Technical Note: Algebraic iterative image reconstruction using a cylindrical image grid for tetrahedron beam computed tomography

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Purpose: To accelerate iterative algebraic reconstruction algorithms using a cylindrical image grid.

Methods: Tetrahedron beam computed tomography (TBCT) is designed to overcome the scatter and detector problems of cone beam computed tomography (CBCT). Iterative algebraic reconstruction algorithms have been shown to mitigate approximate reconstruction artifacts that appear at large cone angles, but clinical implementation is limited by their high computational cost. In this study, a cylindrical voxelization method on a cylindrical grid is developed in order to take advantage of the symmetries of the cylindrical geometry. The cylindrical geometry is a natural fit for the circular scanning trajectory employed in volumetric CT methods such as CBCT and TBCT. This method was implemented in combination with the simultaneous algebraic reconstruction technique (SART). Both two- and three-dimensional numerical phantoms as well as a patient CT image were utilized to generate the projection sets used for reconstruction. The reconstructed images were compared to the original phantoms using a set of three figures of merit (FOM).

Results: The cylindrical voxelization on a cylindrical reconstruction grid was successfully implemented in combination with the SART reconstruction algorithm. The FOM results showed that the cylindrical reconstructions were able to maintain the accuracy of the Cartesian reconstructions. In three dimensions, the cylindrical method provided better accuracy than the Cartesian methods. At the same time, the cylindrical method was able to provide a speedup factor of approximately 40 while also reducing the system matrix storage size by 2 orders of magnitude.

Conclusions: TBCT image reconstruction using a cylindrical image grid was able to provide a significant improvement in the reconstruction time and a more compact system matrix for storage on the hard drive and in memory while maintaining the image quality provided by the Cartesian voxelization on a Cartesian grid. © 2013 American Association of Physicists in Medicine. [<http://dx.doi.org/10.1118/1.4812886>]

Key words: CBCT, TBCT, image reconstruction, SART

1. INTRODUCTION

For undersampled and/or noisy data, algebraic iterative reconstruction methods have been shown to provide better image quality than analytical methods such as the filtered backprojection (FBP) algorithm.¹ They are also very adaptable for use with nonstandard system geometries.^{2,3} Historically, the algebraic reconstruction technique (ART), which was based on Kaczmarz's method for solving a system of linear equations, was the first method used by Hounsfield to reconstruct CT images, and it was one of the first methods to be employed on a commercial machine.⁴ The main drawback of this method is its high computational cost. It required two and a half hours for Hounsfield to complete his first reconstruction using ART. Continual improvement in the image resolution of CT scanners led to a rapid increase in the amount of data that are used for image reconstruction, which made it increasingly

impractical to use the algebraic methods in the clinic. Fourier-based methods, particularly FBP, became the dominant reconstruction methods due to their high computational efficiency and satisfactory image quality.⁵ The continual improvement in the processing capabilities of modern workstations and in the development of more efficient iterative image reconstruction algorithms has made it possible to reintroduce iterative reconstruction methods into the clinic. Most CT vendors now provide various iterative reconstruction methods for their CT scanners.⁶

The system matrix is the key component of iterative reconstruction methods. Constructing it for forward projection of the image and for back projection of the correction image is the most computationally intensive portion of the reconstruction process.⁷ To save time, the system matrix is sometimes created and stored on the hard drive so that it can be loaded into memory instead of calculated during the reconstruction

process.⁸ The size of the system matrices, though, have grown to be prohibitively large due to the increasingly large data sets provided by modern CT scanners, so that saving the matrix onto the hard drive often becomes impractical. The size of the matrix is too large to hold in RAM. If, instead, only a portion of the system matrix is loaded into memory at a time, the total time required to load the matrix during the run could exceed the time it would take to calculate the matrix during runtime.⁹

The system matrix in Cartesian coordinates is rotation-variant, which means that a new geometric model is required for every projection angle. Hence, the dimensions of the system matrix are immense. Various methods to improve the speed and quality of image reconstruction have been proposed that use polar grids combined with various pixelation models for medical^{10–13} and industrial¹⁴ CT and have shown good results in terms of speed and accuracy of the reconstructed images. Using a cylindrical image grid and cylindrical voxelization method along with a circular axial CT scanning geometry would allow us to take advantage of the symmetries of the cylindrical geometry and reduce the size of the system matrix by up to 2 orders of magnitude. At this reduced size, matrix storage on the hard drive or in memory would not be an issue. An additional step consisting of the interpolation of the reconstructed image from the cylindrical voxel space to the regular Cartesian voxel space will be required, but the time required for this operation will be negligible compared to the time saved during the reconstruction.

Tetrahedron beam computed tomography (TBCT) is a new volumetric CT technology that employs a multiple-pixel x-ray source array and a multirow CT detector.¹⁵ It can produce improved image quality compared to cone beam CT (CBCT) as a result of having a system geometry that leads to the efficient rejection of scattered photons and of using a high quality multirow detector. However, when using the FDK reconstruction algorithm, TBCT images still suffer from approximate reconstruction artifacts at large cone angles. Algebraic iterative reconstruction methods have been shown to be able to reduce

these approximate cone artifacts.¹⁶ For the reasons already mentioned, the algebraic iterative reconstruction methods require an excessive amount of time that would be unacceptable in clinical use. Therefore, in this study, we develop an algebraic reconstruction algorithm that utilizes a cylindrical image voxel grid and a cylindrical voxel basis specifically in order to improve the performance of TBCT image reconstruction while avoiding excessively long reconstruction times. Alternatively, this cylindrical voxel model could also be easily modified for use with a cone beam system configuration.

2. METHODS AND MATERIALS

2.A. TBCT

A TBCT system comprises a linear array of x-ray sources that is oriented orthogonally to a linear array of detectors. The beams produced by the x-ray sources are collimated into a stack of fan beams directed toward the detector array. The TBCT system geometry is displayed in Fig. 1. The beams form a tetrahedral volume at each projection angle. At each projection angle, the x-ray sources are activated and deactivated sequentially so that only one source irradiates the patient at a time. Therefore, while the full coverage volume is covered at each projection angle, only a small part of the coverage volume is irradiated at any specific instant. Most scattered photons are deflected out of the fan beam plane and, therefore, go undetected. The detector array is a regular multirow solid-state detector that is similar to those used for diagnostic CT scanners which have high temporal and spatial resolution and high detective quantum efficiency (DQE).

We employed the same geometry that was used for a TBCT benchtop system that was developed in-house.¹⁷ The equiangular detector array was composed of 275 detector elements of size 2.54×2.54 mm. The source-to-detector distance was 145 cm, and the source-to-isocenter distance was 82.5 cm. A total of 360 projections were generated that were spaced at

FIG. 1. TBCT stacked fan-beam geometry: (a) top view, (b) lateral view, and (c) isometric view.

equal 1° intervals. For two-dimensional projection sets, only one x-ray source was used, which provided a full projection set of size 360×275 that was used to reconstruct a 512×512 image having an isotropic pixel size of 0.5 mm. For three-dimensional projection sets, an array of 75 x-ray sources was used with 4 mm spacing between the sources. This created a full projection set of size $360 \times 75 \times 275$ that was used to reconstruct a $256 \times 256 \times 128$ image with an isotropic voxel size of 1 mm.

2.B. SART image reconstruction

During a CT scan, detector measurements at each projection angle are added to the projection set $\mathbf{p} = (p_1, p_2, \dots, p_M)$ where M is the total number of detector measurements at all projection angles. The value of M is equal to the product of the number of detectors and the number of projection angles. A continuous image, $f(\vec{x})$, to be reconstructed is discretized into an image vector $\mathbf{x} = (x_1, x_2, \dots, x_N)$ where N is the total number of image voxels. The CT reconstruction model can now be expressed by

$$\mathbf{p} = \mathbf{A}\mathbf{x}, \quad (1)$$

where $\mathbf{A}$ is the system matrix that has dimensions $N \times M$ in which element a_{ij} relates the j th detector measurement to the i th voxel value. This system of equations suffers from ill-posedness of the problem, noisy data, and immense size so that it cannot be solved directly, but is instead solved iteratively. Given the image $\mathbf{x}$, the j th projection can be calculated using $\hat{p}_j = \sum_i a_{ij}x_i$. This image $\mathbf{x}$ can be iteratively reconstructed by minimizing the difference between the calculated projection $\hat{p}_j$ and the measured projection p_j . The update term for ART is defined by

$$x_i^n = x_i^{n-1} + \lambda \frac{a_{ij}(p_j - \hat{p}_j)}{\sum_{k=1}^N a_{kj}^2}, \quad (2)$$

where n is the update step, i is the image voxel index, and λ is a relaxation factor that controls the convergence during the backprojection process. A full iteration is performed when all M measurements have been used to update the image voxels. Several iterations are needed in order to reconstruct the image.

The simultaneous algebraic reconstruction technique (SART) is an improvement over the ART (Ref. 18) method. It modifies the update term equation so that all the ray sums collected at a specific projection angle are updated simultaneously instead of sequentially. This method has been shown to provide high quality results for cone beam reconstruction.¹⁴ The update term for SART is

$$x_i^{n+1} = x_i^n + \lambda \frac{\sum_{j=A}^B a_{ij} \frac{p_j - \hat{p}_j}{\sum_{i=1}^N a_{ij}}}{\sum_{j=A}^B a_{ij}}, \quad (3)$$

where A and B are the indices of the first and last projection data elements used for the n th update step. These values are defined by $A = (n-1)K$ and $B = nK$ where K is the number of detector measurements that make up a single projection image. For SART, a full iteration has been performed after V

update steps have been completed for each voxel, where V is the number of projection angles.

2.C. System matrix with cylindrical voxel grid and cylindrical voxelization

Conventional image reconstruction discretizes a continuous image $f(\vec{x})$ onto a Cartesian grid. When using a Cartesian grid, the system model is rotation-variant so that the system matrix, $\mathbf{A}$, is huge. For a polar grid, the system model is rotation invariant so that the system matrix needs to be calculated at only one projection angle. This matrix can be reused for each of the V projection angles.

2.C.1. Polar image grid

In 2D, the TBCT geometry becomes identical to that of 2D fan-beam CT. Figure 2 shows a two-dimensional fan-beam CT system consisting of a point x-ray source and a single row of detectors arranged in an equiangular fan-beam geometry. The system rotates around the isocenter. The field-of-view (FOV) of the system is the region that remains within the fan-beam for all projection angles. The continuous image $f(\vec{x})$ within the reconstruction FOV is discretized onto a grid of voxels and is then reconstructed using the SART algorithm. A ray S_j is defined to be the area delimited by the point x-ray source and the two edges of detector element j .

As shown in Fig. 2, the image is discretized into N_θ equally spaced rays proceeding from the isocenter and N_r equally spaced concentric rings. The total number of image voxels, N , is the product of N_θ and N_r . A pixel element S_i is defined to be the area enclosed by two consecutive rays and two consecutive rings. The area of intersection of the pixel element S_i and the ray S_j determines the value of the system matrix element a_{ij} :

$$a_{ij} = \frac{\text{Area of } S_i \cap S_j}{\text{Area of } S_i}. \quad (4)$$

It is difficult to determine the area of intersection analytically, and we, therefore, employed a numerical method to calculate it. For this method, we divided each pixel element into a large number of equally spaced grid points. A line drawn from the x-ray source to the grid point was extended to determine which detector element it would hit, if any. The number of points that fell within the boundaries of detector element

FIG. 2. Image reconstruction in 2D polar image grid. Matrix element a_{ij} is defined in Eq. (4).

FIG. 3. Image reconstructed using polar pixel grid (a) before and (b) after interpolation onto a Cartesian grid.

j divided by the total number of grid points within the pixel determines the value of system matrix element a_{ij} . Although this numerical method is time consuming, the matrix is only calculated once and stored for each system.

In image reconstruction, the polar pixel grid allows us to take advantage of the axial symmetry of the scanning geometry. When the projection angles match the angular divisions, the geometry of the pixel grid relative to the source and detectors will be identical at every projection view. Therefore, the set of system matrix element values will also be identical at every projection view. Only the index of the image pixel that accesses a given system matrix value will change between views. The system matrix, then, needs to be calculated at only one projection angle. Consequently, the system matrix dimensions are reduced from $N \times M$ to $N \times K$, which is a V -fold decrease. Due to symmetries that can be exploited in the Cartesian grid, the actual reduction is by $V/4$.

An image reconstructed on a polar grid cannot be immediately visualized as computer displays have square pixels. The reconstructed image needs to be interpolated onto a Cartesian grid to provide an image that a viewer can interpret properly. The reconstructed image produced using a polar grid is mapped onto a Cartesian grid using piecewise linear inter-

polation on a set of points chosen by using a Delaunay triangulation. Figure 3 gives an example of the image before and after interpolation.

2.C.2. Cylindrical image grid

The TBCT system configuration shown in Fig. 1 was used to model the system geometry for the three-dimensional case. The TBCT system uses the same axial scanning trajectory as in the two-dimensional case. The extension of the polar pixel geometry to the three-dimensional cylindrical space is straightforward. Pixel elements for the polar grid are stretched along the z axis, which is perpendicular to the polar r - θ plane, to cover the lateral FOV. This space is then divided into N_z slices of equal height as can be seen in Fig. 4. A ray can now be pictured as the pyramid-shaped volume that is delimited by a point source and the four corners of a two-dimensional detector element. The definition of the system matrix element a_{ij} would be changed to be the volume of overlap between the i th cylindrical voxel element and the j th ray given by

$$a_{ij} = \frac{\text{Volume of } S_{\theta,r,z} \cap S_j}{\text{Volume of } S_{\theta,r,z}}, \quad (5)$$

FIG. 4. (a) Transverse and (b) sagittal views of the three-dimensional cylindrical geometry.

where S_j is the volume delimited by the source and the boundaries of the two-dimensional detector element j , $S_{\theta,r,z}$ is the volume that falls within the boundaries of the voxel specified by the angular ray index θ , concentric ring index r , and lateral division index z . The cylindrical voxel element i is related to the indices r , θ , and z according to $i = z N_r N_\theta + r N_\theta + \vartheta$.

The same numerical methodology used for calculating the area of intersection of a ray and pixel in two dimensions was used for calculating the volume of intersection for a ray and voxel in three dimensions. Each voxel element was divided into an equispaced three-dimensional grid that was laid wholly within the boundaries of the three-dimensional voxel, and the line passing through a source and grid point was used to determine which detector element, if any, would be hit. The volume of intersection for voxel element S_i and ray S_j is then given by dividing the number of grid points in voxel i projected onto detector j by the total number of grid points.

2.D. Figures of merit (FOM)

As a measure of the fidelity of the reconstruction to an original image, we compared the reconstructed image to a numerical phantom using several FOM. The FOM chosen for this study were the relative root mean square error (RRME) of the reconstructed image, Pearson's correlation coefficient, and Colsher's discrepancy metric (δ), which are defined as

$$\text{RRME} = \sqrt{\frac{\sum_{i \in I} (x_i - x_i^{\text{ref}})^2}{\sum_{i \in I} (x_i^{\text{ref}})^2}}, \quad (6)$$

$$\text{Corr} = \frac{\sum_{i \in I} (x_i - \bar{x})(x_i^{\text{ref}} - \bar{x}^{\text{ref}})}{\sqrt{\sum_{i \in I} (x_i - \bar{x})^2} \sqrt{\sum_{i \in I} (x_i^{\text{ref}} - \bar{x}^{\text{ref}})^2}}, \quad (7)$$

$$\delta = \sqrt{\frac{\sum_{i \in I} (x_i - x_i^{\text{ref}})^2}{\sum_{i \in I} (x_i^{\text{ref}} - \bar{x}^{\text{ref}})^2}}, \quad (8)$$

where x_i^{ref} is the i th image voxel for the reference image, $\bar{x}$ is the average value for the reconstructed image, and $\bar{x}^{\text{ref}}$ is the average value for the reference image. For Pearson's correlation coefficient, the closer the value is to one, the more closely the reference and reconstructed images, while for the RRME and δ values, a value closer to zero indicates a more accurate reconstruction of the reference image. The FOM for the reconstruction using polar or cylindrical coordinates were calculated after the reconstruction was interpolated onto a Cartesian grid.

2.E. Phantom

Tests were performed using simulated projections from the numerical 2D and 3D Shepp-Logan head phantoms, which are commonly used for validating reconstruction algorithms. Ellipse parameters were taken from Kak and Slaney.⁵ The CT values for the phantoms were modified in order to increase

the contrast between the ellipsoids in the phantoms. In addition, the 2D and 3D FORBILD head phantoms (as defined in <http://www.imp.uni-erlangen.de/forbild>) were also used for FOM measurements.

In order to qualitatively evaluate the image quality, we also tested the cylindrical voxelization method using a real patient CT image. We generated the patient projection data by forward projecting the patient CT image using the system matrix created from the TBCT system model.

2.F. Spatial resolution

The modulation transfer function (MTF) was calculated in order to characterize the spatial resolution of the system. For the polar and cylindrical methods, it is expected that the spatial resolution of the system would vary as a function of the radius because the size of the voxels on the grid decreases as the radius decreases. The line-spread function (LSF) was determined using an edge phantom composed of several cylinders that are positioned at varying distances from the center. Since the spatial resolution of the Cartesian method is anisotropic, only the center cylinder was used for the MTF, whereas the MTF was determined at the center, edges, and one point between for the polar and cylindrical methods. The procedure given in the standard ISO 15708-2:2002 was used here for calculating the MTF from a cylinder.

2.G. Computer hardware

All experiments were performed using an Intel Core i7-2600 CPU @ 3.4 GHz with 12 GB of RAM on a 64-bit Windows 7 operating system. Although the CPU has multiple cores, the algorithm only utilizes one thread on a single core.

3. RESULTS

3.A. Polar grid evaluation

For the two-dimensional polar grid, the FOV radius was set to 15 cm, which was then divided into 150 concentric circles spaced at 1 mm intervals. The space was further divided by 720 lines proceeding radially from the isocenter at 0.5° intervals. This gives a total number of pixels in the grid of 108 000. The Cartesian grid contained 512×512 pixels with an isotropic pixel size of 0.5 mm. While the system matrix was precalculated and stored in memory during the reconstruction for the polar pixelation method, the system matrix for the Cartesian pixelation method was instead calculated during runtime. This was done to test the conditions that would exist for the 3D case where the system matrix would be too large to store in memory for the Cartesian method. In Fig. 5, the reconstructions after 1, 5, 10, and 15 iterations are displayed.

These show good visual agreement with the phantom image by the fifth iteration for the regular Cartesian pixel grid and by the tenth iteration for the polar pixel arrangement. From the images shown in Fig. 5, the image reconstructed

FIG. 5. Reconstructions of the (a) 2D Shepp–Logan head phantom after 1, 5, 10, and 15 iterations using (b) Cartesian pixels (top row) and (c) polar pixels (bottom row).

using the Cartesian method does show better uniformity throughout the reconstructed image whereas the image reconstructed using the polar method shows generally higher CT values on the right over the left side. No noticeable artifacts can be seen as a result of the bilinear interpolation method using the noiseless data for the Shepp–Logan phantom. The various interpolation procedures could result in the introduction of artifacts into the reconstructed image when noise is present in the projection data.

Relevant system matrix characteristics and timing results to compare the two pixel configurations are given in Table I. As is shown in the table, the reconstruction using the polar pixel grid was able to achieve a speedup of 2 orders of magnitude after 15 iterations had been reconstructed. This was mainly caused by precalculating the system matrix for the polar method while calculating it during runtime for the Cartesian method. However, the most important result is that the matrix storage size is less than 2 megabytes (MB) for the polar pixelation method compared to over 1 GB for the Cartesian method. As expected, the extra interpolation step that is required when using the polar grid configuration was insignificant compared to the time saved during the reconstruction process.

The images were compared to the original phantom image using the figures of merit described above with the results displayed in Fig. 6. For both the correlation coefficient and the discrepancy metric, the reconstruction using the Cartesian model performed better up to the *ninth* iteration, beyond

which the metrics indicate that the image reconstructed using the polar grid gave a slightly more accurate reconstruction. By contrast, the relative root mean square error for the cylindrical voxel reconstruction was superior to that of the Cartesian pixel reconstruction for all iterations. Also, by the *15th* iteration, the metrics for the polar pixel reconstruction were still converging, which indicates that better image quality could still be achieved and that faster convergence could possibly be achieved by adjusting the relaxation parameter only for the polar grid.

The FORBILD head phantom is more complex than the Shepp–Logan phantom. Instead of using only a set of simple ellipses as in the Shepp–Logan phantom, the FORBILD head phantom consists of objects that have varying shapes, orientations, and contrast. Based off the results from the Shepp–Logan phantom, we reconstructed the FORBILD data using 10 iterations for each of the SART methods. The results are displayed in Fig. 7 where the two images again show good visual agreement. The difference between the two reconstructions is given in Fig. 7(c). The difference values largely fall within 3% of the pixel values for the reconstruction using the Cartesian method, but a slight shading artifact can still be seen. The FOM results for the FORBILD phantom are given in Fig. 8. These results show a greater deal of similarity between the convergence of the Cartesian and polar reconstructions than was seen in the Shepp–Logan results. But both the FORBILD and Shepp–Logan results show that the discrepancy between the final reconstructed image and phantom image for the polar method is similar to that given by the conventional Cartesian method.

The MTF curves are shown in Fig. 9. As expected, the spatial resolution at 50% MTF is highest in the center of the image for the polar grid and decreases as the distance between the voxel and the image center increases. Also shown is that the MTF curve of the voxels midway between the center and edge of the image using the polar method is similar to that provided by the Cartesian method, providing a spatial resolution of roughly 3.9 lp/cm. The spatial resolution at 50% MTF for the center voxels using the cylindrical method was roughly 4.5 lp/cm.

TABLE I. Comparison of system matrix characteristics and running time for the two 2D grid configurations.

	Cartesian grid	Polar grid
Pixel number	262 144	108 000
Storage size	1080 MB	1.64 MB
Reconstruction time/iteration	432.66 s	0.961 s
Interpolation time	...	1.22 s
Total runtime (15 iterations)	6489.85 s	15.63 s
Approximate speedup factor	...	415

FIG. 6. Graphs of the (a) RRME, (b) Pearson correlation coefficient (Corr), and the (c) Colsher discrepancy metric (δ) for Shepp-Logan head phantom.

3.B. Cylindrical grid evaluation

For three-dimensional reconstruction, we extended the polar grid along the z axis and divided the grid into 128 slices that were each 1 mm thick. The Cartesian grid was also extended by the same amount. The dimensions of the transverse slice were modified to be 256×256 voxels with an isotropic voxel size of 1 mm. As is shown in Table II, the reconstruction using the cylindrical voxel grid was able to achieve a speedup factor of nearly 40 after ten iterations. While storage of the system matrix is prohibited by the immense file size for the Cartesian grid, the storage size for the cylindrical system matrix is still under 350 MB, which is reasonable for storage on the hard drive and for loading onto memory during reconstruction.

The results for the FOM are displayed in Fig. 10. For all metrics, the reconstructions that used the cylindrical grid per-

formed better than the reconstructions that used the Cartesian grid. We believe that this was mainly because the CT values were too low at the top and bottom borders of the phantom along the z axis which can be seen in the coronal profile in Fig. 10(d). Both methods converged at nearly the same rate and reached their best results by the sixth iteration. Line profiles were taken through the transverse and coronal images of the reconstructions, and these profiles showed good agreement between the phantom image and the reconstruction using the cylindrical grid, as shown in Fig. 11.

Similar results were obtained for the three-dimensional FORBILD head phantom. Reconstructed coronal and sagittal images are shown in Fig. 12 alongside the original numerical phantom as a reference. In the coronal images for both methods, artifacts appear in the central region that appear to be caused by the ear phantom, but the patterns of these artifacts are different. It is also shown that the bilinear method used to

FIG. 7. Reconstructions of the 2D FORBILD head phantom. The SART reconstructions using (left) Cartesian and (middle) polar pixelation methods are given with an (right) image of the difference between the two images included for reference. The CT number window is [1.04 1.06] in the reconstructions and [-0.03 0.03] in the difference image.

FIG. 8. Graphs of the (a) RRME, (b) Pearson correlation coefficient (Corr), and the (c) Colsher discrepancy metric (δ) for FORBILD head phantom.

interpolate the cylindrical image onto a rectangular grid could have difficulty with small objects as seen with the air cavities in the ear phantom. There also appears to be faint streaking at the center of the image that runs the length of the z axis. This is also seen in the sagittal image. The sagittal images are otherwise similar qualitatively.

Displayed in Fig. 13, the FOM results for this phantom are similar to what was seen for the Shepp–Logan phantom, except that the Cartesian FOM values are slightly better for the first iteration before falling behind the cylindrical results during the second iteration.

The MTF results for the cylindrical voxelization methods shown in Fig. 14 are similar to the polar results where the voxels nearest the center have the highest spatial resolution

at 50% MTF with the spatial resolution gradually decreasing as the voxels move to the edges. The curve for the Cartesian grid is most similar to the MTF curve for the voxels midway between the edge and center using the cylindrical voxelization. The spatial resolution here is a little lower than for the two-dimensional results. The cylindrical voxelization gave a resolution at 50% MTF between 4 lp/cm in the center and 3 lp/cm toward the edges, while the Cartesian method gave a spatial resolution between them of 3.5 lp/cm.

We further tested the reconstruction algorithms using a patient CT image. TBCT projection data were generated using the parameters outlined above. The size of the reconstructed patient images was $256 \times 256 \times 128$ with a voxel size of 2 mm in the transverse plane and 3 mm along the z axis. Since both three-dimensional implementations had reached their best results by the sixth iteration according to Fig. 10, six iterations were used for each of the reconstruction methodologies. Figure 15 shows a comparison of the

FIG. 9. MTF curves for the images reconstructed using each of the pixelation methods.

TABLE II. Comparison of system matrix characteristics and running time for the two 3D grid configurations.		
	Cartesian grid	Cylindrical grid
Pixel number	8 388 608	13 824 000
Storage size	46 080 MB	316.4 MB
Reconstruction time/iteration	4965.28 s	119.61 s
Interpolation time	0 s	86.48 s
Total reconstruction time (10 iterations)	49 652.8 s	1282.6 s
Approximate speedup factor	...	38.7

FIG. 10. Graphs of the (a) RRME, (b) Pearson correlation coefficient (Corr), and the (c) Colsher discrepancy metric (δ) for the 3D reconstructions of the Shepp-Logan head phantom.

transverse and coronal images reconstructed using the cylindrical and Cartesian grids. The speedup factor for the cylindrical method was slightly over 40 for the patient data, which is consistent with the results for the numerical phantom. The

reconstructed image produced using the cylindrical voxel grid provided good image quality that was comparable to the image produced when using the conventional grid. No noticeable artifacts were introduced from the interpolation process.

FIG. 11. (a) Transverse and (b) coronal images of the reconstruction using (left) cylindrical voxel grid and (right) Cartesian voxel grid. Line profiles windowed to [0.8, 1.6] for (c) transverse and (d) coronal views.

FIG. 12. (a) Coronal and (b) sagittal images of the reconstructions of the FORBILD head phantom using the (left) Cartesian and (middle) cylindrical methods as well as the (right) original phantom for reference. The CT number window is [1.04 1.06].

4. DISCUSSION AND CONCLUSION

In this study, we implemented a polar/cylindrical voxelization on a polar/cylindrical image grid for the iterative SART algorithm. The cylindrical grid is a more natural fit for the axial scanning geometry of a volumetric CT system, such as

CBCT and TBCT. By using this framework, 3D reconstruction times while using the SART algorithm improved by a factor of approximately 40, while the accuracy of the reconstructions was comparable to that of the images reconstructed when using the conventional Cartesian voxelization scheme. Although a multicore CPU was used, our algorithm only

FIG. 13. Graphs of the (a) RRME, (b) Pearson correlation coefficient (Corr), and the (c) Colsher discrepancy metric (δ) for the 3D reconstructions of the FORBILD head phantom.

FIG. 14. MTF curves for the images reconstructed using each of the voxelization methods.

utilized a single CPU core. The reconstruction time could be reduced further by implementing this method on a GPU or by using a multithreaded computing method.

The storage size of the system matrices can be reduced further if we take full advantage of the symmetries of the geometry. The cylindrical geometry is symmetric about the x and y axes, and the size of the matrix could be reduced by a factor of 4 if we take advantage of these symmetries. For this study, the matrix size was already small enough to store and load without issue so that the further saving was deemed unnecessary. In a future implementation of this method on a GPU, it may be desirable to further reduce the size of the system matrix so that it can be loaded onto the smaller graphics processor memory. Also, neither the reconstruction methods nor the interpolation procedure for the visualization of the cylindrical image were

optimized for speed, and so the reconstruction times could be reduced further as the algorithms are made more efficient.

As seen in the FORBILD reconstructions, the method used to interpolate the cylindrical data onto a Cartesian grid might need to be modified as the bilinear interpolation method may lead to the introduction of artifacts caused by small objects, particularly those at the edges of the image. Also, shading artifacts appear at the smaller radii when using the cylindrical voxelization method that could result from using voxels that are too small in the center which could lead to problems during the backprojection and interpolation. One possible solution could be to adjust the number of voxels in the center region so that the voxel volume does not fall below some threshold value. The larger voxels would not be as sensitive to errors caused by mismatched data.

In conclusion, the cylindrical reconstruction grid was successfully implemented in combination with the SART reconstruction algorithm. It was able to provide image quality comparable to that provided by the regular Cartesian reconstruction grid. At the same time, it provided a significant improvement in the reconstruction time and a more compact system matrix for storage on the hard drive and in memory.

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FIG. 15. Patient image reconstructions. (Top row) Transverse images reconstructed using (a) cylindrical and (b) Cartesian voxel grids. (Bottom row) Coronal images reconstructed using (c) cylindrical and (d) Cartesian voxel grids.

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