Determination of the electron density in methyl ($\pm$)-(1*S*,2*S*,3*R*)-2-methyl-1,3-diphenylcyclopropanecarboxylate using refinements with X-ray scattering factors from wavefunction calculations of the whole molecule

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The molecule of the title compound, C₁₈H₁₈O₂, is a substituted cyclopropane ring. The electron density in this molecule has been determined by refining single-crystal X-ray data using scattering factors derived from quantum mechanical calculations. Topological analysis of the electron densities in the three cyclopropane C—C bonds was carried out. The results show the effects of this substitution on these C—C bonds.

Keywords: crystal structure; electron density; molecular modelling.

1. Introduction

The large departure of the bond angles in three-membered cyclopropane rings from tetrahedral angles and the resulting strain energy are of great theoretical interest. The research described in this paper is an attempt to extend the theoretical and quantitative information on the molecular structures and electron-density distributions of cyclopropane molecules

available through combining X-ray crystallography and quantum mechanical calculations of a substituted cyclopropane. In their development of invariums, Ditttrich and co-workers have defined a procedure by which Gaussian wave-

functions are used to project theoretical electron densities onto the multipole model (Ditttrich *et al.*, 2005). In this method, theoretical structure factors are generated by the program *TONTO* (Jayatilaka & Grimwood, 2003) from the Gaussian wavefunction, and then least-squares refinements give the populations of the multipoles. We have determined the nature of the electron density in the cyclopropane ring in the title compound, (I), by this method and observed the effects of substitution on the covalent bonding between these C atoms.

2. Experimental

2.1. Synthesis and crystallization

A mixture of *trans*- β -methylstyrene (2.5 mmol) and AgOTf (0.05 mmol) was weighed into a 25 ml one-necked round-bottomed flask covered with aluminium foil to exclude light. The mixture was dissolved in dichloromethane (2 ml) and stirred at room temperature under an atmosphere of argon. Methyl phenyldiazoacetate (0.5 mmol) in dichloromethane (8 ml) was then added to the solution *via* syringe pump over a period of 3 h. After addition, the mixture was stirred for an additional 1 h and then concentrated *in vacuo*. Purification by silica-gel chromatography (using 20:1 *v/v* hexane–Et₂O as the solvent system) gave the product in 64% yield (86 mg) as colorless crystals. ¹H NMR (600 MHz): δ 7.11–7.7.13 (*m*, 2H), 6.76–7.05 (*m*, 6H), 7.02–7.06 (*m*, 2H), 3.65 (*s*, 3H), 3.08 (*d*, 1H, *J* = 7.8 Hz), 2.25 (*dq*, 1H, *J* = 7.2 and 6.0 Hz), 1.47 (*d*, 3H, *J* = 6.0 Hz); ¹³C NMR (100 MHz): δ 172.3, 137.0, 136.4, 131.3, 127.9, 127.8, 127.6, 126.9, 126.0, 52.3, 42.4, 37.7, 27.3, 12.9. The spectroscopic data are consistent with the previously reported results (Thompson & Davies, 2007).

2.2. Calculation of structure factors

Nonspherical scattering factors were obtained from theoretical calculations of (I) by following a procedure developed

Figure 1
The molecular structure of (I), with the atom-numbering scheme. Displacement ellipsoids are drawn at the 50% probability level.

Table 1

Experimental details.

Crystal data	
Chemical formula	C ₁₈ H ₁₈ O ₂
<i>M_r</i>	266.32
Crystal system, space group	Monoclinic, <i>P</i> ₂ ₁ / <i>c</i>
Temperature (K)	173
<i>a</i> , <i>b</i> , <i>c</i> (Å)	16.9049 (7), 7.4168 (3), 11.7545 (5)
β (°)	102.869 (2)
<i>V</i> (Å ³)	1436.76 (10)
<i>Z</i>	4
Radiation type	Mo <i>K</i> α
μ (mm ⁻¹)	0.08
Crystal size (mm)	0.74 × 0.54 × 0.26
Data collection	
Diffractometer	Bruker APEXII area-detector diffractometer
Absorption correction	Empirical (using intensity measurements) (<i>SADABS</i> ; Bruker, 2008)
<i>T_{min}</i> , <i>T_{max}</i>	0.829, 1.000
No. of measured, independent and observed [<i>I</i> > 3 σ (<i>I</i>)] reflections	52859, 11659, 9819
<i>R_{int}</i>	0.038
(<i>sin</i> θ / λ) _{max} (Å ⁻¹)	0.993
Refinement	
<i>R</i> [<i>F</i> ² > 2 σ (<i>F</i> ²)], <i>wR</i> (<i>F</i> ²), <i>S</i>	0.030, 0.027, 1.60
No. of reflections	9819
No. of parameters	199
No. of restraints	0
H-atom treatment	All H-atom parameters refined
$\Delta\rho_{\text{max}}$, $\Delta\rho_{\text{min}}$ (e Å ⁻³)	0.18, -0.34

Computer programs: *APEX2* (Bruker, 2011), *SAINT* (Bruker, 2009), *SHELXS97* (Sheldrick, 2008), *XD2006* (Volkov *et al.*, 2006), *OLEX2* (Dolomanov *et al.*, 2009) and *AIMAll* (Keith, 2012).

by Dittrich *et al.* (2006). Density functional theory (DFT) calculations were carried out using the B3LYP method in the *GAUSSIAN09* suite of programs (Frisch *et al.*, 2009). A Dunning–Huzinaga full double- ζ basis set with 3*df* and 3*dp* polarization functions was used (Dunning & Hay, 1977). Calculations were carried out using 1164 basis functions. The wavefunction was calculated for the nuclear configuration

Figure 2

A molecular graph of (I), with the positions of the bond (3,-1) and ring (3,+1) critical points shown as small spheres.

geometry from the crystal structure. Electron densities were derived by evaluating the Gaussian orbitals using the programs *AIMAll* (Keith, 2012) and *TONTO* (Jayatilaka & Grimwood, 2003). An artificial crystal of (I) (space group *P* $\bar{1}$), with a cubic cell edge of 30 Å and the molecule centered at the position ($\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{4}$), was constructed. Structure factors for this crystal were generated by *TONTO* up to a resolution of (*sin* θ)/ λ = 1.15 Å⁻¹. Thermal smearing was ignored and the contributions of the core electrons were included in structure-factor calculations. Least-squares refinements of the multipole populations parameters with these structure factors, and the fixed geometry from the crystal structure, yielded a multipole model suitable for reproducing the electron density in the molecule. This method provides a means of calculating the atomic electron densities without having to partition individual atoms into pseudo-atoms. These multipole parameters, together with the orientation and symmetry of the local atomic coordinate system, were transferred to the input files for a refinement with the *XDLSM* program of the *XD2006* suite.

2.3. Refinement

Crystal data, data collection and structure refinement details are summarized in Table 1. The program *InvariomTool* (Hübschle *et al.*, 2007) was used to prepare the master and input files for a multipole refinement with the *XDLSM* program of the *XD2006* suite (Volkov *et al.*, 2006). Full matrix least-squares refinements on *F*² using complete multipole expansions were carried out with the program *XDLSM* using statistical weights. Only reflections with intensities *I* > 3 σ (*I*) were included in the refinement. In the final cycles, all atom positions were refined freely. Positional and displacement (anisotropic for non-H atoms) parameters, but not multipoles, were refined. However, a hexadecapolar level of the multipole expansion was used for all atoms. The introduction of the multipole model improved *R*(*F*) from 0.043 to 0.030, using the same weighting scheme [$w = 1/[\sigma^2(F_o^2)]$] as with the spherical atom refinement, and improved the goodness-of-fit value from 2.819 to 1.597.

3. Results and discussion

The molecular structure of (I) (Fig. 1) can be derived from that of cyclopropane by replacing two H atoms on C2 by a methyl formate group and a phenyl ring, one H atom on C3 with a methyl group, and one H atom on C4 with a phenyl ring. The conformation of this compound is of interest. The phenyl rings across the C–C bond give rise to 1,3-steric repulsion, which favors a staggered arrangement in an unbridged C–C bond. The steric repulsion between these groups might influence the electron density in the bond and is likely to have an effect on the geometry of the molecule. The electronic interaction between the substituents and the ring is also of interest. Methyl, phenyl and methyl formate groups might be expected to withdraw electron density and weaken the C–C bonds.

A large number of basis functions for atoms with a large number of electrons are needed to yield accurate results. This

Table 2

Analysis of the electron density at the critical points between C atoms C2—C3, C3—C4 and C4—C2.

Property	X-ray refinement	Gaussian wavefunction
C2—C3		
Bond length (Å)	1.5338 (4)	1.5348
$\rho(r)$ (e Å ⁻³)	1.48	1.54
Bond path (Å)	1.538	1.537
Bond-path angle (°) C4—C2—C3	67.69	67.27
Ellipticity, ε	0.609	0.529
$\nabla^2\rho(r)$ (e Å ⁻⁵)	-4.589	-2.875
λ_1	-10.157	-10.679
λ_2	-6.31	-6.99
λ_3	11.88	7.39
C3—C4		
Bond distance (Å)	1.5019 (4)	1.5016
$\rho(r)$ (e Å ⁻³)	1.62	1.67
Bond path (Å)	1.506	1.504
Bond-path angle (°) C2—C3—C4	67.69	67.27
Ellipticity, ε	0.609	0.529
$\nabla^2\rho(r)$ (e Å ⁻⁵)	-4.589	-2.875
λ_1	-10.157	-10.679
λ_2	-6.31	-6.99
λ_3	11.88	7.39
C4—C2		
Bond distance (Å)	1.5421 (4)	1.54233
$\rho(r)$ (e Å ⁻³)	1.46	1.51
Bond path (Å)	1.545	1.544
Bond-path angle (°) C3—C4—C2	63.85	63.38
Ellipticity, ε	0.67	0.57
$\nabla^2\rho(r)$ (e Å ⁻⁵)	-3.94	-2.74
λ_1	-9.96	-10.53
λ_2	-5.95	-6.69
λ_3	11.97	7.43

compound has 142 electrons, of which 71 are designated β electrons. Calculations were carried out using 1164 basis functions. The nuclear geometry from the crystal structure was used for the wavefunction calculations. Subsequent X-ray refinements with the new scattering factors yielded a geometry-optimized structure. Topological analysis of the resulting densities in the cyclopropane C—C bonds was carried out using *AIMAll* (Keith, 2012) and the values compared with those derived from X-ray refinements using scattering factors derived from the quantum mechanical calculations. This is useful for testing how well the calculated electron densities are projected onto multipole crystal structure refinements and whether the *XD2006* (Volkov *et al.*, 2006) densities represent the theoretical densities faithfully. The results are presented in Table 2. There is a correlation between the densities with some differences, but the match seems to be acceptable.

The C—C bonds are expected to be weakened by strain. The characteristic C—C bond length in cyclopropane is 1.50 Å, shorter than the normal C—C bond length of 1.54 Å (Cambridge Structural Database, Version 5.34; Allen, 2002). The three C atoms in a symmetrically substituted cyclopropane form the geometry of an equilateral triangle [see, for example, Hartman & Hirshfeld (1966)]. In cyclopropane (I), the ring does not have C_3 symmetry but has two normal C—C

Table 3

Selected geometric parameters (Å, °).

C2—C3	1.5338 (4)	C14—C15	1.3941 (5)
C3—C4	1.5019 (5)	C14—H14	0.980 (7)
C4—C2	1.5421 (4)	C5—C10	1.3999 (4)
C2—C1	1.4992 (4)	C10—C9	1.3970 (4)
O1—C1	1.3395 (5)	C10—H10	0.981 (6)
O2—C1	1.2150 (5)	C7—C8	1.3948 (4)
C3—C11	1.5096 (5)	C7—H7	0.980 (7)
C3—H3	1.015 (8)	C15—C16	1.3948 (4)
C12—C13	1.3984 (4)	C15—H15	0.991 (6)
C12—C2	1.4989 (4)	C16—H16	0.974 (7)
C12—C17	1.3961 (4)	C9—C8	1.3915 (5)
C4—C5	1.4880 (4)	C9—H9	0.977 (7)
C4—H4	1.011 (8)	C11—H11A	0.939 (9)
C13—C14	1.3939 (4)	C11—H11B	0.966 (10)
C13—H13	0.978 (7)	C11—H11C	0.962 (9)
C6—C5	1.4026 (4)	C8—H8	0.979 (7)
C6—C7	1.3933 (4)	C18—H18A	0.960 (11)
C6—H6	0.985 (6)	C18—H18B	0.940 (11)
C17—C16	1.3980 (4)	C18—H18C	0.951 (10)
C17—H17	0.973 (6)		
C2—C4—C3	60.50 (2)	C3—C2—C12	119.41 (3)
C3—C2—C4	58.46 (2)	C3—C2—C1	115.91 (3)
C4—C3—C2	61.05 (2)	C12—C2—C1	115.92 (3)
C4—C3—C11	121.26 (3)	C12—C17—C16	120.39 (2)
C4—C3—H3	114.1 (5)	C13—C14—C15	119.67 (2)
C2—C3—C11	124.29 (3)	C4—C5—C6	123.32 (3)
C2—C3—H3	111.3 (6)	C4—C5—C10	118.28 (3)
C11—C3—H3	114.4 (6)	C6—C5—C10	118.24 (2)
C13—C12—C2	119.66 (3)	C5—C10—C9	121.13 (3)
C13—C12—C17	118.96 (2)	C6—C7—C8	120.27 (3)
C2—C12—C17	121.39 (2)	C6—C7—H7	120.0 (5)
C3—C4—C5	122.63 (3)	C14—C15—C16	119.94 (2)
C3—C4—H4	115.6 (6)	C1—O1—C18	115.32 (5)
C5—C4—H4	112.9 (6)	O1—C1—O2	122.61 (4)
C12—C13—C14	120.95 (3)	O1—C1—C2	111.95 (3)
C12—C13—H13	118.4 (4)	O2—C1—C2	125.42 (4)
C14—C13—H13	120.7 (4)	C17—C16—C15	120.07 (3)
C5—C6—C7	120.79 (3)	C10—C9—C8	119.90 (3)
C5—C6—H6	119.2 (4)	C7—C8—C9	119.67 (2)
C7—C6—H6	120.0 (4)		

bonds [1.5421 (4) and 1.5338 (4) Å] and one short bond [1.5019 (4) Å; Table 3]. The shortest C—C bond occurs between the hydride-substituted C atoms distal to the formate group, while the longest bonds are to the C atom bonded to methyl formate. The basicity of the CO₂ group is expected to reduce the electron density in the two C—C bonds attached to this C atom, lengthening these two bonds (and shortening the distal bond). This effect is reflected in the atomic charges and electron densities at the bond-critical points (Table 2). The shortest bond occurs where the phenyl and methyl substituents are *trans*, for which repulsion between these 1,4-bonded atoms will be less. The molecular graph is shown in Fig. 2. Bond-critical points were located between the nuclei. Another two bond-critical points were located that are not normally associated with chemical bonds. The distance between atoms H6 and C12 is only 2.709 (7) Å and a (3,−1) critical point exists between them. The second (3,−1) critical point is associated with a weak intramolecular hydrogen bond (C11—H11A...O2; Table 4). The existence of a (3,−1) critical point is insufficient evidence for chemical bonding between these two atoms (Haaland *et al.*, 2004), but it does show that electron density accumulates preferentially along this direction, thereby stabilizing this short contact.

Figure 3

A deformation electron-density map of (I), in the plane of the C atoms belonging to the cyclopropane group. Contours are drawn at $0.02 \text{ e } \text{\AA}^{-3}$ intervals. Solid lines represent positive contours and dashed lines negative contours. The charge is concentrated in the plane of this ring, with the largest bonding concentrations lying outside the triangle shown. The charge drops off more quickly perpendicular to this plane, resulting in a flattened shape. The local charge concentrations are greatest between atoms C3–C4, distal to the carboxylate group.

A difference electron-density map in the plane of the cyclopropane ring is shown in Fig. 3. It is evident from this plot that there is an accumulation of charge over the entire cyclopropane triangle area and beyond the C–C bonds. Critical points (CP), due to C–C bonding, are displaced by $0.031\text{--}0.041 \text{ \AA}$ from the C–C internuclear axes. The electron densities at the critical points for the C–C bonds, calculated by *XD2006*, are in the range $1.48\text{--}1.61 \text{ e } \text{\AA}^{-3}$. The largest peak,

Figure 4

A contour plot of the negative Laplacian $[-\nabla^2\rho(r)]$ of the charge density in the symmetry plane of the cyclopropane. Solid lines represent positive contours and dashed lines negative contours.

Figure 5

A relief plot of the negative Laplacian $[-\nabla^2\rho(r)]$ of the charge density in the symmetry plane of the cyclopropane. The bonded concentrations between atoms C3 and C4 are the largest.

and the shortest bond path, occur between atoms C3 and C4. The smallest bond-path angle (63.85°) corresponds to the internuclear angle C4–C2–C3 and the longest C–C bonds. These distances indicate that the carboxylate group withdraws some of the valence density from these bonds, thereby lengthening them and decreasing the angles between them. These bond-path angles are greater than the bond angles, but still substantially smaller than the tetrahedral angle normally associated with C–C bonds.

The electron density is at a minimum near the center of the plane of the three C atoms and this point is a (3,+1) critical point of the electron density. The electron density in this ring critical point is very large ($0.190 \text{ e } \text{\AA}^{-3}$), even though this point is the local minimum of the electron density. The bond ellipticities give a measure of how much electron density accumulates normal to the direction of the bond. Single bonds in linear alkanes typically have ellipticities close to 0. The

Figure 6

A positive deformation density isosurface ($0.009 \text{ e } \text{\AA}^{-3}$), viewed in the plane of the ring.

Table 4

Hydrogen-bond geometry (Å, °).

$D-H\cdots A$	$D-H$	$H\cdots A$	$D\cdots A$	$D-H\cdots A$
C4—H4 $\cdots$ O2	1.011 (9)	2.453 (11)	2.8877 (6)	105.2 (6)
C11—H11A $\cdots$ O2	0.939 (11)	2.485 (9)	3.1200 (7)	125.0 (8)
C17—H17 $\cdots$ O2 ⁱ	0.972 (7)	2.530 (6)	3.4020 (6)	149.2 (5)

Symmetry code: (i) $x, -y + \frac{3}{2}, z - \frac{1}{2}$.

bond ellipticities in these three C—C bonds are very large, even considerably larger than those found in ethylene and aromatic systems (Bader, 1983).

A contour and a relief map of the negative Laplacian of the charge density are shown in Figs. 4 and 5, respectively. These plots show the continuous character of the shared interactions between these three C atoms. The electronic charge concentrations spreads out from the three bond paths. The valence shell concentrations of charge between atoms C3 and C4 is larger than those between the other two pairs of C atoms. The bonding electron density has a flattened form in the plane of the ring, perpendicular to the bond paths (Fig. 6). This shape of the electron density is a consequence of the very small bond angles. In ethane, there is a large concentration of electrons along the C—C internuclear axis. In cyclopropane, charge is carried away from between carbon nuclei due to repulsive exchange interactions between the electrons but remains in the internuclear plane. A triple torus of charge density encircles these three C atoms. The close proximity of the C atoms forces the electron density to flatten out away from the bond path angles towards and away from the center of the ring.

Cyclopropyl groups resemble π -systems (Bader, 1983), and the total charge distribution on this fragment of (I) makes it suitable for conjugation with the carboxylate group and the neighboring phenyl rings.

4. Conclusion

These results show a cyclopropane group with an asymmetric and highly flattened bonding charge density. Lengthening of the C2—C4 and C2—C3 bonds and shortening of the C3—C4 bond, due to the withdrawal of electron density by the carboxylate group, is reflected by the electron densities at the bond-critical points. The spread out density is likely to have

been enhanced by the proximity of the two phenyl rings and the carboxylate group.

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Supplementary data for this paper are available from the IUCr electronic archives (Reference: QS3027). Services for accessing these data are described at the back of the journal.

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