in general. The models developed in the references above can be used for designing perforated microstructures in order to control damping. However, for applications where the main role of the holes is to facilitate the fabrication of the device (e.g., reduce the time required to release the micromechanical structure during the etching of sacrificial materials) the existing models may not be appropriate.

The present paper develops a model for characterizing the viscous damping and the spring force for area ratios of perforated microstructures that are designed primarily to speed-up the microfabrication process. In this case, the holes are square and they are arranged in a regular grid. The model uses the periodicity and symmetry of the microstructure to build a simplified domain. The Stokes flow equations are solved, assuming harmonic time dependence, using the finite element method implemented using a commercial software package. In the case of very long perforations, a hybrid solution method that uses a Poiseuille flow solution for the hole region is introduced. In Section 2, the description of the problem domain, the model equations and the formulation of the solution procedure are presented.

For sensors designed for small signal applications, mechanical-thermal noise is a major concern. The thermal noise depends on temperature and the magnitude of mechanical damping (Gabrielson 1993). A microstructure with smaller viscous damping tends to have low mechanical-thermal noise. This is why the problem of designing a perforated microstructure having the smallest damping for a given area ratio A , gap width d and backplate thickness h is very important. The problem was addressed previously using the lubrication approximation in Homentcovschi and Miles (2004; 2007). In Section 3 we analyze this problem for the more general case when the Stokes' system is used instead of the Reynolds equation.

While the periodicity of the perforated microstructures can be used to simplify the analysis, the flow near the edges of the plates may have an important contribution to the viscous damping and spring force of the microstructure as well. In order to adapt the model to the case of real nonperiodic structures, a frame correction is introduced. This is essentially an edge correction where the dimensions of the side cells are chosen so that they have the same damping as an inner cell. This type of correction applies in the design stage. For a physical microstructure where an edge correction has not been applied, a border correction is determined by solving the Stokes' system of equations for the side cell with the appropriate boundary conditions.

2 The periodic problem for the Stokes' system

An illustration of a perforated microstructure is shown in Fig. 1. The upper plate is the perforated backplate (of thickness h) and the lower one is the vibrating diaphragm. The average thickness of the gap is denoted by d . A regular array of square holes is shown. The vertical motion of the diaphragm induces a motion of the air in the gap (defined as the space between the diaphragm and the backplate) and in the holes. The interior, air-filled domain is shown in Fig. 2. The air is treated as a viscous, compressible fluid. Also, owing to the small dimensions of the domain and the range of frequencies of interest, the inertia terms in the momentum equations are negligible and the flow is assumed to be governed by the time-dependent Stokes approximation of the Navier-Stokes equations. For the analysis here, time

periodic forcing is applied and harmonic time-dependence is assumed for the dependent variables,

$$\tilde{\mathbf{v}} = \mathbf{v} \exp(-i\omega t), \quad \tilde{p} = p \exp(-i\omega t) \quad (1)$$

where $\tilde{\mathbf{v}}$ denotes the fluid velocity and p is the pressure. The quantity $\omega = 2\pi f$, where f is the frequency of the harmonic oscillations of the diaphragm. Substitution of the harmonic form of the variables into the continuity equation and time-dependent Stokes equations yields the following set of equations for the amplitudes of the velocity field and pressure:

$$\nabla \cdot \mathbf{v} = \frac{i\omega}{\rho_0 c_0^2} p \quad (2)$$

$$i\omega \rho_0 \mathbf{v} + \mu \nabla^2 \mathbf{v} = \nabla p \quad (3)$$

Here, ρ_0 and μ are the air density and dynamic viscosity, respectively and c_0 is the isothermal speed of sound. The vertical velocity of the diaphragm is also written as a harmonic function $v_z = w^{(d)} \exp(-i\omega t)$.

For a regular arrangement of holes and for time harmonic vertical motion of the diaphragm, periodicity can be used to treat the motion of the air. As a result, the air motion is analyzed in terms of a unit cell consisting of a single square hole (domain D_h) on top of a square-base section corresponding to the air in the gap between the diaphragm and the backplate (domain D_g). A cell is shown in Fig. 3. The length of the side of the holes is $2a$ and the thickness is h . The overall width of the lower part of the cell is defined as $2b$. For air flow in the cell, all the sides of the domain D_g are symmetry planes; therefore, on each symmetry plane the normal component of velocity and the normal pressure derivative are zero. On the bottom surface of the domain D_g (the inlet of the domain), the vertical velocity component is equal to the amplitude of the velocity of the diaphragm. All the velocity components vanish on the other solid surfaces (upper plane surface of D_g outside the hole and the side surfaces of the hole.) Finally, the top surface of the hole is the outlet where the pressure has a prescribed value.

2.1 Analytical formula for the damping coefficient

In Homentcovschi et al. (2010) an analytical formula for the damping coefficient was derived for a similar geometry. The formula was obtained by approximating the lower part of a cell (domain D_g) by a circular cylinder of radius R and height d (with the same cross-sectional area, i.e. $R^2 = 4b^2$) and the hole domain D_h by another circular cylinder of radius r and height h . The “equivalent” radius r was obtained by equalizing the total resistances of the circular and square holes which gives the relationship $r = 1.094a$. For convenience, the main results from the analytical model are summarized below.

The squeeze film damping coefficient (in units of mass/time) is given by the real part of the expression,

$$q_n = \frac{3}{2} \frac{J_1(q_n r) J_2(q_n R)}{J_0^2(q_n R)}. \quad (4)$$

In the above formula, the eigenvalues q_n are determined from the equation

$$J_1(q_n R)=0, \quad n=0, 1, 2, \dots \quad (5)$$

and the expansion coefficients, $A_n^{(0)}$ are given by

$$A_n^{(0)} = \frac{\sinh(q_n d) \cosh(r_n d) - r_n q_n^{-1} \cosh(q_n d) \sinh(r_n d)}{q_n [2 - 2 \cosh(q_n d) \cosh(r_n d) + [q_n r_n^{-1} + r_n q_n^{-1}] \sinh(q_n d) \sinh(r_n d)]}, \quad (6)$$

with

$$r_n = \sqrt{q_n^2 - \frac{i\omega\rho_0}{\mu}}. \quad (7)$$

The total force, F_0 , on the base of the cell can be written as

$$F_0 = B_s w^{(d)} + \pi R^2 p^{(0)}, \quad (8)$$

where the pressure $p^{(0)}$ is obtained by employing the Hagen-Poiseuille solution for fully-developed flow in a circular pipe as

$$p^{(0)} = -8\mu h \frac{R^2}{r^4} w^{(d)}. \quad (9)$$

Finally, the total damping coefficient of a cell is given by the real part of

$$B_T \equiv \frac{F_0}{w^{(d)}} = B_s - 8\pi\mu h \frac{R^4}{r^4}. \quad (10)$$

The formula (10) gives the damping coefficient for small values of frequency where compressibility effects are weak. The analyses by Zuckerwar (1978) and Homentcovschi et al. (2010) show that for the case of microphones in the audio frequency range viscous damping is practically independent of frequency.

2.2 Finite element solution for the damping and spring forces

To obtain the numerical solution for the cell domain shown in Fig. 3, it is possible to further take advantage of symmetry and reduce the calculation domain to 1/8 of the entire cell. The resulting domain is shown in Fig. 4. Thus, the basic computational domain has an inlet (on the lower base corresponding to the diaphragm), an outlet surface (the upper base where the pressure is assumed given, $p=0$), a side plane of the hole and a plane surface corresponding to the lower solid surface of the backplate (where the no slip condition is applied); all other plane surfaces of the boundary are symmetry planes. As mentioned earlier, the air is treated as viscous and compressible and the flow is quasi-steady satisfying the system of equations (2), (3).

Solutions to the governing equations for the basic domain and specified boundary conditions were obtained by modifying the steady-state Stokes flow application model included in the COMSOL 3.5 software (2008) by including the quasi-steady terms and making the variables complex. The software uses the finite element method for spatial discretization of the governing equations. The domain was discretized using tetrahedral elements. A

homogeneous, unstructured mesh was employed. The accuracy of the results was assessed by a thorough mesh convergence study. Starting with approximately 1,000 elements the grid resolution was refined using up to 126,000 elements. In the most refined case, there were 165,000 finite elements. For the damping coefficient results presented in Section 3 (Fig. 7), the change in the computed values was less than four percent.

In the case of very thin long holes, a fully-developed Poiseuille solution for the square hole region is coupled to the computed flow in the gap domain. The computational domain continues from the gap region into the square hole a distance a (i.e., from the connection plane between the two domains D_g and D_h). The parameter, a , showing how high the edge of the Poiseuille domain is from the connection plane between the domains is equal to half of the side of the hole $2a$. In the book by Kinsler, et al. (1984), the end correction for the Poiseuille flow in a circular pipe ($0.85a$), can be considered as a lower limit of this parameter. At the end of the computational domain in the hole region, the pressure value p_1 is specified, where p_1 is obtained from the Poiseuille flow solution. From Homentcovschi and Miles (2008), the resulting force based on Poiseuille flow in a hole of length h with square cross-section (side length $2a$) is given by

$$F^h = \frac{12}{k_1} \mu h \left(\frac{b}{a} \right)^4 w^{(d)}. \quad (11)$$

If the force at the base is given by $F^h = 4b^2 p_1$, then the value of the pressure is

$$p_1 = \frac{b^2}{a^4} \frac{3\mu}{k_1} h_1 w^{(d)}. \quad (12)$$

In the above expression, h_1 is the length of the hole where the Poiseuille solution applies (i.e., $h_1 = h - a$), and k_1 is a constant obtained from the analytical expression for the Poiseuille flow solution in a channel with a square cross-section (see the text by White 1976, equation 3-48 with the cross-section height equal to the width.)

$$k_1 = 1 - \frac{192}{\pi^5} \sum_{n=1}^{\infty} \frac{\tanh((2n-1)\pi/2)}{(2n-1)^5} \approx 0.4217. \quad (13)$$

The value p_1 is used as the outlet pressure in the case of long (and narrow) holes when the computational domain is truncated at the distance a from the connection plane between the two domains D_g and D_h . For shorter holes when the computational domain extends the entire length of the hole, atmospheric pressure is imposed at the outlet.

From the complex set of equations and variables, the viscous damping coefficient is obtained from the real part of the total pressure and the spring force from the imaginary part. From these quantities the total damping force on the microstructure is computed and the cutoff frequency of the system is determined.

3 Minimizing viscous damping

The total viscous damping of a cell consists of the squeeze-film damping due to the motion of the air in the gap and the resistance of the hole. For microstructures having a small number of holes per unit area u^2 , the squeeze-film damping is the dominant part. In order to decrease the total viscous damping, a larger hole density is required. Each additional hole

has resistances (direct+indirect) which add to the total damping. By using the analytical expressions for the squeeze-film damping and the hole resistances, based on the lubrication approximation, see Homentcovschi and Miles (2004; 2007), the existence of an optimal number of holes giving a balance between the two components of the total damping was shown. The optimum configuration provides a perforated microstructure having the smallest total damping for a given area ratio, gap height and backplate thickness. The resulting formulas were used by Lee et al. (2010) and by Cha et al. (2011) for the design of capacitive micromachined ultrasonic transducers. For conditions where the lubrication approximation is not applicable, we show that based on the Stokes' flow model developed here, configurations having a minimum viscous damping can be determined as well.

Using the damping force for a cell F_c obtained from the finite element solution, the damping coefficient for a cell is determined from

$$\tilde{D}_c = \frac{F_c}{w(d)}. \quad (14)$$

For a given area u^2 , the number of cells, N , it contains is given by

$$N = \frac{u^2}{4b^2} \quad (15)$$

Treating the area ratio A , the air gap height d and the plate thickness h as constants, the damping coefficient of a cell depends only upon the length $2b$ of the base. Therefore, the total damping coefficient corresponding to the area u^2 is

$$\tilde{D}_T \equiv N \cdot \tilde{D}_c = C_N u^2, \quad (16)$$

where u^2 is expressed in meters squared. The coefficient C_N is the total viscous damping per square meter; if $u = 1\text{ mm}$ then $D_T = 10^{-6} C_N$, and the coefficient is given by

$$C_N = \frac{2\tilde{D}_c^*(b)}{b^2}, \quad (17)$$

where $\tilde{D}_c^*(b) = \tilde{D}_c(b)/8$ is the damping coefficient corresponding to the basic domain.

The total damping of a cell is the sum of the squeeze-film damping and the resistance of the holes. The total damping can be obtained by summing the pressure on the base surface (square) of the cell and the squeeze-film damping and the indirect resistance of holes, see Homentcovschi and Miles (2007). The total is obtained by adding together the pressure contributions on the perforated surface. The difference of the total pressure on the base surface and the perforated plate of the cell gives the direct hole resistance. For large values of base area and small resistance of the holes, the squeeze-film damping dominates the total damping. If the number of holes on the unit of area is large, then the hole resistance can be the largest contribution to the total damping coefficient. Therefore there is an optimum number of holes which gives an equilibrium between the squeezed-film damping and the resistance of the holes. The minimum damping coefficient is given by

$$C_N^o = \min_b \frac{2\tilde{D}_c^*(b)}{b^2}. \quad (18)$$

By determining the total damping of the cell as a function of the length of the cell base, b , the minimum damping coefficient, and hence the optimum number of holes, can be determined for the structure.

The results of a numerical simulation for the case where the air gap $d = 2\mu\text{m}$ and for different values of area ratio A and plate thickness h are shown in Figs. 5 and 6. Once $b_{\min}$ and C_N are determined, the geometry of the structure having the smallest damping coefficient is obtained using the formulas above. In Fig. 5 the minimum side length as a function of the backplate thickness h is plotted for different values of the area ratio A . In a similar manner, Fig. 6 presents the corresponding minimum damping coefficient as a function of the backplate thickness h for given values of the area ratio.

In Fig. 7, the coefficient C_N is plotted versus number of holes N (per unit of area) for the parameters: $d = 2\mu\text{m}$, $h = 2\mu\text{m}$ and area ratio $A = 0.25$. The plot shows that the coefficient C_N as a function of N has a minimum value for $N = 8260$ holes. The minimum value of the coefficient is $C_N = 428 \text{Ns/m}^3$. This number of holes gives a value of the cell length $b = 5.4\mu\text{m}$. Thus, Fig. 7 supports the results presented in Fig. 5 and Fig. 6.

4 The frame correction

In both the analysis and design stages, in order to completely model the damping of actual devices with perforated microstructures, it may be necessary to apply edge corrections for the cells next to the plate edges. The edge of a microstructure can be a symmetry surface, a solid surface (closed edge) or an outlet surface (open edge). The symmetry condition is a natural condition for a single cell. Consequently, symmetry surfaces do not need any correction. Similar to symmetry conditions, the case of closed edges (bordered by solid surfaces), the normal velocity cancels (no penetration condition). In addition on a solid surface the tangential components of velocity also cancel out. Results from the numerical simulations show that this cancellation has only a small influence on the damping coefficient. As such, the closed surface does not need any correction.

The situation is different on the sides of cells next to open edges. The zero pressure condition applies on an open side of a cell in place of the symmetry condition. This significantly reduces the total pressure of a cell.

4.1 The edge correction

The edge correction applies in the analysis and design stages. In order to preserve the periodicity for the inner cells, the total damping force for the side cell has to be increased to the value corresponding to an inner cell. This can be realized by increasing the area of the surfaces touching the open edge surface as shown in Fig. 8. The extending length b is determined by equating the damping coefficient of an inner cell to that of the modified side cell. The same idea was applied in the paper by Homentcovschi and Miles (2007) in the lubrication approximation where analytical formulas were given for the extended length b . This time the extending length is obtained by the numerical solutions of the Stokes' flow model. The modified cell has a single symmetry plane such that the computation has to be performed for a half of a cell instead of 1/8 of a cell.

We denote by $D^o(b)$ the damping coefficient obtained numerically by considering a side wall of the cell as an open surface at the distance b from the center of the hole. We have

$D^{(o)}(b) < D(b)$ and the function $D^{(o)}(b)$ is increasing with argument b . For trial value b_1 , $D^{(o)}(b_1)$ is the corresponding damping coefficient determined numerically. Also, the damping coefficient $D^{(o)}(b + \delta b)$ corresponds to the length $b + \delta b$. By assuming the three points to be on the same line we obtain the condition

$$\frac{\tilde{D}^{(o)}(b + \delta b) - \tilde{D}^{(o)}(b)}{(b + \delta b) - b} = \frac{\tilde{D}^{(o)}(b_1) - \tilde{D}^{(o)}(b)}{b_1 - b} \quad (19)$$

Finally, by imposing that the value of the damping coefficient be equal to the value for an inner cell: $D^{(o)}(b + \delta b) = D(b)$ yields the value for the correction δb as

$$\delta b = (b_1 - b) \frac{\tilde{D}(b) - \tilde{D}^{(o)}(b)}{\tilde{D}^{(o)}(b_1) - \tilde{D}^{(o)}(b)} \quad (20)$$

If $D^{(o)}(b + \delta b)$ determined numerically equals $D(b)$ (in the limit of required precision) the process is terminated. Otherwise, $b_1 \leftarrow b$, $b + \delta b \leftarrow b_1$ and the iteration continues determining a new δb . Once the edge correction δb is determined, it can be used to design a perforated microstructure that behaves internally as a periodic structure.

We note that for applications where the main role of the holes is to enhance the etching of underlying sacrificial layers in the microfabrication process, the extension surface for edge cells does not impact the etching process since now it also works on the open side surface. Also, in determining the edge correction δb , it is also possible to consider additional secondary effects such as an extension of the base plane by correspondingly modifying the boundary-value problem for the edge cell.

4.2 The border correction

Generally, there are differences between designed devices and physically realized (real) microstructures. For a real structure (no matter if it was designed or not including the edge correction) it is still possible to determine a frame correction (called the border correction) corresponding to the measured value $b + \delta b$. The damping coefficient C is obtained by solving the boundary-value problem outlined in Section 2.2 corresponding to the side cell.

5 Comparison with experiments

In this section a comparison is made between the between damping coefficients measured for some perforated gold microstructures, to those predicted from compact models and to the border correction model presented here. It has been shown that gas damping models adequately predict the measured quality factors for silicon structures with sufficient accuracy (Homentcovschi et al. 2010; Homentcovschi and Miles 2010; Veijola 2009). In the case of a gold structure a similar gas-damping compact model was found to underestimate considerably the damping coefficient. To fix the problem in De Pasquale et al. (2010) an additional pressure frame model was proposed to improve the compact model.

In this section we utilize the measurement results published by DePasquale et al. (2009) DePasquale et al. (2010) where the experimental configuration, the testing procedure and specimen characteristics are given. The test structures are the shape of a central suspended plate with square holes, connected to four lateral clamped supports of small cross-sectional area. The suspended structures have thickness $h = 6.3 \mu\text{m}$ and have a gap height $d = 3 \mu\text{m}$. Six different test structures were fabricated; the first four specimens only differ by the hole size and the last two have the same hole dimensions but a different plate width. Table 1

gives the effective geometrical dimensions of the test structures obtained by profile measurements performed using interferometric microscopy. In order to simplify the comparison of results for different test structures, we use the same notation as in DePasquale et al. (2009 in DePasquale et al. (2010) and refer to the six different test structures by topological index 29-34. The second and third columns contain the dimensions of the plate (the length L and the width W). As noted in the references, small differences between specimen nominal dimensions result from the fabrication process. Column four displays the number of holes ($M \times N$) in the rectangular plates in the length (M) and width (N) directions. The fifth column contains the measured damping coefficients C_M . In columns six and seven of Table 1, we include the parameters b (the half-period of the structure) and $a = s_0/2$ (the half-length of a square hole side). Column eight contains the area ratio A (same as the perforation ratio q) of the structure. Finally, the last column contains the corresponding edge correction determined by the method developed in Section 4.

In De Pasquale et al. (2009), the quality factor is obtained from the experimental curve of displacement versus frequency, and the damping coefficient is then calculated from the quality factor, frequency and the effective mass by the half power bandwidth method. As mentioned above, the values of the measured damping coefficients, labeled as C_M are given in column 5 of Table 1. From the results for the unit cell geometry presented here, the damping coefficient of the whole structure is determined by multiplying the damping coefficient for a cell by the number of cells ($M \times N$) of the microstructure. Figure 9 shows the relative error between predicted values of the damping coefficient and the measured values (listed in Table 1) for each of the experimental geometries denoted by the topological index. The relative error for three different models are shown in the figure. The results obtained from the analytical model C_{AN} , described in Section 2.1, are plotted as open circles. The relative errors of the results given by the finite element solutions, obtained by the method presented in Section 2.2, are shown in Fig. 9 as full circles. Finally, the full squares correspond to the relative errors of the compact model presented by De Pasquale et al. (2009) with respect to the measured values. Until this point Fig. 9 coincides with Fig. 9 in Homentcovschi et al. (2010). We complete these results by considering the frame correction. Table 1 contains the edge correction for every topological index. The use of this edge correction supposes fabrication of new structures having edge correction applied to the side cells. In order to use the measurements for the micro-machined structures published in DePasquale et al. (2009 in DePasquale et al. (2010), which have a frame of width b all around, we introduce a border correction. This is similar to the edge correction but the correction b equals b and, correspondingly, the total damping of a cell next to the frame C will be different from its value C for a center cell. Total damping of the structure is obtained from the following formula:

$$C^* = (M - 2) \times (N - 2) \cdot C + 2 \cdot (M + N) \cdot C' \quad (21)$$

The values obtained for all the topological indexes are plotted as stars and dotted lines in Fig. 9. First, it is important to note that the relative errors of the formulations including the *frame correction* are almost always smaller than the corresponding errors of all the other approaches. The only exception is the case having the topological index 34. The geometry of the basic cell for case 34 is the same as the cases 29 and 33. The number of holes for the case 33 (18×7) is almost double that for the structure 29 which equals (18×7). Correspondingly, the total damping coefficients are respectively $C_{33} = 52.7e^{-6}$ [Ns/m] and $C_{29} = 28.7e^{-6}$ Ns/m which gives $C_{33}/C_{29} = 1.8$. On the other hand, the number of holes of the structure 34 is 18×13 , almost double of holes of the case 33. The corresponding total damping coefficient is $C_{34} = 169.5e^{-6}$ Ns/m which is more than three times larger than that of case 33 (namely, $C_{34}/C_{33} = 3.25$). This may partially explain why

the error for case 34 is somewhat larger. We point out here, that all the model results obtained by different methods (compact, analytical, FEM and those including the *frame correction*) have approximately the same error for case 34. For this case the “frame correction” $9.90\mu\text{m}$ is very close to the “edge correction” $9.65\mu\text{m}$, which may explain the coincidence of the points denoted by the “stars” and the “empty circles.”

These results suggest that, as in the case of silicon microstructures, the gas-damping model alone does a good job of predicting the measured total damping coefficient and hence the quality factors for gold microstructures. The only exception is the microstructure with topological index 34. It is possible that measurement error for this case may be responsible for the difference.

6 Conclusions

Two solution approaches, based on Stokes' flow model, have been presented for analyzing the viscous damping and the spring force in perforated microstructures. The results obtained can be used in the design process for microscale devices. The analytical solution works in the case of small to medium area ratios of the structure and the finite element numerical solutions work in the general case. The results can be used to design perforated microstructures with the lowest thermal-mechanical noise by minimizing the viscous damping through the proper choice of the number of perforation holes. The analysis takes into account the requirements imposed by fabrication techniques for microscale devices to reduce the time required for removing the sacrificial material by the wet etching process.

The models developed utilize the periodic arrangement of the perforations in determining the viscous damping and the spring force. In addition, a frame correction is included to account for the finite size of actual structures. The results from the analysis of the viscous damping were compared to measured data for some perforated gold microstructures published in the literature. The model predictions, including the frame correction, show that viscous damping by itself can explain the experimentally measured quality factors in almost all cases without the need to include any other dissipation mechanisms.

References

1. Bao M, Yang H, Sun Y, French PJ. Modified Reynold's equation and analytical analysis of squeeze-film air damping of perforated structures. *J Micromech Microeng.* 2003a; 13:795–800.
2. Bao M, Yang H, Sun Y, Wang Y. Squeeze-film air damping of thick hole-plate. *Sensors and Actuators A.* 2003b; 108:212–217.
3. Blech JJ. On isothermal squeeze films. *J of Lubrication Technology.* 1983; 105:615–620.
4. Cha BS, Lee SM, Kanashima T, Okuyama M, Tanaka T. Influences of perforation ratio on characteristics of capacitive micromachined ultrasonic transducers in air. *Sensors and Actuators A, Phys.* 2011; 171:191–198.
5. COMSOL 3.5. COMSOL Inc. 2008
6. De Pasquale G, Veijola T, Somà A. Modelling and validation of air damping in perforated gold and silicon MEMS plates. *J Micromechan Microeng.* 2010; 20:015010.
7. De Pasquale G, Veijola T, Somà A. Gas damping effect on thin vibrating gold plates: experiments and modeling. *Proceedings of DTIP Symposium on MEMS and MOEMS, Rome.* 2009:23–28.
8. Gabrielson TB. Mechanical-thermal noise in micromachined acoustic and vibration sensors. *IEEE Trans on Electron Devices.* 1993; 40:903–909.
9. Homentcovschi D, Miles RN. Modelling of viscous damping of perforated planar micromechanical structures. *Applications in acoustics. J Acoust Soc Amer.* 2004; 116:2939–2947. [PubMed: 15603140]
10. Homentcovschi D, Miles RN. Viscous Damping of Perforated Planar Micromechanical Structures. *Sensors and Actuators A, Phys.* 2005; 119:544–552.

11. Homentcovschi D, Miles RN. Viscous microstructural dampers with aligned holes: Design procedure including the edge correction. *J Acoust Soc Amer.* 2007; 122:1556–1567. [PubMed: 17927414]
12. Homentcovschi D, Miles RN. Analytical model for viscous damping and the spring force for perforated planar microstructures acting at both audible and ultrasonic frequencies. *J Acoust Soc Amer.* 2008; 124:175–181. [PubMed: 18646964]
13. Homentcovschi D, Murray BT, Miles RN. An analytical formula and finite element simulations for the viscous damping of a periodic perforated MEMS microstructure outside the lubrication approximation. *Microfluidics and Nanofluidics.* 2010; 9:865–879.
14. Homentcovschi D, Miles RN. Viscous damping and spring force in periodic perforated planar microstructures when the Reynolds' equation cannot be applied. *J Acoust Soc Amer.* 2010; 127:1288–1299. [PubMed: 20329828]
15. Kinsler, LE.; Frey, AR.; Coppens, AB.; Sanders, JV. *Fundamentals of Acoustics.* 3rd. John Wiley & Sons, Inc; New York: 1984.
16. Lee SM, Cha BS, Okuyama M. The influence of acoustic damping on the transformation efficiency of capacitive micromachined ultrasonic transducer in air. *J Appl Phys.* 2010; 108:074512.
17. Mohite SS, Kesari VH, Sonti R, Pratap R. Analytical Solutions for the Stiffness and Damping Coefficients of Squeeze Films in MEMS Devices having Perforated Back Plates. *J Micromech Microeng.* 2005; 15:2083–2092.
18. Mohite SS, Sonti VR, Pratap R. A compact squeeze-film model including inertia, compressibility, and rarefaction effects for perforated 3-D MEMS Structures. *J Microelectromech Syst.* 2008; 17:709–723.
19. Nayfeh AH, Younis MI. A new approach to the modeling and simulation of microstructures under the effect of squeeze-film damping. *J Micromech Microeng.* 2004; 14:170–181.
20. Nigro S, Pagnotta L, Pantano MF. Analytical and numerical modeling of squeeze-film damping in perforated microstructures. *Microfluidics and Nanofluidics.* 2012; 12:971–979.
21. Saad, HM.; Stiffler, AK. *J Eng Ind Trans.* Vol. 97. ASME; 1975. Squeeze film dampers: amplitude effects at low squeeze numbers; p. 1366-1370.
22. Starr, JB. *Tech Digest IEEE Solid State Sensor and Actuator Workshop.* Hilton Head Island, South Carolina: 1990. Squeeze-film damping in solid-state accelerometers.
23. Veijola T, De Pasquale G, Somà A. Experimental validation of compact damping models of perforated MEMS devices. *Microsystem Technology.* 2009; 15:1121–1128.
24. White, FM. *Viscous Fluid Flow.* 2nd. McGraw-Hill; New York: 1976. p. 120
25. Yang, YJ.; Senturia, SD. *Technical Digest of the IEEE Solid State Sensor and Actuator Workshop.* Hilton Head Island, South Carolina: 1990. Numerical simulation of compressible squeezed-film damping.
26. Zuckerwar AJ. Theoretical response of condenser microphones. *J Acoust Soc Amer.* 1978; 64:1278–1285.

Fig. 1.
Illustration of the perforated planar microstructure.

Fig. 2.
The air domain corresponding to the planar microstructure in Fig. 1.

Fig. 3.
Basic cell of the periodic planar microstructure.

Fig. 4.
Computational domain for the basic cell employing symmetry.

Fig. 5. Plot of the length of the cell base b corresponding to the minimum damping coefficient versus backplate thickness, h .

Fig. 6.
Plot of the minimum total damping coefficient versus backplate thickness, h .

Fig. 7.
Plot showing the optimum number of holes for a particular microstructure.

Fig. 8.
Modified edge cell.

Fig. 9. Relative error of the predicted damping coefficients compared to the measured data of De Pasquale, et al. (2009) for gold samples.

Table 1

The geometry of the measured and calculated rectangular gold microstructures.

Topology index	L [μm]	W [μm]	M $\times$ N	[10 ⁻⁶ Ns/m]	b [μm]	a [μm]	A [%]	Corr [μm]
29	376.13	96.66	18 $\times$ 4	28.678	9.90	3.61	12.5	9.65
30	366.47	96.50	18 $\times$ 4	20.023	9.91	4.65	20.7	7.1
31	376.81	96.63	18 $\times$ 4	10.949	9.92	5.35	27.33	5.7
32	376.47	96.61	18 $\times$ 4	7.507	9.91	6.30	38.0	4.4
33	376.44	156.94	18 $\times$ 7	52.135	9.91	3.61	12.48	9.65
34	376.13	276.98	18 $\times$ 13	169.515	9.90	3.60	12.44	9.65