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Possibilities and limitations of the ART-Sample algorithm for reconstruction of 3D temperature fields and the influence of opaque obstacles

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Abstract

The need for the measurement of complex, unsteady, three-dimensional (3D) temperature distributions arises in a variety of engineering applications, and tomographic techniques are applied to accomplish this goal. Holographic interferometry (HI), one of the optical methods used for visualizing temperature fields, combined with tomographic reconstruction techniques requires multi-directional interferometric data to recover the 3D information. However, the presence of opaque obstacles (such as solid objects in the flow field and heaters) in the measurement volume, prevents the probing light beams from traversing the entire measurement volume. As a consequence, information on the average value of the field variable will be lost in regions located in the shade of the obstacle. The capability of the ART-Sample tomographic reconstruction method to recover 3D temperature distributions both in unobstructed temperature fields and in the presence of opaque obstacles is discussed in this paper. A computer code for tomographic reconstruction of 3D temperature fields from 2D projections was developed. In the paper, the reconstruction accuracy is discussed quantitatively both without and with obstacles in the measurement volume for a set of phantom functions mimicking realistic temperature distributions. The reconstruction performance is optimized while minimizing the number of irradiation directions (experimental hardware requirements) and computational effort. For the smooth temperature field both with and without obstacles, the reconstructions produced by this algorithm are good, both visually and using quantitative criteria. The results suggest that the location and the size of the obstacle and the number of viewing directions will affect the reconstruction of the temperature field. When the best performance parameters of the ART-Sample algorithm identified in this paper are used to reconstruct the 3D temperature field, the 3D reconstructions with and without obstacle are both excellent, and the obstacle has little influence on the reconstruction. The results indicate that the ART-Sample algorithm can successfully recover instantaneous 3D temperature distributions in the presence of opaque obstacles with only 4 viewing directions.

Keywords

Tomography; Inverse methods; ART-Sample algorithm; 3D temperature measurement; Holographic interferometry; Obstacle; Opaque obstacle

1. Introduction and background

Processes in nature are complex, unsteady and three-dimensional (3D), and in science and engineering there is an increasing need as well as interest in the better understanding and the modeling of these phenomena. Such problems arise in forced and free convection in nature and in engineered systems. Examples include high speed aerodynamic flows and plasmas, thermal plumes, atmospheric buoyancy driven flows, heat transfer equipment, etc. The approach to the analysis involves both experimental and computational methods that are enabled by the dramatic advances in sensors and instrumentation, computer hardware and software as well as imaging and image processing hardware and software. Due to the complexity of unsteady 3D flow and temperature fields in convection problems, accurate modeling of these processes is still difficult and time-consuming, and experimental data are needed to validate computational models and methods. Therefore, these processes are often investigated experimentally to enable better insight into the physics of the process as well as to generate data for the validation of computational models.

Instrumenting these flows with physical sensors would disrupt the flow and affect the flow and temperature distributions in the system. Optical measurement and imaging methods are therefore ideal candidates for such studies. The advantage of optical methods is that they are noninvasive; they do not affect the flow and have practically no inertia, which makes them uniquely suitable for the analysis of high-speed, unsteady processes [1]. One of the imaging methods frequently used in the study of complex convective heat and mass transfer processes and pressure distributions is holographic interferometry (HI). It is most often used for visualizing two-dimensional (2D) refractive index fields that can be coupled to temperature, pressure, concentration or density distributions (derived variables). In HI the refractive index is integrated along the path of a light beam passing through the test volume (optical path); therefore only average values of a property (refractive index or associated derived variables) can be reconstructed from a single projection. Reconstruction of 3D temperature fields requires multi-directional interferometric data. Interferograms of the temperature field are recorded along several directions; the fringe patterns are digitized and processed using the image processing software. Following these steps, the 3D refractive index field and the temperature field can be reconstructed using inverse methods [2].

3D (also called tomographic) reconstruction techniques involve the process of recovery of a 3D field function from a set of its integrals along well-defined directions along hyperplanes, permitting reconstruction of 3D fields using optical data from 2D projections [2,3]. Tomography was initially introduced into medical applications several decades ago. Nowadays it is also used in numerous engineering fields, such as environmental, chemical and mechanical engineering, to investigate complex, 3D, unsteady (often high-speed) phenomena. The key difference between medical tomography and engineering applications is that in medicine the subject is stationary (the patient can be immobilized or even sedated for the duration of the scan, if needed), whereas convective flow phenomena can be highly unsteady. In medicine the time-consuming sequential irradiation of the test subject from various (usually numerous, 100+) directions is possible. In unsteady convective processes irradiation from different directions has to happen simultaneously, which limits the number of irradiation (viewing) directions. Therefore there are substantial differences between tomographic techniques used in medicine and those suitable for unsteady engineering applications.

In tomography, as shown in Fig. 1, the measurement volume (in this example a cylinder that confines the flow being analyzed) is sliced into a set of parallel 2D planes (one of these planes, plane A, is shown shaded in Fig. 1). Parallel light beams pass through the measurement volume, and the information along the individual 2D planes is analyzed first,

followed by the assembly of the information from many 2D planes into a 3D reconstruction. The theory of tomography is based on the measurement of projection values, which are obtained from the change of the irradiation traversing the measurement volume. One such projection value, a projection image of a thermal plume forming above a circular heater, obtained by HI, is shown on the right hand side of the measurement volume in Fig. 1. The projection of the 2D plane (slice A) is shown as a horizontal line AA in the HI image. The measured fringe order distribution plot for the line AA is shown above the image, and this is the input information used in the tomographic reconstruction algorithm discussed in this paper. The projection values along line AA are used as the input data for the reconstruction of the physical properties of interest within the measurement volume.

HI coupled with tomographic reconstruction methods was shown to be very suitable for the measurement of unsteady 3D temperature fields [1–7] and can yield temperature data at any spatial position and at any time instant. Therefore, many experimental studies relied on this method to visualize and measure 3D temperature fields in engineering thermophysics [8–10]. In order to apply HI to heat transfer measurements, both the investigated fluid and the vessel containing the fluid have to be transparent. By judiciously selecting transparent fluids and solids with matching refractive indices, refractive index measurements in volumes containing solid transparent obstacles become possible.

In practical convective heat transfer applications the fluid is often heated or cooled by an opaque solid structure or surface within the flow field: a heater embedded into a solid, fluid flowing past a shaft of a turbine, or fins or tubes in heat exchangers. In these situations the measurement will be affected by the presence of opaque obstacles (heater, tubes, shaft, fin, etc.). The presence of the opaque obstacle prevents the probing light beams from traversing a portion of the measurement volume. As a consequence, information on the average value of the field variable will be missing in regions that are in the shade of the obstacle. The problem of reconstructing 3D refractive index field distributions (temperature, concentration or pressure) in the presence of opaque obstacles has not been adequately addressed in the literature and is the subject of the present study.

The ART-Sample tomographic reconstruction method has been proven in the study of 3D refractive index fields [1,2], and in this paper its possibilities and limitations are evaluated for reconstruction in the presence of opaque obstacles as well as without them. The accuracy of tomographic reconstruction depends on the number of irradiation (viewing) directions, the number of beams along each viewing direction (discretization determining the mesh size) and the arrangement of the beams. These parameters also determine the experimental and computational effort involved in the reconstruction of 3D fields. In this paper we systematically investigate the influence of these parameters on the accuracy of tomographic reconstruction with and without obstacles. The evaluation of the ART-Sample method is carried out using several phantom test functions and validated using experimental data.

2. Tomographic reconstruction techniques: the ART-Sample method

A large number of tomographic reconstruction techniques has been described in the literature [2], and the selection of the technique for a particular application is going to be affected by the type of measurement data available, the experimental arrangement and the physical process under consideration. In order to keep the computational expenditure and the complexity of the experimental setup as low as possible while enabling the desired accuracy, the ART Sample algorithm (the Sample Method combined with the Algebraic Reconstruction Technique (ART)) [11,12], was selected for this study. The ART-Sample algorithm was found to be suitable for heat transfer and species transport (concentration) measurements [2]. The Sample Method is based on a discrete Fourier transform of the

measurement data. As a result, an underdetermined system of equations is obtained. This system of equations can be solved in an iterative process by the ART method. An iterative algorithm and a computer code that allow the reconstruction of the 3D refractive index and temperature distributions from the fringe patterns recorded in the interferometric visualization experiments (2D projections) was developed at the Heat Transfer Lab of the Johns Hopkins University [12]. In addition to experimental input data, the code accepts data generated by the computational phantom model.

2.1. The Sample Method

In Fig. 2 the steps involved in the tomographic reconstruction of the field function $f(x,y)$ – the unknown temperature field – using the ART Sample algorithm are shown schematically. The 3D (cylindrical) measurement volume is analyzed by subdividing it into several 2D cross sectional planes, one of those is shown in Fig. 2. The selected 2D plane is irradiated from several directions, two of these are shown in Fig. 2, described with the angles θ_m and θ_n relative to the coordinate system defined in Fig. 2. By considering one 2D plane at a time (Figs. 1 and 2) during the tomographic reconstruction process, the data inversion problem becomes manageable. Reconstructed data from individual 2D planes are assembled into 3D field information in the final step of the tomographic reconstruction process.

2.1.1. Measurement of projection values Φ_i —Data obtained by multidirectional interferometry quantify the optical pathlength changes for a collection of rays during their passage through the investigated refractive index field $n(x,y)$. The field function $f(x,y)$ can be expressed as $f(s_j) = n(s_j) - n_0$, which is the refractive index relative to some constant reference refractive index field n_0 (often corresponding to constant ambient conditions). In the HI experiment $f(s_j)$ would describe the change of the refractive index which causes a phase shift between the reference (n_0) and measurement ($n(s_j)$) beams of the interferometric setup [1,2,13]. A ray i irradiating the measurement volume can be uniquely described by the angle θ_i and its distance ρ_i from the origin of the coordinate system (Fig. 2). As ray i passes through the measurement volume, the changes of the physical properties of the fluid, such as the fluid temperature, change the properties of the ray. These changes, integrated along a line, the optical path s_i , are then measured in form of a projection value Φ_i . Two such projection functions are shown in Fig. 2. Mathematically, these projection values can be described as

$$\Phi_i(\rho_i, \theta_i) = P_i \lambda = \int_{s_i} f(s_i) ds \quad (1)$$

In Eq. (1) P_i is the fringe order and λ is the wavelength of the light of the light source (usually a laser). If the field function f is kept constant along the ray path s_i , the value of the field function f and the value of the projection value Φ_i will be identical. This is the case in the study 2D temperature fields by applying HI, which requires one direction of illumination only. Typical interferometric fringe patterns, visualizing the phase shift, for a thermal plume obtained in this way are shown in Figs. 1 and 2.

2.1.2. Discrete Fourier transform—In the next step, the discrete Fourier transform is applied to the projections Φ_i for each direction of illumination. These projection values Φ_i correspond to a one dimensional function in the physical space, as shown in Fig. 2. The result of the Fourier transform will be a one dimensional spectrum in the Fourier space. The same procedure can be applied to another direction of illumination. Consequently, two spectra are obtained in the Fourier domain. These two spectra can be connected in the Fourier domain under an angle $\theta = \theta_m - \theta_n$, the angle between the two directions of illumination shown in Fig. 2. The spectra of many directions of illumination during the

imaging process form a two dimensional function in the Fourier domain. The more spectra are captured, the better the resolution of the function in the Fourier domain. By applying the inverse two-dimensional Fourier transform to this 2D function in the Fourier domain, the field function $f(x,y)$ can be recovered in the spatial domain. As a result of these mathematical manipulations, the following system of equations is obtained

$$\Phi_i(\rho_i, \theta_i) = \sum_m \sum_n w(a_i, b_i) \cdot f(l_x m, l_y n) \quad (2)$$

In Eq. (2), l_x and l_y denote the size of one grid element and m and n are the coordinates of the grid points in the x and y directions, respectively, as illustrated in Fig. 3. One should also note the weight factor $w(a_i, b_i)$ that appears in Eq. (2). The weight factor is a function of the ray i , where a_i and b_i denote the slope and the intercept of ray i (Fig. 3). The mathematical form of the weight factor $w(a_i, b_i)$ as shown in [2,12] is

$$w(a_i, b_i) = \begin{cases} (1+a_i^2)^{0.5} |l_x| \text{sinc}\left(\frac{b_i+a_i l_x m - l_y n}{l_y}\right) & , 0 \leq |a_i| \leq \frac{l_y}{l_x} \\ (1+a_i^2)^{0.5} \left| \frac{l_y}{a_i} \right| \text{sinc}\left(\frac{b_i+a_i l_x m - l_y n}{a_i l_y}\right) & , \frac{l_y}{l_x} < |a_i| < \infty \\ |l_y| \left(\frac{\rho_i + l_x m}{l_x} \right) & , |a_i| = \infty \end{cases} \quad (3)$$

where the sinc-function is defined as

$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \quad (4)$$

The system of Eqs. (2) has $m \times n$ unknowns corresponding to the number of grid points. In order to find a unique solution, $m \times n$ projection values Φ_i are necessary. Sweeney (1974) [14] even recommends working with an overdetermined system of equations to improve the reconstruction accuracy. In medical applications, this requirement is easily satisfied by taking projection values using angle subdivisions of 1° . Apart from a sufficient number of projection values Φ_i , this method also ensures an accurate representation of the two dimensional function in the Fourier domain, since 180 spectra will be available. In engineering applications, it is often not possible to obtain a large number of spectra, in particular in the case of high-speed, unsteady processes.

The Sample Method is a powerful tool to achieve high spatial resolution in 3D measurements. However, in unsteady engineering applications the number of projection values has to be decreased and irradiation from multiple directions has to be achieved simultaneously rather than sequentially as in medical applications, which leaves the system of Eqs. (2) underdetermined. The solution to this problem is the ART algorithm that iteratively determines the values of the field function f . In fact, the ART algorithm allows the reconstruction of the field function with only 4 directions of illumination and 50 rays per direction, for a grid size of 50×50 grid points. When applying the Sample Method alone, 2500 projection values would be necessary instead of 200 (4×50) required for the combination of the ART-Sample methods. The need for fewer number of projections and irradiation directions clearly illustrates the advantages of combining the two methods.

2.2. The ART algorithm

The ART algorithm is the most widely known series-expansion tomographic reconstruction technique. The procedure starts by setting all values of the image matrix to an initial value, usually 0. Then, the value of the element is improved in the following iteration step. The

procedure is based on the comparison of the calculated projection values with measurement data (or known data from phantom test functions) after each iteration step. From the result of the comparison, a correction term for the field function is obtained. The calculated

projection value $\Phi_i^{(k)}$ after the k th iteration step is obtained by applying Eq. (2) from the Sample Method. For the ART algorithm this equation can be written as

$$\sum_{j=1}^{m \cdot n} w_{ij} \cdot f_j^{(k)} = \Phi_i^{(k)} \quad (5)$$

In Eq. (5), the matrix w_{ij} denotes the weight factor for ray i and grid point $j=m \times n$, $f_j^{(k)}$ denotes the field function after the k th iteration step. The weight factor w_{ij} can be evaluated as the contribution of the j th grid element to the line integral of the i th ray. The iterated projection value $\Phi_i^{(k)}$ usually differs from the measured value Φ_i . The deviation $\Delta\Phi_i^k$ can be evaluated as

$$\Delta\Phi_i^k = \Phi_i - \Phi_i^{(k)} = \Phi_i - \sum_{j=1}^{m \cdot n} w_{ij} \cdot f_j^{(k)} \quad (6)$$

The deviation $\Delta\Phi_i^k$ is then used in the correction term to determine the element of the field function in the $k+1$ iteration by adding or subtracting a proportional value of $\Delta\Phi_i^k$ to the iterated value $f_j^{(k)}$. Mathematically, the correction is formulated as

$$f_j^{(k+1)} = f_j^{(k)} + \frac{w_{ij} \cdot \Delta\Phi_i^k}{\sum_{j=1}^{m \cdot n} (w_{ij})^2} \quad (7)$$

Eq. (7) implies that each field element is corrected by using the deviation of the corresponding projection value and the grid element specific weight factor. Sufficient accuracy is achieved in an iteration when no significant changes of the field elements can be detected in the successive iteration step. Due to the fact that in each iteration step the

number of the corrections performed on the iterated field function $f_j^{(k)}$ corresponds to the number of rays, it is not possible to define a unique termination criterion. This means that, for one ray, the criterion for each value f_j of the target matrix may be fulfilled, but this does not necessarily hold for other rays. Therefore, termination criteria that take into account the deviation of all rays have to be applied.

In practice, two termination criteria are used. One is based on the magnitude of the sum of squares of all residuals given by Eq.(6). The argument is that when the deviation between the iterated and measured projection values Φ_i is small enough, the deviation of the values f_j of the field function should be small as well. The second termination criterion prescribes the number of iteration steps.

3. Results: reconstruction of test functions fields using the ART-Sample algorithm

The possibilities and limitations of the ART-Sample algorithm are being explored qualitatively in this section by considering circular and square test volumes as well as

parallel and fan shaped beams. The reconstruction is examined for six analytical test functions, phantoms, by varying the number of directions of irradiation. Using analytical test functions defined by the user in the evaluation of tomographic reconstruction techniques offers the advantage that the exact form of the solution is known, which is not the case with experimental data. Original test functions are compared with tomographic reconstructions.

3.1. Test section shapes and beam arrangements

One of the goals of this study is to better understand the influence of the method of irradiation and the shape of the cross section of the investigated volume on the quality and accuracy of the reconstruction. In Fig. 4 the cross sections and projection methods selected for this study are displayed schematically. In practical applications test sections are typically either cylindrical or square shaped, and these two cases are considered in the paper. The light source in an experiment is typically a point source, and it can be used directly in a fan-shaped beam arrangement to irradiate the test section. The angle between the individual rays is $\Delta\theta$, and the angle θ_0 describes the angle between the x axis of the coordinate system and the individual beam. The fan beam arrangement is easier to implement experimentally, however the disadvantage is the reduction of the spatial resolution with increasing spacing between the rays as the distance from the source increases. The tracking of the optical path is more complex in the tomographic reconstruction for the fan beam arrangement. Another option is to use parallel beam irradiation accomplished by expanding the light beam into a bundle of parallel rays using appropriate optics. The spacing between the beams d is constant along the entire optical path and the resolution remains constant throughout the test section.

The influence of the opaque obstacle on the probing beams is illustrated schematically in Fig. 5. The obstacle prevents the light beams from traversing a portion of the measurement volume. As a consequence, information regarding the projection value will be lost in the regions located in the shade of the obstacle. Since the test section is irradiated from multiple directions, the obstacle will affect each projection. However, depending on the size and location of the obstacle and the shape of the temperature distribution or test function relative to the obstacle, tomographic reconstruction may be possible even with missing information. The influence of the obstacle on the reconstruction quality is investigated qualitatively in Section 3 as well as quantitatively in Section 4 of this paper.

3.2. Selection of the analytical test functions

For the analysis in this paper temperature distributions for characteristic cross sections of a measurement volume were represented through analytical test functions, phantoms, and they serve as input data for the ART-Sample tomographic reconstruction algorithm. Several test functions representative of temperature distributions for realistic physical situations were generated. In order to cover a broad range of possible temperature fields (single and multiple peak functions, moderate and large temperature gradients aligned with the axis or off-axis), six analytical functions, phantoms, summarized in Table 1 are selected to test the ART Sample algorithm. They are plotted in the first column of Figs. 6 and 7.

3.3. Reconstruction of test functions using the ART-Sample algorithm

In this section the reconstruction accuracy is discussed qualitatively first (quantitative assessment is provided in Section 4) based on the visual comparison of the test function and its reconstruction for test functions without obstacles under various conditions (cross section shape, beam type, number of irradiation directions, etc.). One of the goals is to find the best reconstruction conditions – best reconstruction accuracy for minimum computational and experimental effort – and aid the decisions regarding the design of the experimental arrangement. As the results in Figs. 6 and 7 suggest, the reconstructed temperature distributions obtained using the ART-Sample algorithm match the original function

reasonably well for relatively smooth functions with single or multiple peaks for both parallel beam and fan beam projection methods. The reconstructions match the shape of the original function well, with the maximum of the peak being slightly lower in the reconstruction. The domain with imposed zero value around the peak displays nonzero artifacts in the reconstruction as nonzero ‘‘ridges’’, lines along the direction of irradiation.

As illustrated in Fig. 8, the accuracy of the reconstructed function is greatly influenced by the number of irradiation directions. The reconstructions produced by this algorithm improve with the number of irradiation directions, as expected. The reconstruction is unsatisfactory for less than 4 irradiation directions: the magnitude of the peak decreases and the magnitude of the artifacts increases with decreasing number of irradiation directions. For 4 or more irradiation directions, the accuracy of the reconstructed function is satisfactory. The reconstruction fidelity increases only slightly when the number of irradiation directions exceeds 4. For this reason, the case studies in this paper use 4 irradiation directions (Figs. 6 and 7), which was also accepted as the parameter for best reconstruction.

3.4. Effect of the opaque obstacle on the reconstruction

The RF, COSF1, RRF and COSF3 are chosen as the phantoms to test the reconstruction accuracy in the presence of the opaque obstacle. The RF and COSF1 functions are used to evaluate the case with the obstacle located within temperature field. The RRF and the COSF3 test the situation with the obstacle located outside of the temperature field. The radius of the obstacle was 0.2 and 0.1. The test conditions were defined by 20 iteration steps, square cross section, parallel beams and 150 rays per beam.

3.3.1. Obstacle located inside of the temperature field—As shown in Figs. 9–12, the reconstructed temperature distributions in the presence of opaque obstacles match the original function reasonably well for relatively smooth functions with single or multiple peaks. When the obstacle is located within the temperature field, the temperature near the obstacle boundary is lower than in the original and this artifact is caused by the discontinuity of the test function. On the other hand, when the obstacle is located in the unheated region, the temperature near the obstacle boundary will be slightly higher than in the original function (COSF1 with obstacle in the middle, for example). The quantitative analysis of the reconstruction accuracy in Section 4 will provide more accurate information regarding the influence of the number of viewing directions. Algorithms for post processing of the data in the vicinity of the obstacle can be developed to correct for the reconstruction artifacts in this region.

3.3.2. Obstacle in the unheated region—As shown in Fig. 13, the reconstructed temperature distributions with the opaque obstacle in the unheated region match the original function well. There are fewer reconstruction artifacts when the function transitions more smoothly into the unheated region.

4. Quantifying reconstruction accuracy and experimental validation

In order to assess the reconstruction accuracy quantitatively, criteria that characterize the quality of reconstruction have to be identified. The mean square error introduced by Snyder and Hesse-link (1984) [15] as

$$msqr_err = \left\{ \frac{\sum_n \sum_m [f(m,n) - f'(m,n)]^2}{\sum_n \sum_m [f(m,n)]^2} \right\}^{\frac{1}{2}} \quad (8)$$

is used in this paper. In Eq. (8) $f(m,n)$ is the true value and $f'(m,n)$ the reconstructed value of the test field function for the grid point m,n . The $msqr_err$ quantifies the difference between the estimated (reconstructed function) and true value (test function) of the quantity being estimated.

4.1. Accuracy of the reconstruction without obstacle: parametric study

The parameters considered in this quantitative analysis include the (i) number of rays per beam (determines mesh size), (ii) the number of directions of irradiation as well as (iii) the number of necessary iteration steps. The fidelity of the reconstruction was analyzed for 50 rays per beam: the number of directions of irradiation and the number of iteration steps was varied for (iv) circular and (v) rectangular cross sections and (vi) parallel and (vii) fan beam irradiation. Key results of these analyses, expressed in terms of the $msqr_err$ function as the function of the number of iteration steps (between 5 and 30) are summarized in Fig. 14.

For nearly all phantom test functions, the reconstruction accuracy for the square cross section (Fig. 14a, c and e, left column) is slightly higher than for the circular cross section (Fig. 14b, d and f, right column). The square cross section offers advantages in experimental studies as circular cross sections are characterized by variable refraction through the curved wall. Parallel beam irradiation (full line) yields slightly higher accuracy than the fan beam irradiation (dashed line), which can be attributed to the more uniform discretization mesh size throughout the measurement volume for parallel beams. For the multiple peak functions (COSF1, COSF2, for example), the accuracy for the fan beam irradiation is much lower than the parallel beam irradiation. Again, this can be attributed to the variable mesh size (increasing with increasing distance from the light source). For functions with high gradients and sudden transition between unheated (test function zero) and heated (test function non-zero) regions with single or multiple peaks (COSF1, COSF2 and 1PF, for example), the accuracy of the reconstructed function improves greatly with increasing number of irradiation directions (Fig. 14a vs. e and d vs. f). For relatively smooth functions (moderate gradients and smooth transitions) with single or multiple peaks (RF, RRF and COSF3, for example), the accuracy of the reconstructed function improves slightly with increasing number of irradiation directions (from 14a to 14e as well as for 14b to 14f). For all test functions, the reconstruction fidelity increases only slightly when the number of irradiation directions exceeds 4 (14c vs. 14e and 14d vs. 14f). The accuracy changes little when increasing the number of iterations beyond 20 iterations – beyond this value the mean square error remains practically constant.

The results in Fig. 14 suggest that sufficiently accurate results can be achieved for both cross sections (square and circular) and irradiation techniques (parallel and fan beam). This enables measurements of 3D temperature fields in ducts with circular as well as square cross sections. The best reconstruction performance with Original 4 views reconstruction 5 views reconstruction the least computational effort corresponds to 20 iteration steps, the cross section can be either square or circular (determined by the container shape and the nature of the investigated process), 4 views are usually sufficient and parallel beam irradiation yields a slightly better reconstruction. In order to find the effect of the number of rays per view, the above test conditions were maintained while varying the number of rays per view. Based on

the results in Fig. 15, the reconstruction error decreases until the number of rays per view reaches 150, and changes very little beyond this value.

4.2. Effect of obstacle in the heated region on the accuracy of the reconstruction

The presence of the obstacle in the test volume will usually adversely affect the reconstruction accuracy, and a better understanding of the parameters influencing the reconstruction accuracy is essential to optimize the experimental design. In order to analyze the effect of the number of views and the size of the obstacle on the accuracy of the reconstruction, the mean square error was calculated for the RF (single peak, moderate gradient) and COS1 (four peak, pronounced gradients) functions, for the situations shown in Fig. 16 for reconstructions obtained using the ART-Sample algorithm. The results in Fig. 16 indicate that the reconstruction error will be larger both for on-axis and off-axis obstacles than the error without the obstacle (Fig. 14). In most cases shown in Fig. 16, the mean squares error increases with obstacle radius (measured as the ratio of obstacle diameter and the investigated cross section length scale – diameter or side length). Both the location and the size of the obstacle relative to the temperature field, the shape of the temperature distribution, as well as the number of viewing directions will affect the reconstruction accuracy.

As an example, for a simple temperature field (RF) the reconstruction error caused by the obstacle aligned with the axis of the temperature field will decrease with the increase of the obstacle radius (Fig. 16a). This can be attributed to the elimination of the region with the highest gradients and the change in slope: usually the maximum point of a function is reconstructed with lower function values than the original and the region of the maximum is characterized by larger reconstruction errors. Eliminating this region will decrease the total error. The reconstruction error for the obstacle located off-axis relative to the temperature field will increase with increasing obstacle diameter: in this case the central region with larger deviations will remain in the reconstruction (Fig. 16b). For the RF, using 5 views will yield better reconstruction accuracy than using 4 views.

For the more complex temperature field with 4 peaks (COS1, for example), the reconstruction error for the obstacle located in the middle of the test section will increase with increasing radius (Fig. 16a). The reconstruction error will decrease when the obstacle is aligned with the axis of one of the four peaks for the same reason as for the RF function – until the obstacle diameter becomes large enough to extend beyond the region of the individual peak (Fig. 16b). The arrangement with 4 views has better reconstruction accuracy than the arrangement with 5 views. This result suggests that some (test) functions are more sensitive to the direction of irradiation relative to the internal symmetry of the phantom and the overall shape of the test function. Therefore, when designing the experimental setup, it is helpful to have an idea about the shape of the temperature field that is to be investigated.

4.3. Effect of obstacle in the unheated region on the accuracy of the reconstruction

The effect of the number of views and the size of the obstacle on the accuracy of the reconstruction using the ART-Sample algorithm is illustrated in Fig. 17 for obstacle locations in the unheated region. When the obstacle is located in the unheated region, the reconstruction error is larger than the error without obstacle (Fig. 14). This can be attributed to the fact that the region of the peak, which is the region with the highest local errors, is retained in the reconstruction. The results in Fig. 17 suggest that the mean squares error will increase with increasing obstacle radius. This is to be expected, since less data will be available for the reconstruction as the size of the shaded region behind the obstacle increases. The results show that 5 views yield slightly better reconstruction accuracy than 4 views.

4.4. Reconstruction of 3D temperature fields from experimental data obtained by HI

4.4.1. Optical arrangement for tomographic measurements with holographic interferometry—

In order to validate the reconstruction fidelity of the ART-Sample algorithm for a real temperature field, we applied the method to reconstruct the temperature distribution in the thermal plume developing in a fluid-filled enclosure with localized heating from below. The infinite fringe field alignment of real time holographic interferometry, as described in [1,4–7], was used in the study to visualize the thermal plume formed in a fluid over a circular heater. The optical arrangement for holographic interferometry in two dimensions was expanded to allow the simultaneous generation of 4 images from 4 irradiation directions with 45° separation, as illustrated in Fig. 18. The optical components are mounted on a pneumatically elevated optical table. The light source is a 0.6 W Argon ion laser (Lexell Laser Inc., Model 95), operated at a wavelength of 514.5 nm. The exposure time of the laser for making the hologram in the experiment is 0.2 s.

The first beam splitter, BS_1 , in Fig. 18, divides the laser beam into an object beam and a reference beam. With this variable beam splitter, the intensity ratio of the object and reference beams can be adjusted. The second beam splitter BS_2 splits up the object (measurement beam) into two beams of equal intensity. These two beams are next expanded by the beam expanders BE_1 and BE_2 (Newport Model M900). They are collimated by the collimating lenses L_1 and L_2 , respectively, to a parallel beam bundle of a diameter of 38 mm. Each of these beams is then split up into two identical beams using beam splitters BS_3 and BS_4 . The four 38 mm-diameter measurement beams generated in this way pass through the test cell with the thermal plume, as illustrated in Fig. 18. The 4 object beams, which are of the same intensity, transilluminate the test cell from 4 directions perpendicular to the faces of the octagonal walls of the test cell simultaneously, with a separation angle of 45° . After passing through the test cell, two beams are directed onto the holographic plate HP_1 and the remaining two fall onto HP_2 . Two images are formed in this way on each holographic plate by the four measurement beams.

The reference beam is also expanded by the beam expander BE_3 and divided into two beams by the constant beam splitter BS_5 . The two reference beams hit the two holographic plates HP_1 and HP_2 respectively without passing through the test cell. Two simultaneously triggered cameras (Nikon N90s) were used to record the virtual images of the interferograms (two interferograms in each image frame). The illumination procedure for the recording and reconstruction of the holograms and the processing of the holographic plates are identical in tomographic and two-dimensional measurements with holographic interferometry and they are described in detail in the literature [1,4–7].

4.4.2. Test cell—The test cell, shown in Fig. 19, consists of a cylindrical vessel centered inside of an octagonal tank. The double vessel structure is needed to avoid image distortion on the cylindrical walls of the interior vessel in which the thermal plume is formed. For accurate measurements with holographic interferometry it is essential that the four expanded measurement beams are normal to the walls of the test cell (external octagonal container). The height and thickness of the wall of the octagonal tank, which is made of polycarbonate, are 101.6 mm (4 in) and 12.7 mm (0.5 in), respectively. The width of each wall segment is 101.6 mm (4 in). The cylindrical vessel located in the center of the tank is made of Plexiglas acrylic, and has an internal diameter and thickness of 101.6 mm (4 in) and 6.35 mm (0.25 in), respectively.

The octagonal tank and the cylindrical vessel are both filled with sodium iodide (NaI) aqueous solution having the same refractive index as the polycarbonate and the Plexiglas acrylic. In this way refraction errors and distortion on the cylindrical walls of the vessel are eliminated by using liquids and containers with matching refractive indices. Therefore, the

light rays remain straight when they pass through the cylindrical vessel. The NaI liquid inside the cylindrical vessel serves as the working fluid in which the thermal plume is formed.

The refractive index of the sodium iodide aqueous solution depends on the concentration c , the temperature T of the solution and the wavelength of the illuminating light. By varying the sodium iodide solution temperature from 20 to 40 °C (the temperature range for this experiment), the refractive index varies from 1.333 to 1.487 [17]. With the saturated solution as the starting point, one can add water to the solution gradually to reach the concentration point required to match the refractive index with that of Plexiglas acrylic. Narrow et al. [18] developed a correlation for determining the refractive index of sodium iodide solution as:

$$n_{\text{NaI}}(T, c, \lambda) = 1.252 - (2.91 \times 10^{-4} \text{ } ^\circ\text{C}^{-1})T + (0.365)c + (5542 \text{ nm}^2)\lambda^{-2} \quad (9)$$

The solution used in this experiment contains a small amount of sodium thiosulfate ($\text{Na}_2\text{S}_2\text{O}_3$) to avoid discoloration due to I_3^- formation. The properties of the sodium iodide solution used in this experiment are summarized in Table 2.

A 15.9 mm (0.625 in) diameter circular copper block heated by a resistive heater (Energy One, Power Supply Model XP-4) is mounted in the center of the bottom plate of the test cell, and serves as the heat source. The surface temperatures of the copper block, or the reference temperature for the interferogram, are monitored with 6 copper-constantan (T-type) thermocouples made from 0.254 mm (0.01 in) wire (Omega, 5TC-TT-T-30). Thermoelectric potential data are collected by a HP-34401A multimeter. The heater power of the heater is set at 0.5 W, 1.0 W, 1.5 W and 2 W, respectively. The depth of the working liquid in the cylindrical vessel was 40 mm.

4.4.3. Measurement results—The four projection values were visualized by real time holographic interferometry (HI). These four interferograms of the mushroom shaped thermal plume, recorded for four viewing directions, were essentially identical and we were able to validate that thermal plume is axially symmetrical (true 3D field projection data were unavailable and would be less suitable for validation since the exact temperature field is unknown). The four projection data were acquired during the experiment and used as input for the tomographic reconstruction process. One of the four projections (representative of the four interferograms), the interferogram of the mushroom shaped plume, is shown in the top row in Fig. 20. From these images, the 3D temperature distribution – which is expected to be axially symmetrical – is reconstructed for cross sections 1 ($z = 15$ mm, mushroom neck) and 2 ($z = 30$ mm, mushroom cap, Fig. 20). In the row below the interferogram the lines of maximum and minimum fringe intensity are traced and the corresponding fringe order is assigned to each. The fringe orders as a function of location for a projection, the reconstructed 3D refractive index fields and the reconstructed (measured) temperature profile along the line AA' are shown for the two selected cross sections, 1 and 2, underneath the interferograms in Fig. 20.

We applied the best performance parameters of the ART-Sample algorithm obtained in this paper (4 viewing directions, parallel beams, 150 rays per beam, 20 iterations) to reconstruct the instantaneous 3D temperature field in Fig. 20. The steps of the reconstruction procedure are illustrated in the first column of Fig. 20. The full 3D reconstruction of the thermal plume is accomplished by independently reconstructing a series of 2D function fields ($f(s_i) = n(s_i) - n_0$) from the projection data ($i = P_i$) [16] for a series of cross sections (similar to cross sections 1 and 2). By applying the ART-Sample algorithm, the refractive index of the 2D

cross section (cross sections 1 and 2) can be obtained, and the temperature field can be calculated using the known correlation between the temperature and refractive index [4,5]. By comparing the reconstruction result from the original interferogram (unobstructed view) and the one obstructed by a $r = 0.6$ mm cylindrical obstacle, we find that the 3D reconstructions with and without obstacle are both excellent. The obstacle has little effect on the reconstruction. The data also indicate that the ART-Sample algorithm successfully reconstructs the instantaneous 3D temperature distributions in the presence of opaque obstacles. The method described in the paper can therefore be applied to measure instantaneous 3D temperature distributions with only 4 viewing directions.

5. Summary and conclusions

The goal of this paper was to explore the capability of the ART-Sample algorithm to recover instantaneous 3D temperature distributions in the presence of opaque obstacles as well in the unobstructed field. The complexity of the experimental hardware (increasing with the number of viewing directions) and the computational effort should be minimized for best performance of the algorithm. The important conclusions are as follows:

1. A computer code for tomographic reconstruction of 3D temperature fields from 2D projections (that can be obtained by HI, for example) was developed. The reconstruction accuracy was discussed quantitatively both without and with obstacles for various beam arrangements, viewing directions, test function and obstacle shapes and locations. The best reconstruction performance, minimizing the complexity of experimental hardware and computational effort was found to be: 4 viewing directions, parallel beams, 150 rays per beam, 20 iterations.
2. For the smooth temperature field with or without obstacles, the quality of the reconstructions produced by this algorithm is good both visually and as judged by quantitative error measures. The algorithm does not perform well for randomly aligned peaks as well as for step changes in the fluid temperature.
3. The reconstruction error generally increases when the opaque obstacle is present in the measurement volume. When the obstacle is aligned with the axis or near the axis of the temperature field, the obstacle radius has little impact on the magnitude of the mean squares error. Both the location and the size of the obstacle and the number of viewing directions will affect the reconstruction of the temperature field. When the obstacle is located in the unheated region of the measurement volume, the mean square error will increase with increasing obstacle radius. Better reconstruction accuracy can be obtained by increasing the number of viewing directions.
4. When the best performance parameters of the ART-Sample algorithm identified in this paper are used to reconstruct the 3D temperature field, the 3D reconstructions with and without obstacle are both excellent, and the obstacle has little influence on the reconstruction.

The results suggest that the ART-Sample algorithm allows to successfully reconstruct the instantaneous 3D temperature distributions in the presence of opaque obstacles with only 4 views.

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Nomenclature

f	field function
s	ray path through the measurement volume
a	slope of a ray
b	intercept of a ray
d	spacing between the beams
l	size of one grid element
w	weight factor for each ray i and each grid point j
P	fringe order
F	Fourier spectrum
n	refractive index

Greek symbols

μ	projection value of the line integral
θ	angle of irradiation
r	distance between ray and origin
ϕ	angle between the individual rays
λ	wavelength of the illumination light
δ	deviation

Subscripts

i	number of ray
j	number of grid point
m, n	grid parameter for x, y direction
k	number of iterations
x, y, z	coordinates in the Cartesian coordinate system

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Fig. 1. Measurement planes dividing the investigated cylindrical 3D volume into several 2D cross sectional planes; projection image of a thermal plume obtained by HI (bottom right) and fringe order plot for a particular cross section (top right).

Fig. 2. Schematic of the tomographic reconstruction process using the ART Sample algorithm.

Fig. 3.
Subdivision of the reconstruction plane into a grid for use in the Sample Method.

Fig. 4. Projection methods and measurement volume cross sections considered in this study: parallel beams for (a) square and (b) circular cross section. Fan beams for (c) square and (d) circular cross sections.

Fig. 5. Missing information in the shade of the opaque obstacle indicated as a shaded region: (a) parallel beams for square cross section and (b) fan beams for circular cross section.

Fig. 6. Analytical single peak test functions (first column) and the best reconstruction (iteration steps: 20, square cross section, 4 views, number of rays per view: 150) for parallel beam (middle column) and fan beam (third column) irradiation. The arrows indicate the directions of irradiation.

Fig. 7. Analytical test function (first column) and the best reconstruction of the complex multiple-peak test functions (iteration steps: 20, square cross section, 4 views, number of rays per view: 150) for parallel beam (middle column) and fan beam (third column) irradiation. The arrows indicate the directions of irradiation.

Fig. 8.
Influence of the number of irradiation directions on the reconstruction accuracy for the 1PF.
The arrows indicate the directions of irradiation.

Fig. 9.

(a) The RF phantom function with the obstacle aligned with the axis (obstacle radius = 0.2, location: $[0, 0]$) (left) and the reconstruction with 4 (center) and 5 views (right). The arrows indicate the directions of irradiation. (b) Reconstruction along the cross section AA.

Fig. 10.

(a) The COSF1 phantom function with the obstacle in the origin (obstacle radius = 0.2, location: $[0, 0]$) (left) and reconstruction with 4 (center) and 5 views (right). The arrows indicate the directions of irradiation. Reconstructions along (b) AA – midline between the peaks and (c) BB – diagonally across the peaks.

Fig. 11.

(a) The RF phantom function with an off-axis obstacle (obstacle radius = 0.2, location: $[-0.5, 0.5]$) (left) and reconstruction with 4 (center) and 5 views (right). The arrows indicate the directions of irradiation. (b) Reconstruction along the cross section AA as shown under (a).

Fig. 12.

(a) The COSF1 phantom function with an off-axis obstacle (obstacle radius = 0.2, location: $[-0.5, 0.5]$) (left) and reconstruction with 4 (center) and 5 views (right). The arrows indicate the directions of irradiation. (b) Reconstruction along the cross section AA.

Fig. 13.

The (a) RRF and (b) COSF3 phantom functions with the obstacle in the unheated region (obstacle radius = 0.1, location: $[-0.8, 0.8]$) (left) and reconstruction with 4 (center) and 5 views (right). The arrows indicate the directions of irradiation.

Fig. 14.

Mean square error (Eq. (8)) of the reconstruction as a function of the number of iteration steps for the 6 phantom functions (—indicates parallel beams, - - - indicates fan beams) (a) 3 views, square cross section, (b) 3 views, circular cross section, (c) 4 views, square cross section, (d) 4 views, circular cross section, (e) 5 views, square cross section and (f) 5 views, circular cross section.

Fig. 15.

The effect of the number rays per view on the reconstruction accuracy for the investigated test functions.

Fig. 16.

The effect of the number of views and the size of obstacle on the accuracy of the reconstruction (— 4 views, - - 5 views) for (a) obstacle location aligned with the axis and (b) off-axis obstacle location.

Fig. 17.

The effect of number of views and the size of obstacle on the accuracy of the reconstruction with the obstacle in the unheated region (— 4 views, - - 5 views).

Fig. 18.
 Optical arrangement for tomography based on holographic interferometry.

Fig. 19.

(a) Geometry of the test cell with the octagonal external container and cylindrical inside vessel and (b) cross section of the test cell and thermal boundary conditions of the test box.

Fig. 20.

Reconstruction of a 3D temperature field for a thermal plume without and obstacle (left column) and with the obstacle aligned with the axis of the plume (right column) using the ART-Sample algorithm.

Table 1

Analytical test functions, phantoms, for evaluation of the ART-Sample algorithm

	$f(x, y) = \begin{cases} (1 - \cos(4(x - 0.26)))(1 - \cos(4(y - 0.26))), & \sqrt{x^2 + y^2} < 0.3; \\ 0, & \text{otherwise} \end{cases}$
One peak function (1PF)	
	$f(x, y) = (1 - y^2)e^{-2x^2}$
Rayleigh function (RF)	
	$f(x, y) = (1 - \cos(2x))(1 - \cos(2y))$
Cosine function 1 (COSF1)	
Cosine function 2 (COSF2)	$f(x, y) = \begin{cases} (1 - \cos(4x))(1 - \cos(4y)), & \sqrt{x^2 + y^2} < 1, x > 0 \text{ and } y > 0; \\ 0, & \text{otherwise} \end{cases}$
	$f(x, y) = \begin{cases} 2(1 - y^2)e^{-2x^2} - 1, & \sqrt{x^2 + y^2} < 0.6; \\ 0, & \text{otherwise} \end{cases}$
Revised Rayleigh function(RRF)	
	$f(x, y) = \begin{cases} 0.4(1 - \cos(1.4(x - 0.7)))(1 - \cos(1.4(y - 0.7))), & \sqrt{x^2 + y^2} < 0.7; \\ 0, & \text{otherwise} \end{cases}$
Cosine function 3 (COSF3)	

Table 2Properties of the sodium iodide solution ($T = 25.0^{\circ}\text{C}$)

Property	Value
Concentration (c)	62.33% by weight
Density ()	1800 kg/m ³
Specific heat (C_p)	1596.73 J/kg/K
Thermal conductivity (k)	2.382 W/m/K
Kinematic viscosity coefficient ()	$1.1 \times 10^{-6} \text{ m}^2/\text{s}$
Volumetric expansion coefficient ()	0.0002/K
Thermal diffusivity ()	$8.29 \times 10^{-7} \text{ m}^2/\text{s}$