Unsteady numerical simulation of a round jet with impinging microjets for noise suppression

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The objective of this study was to determine the feasibility of a lattice-Boltzmann method (LBM)-Large Eddy Simulation methodology for the prediction of sound radiation from a round jet-microjet combination. The distinct advantage of LBM over traditional computational fluid dynamics methods is its ease of handling problems with complex geometries. Numerical simulations of an isothermal Mach 0.5, $Re_D = 1 \times 10^5$ circular jet ($D_j = 0.0508$ m) with and without the presence of 18 microjets ($D_{mj} = 1$ mm) were performed. The presence of microjets resulted in a decrease in the axial turbulence intensity and turbulent kinetic energy. The associated decrease in radiated sound pressure level was around 1 dB. The far-field sound was computed using the porous Ffowcs Williams-Hawkings surface integral acoustic method. The trend obtained is in qualitative agreement with experimental observations. The results of this study support the accuracy of LBM based numerical simulations for predictions of the effects of noise suppression devices on the radiated sound power. © 2013 Acoustical Society of America.
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I. INTRODUCTION

The use of microjets as a noise suppression device for compressible jets has garnered considerable attention over the past few years. The most notable studies include those by Krothapalli *et al.*,¹ Arakeri *et al.*,² Castelain *et al.*,³ and Alkislar.⁴ These and other experiments show that the inclusion of a circumferential array of microjets may lead to a decrease in overall radiated sound pressure levels (SPLs) between 1 and 3 dB relative to a comparable jet without microjets. Castelain *et al.* studied the impact of the number of microjet nozzles and microjet mass flow rates on the radiated noise. The study by Alkislar shows that a chevron-microjet combination results in a larger decrease in overall radiated sound than a circular and chevron jet alone. Alkislar reported that the turbulence intensities for a jet with microjets is greater within the first jet diameter, but it is lower beyond one jet diameter. Alkislar argues that this is probably why the round jet-microjet combination has the largest impact on noise reduction.

Experimental data alone may not be sufficient to conclusively establish the effectiveness of microjet or chevron noise suppression devices. Despite significant advances in experimental methods, sufficiently detailed experiments to characterize turbulent shear stresses over the entire plume of an aircraft engine jet may not be possible in the foreseeable future. Computational simulations have thus been the primary tool for recent jet noise sound generation studies, despite notable limitations for practical aircraft jet noise problems. Most numerical simulation methods involve the solution of some form of the basic equations of motion using finite difference schemes. With continuous improvements in

computing power, the application of Direct Numerical Simulation (DNS) methods to jet noise prediction is now feasible in some cases.⁵ The approach involves the simulation of the flow dynamics for all the relevant turbulence length scales; it requires no turbulence model. The wide range of time and length scales present in turbulent flows and the limitations of current computational resources hampers the use of DNS for high Reynolds number flows.

Large Eddy Simulation (LES) involves direct computations of the large scales, in conjunction with subgrid scale models. It is assumed that the large scales in turbulence are generally more energetic compared to the small scales and are affected by the boundary conditions directly. In contrast, the small scales are more dissipative, weaker, and tend to be more universal in nature. Most turbulent jet flows that occur in experimental or industrial settings are at high Reynolds numbers, usually greater than 100 000. LES methods for high Reynolds number flows require a fraction of the cost of DNS. One of the first uses of LES as an investigative tool for jet noise prediction was carried out by Mankbadi *et al.*⁶ They performed a simulation of a low Reynolds number supersonic jet and applied Lighthill's analogy⁷ to calculate the far-field noise. Lyrantzis and Mankbadi⁸ have used Kirchhoff's method with LES to compute the far-field noise. Other numerical studies^{9,10} were then carried out by investigators at higher Reynolds numbers. In general, these have been found to be accurate, and in good agreement with experimental results. However, the aforementioned simulations lack the inclusion of a nozzle in the computational domain, which does not allow possible dipole contributions from the nozzle surfaces to be included. Instead, *ad hoc* inflow conditions which typically include random Gaussian or pipe flow simulation output data as forcing are specified to mimic the nozzle exit plume. Although the exclusion of the nozzle reduces computational costs, inflow forcing tends to result in

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higher noise levels in the far-field compared to experiments. The inclusion of the nozzles in LES simulations is rather recent as demonstrated in the work of Shur *et al.*¹¹ and Uzun and Hussaini.¹² The simulation results obtained following the inclusion of the nozzle does improve the far-field noise predictions but at the expense of a significant increase in computational cost. Even if the computational expense accrued with the addition of the nozzle is acceptable, the setup for these simulations includes the arduous task of body-fitted meshing for complex nozzle geometries. Thus, despite recent progress in computational aeroacoustics, detailed LES studies remain largely confined to academic jet configurations, for Reynolds number values that are low relative to that of the actual flows of interest. It is also worth noting that the computational cost is exacerbated for the case of low Mach number flows due to a smaller time step requirement. The only round jet-microjet combination using LES was a study performed recently by Huet *et al.*^{13,14} Huet *et al.* used a MILES approach for LES and studied both cold and hot jets at Mach 0.9. Source terms were included in the Navier-Stokes equations to mimic the effect of microjets. Their setup was similar to the experiments done by Castelain *et al.*³ Huet also studied the effect of continuous and pulsed microjets. Huet's numerical results under-predicted the potential core length by 30% and over-predicted the peak turbulence kinetic energy (TKE) also by 30% compared to experiments. Nonetheless, Huet showed the correct trends compared with experiments, i.e., the far-field sound decreased with the inclusion of microjets. Thus the work of Huet *et al.* highlights the challenges and difficulties of using current LES methodologies for microjet noise studies.

The main aim of this study is an investigation of the noise generated by a compressible turbulent round jet-microjet combination ($M_j = 0.5$) using the lattice-Boltzmann method (LBM). The LBM is chosen because of its robustness in handling complex geometries with minimal mesh generation efforts, and parallel computing scalability.

More details on the basics of the LBM are widely discussed in literature (cf. Chen and Doolen¹⁵ and Shan *et al.*¹⁶), and thus not repeated here. A commercial code, PowerFLOW 4.1c, based on the Lattice-Boltzmann kernel was utilized for this study.

II. COMPUTATIONAL SETUP

The D3Q19 lattice was used for the LBM discretization. The code includes an eddy viscosity turbulence model based on a Renormalization Group turbulence model. However, preliminary simulations with the turbulence model yielded a laminar jet, i.e., the plume exiting the pipe did not exhibit breakup beyond the potential core. Thus no turbulence model was used. The simulations were carried out using an under-resolved grid and no subgrid scale model. This procedure has been argued to be analogous to an LES.^{17–19}

The nozzle geometry considered is the circular jet SMC000 described in previous studies by Bridges and Brown.²⁰ The nozzle has a conical slope of 5° and the exit diameter is $D_j = 0.0508$ m. The addition of a nozzle in the computational domain was intended to eliminate the need

for an artificial forcing mechanism to trip the flow. Artificial forcing techniques used in many LES simulations (see Ref. 21) may cause spurious far-field SPLs. The centerline of the nozzle is along the x axis and y and z are lateral axes, respectively. A total of 18 equally spaced, azimuthal microjets with a diameter of $D_{mj} = 1$ mm were added to the nozzle. The number of microjets is similar to the number identified by Alkislar and Castelain as “optimum” for noise reduction. Each secondary nozzle has an injection angle of 45° with respect to the main jet axis centerline. It is important to note that implementing microjet nozzles in a typical finite difference based jet noise simulation is a very challenging task. In contrast, the LBM approach allowed the microjet geometry to be imported from CAD, and the discretization was relatively simple. Throughout this work, the impinging microjet combination case is referred to as the microjet case. There are no perfectly matched experimental data available at this time. Experimental data for similar microjet setups were performed for different flow conditions. Figure 1 shows the front diagonal view (with grid) of the impinging microjet combination used in this study.

The computational domain was partitioned into several variable resolution (VR) regions, in order to tailor the grid as needed to resolve the flow details, and reduce computational costs. This methodology is similar to grid stretching techniques typically employed in traditional computational fluid dynamics. Each grid cell is called a “voxel.” Hence, each VR region represents one grid resolution level and the VRs cascade outwards from the fine resolution region toward the coarse resolution region. The voxel cell size between each successive VR region differs by a factor of 2. The domain includes a total of 124×10^6 cells. The smallest cells, of size approximately 5×10^{-5} m (0.05 mm), are located in the region of the microjet. Based on the finest grid resolution, the total number of cells across each microjet is 20. The smallest cell size in the main nozzle shear layer is 0.2 mm. The voxel size corresponds to approximately $\Delta r/D_j \approx 0.001$ which is considered very coarse for wall bounded flow studies. The ratio needed to resolve the nozzle boundary layers is at least 1 order of magnitude smaller without the implementation of a wall model. This was deemed prohibitively

FIG. 1. (Color online) An isometric view of the grid setup. Every other cell is shown.

expensive. Although the adopted cell size did not resolve the boundary layer details, it was sufficiently small to supply physical jet inflow conditions without the need for artificial forcing techniques. Figure 1 shows two cut planes illustrating the grid used in this study. Note the fine resolution used in the vicinity of the microjet and jet nozzle shear layer.

III. RESULTS

The physical time scaling or time step for the LBM is 8.41×10^{-8} s. The simulation was evolved for a total of almost 1.3×10^6 time steps to achieve converged statistics. In terms of computational resources used to reach one million time steps, this test case took approximately one week of runtime using 128 processors in parallel on a Dell Xeon cluster (see the Acknowledgments).

A. Near-field flow variables

Figure 2 shows the instantaneous iso-surface streamwise velocity at 100 m/s. Note that the initial shear layers from the single round jet alone, i.e., Fig. 2(a) appears to be laminar, and then become turbulent after approximately one jet radius downstream of the exit. However, the microjet case clearly shows a turbulent shear layer at the jet nozzle exit due to the impingement of the microjets. Figure 3 shows the mean streamwise velocity contours at several streamwise locations downstream of the jets. The microjet case clearly

FIG. 2. (Color online) Instantaneous iso-surface streamwise velocity for the (a) circular jet and (b) microjet cases. Iso-surface velocity of 100 m/s.

FIG. 3. (Color online) Mean streamwise velocity contours at different streamwise stations for both cases. Top: Round jet, Bottom: Microjet.

shows a lobed pattern in the shear layer of the jet within the first two jet diameters downstream of the jet exit. After two jet diameters, the lobed pattern diffuses and then becomes axisymmetric. This observation is consistent with the observations of Alkislar⁴ who performed a microjet experiment for a Mach 0.9 jet.

Figure 4 shows the centerline decay of the mean streamwise velocity along the jet centerline. The round jet result is compared to the experiments of Bridges²² Mach 0.5 jet. The agreement between the round jet case and experiments of Bridges is good up to ten jet diameters. Beyond ten jet diameters, the numerical results start to decay slightly faster. The calculated potential core length for the circular jet is $x_c = 13.34 r_o$. The length of the potential core is defined as the location where the mean jet centerline velocity is reduced to 95% of the inflow jet velocity, $U_c(x_c) = 0.95 U_j$. The decay rate for the round jet is $C = (d/d(x/D_j))(U_o/U_c) = 0.15$, which is in good agreement with the experimental correlation of 0.16 by Zaman.²³ For the microjet case, there seems to be a shift in the streamwise direction. The decay

FIG. 4. (Color online) Mean streamwise velocity decay along the jet centerline.

for the microjet case is initiated slightly faster near the nozzle compared to the round jet but then decays at the same rate as the round jet at roughly $x = 20r_o$. Axisymmetry of the microjet case might have been starting from $x = 20r_o$ which is probably why the two curves coalesce. Huet *et al.*¹⁴ reported a similar behavior for their LES jet; although, their curve coalesces at $x = 16r_o$ for a Mach 0.9 jet. The addition of the microjets causes a shift in the potential core length to $x_c = 14.24r_o$ or about one jet radius longer compared to the round jet. An extension of the potential core length was also observed experimentally by Arakeri *et al.*²⁴ for a Mach 0.9 jet. They measured an extension of almost $3r_o$ although their mass flux ratio of the microjet to the round jet was 1%. The computed decay rate for the microjet case is $C = 0.14$.

Figure 5 shows the root-mean-square (rms) axial turbulent intensity, u'_{rms} , contours on the $z = 0$ plane. The effect of the microjets on the turbulence intensity is clearly seen throughout the flow field. The spread within the shear layer is reduced with the addition of the microjets. The reduction

is more pronounced within the first jet radius after the jet exit. Figure 6 shows the cross section of u'_{rms}/U_j , at $x = 1.5D_j$. Here the turbulence intensity of the microjet case is clearly reduced and the lobed pattern is seen compared to the baseline round jet case. The streamwise turbulence intensity along the jet centerline for both cases is plotted in Fig. 7. The experimental results are from Bridges²² for a round jet only with no impinging microjets. The comparison between the LBM round jet case and experiment is reasonable with the LBM slightly under-predicting the experiments. Nonetheless, there is qualitative agreement between the experiments and LBM. The effect of the microjets is shown to reduce the peak centerline axial turbulence intensity by as much as 6%. There is a cross-over point ($x/D_j \simeq 13$) at which the intensity of the microjet is that of the circular round jet. Figure 8 shows the streamwise turbulence intensity along the jet shear layer or nozzle lip line for the two cases. The peak reduction for u'_{rms} in the shear layer is more pronounced within the first five jet diameters downstream. There is also a cross-over point similar to the centerline

FIG. 5. (Color online) RMS contours of streamwise turbulence intensity u'_{rms}/U_j for the round jet and microjet case.

(a)

(b)

FIG. 6. (Color online) RMS contours of streamwise turbulence intensity u'_{rms}/U_j for the round jet and microjet case at $x = 1.5D_j$.

intensity seen in Fig. 7. The reduction in intensity when microjets are used (at least for this computational setup) is effective up to ten jet diameters. Figure 9 shows the same plot as in Fig. 8 but for $0 \leq x/D_j \leq 6$. Note the energy in the turbulence intensity at approximately $x = 0.2D_j$, which is

FIG. 7. (Color online) RMS u'_{rms}/U_j values along the jet centerline for both cases.

FIG. 8. (Color online) Axial turbulence intensity, u'_{rms}/U_j , plot along the jet lip line for both cases.

where the microjets impinge the shear layer of the main jet. The azimuthally averaged intensity of the spike is roughly $u'_{rms}/U_j = 0.07$ whereas for one microjet, i.e., with no azimuthal averaging, the “spike” intensity was $u'_{rms}/U_j = 0.22$. In Fig. 9 the turbulence intensity is slightly greater for the microjet case compared to the circular jet within the first jet diameter but then this trend is reversed further downstream. Alkisar⁴ reported a similar behavior for a Mach 0.9 jet. The microjets reduce the peak streamwise intensity along the nozzle lip line by approximately 15%.

Figure 10 shows the mean centerline turbulence kinetic energy (TKE), $q = (\overline{u'^2} + \overline{v'^2} + \overline{w'^2})/2$. As for the centerline axial turbulence intensity, the peak TKE value for the microjet case is reduced by roughly 10%. Figure 11 shows the mean TKE along the nozzle lip-line over an axial distance of up to six jet diameters downstream. Similar to Fig. 9, the TKE is greater compared to the baseline round jet case within the first jet diameter. Again significant energy is

FIG. 9. (Color online) Axial turbulence intensity, u'_{rms}/U_j , plot along the jet lip line for both cases. Plot is the same as Fig. 8 but for $0 \leq x/D_j \leq 6$.

FIG. 10. (Color online) Mean turbulent kinetic energy, q/U_j^2 , along the jet centerline.

noticed at $x=0.2D_j$ where the microjet impinges. Beyond one jet diameter the microjet TKE is lower. The peak reduction in TKE along the lip-line is more pronounced with a reduction of up to almost 30% for the microjet case. The experiments of Alkislar⁴ show a somewhat similar trend for Mach 0.9 jet experiments. Alkislar showed the maximum rms and TKE values at each Particle Image Velocimetry cross measurement plane. Hence, Alkislar showed that the rms and TKE values in the first diameter downstream of the jet were greater compared to the maximum value of the baseline case. Beyond one jet diameter, the peak TKE and rms intensities were lower for the microjet case compared to the baseline. So far the trends reported for the centerline and lip-line values are consistent with experimental observations.

B. Far-field acoustics

A modified porous Ffowcs Williams-Hawkins (FWH) surface integral acoustic method developed by Najafi-Yazdi *et al.*²⁵ was used to compute the far-field radiated sound

FIG. 11. (Color online) Mean turbulent kinetic energy, q/U_j^2 , plot along the jet lip line for both cases. Plot is the same as Fig. 10 but for $0 \leq x/D_j \leq 6$.

pressure. The modified FWH methodology, Formulation 1C, includes corrections for mean flow, moving sources, and observers. For simplicity, a continuous stationary control surface around the turbulent jet was used. For details regarding the numerical implementation of the FWH method, the reader is referred to Najafi-Yazdi *et al.*²⁵ Figure 12 shows the FWH control surface used in this study. The funnel-shaped control surface starts about one jet radius upstream of the nozzle exit and has an initial diameter of approximately $8r_o$. It extends streamwise until the near end of the physical domain at which point the diameter of the control surface is approximately $24r_o$. Hence, the total streamwise length of the control surface is $56r_o$. Results are shown for an open control surface, i.e., no surface is present at the end of the physical domain, i.e., $x=56r_o$. Flow field data were gathered on the control surface at every 50 time steps for over a period of 500 000 time steps. Based on the grid resolution around the control surface, and assuming that the LBM requires 12 cells per wavelength to accurately resolve an acoustic wave, the maximum frequency resolved corresponds to a Strouhal number of $Sr = fD_j/U_j \simeq 3$. The overall sound pressure levels (OASPLs) were computed along an arc with a distance of $R = 144r_o$ from the jet nozzle exit. The angle, Θ , was measured relative to the centerline jet axis.

Figure 13 shows the OASPL in the far-field for both jets. The OASPL trend for the round jet case is consistent with the experiments of Tanna *et al.*²⁶ for set point SP03 (herein referred to as Tanna). However, the LBM round jet result over-predicts Tanna's experimental result by about 1 dB for observation angles of 80° and lower. The overall far-field predicted SPL for the microjet case seems to be closer to the experimental results of Tanna. Nonetheless, the microjet case is lower by 1.5 dB than that for the baseline round jet case. The decrease in the far-field OASPL is consistent with the experimental data of Alkislar *et al.*⁴ and Castelain *et al.*³ Since the computed turbulence intensities and TKE showed a reduction in peak values, this resulted in a reduction in the far-field sound. Figures 14 and 15 show the one-third octave band spectra for observation angles $\Theta = 30^\circ$ and $\Theta = 60^\circ$, respectively. For Fig. 14, both cases over-predict Tanna's experimental results up to a Strouhal number of roughly $Sr = 1.2$ and then under-predicts the experimental data of Tanna. The round jet case is noisier

FIG. 12. (Color online) FWH control surface dimensions.

FIG. 13. (Color online) OASPL at $R = 144r_o$ for the jet centerline axis.

compared to the microjet case in the lower frequencies, i.e., up to $Sr = 2.25$ and crossovers beyond. A similar trend is observed for the observation angle $\Theta = 60^\circ$. The spectral shape for the circular jet case is in good agreement with experimental data. The cross-over trend was also reported by Alkislar *et al.*⁴ when they compared a Mach 0.9 round jet and a microjet configuration.

IV. CLOSING REMARKS AND FUTURE WORK

Numerical simulations of Mach 0.5 unheated jets with and without impinging microjets was performed using the LBM. A total of 18 equally spaced microjets each with a diameter of $D_{mj} = 1$ mm were used along a circumferential array. Each microjet had an impingement angle of 45° directed toward the jet centerline axis. The microjet case showed an increase of almost one jet radius in potential core length. This trend is consistent with the reported

FIG. 14. (Color online) One-third octave band spectra at $\Theta = 30^\circ$ and $R = 144r_o$ for both cases.

FIG. 15. (Color online) One-third octave band spectra at $\Theta = 60^\circ$ and $R = 144r_o$.

experimental observations for higher velocity jets. The peak centerline and lip line axial turbulence intensities for the microjet case showed a decrease of approximately 6% and 15%, respectively. The peak centerline and lip line turbulent kinetic energy also showed a trend similar to the axial turbulent intensities.

The far-field sound was computed using a modified porous FWH surface integral acoustic method for both cases. Due to the reduction in the turbulence intensities and turbulent kinetic energy, the far-field sound for the microjet case showed a decrease of approximately 1 dB compared to the baseline round jet case. The trends reported for the microjet case using LBM are consistent with experimental observations. However, further investigation is needed to better understand how the addition of microjets reduces the far-field sound. Noise source investigation methods such as Lighthill's^{7,27} acoustic analogy, possibly vortex sound theory,²⁸ and band-filtered near-field properties could be used.

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¹A. Krothapalli, L. Venkatakrishnan, L. Lourenco, B. Greska, and R. Elavarasan, "Turbulence and noise suppression of a high-speed jet but water injection," *J. Fluid Mech.* **491**, 131–151 (2003).

²V. Arakeri, A. Krothapalli, V. Siddavaram, M. B. Alkislar, and L. Lourenco, "Turbulence suppression in the noise producing region of a Mach 0.9 jet," *J. Fluid Mech.* **490**, 75–98 (2003).

- ³T. Castelain, M. Sunyach, D. Juve, and J. Bera, "Jet noise reduction by impinging microjets: An acoustic investigation testing microjet parameter," *AIAA J.* **46**, 1081–1087 (2008).
- ⁴M. Alkisar, "Aeroacoustics of a Mach 0.9 jet with Chevron-microjet combination," AIAA Paper No. 2008-3041 (2008).
- ⁵J. B. Freund, S. K. Lele, and P. Moin, "Direct numerical simulation of a Mach 1.92 turbulent jet and its sound field," *AIAA J.* **38**, 2023–2031 (2000).
- ⁶R. R. Mankbadi, M. E. Hayder, and L. A. Povinelli, "Structure of supersonic jet flow and its radiated sound," *AIAA J.* **32**, 897–906 (1994).
- ⁷M. J. Lighthill, "On the sound generated aerodynamically: I. General theory," *Proc. R. Soc. London, Ser. A* **211**, 564–587 (1952).
- ⁸A. S. Lyrintzis and R. R. Mankbadi, "Prediction of the far-field jet noise using Kirchhoff's formulation," *AIAA J.* **36**, 1–4 (1996).
- ⁹A. Uzun, G. A. Blaisdell, and A. S. Lyrintzis, "Coupling of integral acoustic methods with LES for jet noise prediction," *Int. J. Aeroacoust.* **3**, 297–346 (2005).
- ¹⁰C. Bogey and C. Bailly, "LES of high Reynolds, high subsonic jet: Effects of the inflow conditions on flow and noise," AIAA Paper No. 2003-3170 (2003).
- ¹¹M. L. Shur, P. R. Spalart, and M. K. Strelets, "Noise prediction for increasingly complex jets. Part I: Methods and tests," *Int. J. Aeroacoust.* **4**, 213–246 (2005).
- ¹²A. Uzun and M. Y. Hussaini, "High frequency noise generation in the near-nozzle region of a jet," AIAA Paper No. 2006-2499 (2006).
- ¹³M. Huet, F. Vuillot, and G. Rahier, "Numerical study of the influence of temperature and microjets on subsonic jet noise," AIAA Paper No. 2008-3029 (2008).
- ¹⁴M. Huet, B. Fayard, G. Rahier, and F. Vuillot, "Numerical investigation of the microjets efficiency for jet noise reduction," AIAA Paper No. 2009-3127 (2009).
- ¹⁵S. Chen and G. D. Doolen, "Lattice Boltzmann method for fluid flows," *Annu. Rev. Fluid Mech.* **30**, 329–364 (2003).
- ¹⁶X. Shan, X.-F. Yuan, and H. Chen, "Kinetic theory representation of hydrodynamics: A way beyond the Navier-Stokes equation," *J. Fluid Mech.* **550**, 413–441 (2006).
- ¹⁷C. Bogey and C. Bailly, "Computation of a high Reynolds number jet and its radiated noise using LES based on explicit filtering," *Comput. Fluids* **35**, 1344–1358 (2006).
- ¹⁸P. Lew, A. S. Lyrintzis, and L. Mongeau, "Noise prediction of a subsonic turbulent round jet using the Lattice-Boltzmann Method," *J. Acoust. Soc. Am.* **128**, 1118–1127 (2010).
- ¹⁹X. Nie, X. Shan, and H. Chen, "A Lattice-Boltzmann/finite-difference hybrid simulation of transonic flow," AIAA Paper No. 2009-0139 (2009).
- ²⁰J. Bridges and C. Brown, "Parametric testing of Chevrons on single flow hot jets," AIAA Paper No. 2004-2824 (2004).
- ²¹P. Lew, A. Uzun, G. Blaisdell, and A. Lyrintzis, "Effects of inflow forcing on jet noise using 3-D Large Eddy Simulation," AIAA Paper No. 2004-0516 (2004).
- ²²J. Bridges, "Effect of heat on space-time correlations in jets," AIAA Paper No. 2006-2534 (2006).
- ²³K. B. M. Q. Zaman, "Asymptotic spreading rate of initially compressible jets—Experiment and analysis," *Phys. Fluids A* **10**, 2652–2660 (1998).
- ²⁴V. Arakeri, A. Krothapalli, V. Siddavaram, M. B. Alkisar, and L. Lourenco, "Turbulence suppression in the noise producing region of a Mach = 0.9 jet," AIAA Paper No. 2002-2523 (2002).
- ²⁵A. Najafi-Yazdi, L. Mongeau, and G. Bres, "An acoustic analogy formulation for moving sources in uniformly moving media," *Proc. R. Soc. London, Ser. A* **467**, 144–165 (2011).
- ²⁶H. K. Tanna, P. D. Dean, and R. H. Burrin, "The generation and radiation of supersonic jet noise. Volume III: Turbulent mixing noise data," Technical Report AFAPL-TR-74-24, Lockheed-Georgia Company, Marietta, GA (1976).
- ²⁷M. J. Lighthill, "On the sound generated aerodynamically. II. Turbulence as a source of sound," *Proc. R. Soc. London, Ser. A* **222**, 1–32 (1954).
- ²⁸M. S. Howe, "Contributions to the theory of aerodynamic sound, with application to excess jet noise and the theory of the flute," *J. Fluid Mech.* **71**, 625–673 (1975).