TECHNICAL REPORT

A mathematical model relating changes of grey values to changes of thicknesses of a stepwedge

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The purpose of this report is to present a mathematical model relating changes in grey values to changes of thicknesses of a stepwedge. Radiographs of a stepwedge were obtained through a limited exposure range on a single Kodak 6100 charge-coupled device detector at 63 kVp and 70 kVp. Grey values from each step were evaluated relative to the corresponding step thickness. All possible regression fits were evaluated based on their coefficient of determination (R^2) value and their mean squared errors (MSEs) relative to the original thickness. For all exposure settings, the fifth-degree polynomial was the best possible fit, with the highest R^2 value and the lowest MSE.

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Introduction

Image processing, and post-processing in general, refers to a broad class of algorithms for modification and analysis of an image. The principal objective of image enhancement is to modify attributes of an image to make it fit for a given task and a specific observer. During this process, one or more characteristics of the image are changed. The choice of characteristics and the way they are changed are specific to a certain task.¹⁻⁴

The most common image enhancement algorithms are based on linear adjustments and non-linear adjustments. Linear adjustments work by adding or subtracting grey values to each pixel of the image (image brightness) or by changing the relative difference between the pixels relative to an arbitrary grey value (image contrast). Non-linear histogram adjustments use exponential and logarithmic intensity modifications to increase the relative difference between the grey values of the pixels (gamma correction).^{1,4-9} Other algorithms are also used: histogram equalization is used to redistribute grey values in a more uniform way; image sharpening utilizes the image high and low frequency to

smooth and sharpen the edges of the image. These algorithms are used to illustrate certain properties of the image more efficiently.^{1,2,5,10,11}

To our knowledge, there is no algorithm based on a specific mathematical model relating the changes of grey values to changes of absorber thicknesses. The objective of this study was to identify a specific model that best fits the relationship between the changes in grey value and the corresponding changes in thickness of an absorber.

Materials and methods

Test objects and test images

The test object was a ten-step aluminium stepwedge with a maximum thickness of 12.5 mm. The size 2 charge-coupled device sensor used was the 8-bit Kodak 6100 (PracticeWorks Systems, Atlanta, GA). The radiographs were exposed with the Planmeca Intra (Planmeca Oy, Helsinki, Finland) intraoral X-ray machine operating at 63 kV, 70 kVp and 8 mA, with exposure time varying from 0.05 s to 0.2 s (a total of 14 exposures). The distance between source and receptor was fixed at 12 inches. The images were transferred as 8-bit tagged image file format files to a personal

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Figure 1 Plot of grey values relative to the step thickness

computer with a Microsoft® Windows® 7 (Microsoft Corporation, Seattle, WA) operating system.

The software used for image analysis was ImageJ® (National Institute of Health, Bethesda, MD)¹² and Microsoft® Excel® (Seattle, WA). ImageJ® provided the numerical values of the histograms from the stepwedge and Microsoft® Excel® provided the regression formulas.

Building the regression formulas

Images were opened in ImageJ®, and from each image the histogram of a 100 × 300 pixel region of interest from each step was evaluated, and the mean grey value from this histogram was exported to Microsoft® Excel® and plotted against the thickness of its corresponding step. This generated a curve representing the change in grey values relative to the increasing step thickness (Figure 1).

All possible regression formulas available in Microsoft® Excel® along with their coefficient of determination (R^2) values were applied to that curve [*i.e.* linear, exponential, power (a^n), logarithmic and up to the sixth-degree polynomials].

Computation of the thickness values

The “Solver” add-in from Excel® provides a reverse calculation, enabling the calculation of a thickness based on its corresponding grey value on the image.

This was applied to all grey values for each and every image based on the corresponding regression formula. The thickness values were then imported and displayed as images in ImageJ®.

Results

Comparing estimated thicknesses to actual thicknesses

The new grey values from the enhanced images represent the calculated thicknesses. The mean calculated thickness from each step on the image was compared with the original thickness of the corresponding step on the stepwedge. The mean squared error (MSE) from each regression model was also evaluated (Table 1). Figure 2 shows the plot of the calculated thicknesses for each regression fit. The linear, exponential and logarithmic fits had an R^2 value less than 0.9 and were disregarded. The power fit had an MSE of 48.64 and therefore was disregarded. The second-degree polynomial fit had an MSE less than 1, but it was larger than the MSE of the third- and fourth-degree polynomial fits, which in turn were larger than the fifth- and sixth-degree polynomial fits. Since the fifth-degree polynomial had an R^2 value of 0.9999 and an MSE of 0.001, it was chosen as the best fit relating the change in the grey values relative to the change in thickness of the stepwedge.

Table 1 Comparison of coefficient of determination (R^2) value and mean squared error (MSE) of all possible regression formulas

	Linear	Exp	Power	Log	Second polynomial	Third polynomial	Fourth polynomial	Fifth polynomial	Sixth polynomial
R^2 value	0.8780	0.5484	0.949	0.6346	0.9901	0.9991	0.9999	0.9999	0.9999
MSE	1.2961	4.6362	48.64	31186	0.37638	0.0400	0.0658	0.0010	0.0011

Figure 2 Calculated thicknesses from all regression fits

Figure 3 Comparison of plot profiles

Figure 4 Images of the stepwedge before and after enhancement

Image enhancement based on the fifth-degree polynomial fit

The plot profile of the grey values from all original images was compared with the plot profile of the enhanced images based on the fifth-degree regression fit. Figure 3 shows that while the plot profile from the original image showed a high difference in densities at the smaller thickness steps and a low difference in density at the higher thickness steps, the plot profile of the enhanced images showed a constant difference in densities for all the steps. Visually, the enhanced images appeared to have more noise than the original images (Figure 4).

Discussion

The polynomial fits less than the fifth-degree polynomials were eliminated because of the relatively larger difference between the original thickness value and the estimated thickness value on any step of the stepwedge. While the fifth-degree polynomial equations have five possible solutions (roots), the only root calculated by the Solver add-on from Excel was within

the range of the stepwedge thickness (*i.e.* between 0 and 13 mm).

The reverse calculation changed the difference in densities between the steps from an irregular difference to a somewhat regular difference; thus, the contrast represented the change in thickness rather than the change in X-ray attenuation. As detection of most common dental lesions (caries, periapical periodontitis and periodontal bone loss) is based on contrast perception,^{13–15} it is expected that having a linear thickness-based contrast might enable better detection of such lesions.

This also resulted in an apparent increase in the noise in the image as the algorithm's purpose was only to recalculate all grey values from each pixel.

Conclusion

This report presented a new algorithm for image enhancement based on a fifth-degree polynomial regression fit. Further studies are required to evaluate the performance of this algorithm and its clinical implications, mainly in detecting caries, periapical lesions and periodontal bone loss.

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