Research on the Mechanical Properties of “Z” Type Double-Decker Ball Bearings

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The mechanical model of a “Z” type double-decker ball bearing under the action of radial load is established in this paper on the basis of the Hertz contact theory. According to the security contact angle theory, the influences of inner and outer bearings' internal clearances on the bearing's static load carrying capacity, radial deformation, radial stiffness, and load distribution of balls are analyzed. This model is verified in both stationary and rotational loading experiments. Moreover, the simulation results show that the static load carrying capacity of Z type bearing is smaller than that of either inner bearing or outer bearing that is contributed to compose the Z type bearing. The static load carrying capacity of a Z type bearing reduces with the increase of the inner and outer bearings' internal clearance. These simulation results also indicate that the contact angle of the maximum loaded ball in the outer bearing easily exceeds its security contact angle compared with the inner bearing, which, as the main factor, may cause the Z type bearing to overload and to fail. In this sense, the investigated Z type bearings are unfit to apply to situations with heavy load, high speed, or high precision. [DOI: 10.1115/1.4024844]

Keywords: Z type double-decker ball bearing, Hertz contact theory, security contact angle theory, radial deformation, radial stiffness, static load carrying capacity

1 Introduction

Rolling element bearings are widely used as basic and important mechanical components. With the development of rotating technology, bearings are expected to apply to high speed machines. However, conventional bearings' relatively low limited rotational speed probably could not satisfy this expectation. In order to settle this issue, a unique bearing consisting of one rolling element bearing and one hydrodynamic sliding bearing was proposed by Anderson [1]. Then, a double-decker high-precision bearing (DDHPB) and its basic concept as well as operating mechanism were developed by Prashad et al. [2–5]. The advantages of a DDHPB over a conventional bearing include the following aspects: higher limited rotational speed, higher damping, lower temperature rise, smaller wear, and longer life. Then a new type of double-decker ball bearing with a simpler structure was presented by Xu [6]. It was composed of two conventional bearings and was easier to be assembled than a DDHPB.

Two types of double-decker ball bearing structures are presented in Fig. 1: as “I” type double-decker ball bearing and a “Z” type double-decker ball bearing (ZTDBB). Both of them consist of two conventional ball bearings and an intermediate ring. In this paper, the smaller bearing is called the inner bearing and the bigger one is called the outer bearing. Moreover, in order to distinguish a double-decker ball bearing from a conventional bearing, each part of it is defined as inner ring, inner ball, middle ring, outer ball, and outer ring.

From the point of kinematics, a ZTDBB has a bigger ratio of the middle ring's rotational speed to the inner ring's than that of an I type bearing [7], which enables the ZTDBB to operate at higher speed. However, the ZTDBB would encounter a problem when a radial force acts on it. As the radial force acts on the inner ring, the reaction force generated on the outer ring is not in line with this radial force, which would produce an additional moment.

And this moment would cause reaction moments in both the inner and outer bearings. Due to these reaction moments, contact ellipses will be generated between the ball and raceways. And they may ride over the shoulders of the raceway grooves and cause damage to the ZTDBB. Most studies concentrated on the mechanical analysis of conventional bearing under radial and axial loads [8–14]. The equilibrium and associated load distribution in ball bearings loaded in five degrees of freedom (DOF) were analyzed by De Mul et al. [15]. Besides the influence of a moment, the effect of the ball centrifugal forces was also considered in their work. The ISO Technical Specification (ISO/TS 16281:2008(E)) [16] specifies the calculation methods on bearing internal load distribution concerning the analysis of bearings of different geometry or for more complex load cases. In Ref. [17], a deviated and inclined trace was observed on the outer ring when an axial load and relative inclination of the inner ring to the outer ring are applied together to a deep groove or angular contact ball bearing.

Based on the mechanical model of conventional bearing under the action of moment, the mechanical model of ZTDBB acted by radial load is first established in this paper according to the security contact angle theory [18]. In addition, four kinds of ZTDBB composed of inner and outer bearings with various internal clearances are fabricated in order to investigate the influences of the radial clearance on their static load carrying capacity, deformation, and stiffness. And the simulation results of the model are verified in relevant experiments on these ZTDBBs.

In Fig. 1, r_{o1} represents the outer radius of the inner bearing; r_{i2} represents the inner radius of the outer bearing; r_{i3} and r_{o3} denote the inner radius and outer radius of the middle ring; and w indicates the width of the middle ring.

2 Deformation Analysis of Deep Groove Ball Bearing

As seen in Fig. 2, a radial force F acts on a deep groove ball bearing at the point O_a . And the axial distance from point O_a to the bearing's center O is d_0 . The analysis of its force system indicates that both a force F' and a moment M_0 act on the bearing. Since the outer race of the bearing is installed into a bearing

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Fig. 1 Two structures of double-decker ball bearing

housing, the center of the outer race remains at rest. The force F' acting on the inner race enables the center of the inner race to move from point O to point O' . Meanwhile, the moment M_0 causes the inner and outer races to form an obliquity θ , which explains why the point O_a moves to point O'_a . Upon distributing the effects of F' and M_0 to all bearing balls, the resultant forces are achieved with various magnitudes and directions. In Fig. 2, r_m is the pitch radius of the ball bearing; δ_{rmax} , δ_{amax} denote the maximum radial and axial displacements of the inner race relative to the outer race; δ_d represents the displacement from point O_a to point O'_a ; α_i denotes the contact angle of the i th ball contacting with the inner race; and Q_i represents the action load from ball. The balls are ordered counterclockwise beginning with the one on the line of action of F .

The moment acted on inner race is

$$M_0 = F \cdot d_0 \quad (1)$$

θ can be estimated through the following equation:

$$\theta = \arctan(\delta_{amax}/r_m) \quad (2)$$

δ_d is given as

$$\delta_d = \delta_{rmax} + d_0 \tan \theta \quad (3)$$

To reduce the analysis complexity, the following major assumptions are made:

Fig. 2 Deformation of deep groove ball bearing

- (1) The deformation of the bearing rings is neglected and only elastic deformation associated with the concentrated contacts in the bearing is considered [15].
- (2) Ignoring the influences of centrifugal force and gyroscopic moment, the contact angle on inner race is the same as that on outer race.
- (3) Assume that there always exists one ball on the line of action of F' .

In order to explore the load distribution of balls in the bearing, the deflection of a single ball considering radial clearance is analyzed in the following part.

Take a radial cross section through the i th ball's center, as shown in Fig. 3; the curvature center of outer raceway $O_e(O'_e)$ remains unchanged during the deflection since the outer race is mounted into a bearing housing. The curvature center of inner raceway moves from O_i to O'_i . And the center of the i th ball moves from O_b to O'_b . Based on the positions of the i th ball center and the relevant curvature centers of inner and outer raceways, the following relationships can be obtained:

$$\delta_{ti} = \xi_2 \cos \alpha_i - \xi_1 \quad (4)$$

Fig. 3 Deflection of a single ball

$$\delta_{ai} = \xi_2 \sin \alpha_i \quad (5)$$

where, δ_{ri} , δ_{ai} are the radial and axial relative displacements between inner and outer races in the position of the i th ball. ξ_1 , ξ_2 represent the distances between the curvature center of inner race and that of outer race before and after deflection:

$$\xi_1 = qD_w - \frac{1}{2}G_r \quad (6)$$

$$\xi_2 = qD_w + \delta_i \quad (7)$$

where, $q = f_i + f_e - 1$; D_w represents the diameter of ball; G_r represents the radial clearance of the bearing; δ_i denotes the total elastic deflection of the i th ball; and f_i , f_e are coefficients of the raceway radius of curvature of inner and outer races.

Considering the relationship between the total deformation of the bearing and the radial and axial displacements of inner race relative to outer race, δ_{ri} , δ_{ai} can also be calculated through the following equations:

$$\delta_{ri} = \delta_{\max} \cos \psi_i \quad (8)$$

$$\delta_{ai} = \delta_{\max} \sin \psi_i \quad (9)$$

where ψ_i is the position angle of the i th ball, $\psi_i = (2\pi/Z)(i-1)$, $i = 1, 2, \dots, Z$; Z is the number of balls in the bearing.

Substituting Eqs. (4)–(7) into Eqs. (8) and (9), the contact angle is

$$\alpha_i = \arctan \left(\frac{\delta_{\max} \cos \psi_i}{qD_w + \delta_{\max} \cos \psi_i - \frac{1}{2}G_r} \right) \quad (10)$$

And the total elastic deflection of the i th ball can be calculated through the following equation:

$$\delta_i = \frac{qD_w + \delta_{\max} \cos \psi_i - \frac{1}{2}G_r}{\cos \alpha_i} - qD_w \quad (11)$$

Negative values here mean loss of contact. The ball contact load Q_i is calculated using the Hertz contact theory [19]:

$$Q_i = \left[\frac{\delta_i}{K} \right]^{3/2} \quad (12)$$

where K is the constant of elastic deflection, which can be determined by the geometric and material parameters of the bearing.

According to Ref. [16], the force and moment equilibrium equations of the inner race are established:

$$\begin{cases} \sum_{i=1}^Z Q_i \cos \alpha_i \cos \psi_i = F \\ \sum_{i=1}^Z Q_i \sin \alpha_i r_m |\cos \psi_i| = M_0 \end{cases} \quad (13)$$

Equation (13) with absolute value indicates that the moment M_0 is reacted by all the balls in the bearing. $\delta_{\max}$ and $\delta_{\min}$ are taken as the two unknowns. Then, the contact angle α_i and deflection δ_i of each ball are obtained through Eqs. (10) and (11). In addition, the least-squares method is used to solve the equilibrium Eq. (13) when F , d_0 , and bearing parameters are given.

3 Security Contact Angle

Contact angles would be formed between the ball and the inner and outer races when a combined load acts on the deep groove

Fig. 4 Security contact angle of ball

ball bearing. As seen in Fig. 4 taking a radial cross section through the ball's center, the contact ellipse would be cut off when it overflows the raceway shoulder [18]. Thus, it would result in stress concentration on the position where the ball contacts with the shoulder. And the ball would probably be damaged. In this sense, a security contact angle is required to prevent this kind of failure. The security contact angle refers to the ultimate contact angle where the contact ellipse does not overflow the raceway shoulder. Meanwhile, the maximum static load carrying capacity of the ZTDBB is obtained. In Fig. 4, A is the intersection of the raceway of outer race and its shoulder; O denotes the center of ball; D_i , D_e are the radius of the bottom of the raceways of the inner and outer races; D_{is} , D_{es} are the radius of the shoulders of inner and outer races; and a , b represent the semimajor axis and semiminor axis of contact ellipse.

The operating contact angle of the ball should satisfy the following inequation:

$$\alpha_0 \leq \gamma - \beta \quad (14)$$

where, γ denotes the angle between line AO and the radial plane through the center of the ball when the ball contacts with the inner and outer races simultaneously:

$$\cos \gamma = 1 - \frac{D_e - D_{es}}{D_w} \quad (15)$$

β is the angle of semimajor axis relative to the ball center:

$$\sin \beta = \frac{2a}{D_w} = \frac{2m_a}{D_w} \sqrt{\frac{3Q}{2\sum \rho} \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)} \quad (16)$$

Therefore,

$$\alpha_0 \leq \gamma - \arcsin \left[\frac{2m_a}{D_w} \sqrt{\frac{3Q}{2\sum \rho} \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)} \right] \quad (17)$$

where, Q is the load acted on the ball; $\sum \rho$ is the curvature sum of the ball and the inner race; E_1 , E_2 are the modulus of elasticity of the ball and the inner race; ν_1 , ν_2 are the Poisson's ratio of the ball and the inner race; and m_a is the parameter related to elliptic eccentricity:

$$m_a = \sqrt{\frac{2E(\chi) a^2}{\pi b^2}} \quad (18)$$

where $E(\chi)$ is the complete elliptic integral of the second kind [16].

From inequation (17), α_0 decreases with the increase of Q . Hence, the contact angle of the maximum loaded ball is required to be smaller than its security contact angle. Otherwise, according

Fig. 5 Deformation of ZTDBB

to Ref. [17], the contact ellipse of this ball would overflow the shoulder of the inner or outer race and this ball could be damaged.

To sum, the contact angle and the load of each ball can be obtained through Eqs. (10)–(13). And on the basis of inequation (17), the maximum force can be achieved when its force arm is d_0 .

4 Mechanical Model of ZTDBB

The inner ring of ZTDBB is mounted to a rotor and the outer ring is installed into a bearing housing when ZTDBB works. As shown in Fig. 5, a y -direction force F_r acts on the inner ring of ZTDBB at point O_1 . The analysis of this force system indicates that ZTDBB is acted by both a force F_r and an x -direction moment M centered at point O_2 . Since the θ -DOF and γ -DOF of inner and outer rings are constrained by the rotor and the bearing housing, the force F_r and the moment M are reacted by all the balls in both inner and outer bearings. In Fig. 5, d represents the axial distance from point O_1 to O_2 ; r_{m1} , r_{m2} denote the pitch radius of the inner and outer bearings; Q_j , Q_k represent the loads acted on middle ring from inner and outer balls; α_j , α_k represent the contact angles of inner and outer balls; and δ_z is the displacement of inner ring, which also represents the deformation of ZTDBB. Both the inner and outer balls are ordered counterclockwise beginning with the one that lies on the line of action of F_r .

According to the mechanical model of deep groove ball bearing mentioned in Sec. 2, the force and moment equilibrium equations of middle ring in ZTDBB can be established:

$$\left\{ \begin{array}{l} \sum_{j=1}^{Z_1} Q_j \cos \alpha_j \cos \psi_j = F_r \\ \sum_{k=1}^{Z_2} Q_k \cos \alpha_k \cos \psi_k = F_r \\ \sum_{j=1}^{Z_1} Q_j \sin \alpha_j \cos \psi_j + \sum_{k=1}^{Z_2} Q_k \sin \alpha_k \cos \psi_k = 0 \\ \sum_{j=1}^{Z_1} Q_j (r_{m1} \sin \alpha_j |\cos \psi_j| - d \cos \alpha_j \cos \psi_j) \\ + \sum_{k=1}^{Z_2} Q_k r_{m2} \sin \alpha_k |\cos \psi_k| = M \end{array} \right. \quad (19)$$

where ψ_j , ψ_k are the position angles of inner and outer balls:

$\psi_j = 2\pi(j-1)/Z_1$, $j = 1, 2, \dots, Z_1$; $\psi_k = 2\pi(k-1)/Z_2$, $k = 1, 2, \dots, Z_2$; Z_1 , Z_2 are the number of balls in inner and outer bearings.

According to Eqs. (2) and (3), the deformation of ZTDBB can be obtained:

$$\delta_z = \delta_{rmax1} + \delta_{rmax2} + d \frac{\delta_{amax1}}{r_{m1}} \quad (20)$$

or

$$\delta_z = \delta_{rmax1} + \delta_{rmax2} + d \frac{\delta_{amax2}}{r_{m2}} \quad (21)$$

where, δ_{amax1} , δ_{amax2} are the maximum axial displacements between the middle ring and the inner and outer rings; and δ_{rmax1} , δ_{rmax2} are the maximum radial displacements between the middle ring and the inner and outer rings.

On the basis of security contact theory, the maximum static load carrying capacity and the load distribution of balls in both the inner and outer bearings are simulated. The method adopted to solve the equilibrium Eq. (19) is consistent with the method to solve the Eq. (13). And the four unknowns are δ_{rmax} and δ_{amax} of inner and outer bearings, respectively. The deformation of ZTDBB is calculated through Eqs. (20) or (21).

5 Simulation Results

Based on the theoretical derivation above, a corresponding calculation program is developed for ZTDBB. The detailed parameters of the inner and outer bearings are shown in Table 1. The parameters of the middle ring are shown in Table 2. In addition, the axial distance $d = 15.5$ mm.

In order to analyze the influences of the inner and outer bearings' radial clearance on the static load carrying capacity of a ZTDBB, load distribution of balls, radial deformation, and radial stiffness of a ZTDBB, four ZTDBB models consisting of inner and outer bearings are simulated. In these models, different radial clearance levels are chosen and permuted for inner and outer bearings. In Table 3, the mean value of each clearance level in ISO 5753-1991 is selected as the simulation parameter of bearing radial clearance. For example, the range clearance values of CN of the inner ball bearing in group A is from $5 \mu\text{m}$ to $20 \mu\text{m}$, then the mean value $12.5 \mu\text{m}$ is used in simulation. In addition, the maximum static load-carrying capacity F_{max} of each group is also given in Table 3.

The load distributions of the balls in these four ZTDBBs' inner and outer bearings under the action of F_{max} are displayed in Figs. 6(a) and 6(b).

According to Table 1, Table 3, and Fig. 6, following conclusions can be drawn:

- (1) The static load carrying capacity of a ZTDBB is relatively small for this specific geometry. From Table 1 and Table 3, its static load carrying capacity is smaller than that of either inner bearing or outer bearing, which is one part to compose it. Take the group A in Table 3 as an example; its static load carrying capacity is 19.8% of the inner bearing's static load rating and 14.6% of the outer bearing's static load rating. This is because the effect of its original force system is equivalent to that of both a radial force and a moment when a radial force acts on it. Thus, contact angles of inner and outer balls could easily overflow their security contact angles which cause ZTDBB to failure.
- (2) As shown in Table 3, with the increase of the radial clearance of inner or outer bearing, the static load carrying capacity of the ZTDBB decreases. The analysis of the inner bearing can be used to illustrate this point. On the one hand, the decrease of angle γ would lead to the reduction of a single ball's load-carrying capacity. On the other hand,

Table 1 Parameters of inner and outer bearings

Parameters	Inner bearing	Outer bearing
Z	12	14
D_w/mm	5	5.556
r_m/mm	16.75	22.5
C_0/kN	7.0	9.5
D_{is}/mm	30.2	41.1
D_{es}/mm	36.8	48.9
f_i		0.515
f_e		0.525
E/GPa		207
ν		0.3

Table 2 Parameters of middle ring

Parameters	Middle ring	Parameters	Middle ring	Parameters	Middle ring
r_{i3}/mm	15	r_{i2}/mm	17.5	w/mm	25
r_{o1}/mm	21	r_{o3}/mm	25		

with the increase of clearances, the initial contact angles [16] of balls increase. Thus, the number of unloaded balls increases. This analysis could also be applied to the outer bearing.

- (3) The balls that are opposite to the radial force direction may be squeezed and loaded. This is because there exists an obliquity deformation between the inner and outer rings in ZTDBB besides radial deformation, which is verified by group C in Fig. 6(a) and group B in Fig. 6(b).
- (4) The maximum loaded ball in the outer bearing is the one whose contact angle reaches its security contact angle first. This is obtained from the simulation results of the four ZTDBB models. Due to the fact that the pitch diameter of the outer bearing is bigger than that of the inner bearing, the maximum axial deformation between the outer and middle rings is bigger than that between the inner and middle rings on the basis of the same obliquity deformation. So the main reason for the failure of the investigated ZTDBBs is that the contact angles of the maximum loaded balls in the outer bearings exceed their security contact angles.

The radial deformations of the four ZTDBB models under various radial loads are simulated and shown in Fig. 7.

According to the radial deformations in Fig. 7, the radial stiffness of the four ZTDBBs under various radial loads (seen in Fig. 8) can be obtained using following equation:

$$K_r = \frac{\delta F_r}{\delta z} \quad (22)$$

As seen in Figs. 7 and 8:

- (1) Both the radial deformation and stiffness of the ZTDBBs increase with the increase of the radial load.

Table 3 Parameters of bearing clearances and maximum static load carrying capacity

Group	Inner bearing		Outer bearing		$F_{\max}/\text{N}$
	Clearance level	Value/mm	Clearance level	Value/mm	
A	CN	12.5	CN	13	1385
B	C3	20.5	CN	13	1384
C	CN	12.5	C3	24	1295
D	C3	20.5	C3	24	1182

(a) Load distributions of inner balls

(b) Load distributions of outer balls

Fig. 6 Load distributions of balls

- (2) The radial deformation increases with the increase of the inner and outer bearings' clearances. However, the radial stiffness hasn't defined correspondence with the bearings' clearances.
- (3) Under a 200 N load, the stiffness of the four ZTDBBs is relatively small and obviously deviate from the approximately linear radial load-stiffness relationships that are indicated from the results under other loads. This can be illustrated as follows. In the early stage of the ZTDBB's deformation, deformation generates when the inner and outer bearings achieve their initial contact angle. And in this stage, the reaction force of the balls is very small. Upon exceeding the initial contact angles, the reaction force becomes larger. And the situation under the 200 N load belongs to the early deformation stage. Accordingly, the radial stiffness in this stage is relatively small.

Fig. 7 Radial deformations under various radial loads

Fig. 8 Radial stiffness under various radial loads

6 Experimental Research

As shown in Fig. 9, a test rig is developed to verify the theoretical analysis and simulation results in this paper. In this test rig, the rotor is supported by a conventional ball bearing at one end and the tested ZTDBBs at the other end. The loading component is installed near the ZTDBB to load various radial forces on the rotor. And at point B of the rotor where it is very close to the ZTDBB,

four displacement sensors are arranged to measure the radial displacement of the rotor. The external radial force F_r is equivalent to act at point D. And the reaction forces are regarded to support the rotor at point A and point C. l_a , l_c are the distances between the points mentioned above.

According to the principle of leverage, the radial force acted on the tested ZTDBBs can be obtained:

$$F_c = \frac{l_a}{l_a + l_c} F_r \quad (23)$$

Considering the stiffness of the rotor is much greater than that of the conventional bearing and the ZTDBBs, the flexural deformation of the rotor can be neglected. And due to the fact that the displacement sensors are arranged very closely to the ZTDBB, the deformation of ZTDBB (δ_z) can be considered to equal the rotor's displacement measured by the sensors.

The measurement system shown in Fig. 10 consists of four displacement sensors, a sensor driver board, a data collection card, and a computer. A frequency converter is used to supply a rotational torque for the motor. The sensitivity of the sensor is 10 mv/ μ m.

The inner bearings and outer bearings used to compose the four ZTDBBs in the experiments are all manufactured by the HRB, China. Their clearances are in accordance with the International Standard No. ISO 5753:1991 MOD and are consistent with the clearance level adopted in the previous simulation.

Fig. 9 Structure of test rig

Fig. 10 Experimental facility

Fig. 11 Experimental results of radial deformations under various radial loads

The experiments can be divided into two parts: In the first one, the experiment is conducted when the rotor is static. A radial force acted on the rotor through the loading component is measured by a spring dynamometer. The relationships between radial deformations of the four ZTDBBs and radial loads are shown in Fig. 11. In the second one, the ZTDBB with group A clearances in Table 3 is tested at 50 Hz rotational frequency. Both a 0 N and a 200 N external centrifugal force (F_r) act on the rotor. The 0 N external centrifugal force represents that the rotor is only stimulated by its inherent unbalance. And the 200 N external centrifugal force is loaded through setting unbalance on the loading component. The axis orbits are shown in Figs. 12(a) and 12(b) with $F_r = 0$ N and $F_r = 200$ N.

According to the radial deformations in Fig. 11 and Eq. (22), the radial stiffness can be obtained, as seen in Fig. 13.

As seen from Figs. 7, 8, 11, and 13, the simulation results keep a nearly consistent trend with the experiments results. Meanwhile, deviation exists and it may be caused by following factors: (1) In experiments, the clearances of the tested bearings are in accordance with the International Standard No. ISO 5753:1991, in which a certain clearance level indicates a range of clearance values. In the simulation, the mean value of each clearance level in ISO 5753:1991 is selected as the simulated bearing radial clearance. So, the true clearances in the experiments might be greater than the clearances of the bearings chosen in simulation, which probably led to the deviations between the results of the experiments and those of the simulation; (2) the calibration errors of the displacement sensors, which would make the measured voltage value become larger than the true value; (3) some bearing parameters

Fig. 12 Rotor orbits

Fig. 13 Experimental results of radial stiffness under various radial loads

such as K , E , and ν are obtained through calculation. And these parameters may not be exactly consistent with the parameters in real bearings; and (4) other errors exist in the experiments.

The displacement of the rotor deviated from its initial center in Fig. 12(b) corresponds with the deformation of group A in Fig. 11 under a radial load $F_r = 200$ N. From Fig. 12, the rotor's inherent unbalance or external force would result in deformation in the ZTDBB. Notably, seen from Fig. 12(a), the rotor's inherent unbalance itself could lead to large deformation. In this sense, the operating ZTDBB-supported rotor not only rotates by itself but also revolves around the center of the device. Thus, the ZTDBB would probably be overloaded to fail when operating at a high rotational speed or under a heavy load. To sum, the investigated ZTDBBs are unfit to apply to situations with heavy load, high speed, or high precision.

7 Conclusions

The mechanical properties of ZTDBBs are analyzed based on the Hertz contact theory and the security contact angle theory. And stationary and rotational loading experiments on ZTDBBs are conducted to verify the simulation results. For the investigated ZTDBBs, the main conclusions are as follows:

- (1) The static load carrying capacity of the ZTDBB reduces significantly compared with that of either inner bearing or outer bearing, which is one part that composes the ZTDBB. In general, the main factor contributed to the risk of failure of the ZTDBB is that the contact angle of the maximum loaded ball in the outer bearing exceeds its security contact angle.
- (2) The static load carrying capacity of the ZTDBB decreases with the increase of radial clearance of the inner or outer bearing.
- (3) The investigated ZTDBBs are unfit to apply in situations with heavy load, high speed, or high precision.

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Nomenclature

- a = semimajor of contact ellipse, mm
- b = semiminor of contact ellipse, mm
- C_0 = basic static radial load rating, kN
- d = axial distance from O_1 to O_2 , mm
- d_0 = axial distance from O_a to O , mm
- D_e = radius of the bottom of the raceway of outer race, mm
- D_{es} = radius of the shoulder of outer race, mm
- D_i = radius of the bottom of the raceway of inner race, mm
- D_{is} = radius of the shoulder of inner race, mm
- D_w = diameter of ball, mm
- E = modulus of elasticity, in GPa

$E(\chi)$ = complete elliptic integral of the second kind
 f_e = coefficient of the raceway radius of curvature of inner race
 f_i = coefficient of the raceway radius of curvature of inner race
 F = load on conventional deep groove ball bearing, N
 F_r = load on ZTDBB, N
 K = the constant of elastic deflection, in millimeter multiplied Newtons power minus two-thirds
 K_r = radial stiffness of ZTDBB, in megaNewtons multiplied meters power minus one
 Q = reaction load from ball, N
 r_i = inner radius, mm
 r_m = pitch radius of the bearing, mm
 r_o = outer radius, mm
 w = width of middle ring, mm
 Z = number of balls
 α = contact angle of ball, deg
 β = angle of semimajor axis relative to the ball center, deg
 δ = total elastic deflection of the i th ball, mm
 δ_d = displacement from O_a to O_a' , mm
 δ_{ai} = axial displacement in the position of the i th ball, mm
 δ_{ri} = radial displacement in the position of the i th ball, mm
 δ_{amax} = maximum axial displacement, mm
 δ_{rmax} = maximum radial displacement, mm
 δ_z = deformation of ZTDBB, mm
 ν = Poisson's ratio
 ξ_1 = two curvature center's distances before deflection, mm
 ξ_2 = two curvature center's distances after deflection, mm
 θ = the obliquity between inner and outer race, deg
 γ = angle between line from the shoulder to the ball center and the radial plane, deg
 ψ = position angle of ball, deg
 $\sum \rho$ = curvature sum, in reciprocal millimeters

Subscripts

i = i th ball of conventional deep groove ball bearing
 j = j th all of inner bearing of ZTDBB
 k = k th ball of inner bearing of ZTDBB
 1 = inner bearing of ZTDBB
 2 = outer bearing of ZTDBB
 3 = middle ring of ZTDBB

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