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3D Segmentation of Maxilla in Cone-beam Computed Tomography Imaging Using Base Invariant Wavelet Active Shape Model on Customized Two-manifold Topology

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Abstract

Recent advances in cone-beam computed tomography (CBCT) have rapidly enabled widespread applications of dentomaxillofacial imaging and orthodontic practices in the past decades due to its low radiation dose, high spatial resolution, and accessibility. However, low contrast resolution in CBCT image has become its major limitation in building skull models. Intensive hand-segmentation is usually required to reconstruct the skull models. One of the regions affected by this limitation the most is the thin bone images. This paper presents a novel segmentation approach based on wavelet density model (WDM) for a particular interest in the outer surface of anterior wall of maxilla. Nineteen CBCT datasets are used to conduct two experiments. This mode-based segmentation approach is validated and compared with three different segmentation approaches. The results show that the performance of this model-based segmentation approach is better than those of the other approaches. It can achieve $0.25 \pm 0.2\text{mm}$ of surface error from ground truth of bone surface.

Keywords

3D segmentation; Active shape model (ASM); Statistical shape model (SSM); Craniomaxillofacial (CMF) surgeries; Cone-beam computed tomography (CBCT)

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1 Introduction

The field of craniomaxillofacial (CMF) surgery involves the correction of congenital and acquired deformities of the skull and face. It includes dentofacial deformities, congenital deformities, combat injuries, post-traumatic defects, defects after tumor ablation, and deformities of the temporomandibular joint. Due to the complex nature of the craniomaxillofacial skeleton, the surgical correction of CMF deformities is among the most challenging. These types of the surgeries usually require extensive surgical planning. The success of these surgeries depends not only on the technical aspects of the operation, but to a larger extent on the formulation of a precise surgical plan. During the past 50 years, there have been significant improvements in the technical aspects of surgery (e.g., rigid fixation, resorbable materials, distraction osteogenesis, minimally invasive approaches, etc). However, the planning methods remain mostly unchanged [1–4]. At present, in CMF surgery, it is clear that many unwanted surgical outcomes are the result of deficient planning.

The advent of computed tomography (CT) and its 3D reconstruction have brought about a revolution in diagnostic radiology since cross-sectional imaging became available [5–7]. 3D rendered visualization provides a surgeon with readily recognizable images of complex anatomic structures. It can exactly record and represent the life-size and the shape of soft tissue and bone for precise surgical planning and simulation. In conjunction with appropriate computer software and hardware, computer-aided surgical simulation (CASS) has been developed and has created a number of options for CMF surgeries [3,8–12].

Cone-beam computed tomography (CBCT) has been adopted rapidly in the past decades and widely used in dentomaxillofacial imaging and orthodontic practices [13–15]. The most important reason is that currently, the effective dose of CBCT for a head scan (from several dozens up to several hundreds μSv) is significantly lower than that of CT for a head scan (from several hundreds up to several thousands μSv) [13,16,17]. Furthermore, reported spatial resolution (voxel resolution) varies from 0.076 to 0.4mm [13]. Although the spatial resolution of a slice in CT can be as small as 0.4mm, the thinnest axial thickness is 0.625mm. CBCT also offers particular accessibility for most of the medical units because of its low cost compared with CT. Commercially available CBCT allows patients to seat or stand vertically during scanning. Natural head position (NHP) can be acquired directly for 3D cephalometry. These advantages have created the possibilities of replacing CT with CBCT for 3D imaging and modeling of CASS [18–21].

Surgical planning and CASS require accurate contours of facial bones¹. The acquisition of a skull model in traditional CT includes two steps: thresholding segmentation and bone surface reconstruction. The thresholding segmentation can easily classify bone voxels because of the calibrated Hounsfield unit (HU) gray values and high contrast resolution (signal-to-noise ratio) of CT. These bone images are then fed to surface reconstruction algorithm such as Marching Cube Algorithm to obtain bone surface. Therefore, accurate segmentation of bone voxels is one of the most essential tasks for 3D rendering and CASS. However, poor image quality becomes its major limitation in establishing skull models in CBCT. Fig. 1 illustrates a skull model obtained by thresholding segmentation and Marching Cube Algorithm in CBCT imaging. The characteristics of degraded images in CBCT imaging can be summarized as follows. First, intensities of CBCT images cannot be accurately represented by standard HU because of no absolute HU calibration. It varies

¹Facial bones includes maxilla, zygomatic bone, lacrimal bone, nasal bone, and vomer in lower part of cranium, and temporal bone, sphenoid bone, parietal bone, and frontal bone in upper part of cranium

between scanners and between scans. In order to reconstruct bone surfaces with reasonable quality, deciding the threshold for segmenting bone images becomes an art (by guessing and trying). Second, when a unique global threshold is applied, some thin bony structures are usually not included. The sella turcica, orbital walls, nasal, surrounding thin bones of maxillary sinus, condyles, and ramus of the mandible are among the most affected bone structures. This is mainly because their image intensities are averaged with those of air during filter-backprojection reconstruction of CBCT. Third, the effect of low contrast resolution in CBCT creates randomly scattered noisy images and the bumpy surfaces of the skull model. It results from any combinations of beam-hardening effect, truncation effect, and Compton scattering. Finally, metal artifact can be the most detrimental in building skull models. It is mostly unavoidable since the dental fillings, implants, surgical plates, and orthodontic appliances are among the most common metallic materials existing inside patients' teeth and heads. Although metal artifacts are reduced in CBCT images compared with those in CT images, its effect is still considerable compared with the other effects described above.

In order to solve the addressed problems above, there are a number of the studies in advanced segmentation and reconstruction approaches of facial bone images in CT and CBCT imaging. They are segmentations based on statistical shape model (SSM) in CT imaging [22–24] and in CBCT imaging [25] and histogram thresholding segmentation in CT imaging [26]. Loubele *et al.* [27] models local histogram as a mixture of Gaussian distributions and determines the thresholds of jawbones (mandible and part of maxilla) and soft tissues in CBCT imaging. However, studies on developing robust segmentation algorithms versatile for most of the facial bones in CBCT imaging are still limited. Facial bones are characterized by complex and inhomogeneous structures (e.g. soft tissues, sinuses, and pieces of soft bones). Voxel-based global and local segmentation based on assumption of homogeneous structures may not be appropriate. Only the trained specialists can manually identify the internal structures of facial bones. This limitation makes it difficult to design segmentation algorithms for the bone voxels and bone surfaces of the internal structures of facial bones. In CASS, the visualization and measurement of simulation relies on the outer surfaces of facial bones. The requirement on accuracy of outer surface of facial bones is more critical than that on accuracy of internal structures of facial bones.

In this study, a new segmentation approach based on wavelet density model (WDM) for a particular interest on left half of anterior maxilla will be proposed. This region consists of thin bones around maxilla sinus and is most likely to be influenced by limitation of CBCT. The procedure of performing segmentation throughout this study is summarized step-by-step in Fig. 2. Table 1 lists all acronyms in the study. This model-based segmentation approach calculates the outer bone surface on left half of anterior wall of maxilla. Fig. 3 shows four different skulls. The marked regions on left half of anterior wall of maxilla represent four different shapes. It is obvious that the shape for SSM is a partial open-surface with closed boundary in a skull model. The outline of this manuscript is listed as follows. First of all, the acquisition and preprocessing of CBCT images will be presented in Section 2. Then, a practical procedure to creating training shapes with regularized landmarks will be proposed in Section 3. It includes extraction and customization of training shapes and landmark digitalization on training shapes. In Section 4, we will describe two statistical models. The first statistical model is wavelet-based SSM called WDM. The second statistical model is IFM. Based on those two statistical models, a new model-based segmentation algorithm BIWASM with a new initialization method CWBI will be proposed in Section 5. Next, BIWASM with CWBI will be validated and compared with three different approaches in Section 6. Section 7 will discuss these approaches, results, and the feasibility of BIWASM in CASS. Finally, Section 8 concludes this study. The notation used in this paper is described as follows. Bold symbols **A** and **a** are represented as a matrix and a column

vector, respectively. $\mathbf{a}^T$ defines matrix transpose of $\mathbf{a}$. $\|\mathbf{a}\| \equiv \sqrt{\mathbf{a}^T \mathbf{a}}$ is the Euclidean norm of a column vector $\mathbf{a}$. A 3D point $\mathbf{a}$ is represented as a 3-tuple column vector $(a_x, a_y, a_z)^T$ in a *Cartesian* coordinate system. The symbol $\mathcal{T}(\bullet)$ with any kinds of subscripts and superscripts denotes spatial transformation of 3D points. $\mathcal{W}$ and $\mathcal{W}^{-1}$ denote wavelet transform and inverse wavelet transform.

2 Data Acquisition and Preprocessing

Nineteen patients were scanned using CBCT (Sirona, Bensheim, Germany) with a voxel resolution of $0.287\text{mm} \times 0.287\text{mm} \times 0.287\text{mm}$, $512 \times 512 \times 512$ voxels, 0° gantry tilt, and 1:1 pitch. Nineteen CBCT volumetric images were acquired. Thresholding segmentation was first applied to each of these volumetric images to obtain bone images. Due to limitation in CBCT imaging, these bone images were recovered by manually editing images slice by slice. Then, Marching Cube Algorithm was applied to each of the recovered bone images to calculate surfaces (meshes) of bone images. Finally, the surfaces of bone images were smoothened using the software (Amira, San Diego, CA) to remove the bumpy surfaces. These bone surfaces are the ground truths for surfaces of physical skeleton. We will use these 19 ground truths of bone surfaces: 1) to customize the shapes and generate landmarks in Section 3; and 2) to validate the proposed segmentation approach in Section 6.

3 Training Shape Customization

The shape information of a target in a subject is characterized by a dense set of points and mesh structures. A SSM of the target is based on shape statistics of customized points on a set of the training datasets. These customized points are called landmarks. Before constructing a SSM, each training shape is remeshed into a new shape with the same number of landmarks and same mesh structures by digitalizing the shape either manually or automatically. Landmarks must be placed at topologically and structurally corresponding positions over all training shapes.

In this section, we will describe approaches to extracting and customizing a shape from the ground truth of bone surface and generating its landmarks for SSM. The shape (the target) in our study is the marked region on the skull (the subject) illustrated in Fig. 3. In the following, we will use the shape in Fig. 3 (a) to illustrate results calculated by each of the steps in the approaches. These steps are summarized in Fig. 4.

3.1 Training Shape Extraction and Patch Decomposition

The training shape of the target is identified by first manually pinpointing anatomical control landmarks on ground truth of bone surface and then determining the boundaries of the training shape. The training shape is the regions defined by these enclosed boundaries. Patch decomposition is performed by dividing the training shape into several patches. It can be implemented similarly by manually pinpointing anatomical control landmarks on the training shape and then determining the boundaries of patches. Each of the boundaries is obtained by calculating a shortest path or a path² composed of several connected shortest paths on the ground truth of bone surface between the anatomical control landmarks. Fig. 5 (a) shows that 8 anatomical control landmarks are used to define the training shape, and one anatomical control landmark is used to define four patches. Fig. 5 (b) illustrates the boundaries. Finally, these patches are extracted from the bone surfaces to form a customized training shape shown in Fig. 5 (c).

²When the shortest path cannot be found, several auxiliary points can be placed in-between and connected by shortest paths form a path.

The shortest path on a mesh is defined and calculated as follows. Assume A and B are two points on the mesh. When the mesh is unfolded onto a plane (by breaking the connection of cells and rotating the cells), A and B are also translated onto the plane accordingly. Let A' and B' be the points corresponding to A and B on the plane, respectively. It is possible to draw a straight line between A' and B' on the plane so that the line passes through cells of this unfolded mesh. This line on the unfolded mesh corresponds to a path on the original mesh. Of the note that this path passes through the valid cells on the mesh instead of only vertices and edges. The shortest path on the mesh is the one with minimum distance (distance of straight line) among all existing paths of that kind. Since calculating shortest paths without any simplifications is computationally expensive, we use Chen & Han's efficient algorithm to calculate shortest paths [28]. Since this searching algorithm uses all the cells in the mesh to find the shortest path, it is necessary to reduce the searching region [29]. It can be done by two steps of precalculation to determine this searching region. The first step of precalculation is that Dijkstra's shortest path algorithm [30] is used to calculate the Dijkstra's shortest path between A and B . This algorithm can quickly find the shortest path of connecting edges. The second step of precalculation is that the searching region can be determined by searching all the neighboring cells in a specific distance to the Dijkstra path. We use 0.2 times of the Dijkstra's shortest path as this specific distance.

3.2 Parameterization

Once the patches of the training shape are obtained, we will define their corresponding planar parameterization domains and the mappings. We use one of the patches (denoting $\mathcal{P}$) shown in Fig. 6(a) to illustrate its planar parameterization domain and mapping shown in Fig. 6(b).

First, the planar parameterization domain of $\mathcal{P}$ is determined as follows. Since $\mathcal{P}$ is defined by anatomical control landmarks, the planar parameterization domain can be defined as a polygon by those anatomical control landmarks. Assume $\{\mathbf{l}_k\}_{k=0}^{K_p-1}$ denotes the K_p anatomical control landmarks forming $\mathcal{P}$, and $\mathbf{l}_k$ and $\mathbf{l}_{(k+1)K_p}$ are the anatomical control landmarks connected by a boundary path of $\mathcal{P}$, where $(k)_{K_p}$ means k modulo K_p . Let $\{T_i\}_{i=0}^{K_p-3}$, $T_i \equiv \Delta(\mathbf{l}_0, \mathbf{l}_{i+1}, \mathbf{l}_{i+2})$ be the planar triangles for patch $\mathcal{P}$. Since $\{T_i\}_{i=0}^{K_p-3}$ are usually not coplanar, we can “unfold” these planar triangles $\{T_i\}$ onto a common plane create a planar K_p -polygon. This new planar K_p -polygon will become the parameterization domain for $\mathcal{P}$. The edges of the planar K_p -polygon are composed of line segments between $\mathbf{l}_k$ and $\mathbf{l}_{(k+1)K_p}$ (distance between two points in 3D space) while the boundaries of $\mathcal{P}$ are formed by the shortest paths on the mesh between $\mathbf{l}_k$ and $\mathbf{l}_{(k+1)K_p}$. Therefore, it can be claimed that $\mathcal{P}$ resembles the planar K_p -polygon.

Second, the parameterization mapping of $\mathcal{P}$ will be calculated using barycentric mapping [31]. Mean Value Coordinate will be used to calculate the spring constants. The planar K_p -polygon is assumed to be convex in order to guarantee the bijectivity of parameterization [31]. The convexity of the K -planar polygon can be achieved by carefully configuring $\{\mathbf{l}_k\}_{k=0}^{K-1}$ when performing patch decomposition. Moreover, before calculating barycentric mapping, the mapping of boundaries between $\mathcal{P}$ and its planar K_p -polygon has to be defined. A simple approach can be done by proportionally projecting the boundary vertices of $\mathcal{P}$ onto the edges of its planar K_p -polygon domain to obtain the boundary mapping.

3.3 3D Landmarks Digitalization Using Calmull-Clark Subdivision

The landmarks of each training shape will be digitalized by the Calmull-Clark subdivision proposed by Catmull *et al.* [32] in several steps. The first step is to use the Calmull-Clark

subdivision algorithm to generate new points on the parameterization domain (i.e. planar K_P -polygon) of $\mathcal{P}$. This planar K_P -polygon is a base mesh at 0-th subdivision. Fig. 7(a) illustrates base meshes of a triangle and a quadrilateral. The vertices of the base meshes denote $\mathbf{v}^0$. After performing one subdivision on the base meshes, it creates new points $\mathbf{f}^1$, and $\mathbf{e}^1$ and the updated vertices $\mathbf{v}^1$ shown in Fig. 7(b). $\mathbf{f}^1$ is called face point corresponding to the cells at the base meshes. $\mathbf{e}^1$ is called edge point corresponding to the edges at the base meshes. $\mathbf{v}^1$ corresponds to the original vertex $\mathbf{v}^0$ at the base meshes. New edges of the supermesh are formed by connecting $\mathbf{f}^1$ and $\mathbf{e}^1$ and by connecting $\mathbf{v}^1$ and $\mathbf{e}^1$. Before performing the next subdivision, all $\mathbf{f}^1$, $\mathbf{e}^1$, and $\mathbf{v}^1$ are relabeled to $\mathbf{v}^1$ as the input of the second subdivision.

Similarly, after the j th subdivision is performed, $\mathbf{v}^j$ (vertex), $\mathbf{f}^j$ (face point), and $\mathbf{e}^j$ (edge point) can be acquired in the same way. The j -th Calmull-Clark subdivision operated on $\mathbf{v}^{j-1}$ is summarized in **Algorithm 1**. n_v is called the valence of vertex $\mathbf{v}^j$ (the number of edges connecting $\mathbf{v}^j$). The average operator $\bar{\mathbf{k}}_m$ means the vertex of type $\mathbf{m}$ is obtained by using the neighboring vertices of $\mathbf{k}$ and calculating the average of them. Fig. 8 summarizes all possible average operators. $\mathbf{m}$ represents one of the corresponding averaged vertex, face

point, and edge point. Of the note that type $\mathbf{m}$ in $\bar{\mathbf{k}}_m^{j-1}$ is calculated by averaging centroid of face ($\mathbf{f}$), midpoint of edge ($\mathbf{e}$), or original vertex ($\mathbf{v}$) in the submesh [the mesh before

performing subdivision] instead. On the other hand, type $\mathbf{m}$ in $\bar{\mathbf{k}}_m^j$ is calculated by averaging $\mathbf{v}^j$, $\mathbf{f}^j$, or $\mathbf{e}^j$ in the supermesh (the mesh after performing subdivision).

At the second step, the patch $\mathcal{P}$ with landmarks are remeshed by inverse mapping the points (generated by subdivision) on the parameterization domains (planar K_P -polygon) onto $\mathcal{P}$. The same number of the subdivisions is applied to each of the patches in each training shape. Finally, the remeshed shape with regularized landmarks can be obtained by stitching these remeshed patches. Since two patches sharing a boundary have the same parameterization mapping on the shared boundary, both the patches will result in the same subdivision on this boundary. Hence, the remeshed patches can be stitched by merging the vertices at boundaries. Fig. (9) illustrates the remeshed and stitched training shape calculated after 0-th, first, third, and fifth subdivision of four base meshes. The training shape corresponding to 0-th subdivision is the mesh formed by anatomical control landmarks. In our study, the number of the subdivisions is 5.

Algorithm 1 Calmull-Clark subdivision algorithm

$$\mathbf{f}^j \leftarrow \bar{\mathbf{v}}_f^{j-1}$$

$$\mathbf{e}^j \leftarrow \frac{1}{2} \left(\bar{\mathbf{v}}_e^{j-1} + \mathbf{f}_e^j \right)$$

$$\mathbf{v}^j \leftarrow \frac{1}{n_v} \left(\bar{\mathbf{f}}_v^j + \bar{\mathbf{v}}_v^{j-1} + (n_v - 2) \mathbf{v}^{j-1} \right)$$

$$\mathbf{v}^j \leftarrow \{ \mathbf{f}^j, \mathbf{e}^j, \mathbf{v}^j \}$$

4 Statistical Models

Two statistical models summarized in Fig. 10 will be built by using N training images and their corresponding N training shapes with regularized landmarks (calculated in Section 3). The first statistical model is WDM using training shapes. It is the multiscale statistical shape model based on PDM (Section 4.1) by using DSWT (Section 4.2). The second statistical model is IFM created by using both training shapes and training images.

4.1 Point Distribution Model

Point distribution model (PDM) is introduced by Taylor *et al.* [33]. Let S_i be the set of n landmarks [illustrated in Fig. 9(d)] in i th training shape. We say S_i consists of landmarks in the image space. All the training shapes are transformed (rotation, translation, and scaling) to a common coordinate of model space using Procrustes Analysis [34] to minimize their squares of distance. Let $\tilde{S}_i$ be the transformed training shape. We say $\tilde{S}_i$ is the i th training shape in the model space. Assume $\mathbf{x}_i$ is the $3n$ -dimensional shape vectors formed by concatenation of the coordinates of all transformed landmarks in $\tilde{S}_i$. $\mathbf{x}_i$ can be expressed by

the mean vector $\bar{\mathbf{x}} \equiv \frac{1}{N} \sum_{i=0}^{N-1} \mathbf{x}_i$ and shape variation vectors $\Delta \mathbf{x}_i \equiv \mathbf{x}_i - \bar{\mathbf{x}}$. The distribution of SSM can be characterized by the covariance matrix of shape variation vectors given by

$$\mathbf{C}_x \equiv \frac{1}{N-1} \sum_{i=0}^{N-1} \Delta \mathbf{x}_i \Delta \mathbf{x}_i^T. \quad (1)$$

In order to capture the dominant shape variations, Principle Component Analysis is applied to the set of shape variation vectors. A set of orthogonal principle axes (also called principle modes) of shape variations can be found by calculating the orthogonal eigenvectors of $\mathbf{C}_x$, and the corresponding eigenvalues are the variances of the distribution in the direction of principle axes. Assume the columns of $\mathbf{P}$ are the orthogonal eigenvectors with the corresponding nonzero eigenvalues λ_k in the descending order. A shape vector $\tilde{\mathbf{x}}$ in the model space can be generated by a shape parameter $\tilde{\mathbf{b}}$:

$$\tilde{\mathbf{x}} = \bar{\mathbf{x}} + \mathbf{P} \tilde{\mathbf{b}}. \quad (2)$$

Any shape vector $\mathbf{x}$ in the model space can be approximated by projecting the shape variation vector $\Delta \mathbf{x} \equiv \mathbf{x} - \bar{\mathbf{x}}$ onto the subspace (spanned by $\mathbf{P}$) of the model

$$\mathbf{x} \approx \bar{\mathbf{x}} + \mathbf{P} \mathbf{b}. \quad (3)$$

Its shape parameter $\mathbf{b}$ can be calculated by the least square solution of minimizing $\|\Delta \mathbf{x} - \mathbf{P} \mathbf{b}\|^2$

$$\mathbf{b} = \mathbf{P}^T \Delta \mathbf{x}. \quad (4)$$

A shape vector $\mathbf{x}$ is said to be defined in the model space if it is expressed by (2) or approximated by (3). A shape vector $\mathbf{y}$ is said to be defined in an image space if its landmark points are located under the coordinate of a volumetric image. Once the PDM is built, a shape vector in the image space can be represented or approximated by a shape vector in PDM. Assume a PDM with shape priors $(\mathbf{P}, \lambda_k)$ is calculated from $\{S_i\}$. Let $\mathbf{y}$ be a shape vector of an arbitrary shape S in the image space. Of the note that the shape S is not among

any of the training shapes S_i . One of the approaches to calculating a shape vector in the model space to best represent $\mathbf{y}$ is to find a transform $\mathcal{T}$ and a shape parameter $\mathbf{b}$ to minimize mean squares:

$$\|\mathbf{y} - \mathcal{T}(\bar{\mathbf{x}} + \mathbf{P}\mathbf{b})\|^2. \quad (5)$$

Algorithm 2 A simple algorithm of calculating suboptimal $\mathcal{T}$ and $\mathbf{b}$ by minimizing (5) and incorporating shape constraints.

$\mathbf{b} \leftarrow \mathbf{0}$

while Until convergence **do**

$\mathbf{x} \leftarrow \bar{\mathbf{x}} + \mathbf{P}\mathbf{b}$; generate a shape in the model space using (2).

Calculate $\mathcal{T}$ by minimizing $\|\mathbf{y} - \mathcal{T}(\mathbf{x})\|^2$ using [35]

$\mathbf{x}' \leftarrow \mathcal{T}^{-1}(\mathbf{y})$; calculate inverse spatial transform of $\mathbf{y}$.

$\mathbf{b} \leftarrow \mathbf{P}^T(\mathbf{x}' - \bar{\mathbf{x}})$; calculate the shape parameters fitting $\mathbf{x}'$ using (3).

Apply shape constraints on $\mathbf{b}$ using (6)

end while

$\mathcal{T}$ is transform consisting of rotation, translation, and scaling of a shape. The eigenvalues λ_k can be used to provide the bounds of the model in order to ensure the shape is plausible. The values of $\mathbf{b}$ are constrained between

$$-a\sqrt{\lambda_k} < b_k < a\sqrt{\lambda_k} \quad (6)$$

where $2 \leq a \leq 3$ generally, and b_k is an element of $\mathbf{b}$. A simple algorithm of calculating suboptimal $\mathcal{T}$ and $\mathbf{b}$ by minimizing mean squares (5) and incorporating shape constraints is summarized in **Algorithm 2**.

4.2 Discrete Surface Wavelet Transform Based on Calmull-Clark Subdivision

In the following, we will perform multiscale analysis on each aligned training shape $\tilde{S}_i$ in the model space. By properly choosing anatomical control landmarks and performing patch decomposition (such as Fig. 5), we can generate landmarks on the training shapes with a rectilinear structure at any level of Calmull-Clark subdivision. Since the rows and columns of the rectilinear training shapes can be well-defined, DSWT of the training shapes can be calculated by performing 2D DWT on rectilinear shapes in x -, y -, and z -coordinate³. However, there are exceptions. When the training shape is decomposed into patches such as the ones in Fig. 14(a) using the same set of anatomical control landmarks, there are two pentagon-like patches and one quadrilateral-like patch forming the training shape [Fig. 14(b)]. After any level of Calmull-Clark subdivision on the patches is performed and the remeshed patches are stitched, the resulting training shape always has an extraordinary landmark (except landmarks on the boundaries) with valence three [32]. Fig. 14(c) shows the remeshed shape with the fifth subdivision. It is obvious that 2D DWT cannot be

³It is equivalent to perform 2D DWT on a 2D image. The image intensity is each of the coordinates (e.g. x -coordinate) of landmarks.

performed on this grid-like mesh since rows and columns of a rectilinear structure are not well-defined.

To overcome this problem, Bertram *et al.* [36] proposed a new construction of wavelet on Calmull-Clark subdivision surfaces of arbitray two-manifold topology by designing a new lifting scheme of biorthogonal wavelet transform. The training shape and its patches can be customized, and its DSWT can be constructed accordingly. The lifting scheme of 2D DWT and Bertram *et al.*'s proposed lifting scheme will be first introduced based on the rectilinear grid. Then, we will discuss how this new lifting scheme can skillfully perform multiscale decomposition on the training shape with a non-rectilinear structure.

Fig. 11 shows the decompositions in 1D DWT. An input $s_{m,J}$ at a resolution of J -scale can be decomposed into wavelet coefficients $w_{m,J-1}$ and scaling coefficients $s_{m,J}$ at $(J-1)$ -scale by a high pass filter (H) and a low pass filter (L) followed by downsamples. $w_{m,J-1}$ represents the signal details of $s_{m,J}$ at $(J-1)$ -scale, whereas $s_{m,J-1}$ represents the content of coarser scale of $s_{m,J}$. It is shown that one decomposition of any classic DWT with finite filters in Fig. 11 can be implemented by starting from the Lazy wavelet transformation (splitting) and then performing a finite number of alternating lifting steps [37]. The DWT using lifting scheme consists of two steps. At the first step of lifting scheme, the scaling coefficient $s_{m,j}$ at j -th scale is split into s -set and w -set:

$$s_{m,j-1} \leftarrow s_{2m,j} \quad (7)$$

$$w_{m,j-1} \leftarrow s_{2m+1,j} \quad (8)$$

At the second step of lifting scheme, a number of consecutive lifting steps are calculated. Only one of ***s*-lifting** (9) and ***w*-lifting** (10) steps is calculated at l -th lifting step:

$$s_{m,j-1} \leftarrow \alpha^{(l)} s_{m,j-1} + \sum_k \alpha_k^{(l)} w_{k,j-1} \quad (9)$$

$$w_{m,j-1} \leftarrow \beta^{(l)} w_{m,j-1} + \sum_k \beta_k^{(l)} s_{k,j-1} \quad (10)$$

The inverse DWT using lifting scheme is simply the analogous operations of (9) and (10) in the reverse order to satisfy perfect reconstruction:

$$s_{m,j-1} \leftarrow \frac{1}{\alpha} \left(s_{m,j-1} - \sum_k \alpha_k w_{k,j-1} \right) \quad (11)$$

$$w_{m,j-1} \leftarrow \frac{1}{\beta} \left(w_{m,j-1} - \sum_k \beta_k s_{k,j-1} \right) \quad (12)$$

Fig. 12(a) shows lifting steps of one decomposition of 2D DWT. 1D DWT is first performed on the rows and then performed on the columns by using lifting steps.

The new lifting scheme designed by Bertram *et al.* is shown in Fig. 12(b). Instead of performing the lifting steps first on the rows and then on the columns, each row lifting step and its corresponding column lifting step will be performed at the same time before moving on the next row and column lifting steps. Since lifting steps are linear operations, the

resulting decomposition of 2D DWT in Fig. 12(b) is equivalent to the one in Fig. 12(a). This new lifting scheme also consists of two steps. At the first step of the new lifting scheme, the splitting is performed on the landmarks of the training shape based on the structure of Calmull-Clark subdivision. As described in Section 3.3, the landmarks of the training shape are obtained by Calmull-Clark subdivision and can be classified into vertex $\mathbf{v}$, face points $\mathbf{f}$, and edge points $\mathbf{e}$ before relabeling in Algorithm 1 (The superscript is ignored for simplicity). $\mathbf{v}$ is split as scaling coefficients similar to (7). $\mathbf{f}$ is split as wavelet coefficients similar to (8). $\mathbf{e}$ can be split as scaling coefficients (when *s-lifting* is performed) or wavelet coefficients (when *w-lifting* is performed). At the second step of the new lifting scheme, the operations of lifting steps in each of row and column liftings) are defined as

$$s_{m,j-1} \leftarrow \alpha s_{m,j-1} + \tilde{\alpha} (w_{m,j-1} + w_{m-1,j-1}) \quad (13)$$

$$w_{m,j-1} \leftarrow \beta w_{m,j-1} + \tilde{\beta} (s_{m,j-1} + s_{m-1,j-1}) \quad (14)$$

The superscript l is ignored for simplicity. It can be shown in Appendix A that the composite *s-lifting* step for each of the vertex $\mathbf{v}$ and each of the edge points $\mathbf{e}$ can be determined by

$$\begin{aligned} \mathbf{v} &\leftarrow \alpha^2 \mathbf{v} + 4\tilde{\alpha}^2 \bar{\mathbf{f}}_{\mathbf{v}} + 4\alpha\tilde{\alpha} \bar{\mathbf{e}}_{\mathbf{v}} \\ \mathbf{e} &\leftarrow \alpha \mathbf{e} + 2\tilde{\alpha} \bar{\mathbf{f}}_{\mathbf{e}} \end{aligned} \quad (15)$$

It can be found that the composite *s-lifting* step is simply an expression of the generalized *s-lifting* step in (9). Similarly, the composite *w-lifting* step for each of the face points $\mathbf{f}$ and each of the edge points $\mathbf{e}$ has the following expression:

$$\begin{aligned} \mathbf{f} &\leftarrow \alpha^2 \mathbf{f} + 4\tilde{\alpha}^2 \bar{\mathbf{v}}_{\mathbf{f}} + 4\alpha\tilde{\alpha} \bar{\mathbf{e}}_{\mathbf{f}} \\ \mathbf{e} &\leftarrow \alpha \mathbf{e} + 2\tilde{\alpha} \bar{\mathbf{v}}_{\mathbf{e}} \end{aligned} \quad (16)$$

After one decomposition of 2D DWT is completed, the resulting $\mathbf{v}$ is the scaling coefficients as the input of the next decomposition of 2D DWT. The resulting $\mathbf{f}$ and $\mathbf{e}$ are the wavelet coefficients of 2D DWT.

It can be found that (15) is involved with the average of four neighboring edge points and the average of four face points for all $\mathbf{v}$. If an extraordinary vertex exists with valence $n_{\mathbf{v}}$, $\bar{\mathbf{f}}_{\mathbf{v}}$ and $\bar{\mathbf{e}}_{\mathbf{v}}$ of (15) can be generalized to the average of $n_{\mathbf{v}}$ neighboring face points and the average of $n_{\mathbf{v}}$ edge points, respectively. Therefore, the composite lifting steps in (15) and (16) can be performed no matter whether rows and columns are well-defined. It simply requires the structure of Calmull-Clark subdivision. The inverse lifting steps can be obtained similarly using (11) and (12).

In this study, the wavelet and scaling filters construction is based on dyadic refinement of linear B-spline scaling function. The lifting scheme of this wavelet construction based on linear B-spline scaling function is composed of only two composite lifting steps in each of 2D DWT decomposition: one composite *s-lifting* step followed by one composite *w-lifting*

step. The parameters in (15) and (16) are $\alpha = 1$, $\tilde{\alpha} = -\frac{1}{2}$, $\beta = 1$, and $\tilde{\beta} = \frac{1}{4}$. Fig. 13 shows the scaling coefficients after one, three, and five decompositions of Fig. 9(d) by using this new lifting scheme. Fig. 14(d)–(f) shows the scaling coefficients of another customized shape shown in Fig. 14(c), in which there is an extraordinary landmark with valence three. This

example (shown in Fig. 14) demonstrates that DSWT of arbitrarily customized training shapes can be constructed accordingly.

4.3 Wavelet Distribution Model

Since the rank of covariance matrix $\mathbf{C}_x$ in PDM is at most $N-1$, the number of the valid principle axes corresponding nonzero eigenvalues is at most $N-1$. Typically, it is the case that $n \gg N$ in the 3D SSM. The number of principle axes may not be sufficient enough to well-represent a shape with a large number of landmarks. Moreover, since each shape in PDM is a global linear combination of principle axes, PDM may not be able to capture fine shape details. Davatzikos *et al.* [38] first proposes multiscale SSM to solve this problem. Several studies applying different multiscale analysis to SSM are proposed such as spherical wavelet on spherical topology [39], subdivision based surface wavelet [40] on spherical topology, and diffusion wavelet on arbitrary surface topology [41].

In the following, DSWT will be incorporated with PDM to form a multiscale SSM. The training shapes in the model space are decomposed into multiscale representations, and a PDM is built in the subspace associated with each scale. This wavelet-based SSM is called wavelet density model (WDM). For simplicity of notation, all the scaling coefficients at 0-scale will be viewed as wavelet coefficients at (-1) -scale. Let $\widehat{\mathbf{w}}_{i,k,l}$ be DSWT of $\tilde{S}_i$ at scale l , spatial location k , shape i . Define a collection of wavelet coefficients $B_{i,l} = \{\widehat{\mathbf{w}}_{i,k,l}, \forall k\}$ at each scale, and concatenate them to form $\mathbf{w}_{i,l} \equiv (\widehat{\mathbf{w}}_{i,0,l}^T, \widehat{\mathbf{w}}_{i,1,l}^T, \dots)^T$. Finally, PCA is performed on $\mathbf{w}_{i,l}$ at each scale over all the training shapes to obtain the matrices $\mathbf{P}_l$ of eigenvectors. A set of wavelet coefficients $\tilde{\mathbf{w}}_l$ of a shape at specific scale l can be generated by a shape parameter $\tilde{\mathbf{b}}_l$

$$\tilde{\mathbf{w}}_l = \bar{\mathbf{w}}_l + \mathbf{P}_l \tilde{\mathbf{b}}_l \quad (17)$$

Similarly, the wavelet coefficients $\mathbf{w}_l$ of any shape at scale l can be approximated by projecting $\mathbf{w}_l$ onto the subspace of $\mathbf{P}_l$

$$\mathbf{w}_l \approx \bar{\mathbf{w}}_l + \mathbf{P}_l \mathbf{b}_l \quad (18)$$

with

$$\mathbf{b}_l = \mathbf{P}_l^T (\mathbf{w}_l - \bar{\mathbf{w}}_l) \quad (19)$$

It can be found that the total number of the eigenvectors in the model is increased by around J times. Similarly, the algorithm of minimizing (5) can be developed and is summarized in **Algorithm 3**.

Algorithm 3 A similar algorithm analogous to **Algorithm 2** when WDM is used.

$\mathbf{b}_l \leftarrow \mathbf{0}$

while Until convergence **do**

$\mathbf{w}_l \leftarrow \bar{\mathbf{w}}_l + \mathbf{P}_l \mathbf{b}_l$; generate wavelet coefficients in the model space using (17).

$\mathbf{x} \leftarrow \mathcal{W}^{-1}(\mathbf{w}_l)$; Calculate inverse DSWT of $\mathbf{w}_l$.

Calculate $\mathcal{J}$ by minimizing $\|\mathbf{y} - \mathcal{T}(\mathbf{x})\|^2$ using [35]

$\mathbf{x}' \leftarrow \mathcal{T}^{-1}(\mathbf{y})$; calculate inverse spatial transform of $\mathbf{y}$
 $\mathbf{w}_l \leftarrow \mathcal{W}(\mathbf{x}')$; Calculate DSWT of $\mathbf{x}'$
 $\mathbf{b}_l \leftarrow \mathbf{P}_l^T (\mathbf{w}_l - \bar{\mathbf{w}}_l)$; calculate the shape parameters fitting $\mathbf{w}_l$ using (19).
 Apply shape constraints on $\mathbf{b}_l$ analogous to (6)
 end while

4.4 Image Feature Model

In addition to WDM, IMF is built by using training images and shapes (before performing Procrustes alignment) to describe the image features embedded on the landmarks in the image space. For each of the landmarks of S_i in the image space, a line $\mathcal{L}$ passing through it and perpendicular to the shape is calculated [Fig. 15(a)]. Then $\mathcal{L}$ is uniformly sampled to obtain a set of ordered $2M + 1$ points $\{z_m\}_{m=-M}^M$ (M points on each side). Image intensities along the line can be interpolated at each point z_m . Fig. 15(b) illustrates a set of the sampled points along $\mathcal{L}$ and their image intensities. An image feature vector $\mathbf{g}$ (such as first order derivative of image intensity) can be calculated using the interpolated image intensities. Let $\bar{\mathbf{g}}$ and S_g be the mean and covariance matrix of $\mathbf{g}$ over all the training images. The IFM is characterized by $\bar{\mathbf{g}}$ and S_g .

In our study, three kinds of image features are calculated: zero order ($\mathbf{g}^{(0)}$), first order ($\mathbf{g}^{(1)}$), and second order ($\mathbf{g}^{(2)}$) derivatives of image intensity. $\mathbf{g}^{(i)}$ is normalized by $\|\mathbf{g}^{(i)}\|$ in order to avoid the effect caused by non-calibration in CBCT imaging.

5 Segmentation

Once the statistical models are built, we will calculate the outer surface of anterior wall of maxilla in a CBCT image by using statistical priors of the statistical models. In the following, we propose a novel model-based algorithm called base invariant wavelet active shape model (BIWASM) based on WDM with an initialization method called customized wavelet base initialization (CWBI) to calculate it. A brief flowchart is summarized in Fig. 16.

5.1 Preliminary Studies

In the following, we will introduce two model-based segmentation algorithms ASM and WASM and one of the most common initialization methods called RBI. ASM is introduced by Cootes *et al.* [42] and is an active contour iterative searching algorithm using PDM. WASM is the variation of ASM by applying multiscale analysis and is introduced in [38] using WDM. The final shape can be calculated by RBI followed by two-step iterative shape searching of WASM (ASM) based on WDM (PDM).

RBI can be one of the most simple initializations for ASM and WASM. The mean shape $\mathcal{W}^{-1}(\{\bar{\mathbf{w}}_l, \forall l\})^4$ [See (17)] in the model space can be viewed as the most probable shape for initialization. The initial shape can be obtained by transforming the mean shape from the

⁴It can be shown that mean shapes of PDM and WDM are exactly the same given the same training shapes are used, i.e. $\bar{\mathbf{x}}$ [See (2)] = $\mathcal{W}^{-1}(\{\bar{\mathbf{w}}_l, \forall l\})$

model space to the image space. Therefore, RBI can be performed in two steps. The first step of RBI is to use user interaction to select a number of the points in the image space described in Appendix B. They correspond to control landmarks of the mean shape. The control landmarks of a shape are defined based on the construction of the training shape and are described as follows. In Fig. 9(a), 9 anatomical control landmarks of the training shape form a mesh at 0-th subdivision. After any level of subdivision is performed, new landmarks are added. It can be observed that these 9 landmarks exist in the remeshed training shape at each level of subdivision. We define these 9 landmarks as control landmarks of a shape. Fig. 17(a) shows these enumerated 9 control landmarks [they correspond to anatomical 9 control landmarks of the mesh at 0-th subdivision in Fig. 9(a)]. Therefore, there are 9 corresponding control landmarks in the mean shape. The selection of the points in RBI is based on the assumption: if we were able to know the true shape⁵ (in the image space) composed of regularized landmarks with the same mesh structure as the shape in SSM, each of these selected points (in the image space) is assumed to be exactly one of the control landmarks on the true shape (in the image space). Furthermore, the selection criterion in RBI is the same as the configuration of anatomical control landmarks for training shapes demonstrated in Fig. 5(a). However, the datasets used for selection of the points are different: the points selected for calculation of initial shape are pinpointed on slices of the image using user interaction (Appendix B); and the points selected for patch decomposition and shape extraction in Section 3.1 are pinpointed on the ground truth of bone surface. Two examples of point selection for calculation of initial shape in RBI are shown in Fig. 17(a) and Fig. 17(b). The first example is that we can select 4 points corresponding to 4 of the control landmarks (such as the corner control landmarks) of the mean shape (in the model space). The second example is that we can select 9 points corresponding to 9 control landmarks of the mean shape (in the model space). At the second step of RBI, a transformation from the model space to the image space can be calculated by registering these selected points (in the image space) and their corresponding control landmarks of the mean shape (in the model space). The initial shape calculated by RBI using 9 selected points is illustrated in Fig. 18(a)

The first step of iterative shape searching of WASM (ASM) is that given a shape $\tilde{\mathbf{y}}^{(k)}$ in the image space at the k th iteration, each candidate landmark is examined along the line passing through it and perpendicular to the evolving shape $\tilde{\mathbf{y}}^{(k)}$ by using IFM. Similar to the notation in Section 4.4, a set of ordered $2K + 1$ points $\{z_m\}_{m=-K}^K$ can be acquired along the line, where $K > M$. The image feature vector $\tilde{\mathbf{g}} \equiv (g_{-K}, g_{-K+1}, \dots, g_K)$ with length $2K + 1$ can be calculated. A searching window defining a temporary image feature vector $\mathbf{g}_m \equiv (g_{-M+m}, g_{-M+m+1}, \dots, g_{M+m})$, $m = -(K-M), \dots, (K-M)$ with length $2M + 1$ scans the image feature vector $\tilde{\mathbf{g}}$. The candidate landmark $z_{\hat{m}}$ can be selected among $\{z_m\}_{m=-(K-M)}^{(K-M)}$ so that

$$\hat{m} = \arg \min_m f(\mathbf{g}_m) \quad (20)$$

Where $f \equiv (\mathbf{g} - \tilde{\mathbf{g}})^T S_{\tilde{\mathbf{g}}}^{-1} (\mathbf{g} - \tilde{\mathbf{g}})$ is Mahalanobis distance evaluated at $\mathbf{g}$. The landmarks calculated by 20 form a new evolving shape $\mathbf{y}^{(k)}$.

Algorithm 4 WASM (ASM)

while Until convergence **do**

⁵The true shape means the most ideal shape

$\mathbf{y}^{(k)} \leftarrow \tilde{\mathbf{y}}^{(k)}$; calculate a candidate shape by examining the neighboring region of each of the landmark points.

$\tilde{\mathbf{y}}^{(k+1)} \leftarrow \mathbf{y}^{(k)}$; given $\mathbf{y}^{(k)}$, calculate $\tilde{\mathbf{y}}^{(k+1)}$ by minimizing $\|\tilde{\mathbf{y}}^{(k+1)} - \mathbf{y}^{(k)}\|$ using **Algorithm 3** (for WASM) or **Algorithm 2** (for ASM).

end while

It can be observed that when the distribution of $\mathbf{g}$ is assumed to be jointly Gaussian, (20) is equivalent to finding a image feature vector maximizing the probability of a joint Gaussian distribution and then assigning the center point of the optimal image feature vector $\mathbf{g}$ to be the candidate point. However, this assumption is not appropriate in our study. Therefore, we set the covariance of image feature vector to be identity, i.e. $\mathbf{S}_g = \mathbf{I}$. Furthermore, a new evaluation function in (20) for candidate landmarks is defined as $\tilde{f} \equiv f^{(0)} + f^{(1)} + f^{(2)}$ in our study, where $f^{(i)}$ is Mahalanobis distance for image feature vector $\mathbf{g}^{(i)}$. The second step of iterative shape searching of WASM (ASM) is to generate a shape in the model space and transform it to obtain a shape $\tilde{\mathbf{y}}^{(k+1)}$ in the image space in order to fit $\mathbf{y}^{(k)}$. $\tilde{\mathbf{y}}^{(k+1)}$ can be calculated by minimizing $\|\tilde{\mathbf{y}}^{(k+1)} - \mathbf{y}^{(k)}\|^2$ using **Algorithm 3** (**Algorithm 2**). The algorithm of WASM (ASM) is summarized in **Algorithm 4**.

5.2 Customized Wavelet Base Initialization

Although RBI incorporated with user interaction can quickly generate an initial shape, this initial shape contains only information of the mean shape, i.e. $\mathbf{b}_l = \mathbf{0}$ (in WASM) or $\mathbf{b} = \mathbf{0}$ (in ASM). We will exploit the information of selected points to design a customized initial shape by using WDM.

Let S_b be the mesh formed by 9 selected points by using user interaction on slices of images illustrated in Fig. 17(c). Assume $\mathbf{y}_b$ is the 27×1 vector formed by concatenation of the coordinates of all the points in S_b . Define

$$f(\mathcal{T}, \mathbf{b}_l, \forall l) \equiv \|\mathbf{y}_b - \mathcal{T}(\tilde{\mathbf{x}}(\mathbf{b}_l, \forall l))\|^2 \quad (21)$$

where

$$\tilde{\mathbf{x}}(\mathbf{b}_l, \forall l) \equiv \mathcal{C}\left(\mathcal{W}^{-1}\left(\bar{\mathbf{w}}_l + \mathbf{P}_l \mathbf{b}_l, \forall l\right)\right). \quad (22)$$

$\mathcal{C}(\bullet)$ is the operator identifying the corresponding control landmarks from a shape in the model space and concatenating the coordinates of all the control landmarks to form a 27×1 vector (same formation as $\mathbf{y}_b$). Therefore, $\tilde{\mathbf{x}}(\mathbf{b}_l, \forall l)$ is a 27×1 vector.

To generate a better initial shape, we need to calculate optimal $\mathcal{T}^*$ and $\mathbf{b}_l^*$, $\forall l$ so that (21) is minimized:

$$(\mathcal{T}^*, \mathbf{b}_l^*, \forall l) = \arg \min_{\mathcal{T}^*, \mathbf{b}_l^*, \forall l} f(\mathcal{T}, \mathbf{b}_l, \forall l) \quad (23)$$

The initial shape can be determined by

$$\tilde{\mathbf{y}}^{(0)} = \mathcal{T}^*\left(\mathcal{W}^{-1}\left(\bar{\mathbf{w}}_l + \mathbf{P}_l \mathbf{b}_l^*, \forall l\right)\right) \quad (24)$$

It can be observed that $f(\mathcal{T}, \mathbf{b}_l, \forall l)$ is a non-linear function. We can use simulated annealing approach to solve it. However, this optimal searching approach is computationally expensive. Therefore, in this study, we propose a simple and efficient approach (CWBI) to estimate an initial shape as follows.

There are two observations for Calmull-Clark subdivision and DSWT of a training shape. The first observation is as follows. Each of the training shapes are remeshed using Calmull-Clark subdivision to generate landmarks [Fig. 9(d)]. To construct WDM, each of the remeshed shapes with regularized landmarks is aligned to the model space using Procrustes alignment and decomposed into scaling coefficients and wavelet coefficients by DSWT. Based on the construction of DSWT described in Section 4.2, the scaling coefficients at each scale correspond to the landmarks at each level of subdivision (i.e. they have same mesh structure). For example, when the scale coefficients at coarsest scale [Fig. 13(c)] is reached, they correspond to the mesh at 0-th Calmull-Clark subdivision [Fig. 9(a)]. Although the mesh of the scaling coefficients at each scale and the remeshed training shape at its corresponding level of Calmull-Clark subdivision have the same mesh structure, they are defined in different ways: the former is in wavelet domain, and the latter is in the model space. The second observation is as follows. In each training shape, there are 8 vertices at the boundaries of the base mesh and one vertex at the central area of the base mesh [Fig. 9(a)]. Since the operation of Calmull-Clark subdivision on these 8 corresponding vertices at the boundaries of the training shape cannot be well-defined using **Algorithm 1** at each level of subdivision, these vertices at the boundaries of the training shape are unchanged at each level of subdivision. Similarly, when the remeshed training shape is decomposed into wavelet coefficients and scaling coefficients, the corresponding 8 scaling coefficients at the boundaries of the scaling mesh cannot be well-defined using (15) at each scale. These 8 corresponding scaling coefficients at the boundaries of scaling meshes are kept unchanged during wavelet decomposition at each scale of DSWT. Therefore, the values of these 8 corresponding vertices at the boundaries of the training shape are unchanged at all levels of Calmull-Clark subdivision and the same as the values of the corresponding 8 scaling coefficients of scaling meshes at all scales of DSWT. For example, the values of 8 vertices at the boundaries of the mesh illustrated in Fig. 13(a) are exactly the same as the corresponding 8 scaling coefficients at the boundaries of the coarsest scaling mesh illustrated in Fig. 9(c). However, the values of the vertex at the central area of the mesh are usually not equal to the values of its corresponding scaling coefficient at the central areas of the coarsest scaling mesh.

Algorithm 5 Customized Wavelet Base Initialization (CWBI)

$\mathcal{T}_b \leftarrow \left(\mathbf{y}_b, \mathbf{P}_{-1}, \bar{\mathbf{w}}_{-1} \right)$; use **Algorithm 2** to calculate the suboptimal transformation $\mathcal{T}_b$ of (26) without shape constraints.

$$\tilde{\mathbf{x}}_b \leftarrow \mathcal{T}_b^{-1}(\mathbf{S}_b)$$

$\mathbf{w}_{-1,b} \leftarrow \tilde{\mathbf{x}}_b$; define a new scaling coefficients $\mathbf{w}_{-1,b}$ at the coarsest scale based on the assumption of (28) and set it equal to $\tilde{\mathbf{x}}_b$.

$$\mathbf{x} \leftarrow \mathcal{W}^{-1} \left(\mathbf{w}_{-1,b} \cup \left\{ \bar{\mathbf{w}}_l, l \geq 0 \right\} \right)$$

$$\hat{\mathbf{y}}^{(0)} \leftarrow \mathcal{T}_b(\mathbf{x})$$

Based on two observations above, we claim that there exists $\mathbf{b}_l, l \geq 0$ so that the value of each element in $\tilde{\mathbf{x}}(\mathbf{b}_l, \forall l)$ [see (22)] can be approximated by the value of each element in $\bar{\mathbf{w}}_{-1} + \mathbf{P}_{-1}\mathbf{b}_{-1}$ given $\mathbf{b}_{-1}$:

$$\tilde{\mathbf{x}}(\mathbf{b}_l, \forall l) \approx \bar{\mathbf{w}}_{-1} + \mathbf{P}_{-1}\mathbf{b}_{-1} \quad (25)$$

Based on the assumption in (25), we reduce complexity of the problem in (23) by calculating

$$(\mathcal{T}^*, \mathbf{b}_{-1}^*) = \arg \min_{\mathcal{T}, \mathbf{b}_{-1}} \|\mathbf{y}_b - \mathcal{T}(\bar{\mathbf{w}}_{-1} + \mathbf{P}_{-1}\mathbf{b}_{-1})\| \quad (26)$$

(26) can be solved (approximately) using **Algorithm 2** without applying shape constraints. Let $\mathcal{T}_b$ be the suboptimal transformation of (26). It will be used to define a transformation between the model space and image space and will play an essential role in the proposed BIWASM. A new base in the model space is defined by

$$\tilde{\mathbf{x}}_b \equiv \mathcal{T}_b^{-1}(\mathbf{y}_b) \quad (27)$$

We define a new scaling coefficient $w_{-1,b}$ at the coarsest scale by assigning the values of $w_{-1,b}$ to be $\tilde{\mathbf{x}}_b$. Based on the similar assumption in (25), we claim that there exists $\mathbf{b}_l, l \geq 0$ so that

$$\tilde{\mathbf{x}}_b \approx \mathcal{C}\left(\mathcal{W}^{-1}\left(\mathbf{w}_{-1,b} \cup \left\{\bar{\mathbf{w}}_l + \mathbf{P}_l \mathbf{b}_l, \forall l \geq 0\right\}\right)\right) \quad (28)$$

By combining local details using the mean wavelet coefficients $\left\{\bar{\mathbf{w}}_l, l \geq 0\right\}$ of WDM, the initial shape can be constructed by

$$\tilde{\mathbf{y}}^{(0)} = \mathcal{T}_b\left(\mathcal{W}^{-1}\left(\mathbf{w}_{-1,b} \cup \left\{\bar{\mathbf{w}}_l, l \geq 0\right\}\right)\right) \quad (29)$$

Algorithm 5 summarizes CWBI. Fig. 17(b) illustrates the initial shape generated by the proposed CWBI.

5.3 Base-Invariant Wavelet Active Shape Model

In most of the studies, the shapes of subjects are closed surfaces such as spherical topology [23, 39, 43–46], open surfaces representing most of the whole subject [41, 47], and tubular topology [48]. The shape in our study is different. It is a partial surface in skull model and is a shape-customized open surface with closed boundary. We observe that WASM and ASM may become unreliable in order to recognize the corresponding partial surface in the skull model. At the first step of WASM and ASM, they recognize the topology of the true partial shape by searching for the topology of neighboring candidate landmarks. The candidate landmarks at the boundary of this open surface are most likely to be out of the true shape (such as the parts of maxilla near nasal bones and orbits). Once the candidate landmarks out of the true shape, the evolving shape at the second step of WASM and ASM will erroneously fit these candidate landmarks. This is because WASM and ASM are involved with update of $\mathcal{T}$ from the model space to the image space. Therefore, it is necessary to incorporate selected points S_b defined in Section 5.2 to constrain the evolving shape.

Based on the concept in Section 5.2, we design a new model-based algorithm (BIWASM) based on WDM to overcome this problem. It is similar to WASM, but the differences are described as follows. First, the transformation between model space and image space is no

longer be calculated during the iterative steps. $\mathcal{T}_b$ in (27) is used to define this invariant transformation. Second, the coarsest scaling coefficients $w_{-1,b}$ will be used instead of generating a new coarsest scaling coefficients w_{-1} using (17). Third, to keep the evolving shape constrained by the selected points S_b , the control landmarks of the evolving shape corresponding to S_b in the image space will be unchanged during the iteration of BIWASM. The proposed BIWASM is summarized in **Algorithm 6**.

6 Validations and Results

Nineteen sets of CBCT images were used for the validation. Their ground truths of bone surface were manually established as described in Section 2. They served as a control group. The outer surfaces of anterior wall of maxilla, illustrated in Fig. 3, were segmented using our BIWASM with CBWI. The same images were also segmented using ASM with Registration-Based Initialization (RBI), WASM with RBI, and WASM with CWBI, respectively. They all served as an experimental group.

Algorithm 6 Base Invariant Wavelet Active Shape Model (BIWASM)

$(\tilde{\mathbf{y}}^{(0)}, \mathcal{T}_b, \mathbf{w}_{-1,b}) \leftarrow (\mathbf{y}_b, \mathbf{P}_{-1}, \bar{\mathbf{w}}_{-1})$; use **Algorithm 5** to calculate the initial shape, the transformation, and scaling coefficients at the coarsest scale.

while Until convergence **do**

$\mathbf{y}^{(k)} \leftarrow \tilde{\mathbf{y}}^{(k)}$; calculate a candidate shape by examining the neighboring region of each of the landmark points, and the corresponding control landmark points in $\mathbf{y}^{(k)}$ are replaced with S_b .

$\mathbf{x}' \leftarrow \mathcal{T}_b^{-1}(\mathbf{y}^{(k)})$; calculate inverse transform of $\mathbf{y}^{(k)}$

$\mathbf{w}_l \leftarrow \mathcal{W}(\mathbf{x}')$; DSWT.

$\mathbf{b}_l \leftarrow \mathbf{P}_l^T (\mathbf{w}_l - \bar{\mathbf{w}}_l), l \geq 0$; calculate the shape parameter fitting $\mathbf{w}_l$ using (19).

Apply the constraints on $\mathbf{b}_l, l \geq 0$ using (6)

$\mathbf{w}_l \leftarrow \bar{\mathbf{w}}_l + \mathbf{P}_l \mathbf{b}_l, l \geq 0$; generate wavelet coefficients using (17)

$\mathbf{x} \leftarrow \mathcal{W}^{-1}(\mathbf{w}_{-1,b} \cup \{\mathbf{w}_l, l \geq 0\})$; inverse DSWT.

$\tilde{\mathbf{y}}^{(k+1)} \leftarrow \mathcal{T}_b(\mathbf{x})$

end while

6.1 Data Preparation

The segmentation dataset, referred as the datasets to test segmentation approaches, were labeled $D_i, i = 1, 2, \dots, 19$. Each of the segmentation datasets D_i consisted of a set of CBCT volumetric images (target datasets) and its corresponding ground truth of bone surface. The model datasets, referred as the training datasets in the base invariant active shape model, were defined by 19 shapes and 19 sets of CBCT volumetric images. They were labeled $M_i, i = 1, 2, \dots, 19$ in the same order. Those 19 shapes were extracted from 19 ground truths of bone surface and were decomposed into patches using the approaches described in Section 3.1. Nine anatomical control landmark points were pinpointed to form four patches in each

of the shapes as illustrated in Fig. 5(a) and (b). The patches in each of the shapes were parameterized (Section 3.2), subdivided five times, remeshed, and stitched to form a remeshed shape (Section 3.3) with 4225 regularized landmarks (illustrated in Fig. 9(d)).

Once N model datasets were built as training datasets, one segmentation (target) dataset, other than N model datasets, was used to compare our developed approaches to the three traditional approaches. The preparation of target dataset was completed in the following four steps and summarized in Fig. 19.

Step 1 was landmark digitization. Nine control landmarks were digitized interactively as described in Section 5.1 for initialization. They were selected based on the same criterion as control landmarks illustrated in Fig. 5(a). A set of 9 landmarks was digitized in each segmentation dataset. Once digitized, these landmarks were used for all the experiments.

Step 2 was initialization. The digitized control landmarks were used to create two initial shapes using two initialization methods: RBI and our newly developed CWBI. RBI was used to register these selected control landmarks of the shapes in the image space and their corresponding control landmarks of mean shape in the model space, and to transform the mean shape in the model space into the image space. The resulted initial shape served as the input of ASM and WASM. CWBI was used to calculate a transformation between the image space and the model space by using those 9 selected points. It was also used to add the mean local details using WDM to obtain an initial shape. The resulted initial shape served as the input of WASM and BIWASM.

Step 3 was to calculate the final shapes. The shapes initialized by RBI were fed into ASM and WASM approaches respectively, resulting in ASM-RBI and WASM-RBI final shapes. They represented the final shapes calculated by the traditional approaches. In addition, the shapes initialized by CWBI were fed into WASM approach and our newly developed BIWASM approaches respectively, resulting in WASM-CWBI and BIWASM-CWBI final shapes. While WASM-CWBI shape represented the outcome generated using our initialization and the traditional segmentation approaches, the latter represented the outcome generated using both our developed initialization and final segmentation approaches. During the computation, the number of the iterations in ASM and WASM approaches (**Algorithm 4**) is 20. The number of the iterations in BIWASM (**Algorithm 6**) is 20. The maximum number of the iterations in **Algorithm 2** is 40, and 10 in **Algorithm 3**. We intentionally reduce the maximum iterative number in **Algorithm 3** to 10 instead of 40. This was because DSWT was computationally expensive. In all of ASM, WASM, and BIWASM approaches, 23 points ($K = 12$) were sampled along a line perpendicular to landmarks in order to examine neighboring region and find candidate landmarks. The sample distance was 0.2mm. In addition, the shape constraints were $a = 3$ for both b in PDM and b_1 in WDM. This step produced four kinds of final shapes: ASM-RBI, WASM-RBI, WASM-CWBI, and BIWASM-CWBI.

Step 4 was to compare the ground truth to the final shapes generated by different approaches. It was done by calculating surface deviations, the closest distances, and Hausdorff distance between the ground truths and the final shapes generated in step 3. Surface deviation was a set of the closest distances between the landmarks of the ground truth and the final shape. Hausdorff distance was also calculated. Therefore, there were 4225 surface distances and one Hausdorff distance produced from each target dataset.

The validation was achieved by two sets of comparisons. In the following, the first set of the comparisons was to detect the variabilities amongst 4 approaches when the number of training datasets was static. The second set of comparisons was to detect the variabilities

amongst 4 approaches when the number of training datasets was dynamic. In addition, the computational times were also compared amongst the 4 approaches.

The first set of comparisons was conducted using leave-one-out arrangement (cross validation). Six groups (total of 69) leave-one-out experiments were conducted: $i = 1, 2, \dots, 19$ (19 experiments), $i = 1, 2, \dots, 16$ (16 experiments), $i = 1, 2, \dots, 13$ (13 experiments), $i = 1, 2, \dots, 10$ (10 experiments), $i = 1, 2, \dots, 7$ (7 experiments), $i = 1, 2, \dots, 4$ (4 experiments). The dataset was randomly selected using SPSS software. In each experiment, the target dataset was excluded from the training datasets. For example, in the second group, the experiment of the datasets $i = 1, 2, \dots, 16$ was conducted using M_i , $i = 1, 2, \dots, 11, 13, \dots, 16$, and the target D_{12} dataset was excluded from the training dataset. After final shapes were generated by four approaches (ASM-RBI, WASM-RBI, WASM-CWBI, BIWASM-CWBI), they were compared to their ground truths. In each of the six groups of experiments, the mean and standard deviation of surface distances were calculated over 80275 (19×4225), 67600 (16×4225), 54925 (13×4225), 42250 (10×4225), 29575 (7×4225), and 16900 (4×4225) surface distances, respectively. The mean Hausdorff distances were also calculated over the 19, 16, 13, 10, 7 and 4 final shapes, respectively. The results are shown in Fig. 20 (a)–(c).

The second set of comparisons was conducted by using 13 segmentation datasets D_i , $i = 1, 2, \dots, 13$ and varying the number of model datasets by 12, 15, and 18. The dataset was also randomly selected using SPSS software. Three groups of the datasets were used to conduct 39 experiments. Again, the target dataset was excluded from the training datasets. The first group of the datasets were D_i , $i = 1, 2, \dots, 13$ and M_i , $i = 1, 2, \dots, 13$. Each experiment was conducted by choosing one segmentation dataset from D_i , $i = 1, 2, \dots, 13$, and the rest 12 model datasets from M_i , $i = 1, 2, \dots, 13$. The second group of the datasets were D_i , $i = 1, 2, \dots, 13$ and M_i , $i = 1, 2, \dots, 16$. Each experiment was conducted by choosing one segmentation dataset from D_i , $i = 1, 2, \dots, 13$, and the rest 15 model datasets from M_i , $i = 1, 2, \dots, 16$. The third group of the datasets were D_i , $i = 1, 2, \dots, 13$ and M_i , $i = 1, 2, \dots, 19$. Each experiment was conducted by choosing one segmentation dataset from D_i , $i = 1, 2, \dots, 13$, and the rest 18 model datasets from M_i , $i = 1, 2, \dots, 18$. In each group, 13 experiments were conducted using each segmentation approach, respectively. Means and standard deviations of surface distances were calculated over 54925 (13×4225) surface distances, respectively. The mean Hausdorff distances were also calculated over the 13, 13, and 13 final shapes, respectively. The results are shown in Fig. 20 (d)–(f).

6.2 Results

The results (Fig. 20) showed that our BIWASM-CWBI approach outperformed the others in each of six groups (the first set of comparisons) and in each of three groups (the second set of comparisons). It also indicated that the more accurate result was achieved with more training dataset. The results indicated that our BIWASM-CWBI approach was capable of capture the outer surface of thin bones (1mm) in the skull model. Fig. 21 shows the visualization of the evolving shapes and the growth truth in a single experiment. This single experiment was in the first set of experiment based on D_i and M_i , $i = 1, 2, \dots, 16, 18, 19$.

Finally, the computational times of the 4 approaches are presented in Table 2. This was calculated in the first set of experiment based on D_i and M_i , $i = 1, 2, \dots, 16, 18, 19$. The computer was Intel i7 2.8Hz with 4G RAM. The result revealed that the computational time of our approach was comparable with that of ASM-RBI and significantly shorter than WASM-RBI and WASM-CWBI.

7 Discussion

The correspondence of landmarks over all the training shapes can be properly constructed structurally and geometrically by using patch decomposition and mesh subdivision. In Section 3, the training shapes are extracted from ground truths of bone surfaces. Each of the training shapes is partitioned into several polygon-like patches. These patches can be customized according to anatomical structures of the training shapes. When the boundaries of patches are chosen along high curvature ridges and edges of the training shapes, structural shape correspondence is created. When each of the patches is characterized by smooth surface and barely has prominent features, the correspondence of landmarks among the corresponding patches can be constructed geometrically by regular subdivision. The shape correspondence constructed by patch decomposition and regular subdivision can be done in a short time compared to the description length approach for model building in [46]. This approach may take hours to days to build a 3D SSM.

The shortest path used to create boundaries of patches is different from other studies. In [43,49–51], they use Dijkstra's algorithm to define patches. It will lead to zigzag boundaries along the patches. Barycentric mapping used to calculate parameterization of patches is simple and fast. It requires the boundary of a patch to be fixed onto a convex topology. However, if the boundary of the convex topology cannot reflect the 3D boundary of the patch, high distortion on parameterization mapping will occur near the boundary [52]. Therefore, the shortest path calculation we apply in the study can prevent it since the patch boundary resembles straight lines on the mesh. Regarding the computational time of shortest path calculation in our study, the calculation of each shortest path in Fig. 5 (b) requires from 2s to 5s. Hence, patch decomposition on each of the training shapes used in our study can be completed in less a minute. WDM provides more dimensions and captures more local features to model a shape than PDM. However, it does not mean WASM can always outperform ASM. We will have two observations from the results to demonstrate it. First, it can be found in the studies of Davatzikos *et al.* [38] and Nain *et al.* [39] that there are no significant differences of mean distance between ASM and WASM in occasional circumstances when the number of the models are increased. Our results also demonstrate the point. The mean distances in ASM-RBI and WASM-RBI have no significant differences when at least 15 models are used [shown in Fig. 20 (a) and (d)]. Second, the STD distance and Hausdorff distance in WASM-RBI are larger than that in ASM-RBI. By visualizing the results on a single case in Fig. 21, the shape at the boundary is distorted in both WASM-RBI and ASM-RBI. WASM-RBI has higher dimensions to capture this distortion while ASM-RBI has fewer dimensions to capture it (instead, ASM-RBI constrains it). Therefore, ASM-RBI has smaller Hausdorff distance than WASM-RBI. Since WASM-RBI still captures local details better than ASM-RBI, it is explained that the range (STD) of surface distance in WASM-RBI can be larger than that in ASM-RBI.

According to the results shown in Fig. 20, our approach BIWASM-CWBI can achieve mean surface distance as low as 0.25 mm and standard deviation of surface distance less than 0.2 mm . Due to the limitation of CBCT, transition of image intensities from soft tissues to bone is very smooth. The thinnest visible anterior wall in CBCT image can be as small as 1 mm ⁶. It means that our approach can capture the outer surface of maxillary thin bones without significant loss of the shapes in the skull model. Quantitatively, we can expect our BIWASM-CWBI incurs 0.25 ± 0.2 surface errors when performing any kinds of surgical planning and simulations in CASS system. Based on our clinical experience and published literature [53–56], there would be no clinical significance if the surface error is less than

⁶This measurement is based on our experiences on CBCT datasets. The real bone thickness of anterior maxillary walls can be smaller than 1 mm .

0.5mm. Therefore, we demonstrate that our approach BIWASM-CWBI is robust in building CBCT skull model for CASS.

8 Conclusion

We develop a segmentation approach BIWASM and an initialization approach CWBI to calculate the outer surface of the anterior wall in maxilla. 19 CBCT datasets are used to test these approaches. We design two sets of the experiments to validate our segmentation approach by using 19 CBCT datasets. Three other model-based segmentation approaches are also applied in the experiments to compare their results. Our proposed approach outperforms the other three approaches in both sets of the experiments. It achieves 0.25 ± 0.2 surface errors. This small degree of deviation does not have clinical significance.

A Composite Lifting Step

Assume the rectilinear grid is applied. Fig. 22 shows an example of the grid. r and c denote the row and column indices. The s -lifting step in the row using (13) is

$$\mathbf{v}_{r,c} \leftarrow \alpha \mathbf{v}_{r,c} + \tilde{\alpha} (\mathbf{e}_{r,c} + \mathbf{e}_{r,c+1}) \quad (\text{A.1})$$

$$\tilde{\mathbf{e}}_{r,c+i} \leftarrow \alpha \tilde{\mathbf{e}}_{r,c+i} + \tilde{\alpha} (\mathbf{f}_{r+i,c} + \mathbf{f}_{r+i,c+1}) \quad (\text{A.2})$$

Similarly, the s -lifting step in the column using (13) can be written by

$$\mathbf{v}_{r,c} \leftarrow \alpha \mathbf{v}_{r,c} + \tilde{\alpha} (\tilde{\mathbf{e}}_{r,c} + \tilde{\mathbf{e}}_{r+1,c}) \quad (\text{A.3})$$

$$\mathbf{e}_{r+i,c} \leftarrow \alpha \mathbf{e}_{r+i,c} + \tilde{\alpha} (\mathbf{f}_{r,c+i} + \mathbf{f}_{r+1,c+i}) \quad (\text{A.4})$$

Incorporated with (A.1), (A.3) can be written as

$$\mathbf{v}_{r,c} \leftarrow \alpha^2 \mathbf{v}_{r,c} + 4\tilde{\alpha}^2 (\bar{\mathbf{f}}_{r,c}) + 4\alpha\tilde{\alpha} \bar{\mathbf{e}}_{r,c} \quad (\text{A.5})$$

where $\bar{\mathbf{e}}_{r,c} \equiv \frac{\tilde{\mathbf{e}}_{r,c} + \tilde{\mathbf{e}}_{r+1,c} + \mathbf{e}_{r,c} + \mathbf{e}_{r,c+1}}{4}$ and $\bar{\mathbf{f}}_{r,c} \equiv \frac{\mathbf{f}_{r,c} + \mathbf{f}_{r+1,c} + \mathbf{f}_{r,c+1} + \mathbf{f}_{r+1,c+1}}{4}$. (A.5) is the composite s -lifting step on vertex point. (A.2) and (A.4) become the s -lifting step of edge points around the vertex $\mathbf{v}_{r,c}$.

B User Interaction for Initialization

The user interaction can be implemented by incorporating CBCT volumetric image and its skull shape [Fig. 23(a)]. This skull shape is calculated by the simplest approach: thresholding segmentation and March Cube Algorithm. It is only used to provide reference for user interaction. Only a slice of the volumetric image is visualized in 3D space each time. It can be switched to the previous slice or the next slice. The skull shape can be temporarily hidden to visualize the internal content of the slice. Therefore, we have two objects to help identify a desired landmark point: the rough skull model and slices of the volumetric image. Landmark points are pinpointed on the slice of the image, and their coordinates are recorded.

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Fig. 1.
A skull model reconstructed by Marching Cube Algorithm after thresholding segmentation is performed on a 3D CBCT volumetric image.

Fig. 2.

The procedure of performing segmentation in the study. The statistical models are acquired off-line (enclosed by dashed lines) by using training CBCT datasets. Once the statistical models are built, a new CBCT image (other than the training datasets) is segmented based on the information of statistical models.

Fig. 3. Four different skull models are illustrated in (a), (b), (c), and (d). The marked regions are anterior maxilla in the left half of skull and will be the particular interest in the study.

Fig. 4.

The procedure of generating training shapes with digitalized landmarks. These training shapes will be used to build SSM in Section 4.

Fig. 5. Training shape extraction and patch decomposition. (a) 9 anatomical control landmarks. (b) Patch decomposition using shortest paths as boundaries. (c) Extraction of the training shape and its patches.

Fig. 6.
(a) One of four patches in Fig. 5(c). (b) The polygonal domain and its parameterization mapping.

Fig. 7. Calmull-Clark subdivision. (a) Triangle base mesh and quadrilateral-like base mesh. (b) The supermeshes generated by one subdivision of the base meshes.

Fig. 8.

The average operators $\mathbf{k}_m^{-j}$ without superscript index j .

Fig. 9.
The remeshed shape with landmarks after different number of Calmull-Clark subdivisions.
(a) 0-th subdivision. (b) First subdivision. (c) Third subdivision. (d) Fifth subdivision.

Fig. 10.
The flowchart of creating WDM and IFM.

Fig. 11.
The decompositions in 1D DWT.

(a)

(b)

Fig. 12.

(a) One decomposition of 2D DWT using lifting steps (b) One decomposition of 2D DWT using composite lifting steps.

Fig. 13.

DSWT of Fig. 9(d) (a) Scaling coefficients after one decomposition of DSWT. (b) Scaling coefficients after three decomposition of DSWT. (c) Scaling coefficients after five decompositions of DSWT

Fig. 14.

(a) Different patch decomposition applied to the same shape in Fig. 5. (b) The extracted shape (c) The shape at five Calmull-Clark subdivisions. (d) Scaling coefficients after one decomposition of DSWT. (e) Scaling coefficients after three decomposition of DSWT. (f) Scaling coefficients (base) after five decompositions of DSWT

Fig. 15.

(a) At each of the landmarks, a line $\mathcal{L}$ (dashed line) perpendicular to the training shape in the image space is defined for calculating the image feature. (b) The interpolated image profile on sample points along line $\mathcal{L}$. Without loss of generality, z_0 is the landmark.

Fig. 16.
The flowchart of performing the proposed CWBI and BIWASM.

Fig. 17.

(a) Nine control landmarks of a shape are defined and derived from anatomical control landmarks of the mesh at 0-th subdivision in Fig. 9(a). (b) Four selected points using user interaction (Appendix B). These points with labeled numbers are assumed to be exactly the same as the corresponding (See the corresponding numbers) control landmarks of the true shape. (c) Nine selected points correspond to all the control landmarks of the true shape.

(a)

(b)

Fig. 18.

(a) The initial shape (accompanied with a slice in the image space) calculated by RBI using 9 selected points. (b) The initial shape (accompanied with a slice in the image space) calculated by CWBI using 9 selected points.

Fig. 19.
The flowchart of data preparations.

Fig. 20.

(a), (b), and (c) are mean surface distances, standard deviation of surface distances, and mean Hausdorff distances in the first set of the experiments. (d), (e), and (f) are mean surface distances, standard deviation of surface distances, and mean Hausdorff distances in the second set of the experiments.

Fig. 21.

The initial and final shapes (blue meshes) calculated by four approaches using the segmentation dataset (D_{17}) and 18 model datasets. (a) The left half of the anterior surface of the skull is segmented and extracted manually from the ground truth of bone surfaces to validate the results. It is visualized as the red mesh in the rest of the subfigures. (b) and (e) are the initial and final shapes in ASM-RBI, respectively. (c) and (f) are the initial and final shapes in WASM-RBI, respectively. It can be found (b) and (c) are the same when they are calculated using RBI under PDM and WDM. (d) is the initial shape in both WASM-CWBI and BIWASM-CWBI. The final shapes in WASM-CWBI and BIWASM-CWBI are shown in (g) and (h), respectively.

Fig. 22.
An example of a rectilinear grid.

Fig. 23.

(a) Selection of landmark points for initialization using user interaction. (b) The same user interaction but the skull shape is hidden to visualize the internal context of the slice.

Table 1

List of Acronyms

ASM	Active Shape Model
BIWASM	Base Invariant Wavelet Active Shape Model
CASS	Computer-aided Surgical Simulation
CBCT	Cone-beam Computed Tomography
CMF	Craniomaxillofacial
CT	Computed Tomography
CWBI	Customized Wavelet Base Initialization
DSWT	Discrete Surface Wavelet Transform
DWT	Discrete Wavelet Transform
HU	Hounsfield Unit
IFM	Image Feature Model
RBI	Registration-based Initialization
SSM	Statistical Shape Model
PDM	Point Distribution Model
WASM	Wavelet Active Shape Model
WDM	Wavelet Distribution Model

Table 2

The Computation Times in the First Experiment

ASM-RBI	WASM-RBI	WASM-CWBI	BIWASM-CWBI
164s	575s	618s	205s