Viscous effects in a static physical model of the uniform glottis

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The classic work on laryngeal flow resistance by van den Berg *et al.* [J. Acoust. Soc. Am. **29**, 626–631 (1957)] is revisited. These authors used a formula to summarize their measurements, and thus they separated the effects of entrance loss and pressure recovery from those of viscosity within the glottis. Analysis of intraglottal pressure distributions obtained from the physical model M5 [R. Scherer *et al.*, J. Acoust. Soc. Am. **109**, 1616–1630 (2001)] reveals substantial regions within the glottis where the pressure gradient is almost constant for glottal diameters from 0.005 to 0.16 cm, as expected when viscous effects dominate the flow resistance of a narrow channel. For this set of glottal diameters, the part of the pressure gradient that has a linear dependence on the glottal volume velocity is isolated. The inverse cube diameter of the Poiseuille expression for glottal flows is examined with the data set provided by the M5 intraglottal pressure distributions. The Poiseuille effect is found to give a reasonable account of viscous effects in the diameter interval from 0.0075 to 0.02 cm, but an inverse 2.59 power law gives a closer fit across all diameters.

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I. INTRODUCTION

It has long been recognized that reliable information about glottal pressure-flow relationships and about their dependence on the parameters of glottal geometry is a crucial ingredient for the understanding of phonation. One of the early papers in the development of this subject is that by van den Berg *et al.*,¹ who constructed a physical model of the human larynx with a uniform glottis made to scale from dental material. They measured pressures at the locations of four pressure taps, one at the glottal entrance, one at the glottal exit, and two inside the glottis, and their work established the general characteristics of intraglottal and subglottal pressures. They presented a formula that summarized the results of their experiments and did not present the related data. The essential features of their formula may be conveyed in the general form^{2,3} for the transglottal pressure drop ΔP_{tr} , that is

$$\Delta P_{tr} = \frac{(k_{ent} - k_{ex}) \rho U_g^2}{2 l_g^2 d^2} + 12 \frac{\mu \Delta x}{l_g d^3} U_g, \quad (1)$$

where ρ and μ are the density and viscosity^{4,5} of the air, respectively, U_g is the glottal volume flow rate, and l_g and d are the glottal anterior–posterior length and diameter, respectively. In the original work of van den Berg *et al.*, the quantity Δx represented the glottal thickness, the axial inferior–superior length of the glottis. Below it will play a

slightly different role, namely, the axial distance interval for which the dependence of the intraglottal pressure on this distance is approximately linear. In many cases this will be the distance between taps six and 10 in Fig. 1 below. The pressure coefficient k_{ent} describes the effects responsible for the loss of energy at the glottal entrance,^{1–4,6,7} and the exit coefficient k_{ex} describes the effects of pressure recovery.^{1–3,6,7}

van den Berg *et al.* summarized entrance losses with a coefficient of 1.375 and chose the value $k_{ex} = 0.5$ to describe the effects of pressure recovery. They stated that the formula of Eq. (1) reproduced their measurements of the flow resistance (ratio of ΔP_{tr} to U_g) to a precision of 10% for glottal diameters between 0.01 and 0.2 cm. The last term of Eq. (1) with its linear dependence on the volume velocity U_g and inverse cube dependence on the glottal diameter d is that expected if Poiseuille's formula⁵ is a valid description of viscous effects within the glottis. The validity of Poiseuille's formula assumes that the airflow within the glottis is laminar and fully developed into a parabolic velocity profile across the glottis. In their seminal work on the two-mass model of the vocal folds, Ishizaka and Flanagan^{2,3} adopted the value $k_{ent} = 1.375$, but replaced the generic value of k_{ex} by a formula derived from momentum conservation at the glottal exit. This formula involves a ratio of the area at the glottal exit to that of the hypolarynx and thus depends on the glottal diameter d . Its values are considerably smaller than 0.5 for realistic values of d , and it does not depend upon the glottal flow rate U_g . Ishizaka and Flanagan³ also chose the inverse cube dependence of the second term of Eq. (1) to represent viscous effects within the glottis.

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FIG. 1. (Color online) Schematic diagram of the medial surfaces of the uniform glottis and the subglottal and supraglottal regions used for the experiments with model M5 for a glottal diameter $d = 0.04$ cm. These surfaces define the constriction between the sections of the wind tunnel that represent the trachea and the vocal tract of model M5. Locations of the 16 pressure taps are shown. Tap six lies at the glottal entrance and taps 13 and 14 lie on the superior surface of the vocal folds near the glottal exit. The broken line between taps 15 and 16 indicates that tap 16 is located much farther downstream than the diagram would indicate (at an axial distance of 3.56 cm). The radii of the rounded surfaces near the glottal entrance and the glottal exit¹³ were chosen to represent those of the normal male larynx as accurately as possible.

Another phase in the experimental verification of aerodynamic formulas such as Eq. (1) began with the construction of two polyester resin models MI and MII by Scherer *et al.*⁸ These early models had rectangular glottal shapes and fixed minimal glottal diameters of 0.104 and 0.158 cm, respectively, and they were constructed to examine the existing pressure-flow relations. The laryngeal models of Gauffin *et al.*⁹ and of Scherer and Titze¹⁰ were two of the earlier models to use the principle of dynamic similitude in fluid mechanics⁴ to collect pressure and flow data with scaled-up models (by a factor of 5) that could be interpreted as data for the human larynx. In model M5, the scale factor was increased to 7.5, which allowed the introduction of more pressure taps along the vocal fold surfaces.^{11–16} A schematic diagram of the medial surfaces of the Plexiglas vocal fold inserts used to define the glottal constriction of M5^{13,14,16} is shown in Fig. 1. It specifies the locations of 16 pressure taps along one side of the glottis. The glottal entrance is located at tap 6, and the glottal exit plane is located along the superior vocal fold surface where taps 13 and 14 are placed. The first 5 taps are located along the inferior vocal fold surface, and taps 15 and 16 are in the vocal tract part of the wind tunnel. The glottal length l_g is 1.2 cm (direction perpendicular to the plane of Fig. 1) for model M5. Long shims and feeler gauges^{13,16} were used between the vocal folds and the wind tunnel wall to adjust the glottal diameter (separation between the medial surfaces of Fig. 1), so that pressure distributions could be measured at minimal diameters of $d = 0.005$, 0.0075, 0.01, 0.02, 0.04, 0.08, 0.16, and 0.32 cm. At each diameter a number of pressure distributions were collected at transglottal pressures ranging from 1 to 25 cm H₂O. A comprehensive account of the design of Model M5, a schematic of the geometry of the Plexiglas model, a schematic of the

experimental setup, and the procedures used to obtain the pressure distributions are given in Ref. 13.

These pressure distributions were used in Ref. 7 to determine a table of entrance loss coefficients. Entries in this table range from 17.5 to 0.687, and the variations of the coefficients with transglottal pressure for a given glottal diameter are especially large when the glottal diameters are very small. However, for intermediate diameters (0.02–0.08 cm), ranges of subglottal pressures can be identified where the entrance loss coefficient is within 10% of the value of 1.375. Further, the exit coefficients calculated from the M5 pressure distributions⁷ do not support the increasing trend with minimal diameter predicted by the momentum-conservation-based formula of Ishizaka and Flanagan,³ and the variation of these coefficients with transglottal pressure is comparable to those found by Gauffin *et al.*⁹

The importance of revisiting the classic papers of van den Berg *et al.*,¹ of Ishizaka and Flanagan,³ and of Titze's work with small amplitude oscillations and the surface wave model¹⁷ was emphasized by some recent developments reported in Refs. 18 and 19. In Refs. 18 and 19 the often-used assumption of a constant (or almost constant) entrance loss coefficient not too different from one (1.1, 1.2, or 1.375) was shown to be inconsistent with measurements of this coefficient with model M5 at small diameters.⁷ In particular, the large variation of the entrance loss coefficient with diameter was shown to be consistent with an inverse dependence of this coefficient on the glottal diameter. Furthermore, incorporating this inverse diameter dependence in Titze's formula for the phonation threshold pressure introduced a qualitatively new behavior into the phonation threshold pressure at small diameters, namely, the threshold pressure approached a constant at small glottal diameters instead of zero, the result of Titze's original formula. This important observation proved to be the key development in clearing up a troubling discrepancy between the predictions of Titze's surface wave model for the threshold pressure at small diameters and the 2006 measurements of Chan and Titze,²⁰ since the 2006 experiments showed clearly that phonation threshold pressure approached a constant at small glottal diameters, in marked contrast to the original predictions of Titze's surface wave model.¹⁷

The representation of viscous effects within the glottis by the last term of Eq. (1) occurs fairly often in the literature, as Refs. 3 and 21–27 attest. However, experimental data that support the inverse cube dependence of this term through channels with shapes similar to that of the human larynx are sparse. A recent attempt to validate the Poiseuille approach in a static model with a uniform constriction and steady flow by Cissoni *et al.*²⁸ ran into several difficulties. For example, in Fig. 6 of their paper for a transglottal pressure of 1000 Pa (10.2 cm H₂O), one of the calculated pressures agreed with the measured pressure, but the second calculated pressure was about 40% below the measured value. Their calculations of the volume velocities at this pressure were even more inaccurate; one was about 30% below the measurement, and the other about a factor of 5 below the measurement. Further, a recent calculation reported in Ref. 18 (see Figs. 10 and 11), which was based on Titze's surface wave model,¹⁷

showed that the inverse cube behavior of the last term of Eq. (1) was not consistent with Chan and Titze's 2006 phonation threshold experiments²⁰ at small glottal widths. *The pressure distributions described in the next section represent one of the first good opportunities to examine thoroughly the question of whether Poiseuille's formula is an adequate representation of viscous effects within the glottis and what might be done to improve it.*

Not only were the entrance loss coefficients calculated in Ref. 7 for $d = 0.005, 0.0075, 0.01$, and 0.02 cm substantially larger than the value 1.375 that van den Berg *et al.*¹ adopted as a useful average, but also those calculated for $d = 0.16$ cm were from 20% to more than 50% smaller than the 1.375 value. Since the geometry used by van den Berg *et al.* is similar to that of model M5, one would expect similar behavior for the entrance loss coefficients in their experiments. Thus, one should expect fairly substantial corrections to their useful average of 1.375 over parts of the range 0.01 to 0.2 cm, for which van den Berg made their claim of agreement of their measurements within 10% of the results of their formula. A likely explanation of the 10% accuracy claim by van den Berg *et al.*, is a compensation of the errors in the two terms of their flow resistance formula. This would mean that they were not able to test the validity of the Poiseuille term for viscous flow resistance to an accuracy of 10%.

To build a context for discussing viscous effects within the glottis, we consider Fig. 2, a schematic diagram summarizing results obtained by Ishizaka and Matsudaira,² who treated the development of the velocity profile and the boundary layers along the axial direction. Their work was based on a momentum approach to the Navier Stokes equation, and they obtained an analytic expression for the development of the velocity profile by using Pohlhausen's polynomial representation²⁹ of this profile within the boundary layers. In Fig. 2, a uniform airflow enters the glottis from the left, where the boundary layers starts to develop. In the boundary layers the flow is assumed to be laminar and to present a velocity profile characteristic of viscous flow.

FIG. 2. (Color online) Schematic diagram of the velocity profile and boundary layers along the direction of flow. This diagram is based on Ishizaka and Matsudaira's solution² to the Navier Stokes equation. A uniform velocity field enters from the left side of the diagram, where development of the boundary layers begins (dotted lines). The boundary layers grow as the flow moves further into the glottis. If the channel is long enough, the boundary layers from the opposing surfaces merge, and eventually the flow takes the fully developed parabolic form characteristic of Poiseuille flow.

These layers are separate from the central region where there is no viscosity effect, and thus the velocity field across the central region is uniform. As the airflow proceeds further into the glottis, the boundary layers grow, and if the channel is long enough, the two boundary layers merge. At this point the flow in the entire channel is viscous, and the velocity profile eventually takes on the parabolic shape characteristic of fully developed laminar flow. Once the flow reaches this stage, the axial pressure gradient ($\partial P / \partial x$) within the glottis for a given glottal diameter becomes a constant with the inverse cube dependence on diameter shown in the second term of Eq. (1), which is characteristic of Poiseuille flow.

A complication that Ishizaka and Matsudaira did not consider is the effect of the converging channel of the subglottal region on the velocity profile entering the glottis. Earlier work by Beavers *et al.*³⁰ examined this question in a variety of rectangular ducts with different aspect ratios. They found that the development of the boundary layer began within the channel leading to their rectangular ducts. Thus, in the case of M5 the boundary layer would be expected to begin in the subglottal region somewhere between taps one and six. A consequence of this observation is that one would not expect the velocity profile of the flow at the glottal entrance (tap six in Fig. 1) to have the uniform shape of the initial profile of Fig. 2. This pre-entrance boundary layer development is quantified in the entrance loss coefficient.

In Sec. II representative pressure distributions collected with model M5 are presented and those regions within the glottis where the pressure gradient is almost constant are isolated. In Sec. III a generalization of the parabolic flow model is presented that approximates the flow development in Fig. 2 by finding an appropriate average for the entire glottal channel. In Sec. IV the part of the pressure gradient that depends linearly upon the volume velocity U_g for each diameter is obtained, and the dependence of this part of the pressure gradient on the glottal diameter is examined. This diameter dependence is also compared with the inverse cube behavior of the second term of Eq. (1). Conclusions are given in Sec. V.

II. PRESSURE DISTRIBUTIONS AND VISCOSITY EFFECTS

In Fig. 3 a subset (glottal diameters of $0.0075, 0.01, 0.08$, and 0.32 cm) of the pressure distributions measured at the taps of model M5 is presented. The pressure data there are presented as pressure drops (because of the decreases) from an upstream reference (tap 0, not shown) to each of the 16 taps at the axial locations shown in Fig. 3(A). The transglottal pressure drop in cm of H_2O is listed for each case. For the two largest diameters in Fig. 3, the flow tends to form a jet¹³⁻¹⁵ which adheres to one side of the glottis, the flow wall (FW), and separates from the opposite side, the non-flow wall (NFW). The tendency of the jet to attach to a wall is often referred to as the Coanda effect.^{15,31} As explained in Ref. 13, the flow is stable for the jet directed to either side and may be switched from one side to the other

FIG. 3. (Color online) Pressure distributions collected with model M5 within and near the glottis for minimal glottal diameters of (A) 0.0075, (B) 0.01, (C) 0.08, and (D) 0.32 cm. The pressure drop is defined as the difference between an upstream reference (tap 0, not shown) and the pressure tap at a given axial location (A). The transglottal pressure drops are shown for each pressure distribution. For the larger diameters, the pressure distributions measured on the side from which the flow jet has separated are designated as the nonflow wall (NFW), and the pressure distributions measured on the side to which the jet adheres is designated as the flow wall (FW).

(where it remains) by the use of a paper guide. This dichotomy breaks the symmetry of the pressures on the two walls and requires that two sets of measurements be made for all diameters larger than 0.01 cm. For $d = 0.02$ cm and $d = 0.04$ cm, the cross-channel differences are very small. They become as large as 5% of the transglottal pressure for $d = 0.08$ cm and as large as 10% for $d = 0.16$ cm. However, their effect on the determination of viscous effects within the glottis is smaller than these numbers would suggest, since the most of the pressure decrease associated with viscosity occurs within the glottis upstream of the largest cross-channel pressure differences. For example, the 5% difference between the FW and the NFW pressures for the 15 cm H₂O case of Fig. 3(C) occurs at tap 14 on the superior glottal surface, whereas the difference between the FW and the NFW pressures at tap 10 is only about 1%. Tap 10 marks the end of the region of linear behavior of the pressure within the glottis, which is used to compute the pressure gradients of Fig. 5 below. Thus in this case, it is this 1% difference that should be used to measure the effect on flow separation on the determination of intraglottal viscous effects and not the 5% difference. Accordingly, the average of the FW and the NFW pressures is used for all the calculations below for which the diameter is 0.02 cm or greater.

Effects of the pressure drop between the upstream reference (tap 0) and the glottal entrance may be quantified with

an entrance loss coefficient,^{7,17,18} and our concern in the present work is the behavior of the pressure distribution after the flow has passed the glottal entrance. For the two smallest diameters of Fig. 3, the pressure within the glottis undergoes a nearly uniform decrease (since the pressure drop is increasing almost linearly) between taps six (axial location = 0.211 cm) and 10 (axial location = 0.373 cm), with slight deviations from linear behavior beginning at tap 11, the start of glottal curvature. For a given transglottal pressure, the pressure distribution within the glottis decreases more rapidly for $d = 0.0075$ cm than for $d = 0.01$ cm. This is the kind of behavior one would expect for a narrow channel where the pressure decrease is dominated by viscous effects. There is a good indication of a linear decrease [Fig. 3(C)] for $d = 0.08$ cm between taps 7 (axial location = 0.252 cm) and 10, but there is no indication of such behavior at $d = 0.32$ cm, which gives a pressure distribution resembling that of a divergent glottis. Thus, one would not expect Fig. 3(D) to be useful in looking for geometrical parameters that describe viscous effects.

One also sees clearly defined regions of linear decrease of the pressure (as the pressure drop increases) within the glottis for $d = 0.005$ cm and 0.02 cm [Figs. 2(A) and 2(B) of Ref. 7] between taps six and 10. For $d = 0.04$ cm (Fig. 2 of Ref. 14), the axial region of linear decrease is also between taps six and 10. There is also an indication of linear behavior

FIG. 4. (Color online) Dependence of intraglottal pressure drops on the axial distance from taps 6 to 10 at $d = 0.01$ cm (A) for the five transglottal pressure drops shown in Fig. 3(B). The R^2 coefficients measure the quality of the straight line fits as a functions of the axial distance (x in the trend line equations). In part (B) the R^2 coefficients for the 32 pressure distributions discussed in Sec. II are shown as functions of glottal diameter.

at $d = 0.16$ cm [Fig. 2(C) of Ref. 7] between taps eight and 11. Thus, it is of interest to analyze the intraglottal parts of the pressure distributions to ascertain those locations where the pressure drops are adequately represented by a straight line. This analysis for the case $d = 0.01$ cm is shown in Fig. 4(A) for the five values of transglottal pressure at which data were taken. There it is apparent that straight lines give excellent representations of the dependence of the intraglottal pressures³² on the axial distance between taps six and 10 by inspection. This conclusion is reinforced by the calculation of the squares of the linear correlation coefficients (R^2) for the trend lines supplied by the EXCEL software, since all of these are much closer to unity than 1%. An estimate of the p values using the formula given in Appendix C of Taylor's *An Introduction to Error Analysis*³³ gives p values much less than 1%, the threshold for a highly significant correlation. The slopes of the trendlines may be read from the linear fits to the pressure drops, in effect rendering the ratio of $\Delta P/\Delta x$ (in units of Pa/cm) within the glottis, that is, determining the pressure gradient for each of the five transglottal pressures of Fig. 4(A). This same analysis has been done for each of the other six diameters (0.005, 0.0075, 0.02, 0.04, 0.08, and 0.16 cm) discussed above. From the linear fits to the dependence of the pressure drop on axial distance, one obtains the pressure gradients within the glottis for each of the pressure

distributions at these diameters. The R^2 values determined for all 32 pressure distributions are shown in Fig. 4(B). For the six smallest diameters (0.005–0.08 cm), all of the R^2 values are within 2% of unity, reinforcing the selection of the axial intervals within the glottis dominated by a linear connection between pressure drop and axial distance. All of the p values for this set of six diameters are less than 5%, the level associated with a significant correlation.³³ The pressure gradients derived from the slopes of the pressure drop curves for this set of pressure distributions will be the focus of the analysis carried out below. The behavior of the $d = 0.16$ cm distributions near the glottal entrance differs somewhat from that of most of the other distributions. As described above, this prompted a different choice of the linear region (taps eight to 11) for the EXCEL trend lines used to determine the pressure gradients. The quality of the linear fits was not as good in this case, since the R^2 values ranged from 0.968 to 0.901. Nevertheless, these values seem reasonable, and thus, the pressure gradients obtained in this case are also included in the analysis below.

III. VELOCITY PROFILES WITHIN THE GLOTTIS

The Navier Stokes equation for one-dimensional flow³⁴ with velocity v_x along the glottal axis x shown in Fig. 2 is given by

$$\rho v_x \frac{\partial v_x}{\partial x} = -\frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} \right), \quad (2)$$

where y represents the transverse direction. Fully developed flow is described by Eq. (2) by assuming that v_x does not depend upon x and that its dependence upon the transverse coordinate y is given by the parabolic form $v_x(y) = A(y/d)(1 - y/d)$ with a maximum $A/4$ at the center of the channel where $y = d/2$. For such a velocity profile the volume flow rate may be obtained by integrating over the area perpendicular to the axial direction. It is given by $U_g = A l_g d / 6$, and one readily obtains

$$\frac{\partial P}{\partial x} = -\frac{12 \mu U_g}{l_g d^3}, \quad (3)$$

from Eq. (2) for the pressure gradient. This equation gives a pressure gradient that is constant within the glottal channel and that is the negative of the gradient that may be obtained from the last term of Eq. (1). This sign difference is expected since Eq. (1) expresses the pressure drop measured from tap 0 upstream, whereas the pressure P is measured relative to atmospheric pressure (taken as zero) that pertains downstream from tap 16.

In this section we explore the consequences of allowing corrections to the parabolic form for the velocity profile. In particular, the parameter α is introduced to consider corrections of the form

$$v_x(y) = A[(y/d)(1 - y/d) + \alpha(y/d)^2(1 - y/d)^2]. \quad (4)$$

This form accommodates the no-slip boundary conditions on the two medial surfaces, namely, $v_x(0) = v_x(d) = 0$. In the

limit of $\alpha = 0$, the Poiseuille formula of Eq. (3) emerges. For the profile of Eq. (4) the volume velocity may also be found by integration. It is given by

$$U_g = \frac{A l_g d}{6} (1 + \alpha/5). \quad (5)$$

Taking the two partial derivatives of Eq. (4) with respect to y , as required by the last term of Eq. (2), and using Eq. (5) to substitute for the constant A in Eq. (4) yields

$$\frac{\partial P}{\partial x} = -\frac{12 \mu U_g}{l_g d^3} \left(\frac{1 - \alpha}{1 + \alpha/5} \right), \quad (6)$$

for the pressure gradient at either medial surface. The term in the large parenthesis of Eq. (6) represents the correction due to generalization of the parabolic form described by the second term of Eq. (4). In the next section, we determine an explicit formula for α that is derived from the behavior of the pressure gradients determined from the M5 data set. This formula expresses the dependence of α on the glottal diameter d .

IV. INTRAGLOTTAL PRESSURE GRADIENTS AND VOLUME VELOCITIES

The form of Eq. (6) suggests that it would be worthwhile to examine the pressure gradients obtained from the M5 pressure distributions to see if there are corrections to the Poiseuille formula that are linear in the volume velocity. Thus, Fig. 5 shows the dependence of the absolute value of the pressure gradient at each minimal diameter on the volume velocity. Each of the curves presented there is almost a straight line, suggesting that a linear relationship between these two variables would account for the most of the variation with volume velocity. The pressure gradients for each diameter are fit with a polynomial of degree 2 that is required to pass through the origin, in order to preserve the condition of no pressure gradient when the volume velocity is zero. All of the R^2 values listed in Fig. 5 for the six smallest diameters are closer than 1% to the value of unity, and the R^2 coefficient for the $d = 0.16$ cm case is also within 1% of unity. The latter observation is an indication that the smaller glottal region (taps eight to 11) used to determine the pressure gradient for $d = 0.16$ cm is also dominated by the viscous effects. Thus, our conclusion from Fig. 5 is that the polynomial fits to the pressure gradients there are excellent and that one should be able to decide if corrections to the Poiseuille formula of the form proposed in Eq. (6) are present in the M5 data set. The linear coefficients of Fig. 5 (in units of Pa s/cm^4), which may be read directly from the two parts of this figure, are the focus of this paper. Presumably, the quadratic dependence of the pressure gradients on the flow rate has its origin in some of the nonlinear complexities present in the Navier Stokes equation of Eq. (2). This suggestion would be a candidate for study in a more comprehensive approach to the Navier Stokes equation.

In Fig. 6 the coefficients of the dominant linear terms of Fig. 5 are presented as a function of the diameter on a log-log plot. Such a plot is clearly indicated since the dependent variable varies by a factor of almost ten thousand. There it is seen that this relationship is a good candidate for a power law fit,

FIG. 5. (Color online) Absolute value of the intraglottal pressure gradients as functions of the volume velocity for small (A) and large (B) glottal diameters. In this figure the trend line variable x represents the volume velocity in cm^3/s .

since the values derived from the M5 data form a trend that is approximately consistent with a straight line. To make this statement more precise, the results of a power-law fit are also shown in Fig. 6, and the value of R^2 there reinforces our observation about the adequacy of a power law fit. The power law fit is compared with the dashed line that represents the Poiseuille formula with its characteristic inverse cube behavior. The two curves intersect near $d = 0.01$ cm, where both are close to the results derived from the M5 data, and hence there

FIG. 6. (Color online) Linear coefficients from the fits of the pressure gradients of Fig. 5. These are presented as a function of the minimal glottal diameter. The solid curve represents the power law fit to the linear coefficients of Fig. 5 as a function of the glottal diameter (the trend line variable x in the power law equation). The dashed curve is based on the Poiseuille formula.

is a range of glottal diameters (0.0075–0.02 cm) for which the Poiseuille formula would be a reasonable approximation. At diameters smaller than this range, the curves of Fig. 6 show that the Poiseuille formula overestimates the effects of viscosity within the glottis and for diameters larger than this range, the Poiseuille law underestimates the effects of viscosity. From the power law of Fig. 6 one would infer that a better approximation to the gradient of the pressure drop over the range of diameters covered by the relevant part of the M5 data set (0.005–0.16 cm) would be

$$\frac{\Delta P_{\text{drop}}}{\Delta x} = \frac{12 \mu}{l_g d^3} \left(\frac{d}{d_{\text{Poise}}} \right)^{\lambda} U_g, \quad (7)$$

where $\lambda \cong 0.41$, the difference between the exponent of the power law fit of Fig. 6 and the inverse cube dependence of the Poiseuille law, and $d_{\text{Poise}} \cong 0.01$ cm is the diameter at which the Poiseuille formula is the best approximation to the linear coefficients of Fig. 6. To get a better understanding of the difference between the power law of Eq. (7) and the Poiseuille formula, the percentage errors that each formula gives in accounting for the linear coefficients of Fig. 6 are calculated. These are presented in Fig. 7. For the Poiseuille formula, the percentage error ranges from 39.9% at $d = 0.005$ cm to -67.4% at $d = 0.16$ cm. For the power law of Eq. (7) the percentage error ranges from 22.7% at $d = 0.02$ cm to -16.3% at $d = 0.08$ cm. The average of the absolute value of the errors for the Poiseuille formula is 32.1%, and the average of the absolute value of the errors for the power law is 9.2%. Thus, the power law of Eq. (7) enjoys a factor of about 3 advantage in accounting for all of the M5-derived linear coefficients of Fig. 5. Both the power law and the Poiseuille formula give their smallest errors (2.6%) at $d = 0.01$ cm, where these two curves intersect. Nevertheless, in the interval (0.0075–0.02 cm) where it best accounts for the linear coefficients, the Poiseuille law gives more accurate results than the power law.

It is worth noting that these results are more or less what one would expect from the analysis of Ishizaka and Matsudaira.² Their analysis of the development of the boundary layers (Fig. 1.2 of Ref. 2) shows that they expect the boundary layers on each side of the glottis to merge for some small diameter. This small diameter should be in the

FIG. 7. (Color online) Percentage errors of the power law (solid curve) and the Poiseuille (dashed curve) fits to the linear coefficients of Fig. 5.

FIG. 8. (Color online) Average velocity profiles within the glottis for the seven diameters analyzed from the M5 experiments.

region of glottal diameter of Fig. 6 where the Poiseuille formula is shown to be a reasonable approximation.

The formula for the gradient of the pressure drop in Eq. (7) allows us to determine the parameter α by comparing the form of Eqs. (6) and (7). This relationship takes the form

$$\alpha(d) = \frac{1 - (d/d_{\text{Poise}})^{\lambda}}{1 + \frac{1}{5} (d/d_{\text{Poise}})^{\lambda}}. \quad (8)$$

Equation (8) shows that $\alpha = 0$ when $d = d_{\text{Poise}}$, that it is negative for $d > d_{\text{Poise}}$, and that it is positive for $d < d_{\text{Poise}}$. The velocity profiles calculated from Eq. (4) for the values of α determined by Eq. (8) are shown in Fig. 8. For the two smallest diameters, the profiles have a curvature near the center of the glottis that is sharper than the parabola of the $d = 0.01$ cm case. As the glottal diameter increases beyond $d = 0.01$ cm, the profiles near the center flatten out, a behavior reminiscent of profiles during flow development in Fig. 2. Thus, the profiles of Fig. 8 provide one means of understanding that as the diameter increases beyond $d = 0.01$ cm, the flow makes less progress towards full development.

V. SUMMARY AND CONCLUSIONS

The investigation of the glottal pressure-flow relations carried out by van den Berg *et al.*¹ more than 50 years ago is re-examined with pressure distributions measured by model M5. The reason for this further examination is to provide greater clarity of the viscous flow contribution in the uniform glottis made available by the “greater resolution” of the intraglottal pressures in the more recent model. Earlier publications^{7,18,19} have already described how to incorporate more detailed information about entrance loss coefficients into the formula that van den Berg *et al.* used to summarize their measurements of glottal entrance coefficients, and thus this paper focuses on their claim that the inverse cube Poiseuille formula is an adequate summary of viscous effects within the uniform glottis.

In Fig. 5 it was shown that the major part of the magnitude of the pressure gradient within the glottis can be accounted for by a formula that is directly proportional to the first power of the volume velocity, although evidence for a

quadratic term is also present. In Sec. III it was shown that one would expect such a linear term from an analysis of the viscosity contribution to the one-dimensional Navier Stokes equation. Moreover, a consideration of possible corrections to the parabolic velocity profile required for the Poiseuille formula shows that such corrections may modify how the pressure gradient depends upon the glottal diameter. In Fig. 6 the limitations of the inverse cube diameter dependence of the Poiseuille formula were explored, and it was shown that a more accurate overall fit to the linear coefficients derived from the M5 pressure distributions is given by an inverse 2.59 power law. The accuracy of the new power law and the Poiseuille formula were compared in Fig. 7, where it was shown that the new power law has an advantage of a factor of about 3 in calculating the linear coefficients derived from the M5 pressure distributions. From values of the parameter α determined from Eq. (8) one may calculate average velocity profiles for the flows within the glottis at the different diameters used in the M5 experiments (see Fig. 8). These profiles allow one to picture how larger values for the glottal diameter hinder the full development of the parabolic form of the flow within the glottis, which is necessary for the validity of the Poiseuille formula. It is interesting to note that there is a relatively small range of glottal diameters (0.0075–0.02 cm) where the Poiseuille formula gives results more accurate than the new power law, although the difference is not large.

A future calculation of the transglottal flow resistance (ratio of ΔP_{tr} to U_g) comparing van den Berg *et al.* formula¹ to one that substitutes the power law of Eq. (7) for the Poiseuille part of the formula of Eq. (1) is planned.

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