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2-D Fused Image Reconstruction approach for Microwave Tomography: a theoretical assessment using FDTD Model

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Abstract

This paper describes an efficient two-dimensional fused image reconstruction approach for Microwave Tomography (MWT). Finite Difference Time Domain (FDTD) models were created for a viable MWT experimental system having the transceivers modelled using thin wire approximation with resistive voltage sources. Born Iterative and Distorted Born Iterative methods have been employed for image reconstruction with the extremity imaging being done using a differential imaging technique. The forward solver in the imaging algorithm employs the FDTD method of solving the time domain Maxwell's equations with the regularisation parameter computed using a stochastic approach. The algorithm is tested with 10% noise inclusion and successful image reconstruction has been shown implying its robustness.

Keywords

Microwave Imaging; Finite Difference Time Domain Methods; Image reconstruction; Iterative methods

Introduction

Microwave tomographic imaging is an emerging technology to non invasively image electromagnetic properties of materials, for instance in medical imaging to characterise tissue inhomogeneities based on its electrical properties [Bindu 2012, Semenov et al(1) 2011, Semenov et al(2), 2011]. Various image reconstruction algorithms had been developed in the last decades to solve the complex ill posed microwave imaging problem [Sourov et al 1998, Chew 1998, Fang et al 2004, Bindu et al 2009, Mojabi et al 2011]. Linearising the inverse problem using Born Approximation has been proven to be a successful approach for objects with low contrast in dielectric properties. However, for high contrast objects in noisy environments it is necessary to resort to more computationally demanding iterative algorithms based on optimisation [Meaney et al 1998, Paulsen et al 1999]. While frequency domain algorithms are considered to have simultaneous interaction of the entire medium with the incident field, the time domain approach resorts to a causal behaviour [Hagness et al 1999, Xu et al 2001]. The former generates a large number of unknown parameters while the latter could explore causality to limit the region of inversion. Many models have been developed for reliable image reconstruction by combining time domain and frequency domain approaches [Popovic et al 2004, Fhager et al, 2009, Shea et al 2010]. Finite Difference Time Domain (FDTD) methods and non linear optimisation techniques have been formulated to develop a 2D inverse scattering scheme for remote sensing of inhomogeneous lossy layered media [Popovic et al, 2004] where the convergence

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and robustness of the algorithm with respect to broadband Gaussian noise was found to be dependent on the shape and duration of the illuminating electromagnetic wave. Time reversal algorithm based on FDTD method for ultra wide band microwave breast cancer detection was proposed to localise the tumour target [Shea et al, 2010]. Initial studies on time domain inverse scattering for 2D homogeneous dielectric cylinder buried in a half space using FDTD and dynamic differential evolution [Chien et al, 2010] shows promising results. Born Iterative method is reported to yield accurate inversions and convergence using time domain data when the scatterer is electrically large with high contrast without significantly affected by background noise [Moghaddan et al, 1993].

We have previously reported [Semenov et al(1) 2011, Semenov et al(2) 2011] a dedicated 2D system and functional fused imaging results. The reconstructed images, while not perfect, did indicate the great potential for this technology in functional imaging. In this paper we study the efficacy and robustness of 2D differential fused imaging algorithm on realistic FDTD modelling of the experimental system with the antenna arrays modelled using thin wire approximations and resistive voltage sources. The tomographic images were reconstructed from the scattered electromagnetic fields computed from the FDTD simulations.

Materials and Methods

FDTD formulation of the problem

FDTD numerical method using XFDTD 7 simulation package from Remcom Electromagnetic Solutions, USA is used to simulate the 2D microwave tomographic imaging scenario. The models developed for our analysis are shown in Figure 1. These models were identical to the measurement system and samples used in our experiments [Semenov et al(1) 2011, Semenov et al(2), 2011]. Model b was designed to show the contrast in the tissue parameters considered compared to Model a, and to check the effectiveness of the developed fused imaging algorithm. However due to the limitation of the available computer resources the studies were restricted to these two models alone.

The models consisted of an imaging chamber of diameter 21 cm and height 40 cm modelled as perfect electric conductor. The dielectric parameters of the objects under study are tabulated in Table 1. The chamber is filled with a coupling medium of permittivity $57+j22$. In the permittivity notations used in the paper the real part represents the dielectric constant and imaginary part represents the dielectric loss. Model a as shown in Figure 1 is treated as the Base Line model with two inclusions of dielectric permittivities $55+j21$ and $57+j22$ axisymmetric along the vertical line embedded inside the muscle tissue of permittivity $54+j20$. A high contrast bone tissue is modelled at the centre with permittivity $17+j5$. Model b is simulated with inclusions axisymmetric both on the horizontal and vertical directions with variations as mentioned in Figure 1 (b) with respect to the permittivity value of the muscle tissue. The goal of this extremity modelling was to test the robustness of the differential imaging algorithm in fused imaging to distinguish the physiology of high contrast and low contrast tissues in terms of its electrical properties. The inclusion of the coupling medium was to get the best impedance matching possible between the antennas and tissues and to decrease the unwanted reflections of the electromagnetic field from the boundaries of the imaging domain.

Twenty four equidistantly placed monopole antennas of length 1 cm each located at a distance of 0.8 cm from the perimeter of the horizontal cross-section of the chamber and 6.5 cm distant from the bottom of the chamber were used as transceivers. The antenna array was modelled with thin wire approximation and a resistive voltage source of 50 ohms, with the field values sampled directly at the receiver locations. In each simulation one antenna in the

array is sourced by exciting the electric field with a ramped sinusoid waveform at 1 GHz and the time domain scattered fields were measured for the rest 23 antennas in the array.

The 3D FDTD computational domain is composed of 0.5 mm cubical grid cells and terminated with seven layers of perfectly matched layer boundary conditions. The time domain data is then converted to phasor form by fast fourier transform. To nullify the mutual coupling effects between the antennas, the recorded electric field values in the presence of inclusions were normalised with respect to that of the field values measured with the chamber having only the coupling medium and no inclusions. A difference in resolution between the 0.5 mm FDTD cell size and the 1.5 mm imaging grid is also believed to be compensated by this correction method.

Image Reconstruction

Many non linear inverse scattering theories have been developed to deal with problems involving substantial multiple scattering effects [Chew, 1998, Sourov et al 1998, Fang et al, 2004, Semenov et al(1), 2011]. These methods are usually iterative and forward scattering solver is needed for each iteration. The Born and Distorted Born iterative methods have been tried for solving non linear inverse scattering problems for dielectric objects. Distorted Born Iterative Method (DBIM) is reported to be superior in convergence for objects with large contrast while Born Iterative Method (BIM) offers better performance in presence of noise [Chew, 1998]. In this work we used DBIM to reconstruct absolute images from the Base Line data and used BIM for differential imaging to reconstruct the regions of low dielectric contrast. The DBIM uses the simulated field values of Model a as the $E_{scat,b}(r)$ whereas for the BIM the difference in the simulated field values of Models a and b is treated as the differential scattered field data $E_{scat}(r)$.

Born Iterative Methods

In formulating DBIM the integral equation describing scattering is derived from the partial differential equation as,

$$(\nabla^2 + K_b^2(\vec{r}))g(\vec{r}, \vec{r}') = -\delta(\vec{r} - \vec{r}') \quad (1)$$

where the wave number $k_b(r)$ represents the spatial frequency of the plane wave in the inhomogeneous medium and is a function of the wavelength or $k_b = 2\pi/\lambda = \omega \sqrt{\mu\epsilon_b}$.

A solution to this equation is given by a plane wave as

$$E(\vec{r}) = e^{j\vec{k}_b \cdot \vec{r}} \quad (2)$$

which on rewriting the including the Green's function becomes,

$$E(\vec{r}) = E_{inc,b}(\vec{r}) + \omega^2 \mu \int_S dS g_b(\vec{r}, \vec{r}') \delta\epsilon(\vec{r}') E(\vec{r}') \quad (3)$$

$E(r)$ is the total field, $E_{inc,b}(r)$ is the incident field in the presence of the background inhomogeneity, ω represents the angular frequency and μ the permeability. As biological materials are non-magnetic, the relative permeability is unity and Hence, $\mu = \mu_0$, the permeability of free space.

The $\delta\epsilon(r)$ is defined as

$$\delta\epsilon(\vec{r}') = \epsilon(\vec{r}') - \epsilon_b(\vec{r}') \quad (4)$$

where $\epsilon(r)$ is the complex relative permittivity of the object and $\epsilon_b(r)$ is the complex relative permittivity of the background medium. $g_b(r, r')$ is the Green's function. The integral term in equation 3 is the scattered field due to the dielectric contrast of the scatterer. Hence,

$$E_{scat,b}(\vec{r}) = E(\vec{r}) - E_{inc,b}(\vec{r}) \quad (5)$$

Therefore

$$E_{scat,b}(\vec{r}) = \omega^2 \mu \int_S dS g_b(\vec{r}, \vec{r}') \delta\epsilon(\vec{r}') E(\vec{r}') \quad (6)$$

This scattered field is a non-linear functional of $\epsilon(r)$ because $E(r)$ in the integral is also a function of $\epsilon(r)$. To linearize the non-linear relation between the scattered field and the object function, it is assumed that $\epsilon(r)$ is close to $\epsilon_b(r)$ [Sourov et al, 1998]. Hence, $E_{scat,b}(r) \approx E_{inc,b}(r)$. Therefore $E(r)$ in equation 6 is approximated with $E_{inc,b}(r)$ and the scattered field is written as

$$E_{scat,b}(\vec{r}) = k_0^2 \int_S dS g_b(\vec{r}, \vec{r}') (\epsilon_r(\vec{r}') - \epsilon_{rb}(\vec{r}')) E_{inc,b}(\vec{r}') \quad (7)$$

where $k_o^2 = \omega^2 \mu \epsilon_o$. This is solved numerically to obtain the value of $\epsilon(r)$. $\epsilon(r)$ can be calculated from $\epsilon_b(r)$ and this value of $\epsilon(r)$ is then used as the new $\epsilon_b(r)$ and using this the Green's function is updated.

The entire procedure is repeated by first obtaining the inverse scattering solution and then the forward scattering solution. The iteration is continued until convergence is reached. We considered the termination criteria as the difference between the estimated and the measured scattered field is less than 5%.

In the BIM the scattered field $E_{scat}(r)$ is assumed small compared to the incident field $E_{inc}(r)$, Hence, the scattered field is approximated as,

$$E_{scat}(\vec{r}) = \omega^2 \mu \int g_o(\vec{r} - \vec{r}') \delta\epsilon(r') E_{inc}(r') dS \quad (8)$$

which constitutes the first order Born Approximation.. This represents radiation from a two-dimensional impulse source and the Green's function is considered to represent the field from a single point scatterer. In terms of Hankel function the incident field can be written as,

$$E_{inc}(\vec{r}_T, \vec{r}_T') = \frac{j}{4} H_o^1(k_o(\vec{r}_T, \vec{r}_T')) \quad (9)$$

It is assumed that the medium is homogeneous and Hence, Green's function is known in the closed form. An improved profile is obtained by comparing the differences between the estimated and the measured scattered fields. This process is repeated until a convergent solution is reached.

While the Born iterative method is simple to implement, it does not offer second order convergence and Hence, not suitable to solve non linear inverse scattering problem for objects with large contrasts [Chew, 1998]. We adopted this iteration scheme to develop the differential images of low contrast while Distorted Born Iterative Method is developed for the image reconstruction of Base Line images.

Discretization of the integral equation

The $E_{scat,b}$ on discretization takes the form

$$E_{scat,b} = \overline{M} \cdot \delta \overline{\varepsilon} \quad (10)$$

$[E_{scat,b}]_m$ represents the data measured for all transmitter, receiver and frequency combinations. That is

$$[E_{scat,b}]_m = E_{scat,b}(\bar{r}_i, \bar{r}_{T_j}, \omega_k) \quad (11)$$

$$[\overline{M}]_{mn} = \Delta s \omega_k^2 \mu g_b(\bar{r}_i, \bar{r}_n, \omega_k) E_{inc,b}(\bar{r}_n, \bar{r}_{T_j}, \omega_k) \quad (12)$$

$$[\delta \overline{\varepsilon}]_n = \delta \varepsilon(\bar{r}_n) \quad (13)$$

where index m represents all combinations of (i, j, k) .

$E_{scat,b} - \overline{M} \cdot \delta \overline{\varepsilon}$ is solved using an optimization technique where the error is minimized as well as the norm of the solution. Hence, an error functional I for equation 10 could be defined as,

$$I = \|E_{scat,b} - \overline{M} \delta \overline{\varepsilon}\|_{\frac{1}{\delta}}^2 + \delta_t \|\delta \overline{\varepsilon}\|_{\frac{1}{\delta}}^2 \quad (14)$$

where δ_t is the Tikhonov regularization parameter. This is solved iteratively to obtain $\delta \overline{\varepsilon}$.

The regularization parameter δ_t is computed using a stochastic approach [Qin et al, 1994]. If δ_t is too small, the numerical stability may be compromised and if it is too big, too much useful information may be lost.

$$\delta_t = \rho_N^2 / \rho_o^2 \quad (15)$$

where

$$\rho_N = (1/mn) \|\overline{E}_{scat} - \overline{M} \cdot \delta \overline{\varepsilon}\|^2 \quad (16)$$

$$\rho_o = n \cdot \sigma^2 \left[\text{tr}(\overline{M}^\dagger \cdot \overline{M}) \right]^{-1} \quad (17)$$

and

$$\sigma = (1/mn) \|\bar{E}_{scat} - \bar{E}_{sca, mean}\|^2 \quad (18)$$

To avoid too small values of σ as the iterative process progresses, when at a certain iteration step the relative error becomes less than an imposed value (10^{-3}), the value corresponding to that step is used for the remaining iterations.

Results and Discussions

The distribution of electric fields inside the imaging chamber in the presence of the coupling medium but in the absence of the object, computed from FDTD simulation is shown in Figure 2. Over the circumference the electric field decreases towards 180° and then gets back to the initial value.

2D microwave tomographic images of the models were reconstructed from the electric field data computed from FDTD simulations. Due to the limitation of the BIM in imaging high contrast objects we have adopted distorted BIM to reconstruct high contrast, absolute images using the computed scattered field $E_{scat}(tk)$ (Model a) [Chew, 1998]. The BIM was used for the reconstruction of low contrast differential images using the differential scattered field computed as $E_{scat}(tk) - E_{scat}(tm)$ (Model b) where tk and tm represent the times of acquisition of the frames k and m .

The imaging results are presented in Figures 3 and 4 as fused images. The background images in the gray palette are the absolute images of the 2D cross section of the Base Line model (Model a). The time-differential images in colour are superimposed with the background images. These colour images represent the induced interventions simulated in Model b. The degree of such changes obtained from reconstructed images is represented by the colour bar as percentile of changes; $(_Model\ b - _Model\ a) / _Model\ a (\%)$

Figure 3 shows the fused images without noise inclusion with the left column represents fused images (absolute and time differential) of ϵ' and the right column represents corresponding fused images of ϵ'' . The reconstructed absolute images correlate well with the simulated dielectric properties and its location, and clearly reveal the boney area with low dielectric properties (dark coloured) and the soft tissue component with higher dielectric properties (bright coloured). The absolute value of the percentile changes in the dielectric parameters of the inclusions vary from 4–15% which agrees with the tabulated values provided in Table 1.

In Figure 3, an absolute value of 4.3% (percentile) change in ϵ' is observed for the vertical axisymmetric intervention (Figure 3 a) and with no % variation in ϵ'' (Figure 3b), complements well with the dielectric parameters considered for the simulation (Figure 1 and Table 1). However, the very closely located horizontal axisymmetric interventions with an edge to edge actual spacing of 2 mm are observed as a single large image. It has to be noted that within microwave imaging region the average dielectric properties over a certain volume are reconstructed [Semenov et al(1), 2011]. In this case, it includes $\pm 5\%$ variation of dielectric properties from the muscle tissue. In our early imaging experiments and computer simulations, we suggested that in spite of the fact that microwave imaging technology is in its early development stage, it is estimated that this technology is able to reconstruct even about 1% change in overall cross sectional variation in the dielectric properties [Semenov et al(1), 2011, Semenov et al(2), 2011]. A further in-depth study is under way to ascertain the minimum extremity separation for an accurate determination of the dielectric properties. This will allow the direct reliable application of this imaging modality in clinical settings.

Figure. 4 shows the image reconstruction using the FDTD data corrupted with 10% Gaussian noise. The algorithm performs well in the presence of noise with results in good agreement to Figure 3 as well as to the dielectric parameters tabulated in Table 1. In our previous studies [Semenov et al(1), 2011] on 2D fused imaging using experimental data we had compared Born Iterative methods with Newton method and could obtain comparable results. Even though the reconstructed absolute images correlated well with animal anatomy, the reconstructed values of dielectric properties by both BIM and Newton methods were not within a range of expected/tabulated properties of the tissues. The distribution of EM field in a small chamber in real situations is more complex unlike the idealistic model with perfect boundary conditions considered here. Compared to the computation of the regularisation parameter by trial and error method used in our previously reported paper [Semenov et al(1), 2011, Sourov et al, 1998], the stochastic approach adopted here offers a better convergence at the global minimum and Hence, more precise image reconstruction is obtained. However, the technique is computationally intense and Hence, considerably increases the computational time. The error in minimising the cost functional is shown in Figure 5. No local minima are visible and the error function converges to global minimum even in the presence of 10% noise which shows the robustness of the algorithm.

The need for further improvement of reconstructed images is applicable to both absolute and time-differential portions of fused images. Development of 3D image reconstruction techniques, optimization of the number of transmitting and receiving antennas are just a few improvements required to obtain better images. Though it is understood that a 3D approach provides access to wider range of viewing planes, this 2D approach allows a clear demonstration that fused imaging technique using differential imaging could well be implemented in microwave tomography.

Conclusion

A 2D fused imaging approach has been introduced with a differential imaging reconstruction algorithm for microwave tomography with realistic modelling of the antenna system using FDTD method. Good reconstruction accuracy was obtained. In spite of the fact that microwave tomography technology is in its early development stage, it is estimated that this technology is able to reconstruct functional extremity images in terms of differential dielectric properties. This will allow for direct applications of this novel imaging modality in clinical settings.

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References

- Nair, Bindu G. Active Microwave Imaging for Mammography. Lap Lambert Academic Publishers; Germany: 2012.
- Bindu G, Mathew KT. Characterization of benign and malignant breast tissues using 2-D microwave tomographic Imaging. Microw Opt Technol Lett. 2009; 49:2341–2344.
- Chew, WC. Imaging and inverse problems in electromagnetics. In: Taflove, A., editor. Advances in Computational Electrodynamics: The Finite Difference Time Domain Method. Vol. Chapter 12. Norwood, MA: Artech House; 1998.
- Chien, W.; Chiu, C.; Huang, C.; Fan, Y. Microwave imaging of buried 2D homogeneous dielectric cylinder using dynamic differential evolution. Proc IMECS; Hong Kong. 2010.

- Fang Q, Meaney PM, Paulsen KD. Microwave image reconstruction of tissue property dispersion characteristics utilizing multiple frequency information. *IEEE Trans Microw Theo Tech*. 2004; 52:1866–1875.
- Fhager A, Padhi SK. 3D Image reconstruction in microwave tomography using an efficient FDTD model. *IEEE Ant Wireless Propag Lett*. 2009; 8:1353–1356.
- Hagness SC, Taflove A, Bridges JE. Three-Dimensional FDTD analysis of a pulsed microwave confocal system for breast cancer detection: design of an antenna-array element. *IEEE Transactions of Antennas and Propagation*. 1999; 47:783–791.
- Meaney PM, Paulsen KD, Chang JT. Near-field microwave imaging of biologically based materials using a monopole transceiver system. *IEEE Transactions on Microwave Theory and Techniques*. 1998; 46:31–44.
- Moghaddam M, Chew WC. Study of some practical issues in inversion with the Born iterative method using time domain data. *IEEE Trans Ant Propag*. 1993; 2:177–184.
- Mojabi P, LoVetri J. A Pre-Scaled Multiplicative Regularized Gauss-Newton Inversion. *IEEE Trans on Antennas and Propagation*. 2011; 59(8):2954–2963.
- Paulsen KD, Meaney PM, Chang JT, Fanning MW. Initial microwave imaging experiments in ex-vivo breast tissue. *Proc of First Joint IEEE BMES/ EMBS Conference Serving Humanity; Advancing Technology*. 1999; 1130:1999.
- Popovic M, Taflov A. Two-dimensional FDTD inverse scattering scheme for determination of near surface material properties at microwave frequencies. *IEEE Trans Ant Propag*. 2004; 52:2366–2373.
- Qin YM, Ciric IR. Method of selecting regularisation parameter for microwave imaging. *Elect Lett*. 1994; 24:2028–2029.
- Semenov S, Kellam J, Bindu G, Williams T, Quinn M, Sizov Y, Nazarov A, Pavlosky A. Microwave tomography of extremities: 2. Functional fused imaging of flow reduction and simulated compartment syndrome. *Phy Med Biol*. 2011; 56:2019–2030.
- Semenov S, Kellam J, Sizov Y, Nazarov A, Bindu G, Williams T, Quinn M, Pavlosky A. Microwave tomography of extremities: 1. Dedicated 2D system and physiological signatures. *Phy Med Biol*. 2011; 56:2005–2017.
- Shea JD, Kosmas P, Hagness, Veen BDV. Three-dimensional microwave imaging of realistic numerical breast phantoms via a multiple frequency inverse scattering technique. *Med Phy*. 2010; 37(8):4210–4226.
- Souvorov AE, Bulyshev AE, Semenov S, Svenson RH, Nazarov AG, Sizov YE, Tatsis GP. Microwave Tomography: A two-dimensional Newton iterative scheme. *IEEE Trans Microwave Theo Tech*. 1998; 46:1654–1658.
- Li, Xu; Hagness, SC. A confocal microwave imaging algorithm for breast cancer detection. *IEEE Microwave and Guided Wave Letters*. 2001; 11:130–132.

Figure 1. Models and the dielectric parameters considered for the FDTD simulation and MWT imaging. a) Base Line model, b) Model with induced tissue variations simulating different physiological conditions within extremities soft tissues

Figure 2.
Distribution of electric field inside the imaging chamber in the presence of coupling medium.

Figure 3. Fused reconstructed images of the FDTD simulated data without noise, a) and b) , with absolute value of percentile changes.

Figure 4. Fused reconstructed images of the FDTD simulated data with 10% noise, a) and b) , with absolute value of percentile changes.

Figure 5.
Norm of the error in minimising the error function.

Table 1

Dielectric parameters considered for the FDTD simulation

Name	Model a	Model b
Coupling medium	57+j22	57+j22
Muscle	54+j20	54+j20
Bone	17+j5	17+j5
Vertical axisymmetric inhomogeneity 1 (up)	55+j21	48.6+j18
Vertical axisymmetric inhomogeneity 2 (down)	57+j22	59.4+j22
Horizontal axisymmetric inhomogeneity 1 (left most)	No inhomogeneity modelled	51.3+j19
Horizontal axisymmetric inhomogeneity 2	No inhomogeneity modelled	56.7+j21