RESEARCH

Object position and image magnification in dental panoramic radiography: a theoretical analysis

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Objectives: The purpose of our study was to investigate how image magnification and distortion in dental panoramic radiography are influenced by object size and position for a small round object such as a ball bearing used for calibration.

Methods: Two ball bearings (2.5 mm and 6 mm in diameter) were placed at approximately the same position between the teeth of a plastic skull and radiographed 21 times. The skull was replaced each time. Their images were measured by software using edge detection and ellipse-fitting algorithms. Using a standard definition of magnification, equations were derived to enable an object's magnification to be determined from its position and *vice versa* knowing the diameter and machine parameters.

Results: The average magnification of the 2.5 mm ball bearing was 1.292 (0.0445) horizontally and 1.257 (0.0067) vertically with a mean ratio of 1.028 (0.0322); standard deviations are in parentheses. The figures for the 6 mm ball bearing were 1.286 (0.0068), 1.255 (0.0018) and 1.025 (0.0061), respectively. Derived positions of each ball bearing from magnification were more consistent horizontally than vertically. There was less variation in either direction for the 6 mm ball bearing than the 2.5 mm one.

Conclusions: Automatic measurement of image size resulted in less variation in vertical magnification values than horizontal. There are only certain positions in the focal trough that achieve zero distortion. Object location can be determined from its diameter, measured magnification and machine parameters. The 6 mm diameter ball bearing is preferable to the 2.5 mm one for more reliable magnification measurement and position determination.

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Introduction

Panoramic radiography is a common imaging technique in general dental practice as it provides a view of the teeth and surrounding structures. Millions of these radiographs are taken annually for diagnostic¹ and treatment purposes.² Many dentists only use panoramic radiography for dental implant assessment, despite the improved measurement accuracy given by three-dimensional imaging methods such as cone beam CT. In a study by Sakakura et al,³ nearly two-thirds (63.8%) of dentists attending a Brazilian implant

meeting only prescribed panoramic radiography. This may be related to the cost of cross-sectional imaging techniques, their less familiarity with interpreting these images and the limited availability of these techniques in some countries. Panoramic radiography does not provide information about the thickness of host bone, but, despite this, Vazquez et al⁴ found it to be a safe method of evaluating posterior mandibular implant placement as no permanent sensory disturbances were reported in 2584 implant placements in this region. However, surgeons may be accounting for image magnification by placing implants at less than optimal depth. It is important that the clinician is fully aware of the image distortion in surgical planning.

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One disadvantage to the use of panoramic radiography is the magnification and distortion that occurs when the patient's jaws are not positioned near the focal trough of the X-ray beam. In a study of panoramic dental radiographs taken in general dental practices, one-third had low density and contrast due to poor patient positioning⁵ and less than 1% had excellent quality. Post-image processing could enhance some digital images and improve their diagnostic quality, but only to some extent.

Even when properly taken, dental panoramic radiography images are associated with enlargement of the actual object size by about 15–25%, and distortion occurs when horizontal magnification differs from vertical magnification with poor patient positioning.⁶ Magnification may also be influenced by variation in a patient's jaw shape and size,⁷ and even within a particular jaw distortion and magnification are greatest in the canine and premolar regions and least in the third molar region.⁸

This magnification is not easy to predict clinically. Despite using aiming lights to aid positioning the patient correctly, the clinician remains unaware of the true position of the focal trough of the X-ray beam. Many design features of the radiographic machine, such as the speed of movement of the digital or film receptors relative to the X-ray beam and the number of rotation centres, vary between different models. Wear of moving components in the same machine model can also cause slight variations in the position of the focal trough. This results in uncertainty in an ability of the clinician to position the patient's teeth and jaws at the focal trough. We disagree with the conclusion of Freedman and Matteson⁹ that "image magnification is constant at the plane of focus". It has been assumed that, when the patient's teeth are placed in the focal trough, distortions of the image are avoided. This is not true for all vertical positioning. In addition, even small variations in horizontal position cause image distortion.

Two different approaches have been used historically to try to circumvent the problems associated with image magnification. The first was to develop^{10–14} mathematical theory of dental panoramic imaging, but surprisingly no ready answers exist in the literature. The second, practical approach was to use calibration objects with which to calculate magnification at some points on the image. We combined both approaches by developing a set of equations that are precise yet simple enough to be solved easily for magnification from object position, and *vice versa*, for calibration ball bearings and other small round objects.

The aim of our work was to investigate image magnification and distortion in panoramic radiography to quantify how it depends on object position. A lot of mathematical theory of dental imaging exists in the literature,^{10–14} but the definitions and equations are overly complicated. We provide a set of equations that are precise yet simple enough to be solved easily

for magnification from object position, and *vice versa*, for calibration ball bearings and other small round objects.

Methods

Experimental set-up and automatic image measurement

A spherical steel ball bearing, either 2.5 mm or 6 mm in diameter, was placed between the premolar teeth of a skull. The skull was encased in a plastic tissue equivalent and underwent a dental radiographic examination using a Proline PM 2002 CC dental panoramic machine (Planmeca OY, Helsinki, Finland). The plastic surrounding the bony skull simulated soft tissue on the radiographic image. The purpose of the plastic skull was to simulate the typical variation in object placement that would be observed in clinical practice and to observe the corresponding effect on the magnification and distortion. The plastic skull was positioned with the aid of light beam markers placed on equivalent patient landmarks (Frankfort plane, sagittal plane and between the left maxillary canine and lateral incisor). Care was taken that the ball bearing was in the focal trough of the beam by aligning the skull correctly. The skull was removed from the machine and replaced in the same position on each occasion. This procedure was undertaken 21 times with each ball bearing, to simulate repeated radiographic examination of patients. The skull was irradiated at 70 kVp and 8 mA. The cassettes used were Kodak GP Storage Phosphor screens (Carestream Health Inc., Rochester, NY) of size 15 × 30 cm. The irradiated phosphor plates were read in a Kodak DirectView CR 850 reader system (Carestream Health Inc.). The files were stored in digital imaging and communications in medicine format in the hospital picture archiving and communication system for later access. The target anode to focal trough distance for the Proline was 400 mm and the distance from the focal trough to the film was 80 mm, provided by the manufacturer.

To reduce variation in the measurements due to subjective assessment, we used image-processing techniques¹⁵ to find edge points, then used a numerical algorithm to fit an ellipse to the edge points with subpixel precision.¹⁶ For software edge detection, a Sobel filter was used with automatic choice of threshold in MATLAB. An ellipse was fitted by modifying the algorithm and MATLAB code in Halir and Flusser¹⁷ to leave out the xy term. The size of each image was determined from the fitted ellipse parameters. The smaller ball bearing images were enhanced before the ellipse fitting to bring out more details around the edges. The large ball bearings did not require any further enhancement to make the edge strength similar to that of the small ball bearings. Further enhancement of the large ball bearings would not significantly improve the edge strength and was therefore unnecessary.

Definitions of magnification

We define

$$\alpha = \frac{\text{Image width}}{\text{BB diameter}} \text{ and } \beta = \frac{\text{Image height}}{\text{BB diameter}} \quad (1)$$

as the horizontal and vertical magnification. This is a standard definition in the form of image size/actual size; see, for example, Burger et al.¹⁸ More elaborate definitions were used in Welander,¹⁰ Tronje et al.^{11–13} and Welander et al.¹⁴ to describe what happens *in situ* while the object is being radiographed. In particular, Tronje et al.¹³ studied magnification of a spherical object. Their vertical magnification accounts for part of the image above the projection line through the centre. However, the lower part of the image will be slightly smaller because of the elevation of the object. Their definition of horizontal magnification uses a curved distance in the denominator. They gave formulae for calculation of their magnification from the position of the object and various machine-specific parameters. We provide a simpler set of equations for our definition and give a proof in Appendix A. The advantage of using α and β as defined above is that they are directly calculable from off-line measurements of object and image sizes without knowing the object position.

We define the overall magnification as

$$\gamma = \sqrt{\alpha\beta} = \frac{\sqrt{\text{Image width} \times \text{Image height}}}{\text{BB diameter}} \quad (2)$$

which is the ratio of the diameter of a circle with the same area as the ball bearing image (approximated by an ellipse) to the diameter of the ball bearing.

Theoretical dependence of magnification on object position and size

Figure 1 illustrates a ball bearing undergoing panoramic radiography. The X-ray beam moves around a vertical rotation centre and is effectively fan shaped with zero width. The image receptor (film or digital) moves relative to the X-ray beam. This results in a sharply depicted layer at a position between the rotation centre and the film/sensor.¹¹

Using the definition in Equation (1), the exact horizontal (α) and vertical (β) magnifications are given by

$$\alpha = \frac{\phi d}{r s} (s - c), \text{ where } \phi = \sin^{-1} \left(\frac{r \cos \theta}{a - c} \right) \quad (3)$$

$$\beta = \frac{d \sqrt{a^2 + b^2 - r^2}}{a^2 - r^2} \quad (4)$$

where a is the horizontal position (and b is the vertical position) of the centre of the ball bearing (of radius r) relative to the radiation source, c is the horizontal distance from the rotation centre to the radiation source, d is the horizontal distance from the film to the source of radiation and s is the horizontal distance from the sharply depicted layer to the source of radiation.

Figure 1 Projection geometry of a ball bearing in a panoramic radiograph. The plane containing the solid lines and circle (for image width) is perpendicular to that containing the dotted ones (for image height). The positions of the sharp layer (not depicted) should be somewhere close to a

The angle θ as depicted in Figure 1 is that of the ray going through the centre of the ball bearing. For the Proline panoramic machine that we used, it was given that $c = 305$ mm, $d = 480$ mm and $s = 400$ mm.

Letting $r \rightarrow 0$ in Equations (3) and (4), we have in the limits

$$\alpha = \frac{d(s-c)}{s(a-c)} \cos \theta, \beta = \frac{d}{a \cos \theta} \quad (5)$$

The above can be used as the definition of the magnification of a dot (with radius $r = 0$) or as an approximation when r is positive but small relative to a and $a - c$. It follows that the overall magnification

$$\gamma = \sqrt{\alpha\beta} = \sqrt{\frac{d(s-c)}{s(a-c)} \cos \theta \frac{d}{a \cos \theta}} = \sqrt{\frac{d^2(s-c)}{as(a-c)}} \quad (6)$$

does not depend on the vertical position b . This can be solved easily to find a from γ . It will be a good approximation when r is small. The value of b can then be found from Equation (4). The solutions are

$$a = \frac{c}{2} + \sqrt{\frac{d^2(s-c)}{s\alpha\beta} + \frac{c^2}{4}}, b = \frac{a}{d} \sqrt{a^2\alpha^2 - d^2} \quad (7)$$

For example, with $\alpha = \beta = d/s$ we get $\alpha = s$ and $b = 0$. Thus a dot positioned at the centre of the sharp layer with zero elevation will have the nominal magnification d/s , and it is the only position with this magnification. For the machine that we used $d/s = 480/400 = 1.2$.

Moving rotation centre

Modern machines have a moving rotation centre and thus the distance c from the rotation centre to the radiation source may change during the excursion of the beam from beginning to end. However, as a reasonable approximation, the rotation centre and thus the distance c may be regarded as fixed during the exposure of the ball bearing owing to its positioning and size resulting in a small amount of change relative to the distances a , b , d and s . Therefore Equations (3–6) still apply as a reasonable approximation.

Determining ball bearing position from magnification

Using the exact Equations (3) and (4) for $r > 0$, we can find a by solving the sixth-order polynomial equation

$$a^2 = \frac{\sin^2 \phi}{r^2} (a-c)^2 \left[\frac{\beta^2}{d^2} (a^2 - r^2)^2 + r^2 \right] \quad (8)$$

where $\phi = \frac{\alpha r s}{d(s-c)}$ is known. We can then find b from

$$b^2 = \frac{\beta^2}{d^2} (a^2 - r^2)^2 - (a^2 - r^2) \quad (9)$$

provided $a^2 > \frac{d^2}{\beta^2} + r^2$ to ensure that the right-hand side of the equation is greater than zero. For example, for a

ball bearing of radius $r = 3$ mm to have the nominal magnification 1.2 on our machine, it can only be placed at $a = 400.01$ mm and $b = 1.71$ mm.

A consequence of calculating b from a using Equation (9) is that variation in a due to measurement error will be reflected in variation in b , and the latter can be much larger. For example with $a \approx 390$, $\beta \approx 1.26$, $d = 480$, $r = 1.25$ or 3, simulation results show that the standard deviation of b will be about five times that of a .

Results

Theoretical dependence of magnification on object position and size

Using Equations (3) and (4), image magnification can be determined from the position and size of the ball bearing. Figure 2 shows contour plots of horizontal and vertical magnification of a ball bearing of radius $r = 3$ mm as a function of horizontal and vertical position. It can be seen that slight movement of the object in a horizontal direction has a more profound effect on the horizontal magnification than the vertical magnification in the ratio 4:1 (dotted horizontal line), whereas moving vertically has about the same magnitude of effects (dotted vertical line). Distortion, or disagreement between horizontal and vertical magnification, is zero when the object is placed where identically labelled lines in Figure 2 intersect (marked by asterisks). Moving the object away from the starred points results in distortion.

The horizontal and vertical magnifications will also depend on the radius r of the ball bearing, but from our calculations the effect will be small for any reasonably sized object used clinically ($r < 20$ mm). Figure 3 is a plot of horizontal and vertical magnification against the radius of the ball bearing. For a fixed vertical position of the ball bearing at $b = 85$ mm and different horizontal

Figure 2 Contours of horizontal and vertical magnification for a ball bearing of radius $r = 3$ mm. Solid lines, horizontal magnification; dashed lines, vertical magnification

Figure 3 Horizontal and vertical magnification plotted against the radius of a ball bearing for three different horizontal ball positions (395 mm, 400 mm and 405 mm)

positions at $a = 395$ mm, 400 mm and 405 mm, there was only a slight slope in the horizontal magnification curves whereas the vertical magnification remains almost the same as r increases. Among the three settings of position, distortion is the smallest when $a = 395$ mm and $b = 85$ mm.

Experimental data for magnification

We analysed data from the experiment as described in the Methods section. Summary statistics of the vertical and horizontal magnifications for the 2.5 mm and the 6 mm diameter ball bearings are shown in Table 1. Standard deviations in image sizes were 0.111 mm horizontally and 0.017 mm vertically for the 2.5 mm ball bearing, and 0.041 mm and 0.011 mm, respectively, for the 6 mm one. The mean values for magnification of the large and small ball bearings were very similar, but the variation in the magnification values for the small ball bearing was much greater than that for the larger ball bearing. The small and large ball bearing images had similar values for the ratio of horizontal to vertical magnification, but this was again more variable for the small ball (variance ratio = 28.05, $p < 0.001$). The mean ratio for the large ball bearing was significantly different from unity (t -statistic = 18.78, $p < 0.0001$), as was the small bearing (t -statistic = 3.928, $p = 0.0008$).

Derived position of the ball bearing

We used Equations (8) and (9) to derive the ball bearings position (a , b) from measured magnification (α , β). The results are shown in Figure 4. As explained earlier, as a result of calculating b from a , more variation in derived position can be expected vertically than horizontally. The actual positions varied in the

Figure 4 Positions of the ball bearings (BB) (mm) derived from their measured vertical and horizontal magnifications. Circle, 6 mm diameter ball bearing; dot, 2.5 mm diameter ball bearing

experiment but the amount of variation should have been similar in both directions and for both ball bearings. The amount of variation in the calculated positions is less for the larger ball bearing in either direction. The larger diameter of the ball bearing is therefore a contributing factor, which enabled accurate edge detection without image enhancement.

Simulation results in Figure 5 show that, even if the ball bearings were placed at exactly the same position with the same accuracy of measurement, the larger ball bearings would give more consistent values of the position. In the simulation, a ball bearing was hypothetically placed at (a , b) = (390, 85). The size of the image was calculated using Equations (3) and (4), to which small amounts of random variation were added. The distribution was assumed to be normal with standard deviation 0.02875 mm or one-quarter of a pixel. Positions were then derived from the imperfect values of magnification, for both ball bearing sizes $r = 1.25$ mm and 3 mm.

Discussion

We have shown theoretically how image magnification changes with object position and size so that magnification can be calculated from known position and machine parameters before a radiograph is taken. A contour map (Figure 2) has been provided to show how magnification changes with ball bearing position and more can be produced for looking up values of magnification. One of our findings is that the nominal magnification provided by manufacturers is only achieved when the object is positioned at certain locations.

Table 1 Results of automatic edge detection and ellipse fitting. Standard deviation (SD) in brackets and ratio h/v = horizontal magnification/vertical magnification

Ball bearing size	Vertical magnification (SD)	Horizontal magnification (SD)	Ratio h/v (SD)
Small ball (2.5 mm)	1.257 (0.0067)	1.292 (0.0445)	1.028 (0.0322)
Large ball (6 mm)	1.255 (0.0018)	1.286 (0.0068)	1.025 (0.0061)

Figure 5 Simulation results of calculated positions for ball bearings (BBs) hypothetically placed at the same position, whose image measurements were subject to random error

For a tiny object with $r \approx 0$ placed at the sharp layer ($a=s$), Equation (5) implies that the vertical and horizontal magnification are identical ($\alpha=\beta$) only if its vertical elevation is zero ($\cos \theta=1$) relative to the tubehead. For a larger object with $r=3$ mm, the contour map in Figure 2 shows that, at the sharp layer ($a=400$ mm), the horizontal and vertical magnification are equal only when there is nearly zero vertical elevation. If the upward elevation is $\theta=10^\circ$, the vertical position will be $b=70.5$ mm (Figure 1). Reading from the contour map (Figure 2, where the cross is), the corresponding vertical and horizontal magnification would differ at 1.18 and 1.22, respectively. The nominal value is 1.20 in both directions.

Vazquez et al¹⁹ found that 10 mm implants provided values of vertical magnification smaller than that specified by manufacturers (1.30), varying between 1.245 and 1.295 among observers. We find that object size has little effect on magnification (Figure 3) and a ball bearing with radius $r=5$ mm would have vertical magnification between 1.21 and 1.24 on our machine when placed at 85 mm vertically, on either side of the sharp layer by 5 mm.

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Scarfe et al²⁰ found an extensive region of the sharp layer with constant vertical and horizontal magnifications, but which was not observed in another study using a similar machine.²¹ From Figure 2 it can be observed that the contours for horizontal and vertical magnification are not parallel, but this corresponds to a single position of the tubehead. The trajectory of the moving rotation centre is machine specific and we do not have a universal conclusion.

Choi et al⁷ reported that among 80 patients with 210 implants the average magnification of the implants varied by anatomic location from 1.20 to 1.32 horizontally and from 1.23 to 1.31 vertically. Gomez-Roman²² found a similar range of vertical image magnification values. Schulze et al²³ found that vertical measurements were less reproducible than horizontal measurements in dental panoramic images. They argued that the difference was due to the observer finding the vertical measurements more difficult to perform than the horizontal measurements, because the measurement device used had identical precision in both directions. Vertical length measurements are routinely made on panoramic radiographs to determine whether sufficient bone is available prior to surgical implant insertion. We have demonstrated by using automatic measurements to reduce subjective assessment error that less variation can be expected in the vertical than in the horizontal direction of an image when an object is placed in random positions about the sharp layer.

We have also demonstrated that object position can be derived from measured image magnification. Comparing this with known position would allow machine calibration as any discrepancy may be an indication of misalignment. In conclusion, the 6 mm diameter ball bearing has the advantage that its images are easier to measure, resulting in less variation in magnification than the 2.5 mm one. It is thus recommended in routine use to provide more consistent results. Our future work will explore the possibility of automatically correcting panoramic images for distortion using automatic registration of intraoral calibration images.

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Appendix A

A mathematical proof

Now we give a proof of the formulae in Equations (3) and (4). When the X-ray beam goes through the centre of the ball bearing, its intersection with the surface is a circle represented by $(x - a)^2 + (y - b)^2 = r^2$. A ray in the beam is $y = kx$. Solving the two equations together gives intersections of the line with the circle. The equation for x is quadratic

$$(1 + k^2)x^2 - 2(a + kb)x + a^2 + b^2 - r^2 = 0 \quad (\text{A1})$$

which has a unique solution when

$$(a + kb)^2 - (1 + k^2)(a^2 + b^2 - r^2) = 0 \quad (\text{A2})$$

or equivalently

$$(a^2 - r^2)k^2 - 2abk + b^2 - r^2 = 0 \quad (\text{A3})$$

which itself is a quadratic equation in k . The two solutions k_1 and k_2 ($k_1 > k_2$) satisfy

$$\begin{aligned} k_1 - k_2 &= \frac{\sqrt{4a^2b^2 - 4(a^2 - r^2)(b^2 - r^2)}}{a^2 - r^2} \\ &= \frac{2r\sqrt{a^2 + b^2 - r^2}}{a^2 - r^2} \end{aligned} \quad (\text{A4})$$

They correspond to two tangent lines. The projected image has top and bottom at k_1d and k_2d , respectively, thus the vertical size of the image is $(k_1 - k_2)d$ and vertical magnification is

$$\beta = \frac{(k_1 - k_2)d}{2r} = \frac{d\sqrt{a^2 + b^2 - r^2}}{a^2 - r^2} \quad (\text{A5})$$

Now consider the ray of the beam that goes through the centre of the ball bearing. When this is rotated with the beam while maintaining an angle θ with the horizontal plane, it sweeps through the ball bearing spanning an angle 2θ , which is the angle between the two tangent lines. It follows that

$$\sin \phi = \frac{r}{(a - c)/\cos \theta} \quad (\text{A6})$$

so that

$$\phi = \sin^{-1} \left(\frac{r \cos \theta}{a - c} \right) \quad (\text{A7})$$

The speed of the film relative to the rotating beam is^{10,24}

$$v = \frac{d}{s}(s - c)\omega \quad (\text{A8})$$

where ω (radians/s) is the angular speed of the beam. Using the argument of Welander,¹⁰ the time it takes to rotate by 2θ is $2\theta/\omega$, thus the horizontal size of the projected image is

$$\frac{2\phi}{\omega} v = \frac{2\phi d}{\omega s}(s - c)\omega \quad (\text{A9})$$

which does not depend on ω . Dividing the above by $2r$, the horizontal magnification is then

$$\alpha = \frac{\phi d}{r s}(s - c) = \frac{\phi}{\sin \phi} \frac{d s - c}{s a - c} \cos \theta \quad (\text{A10})$$

where ϕ is given by Equation (A7).