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A robust general phase retrieval method for medical applications

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Abstract

From medical imaging perspective the robustness of a phase retrieval method is of critical importance. In this presentation we compare the robustness of two general phase retrieval methods, namely the transport of intensity equation inversion (TIE-inversion) method and the attenuation partition based (AP-based) method. We showed that the TIE-inversion method, regardless if being assisted with the Tikhonov regularization, failed to retrieve the phase maps in two experimental studies. The failure exposes this method's weakness as being unstable against the noise. In contrast, the sample phase maps are retrieved successfully by using the AP-based method. The stark performance differences of the two methods are rooted in their different techniques dealing with the singularity problem. This comparison shows that the robust AP-based phase retrieval method will be superior to the TIE-inversion method for medical imaging applications where radiation doses are stringently limited.

Keywords

Medical-image reconstruction methods and algorithms; computer-aided so; X-ray radiography and digital radiography (DR)

1 Introduction

The phase retrieval is an important task in the inline phase contrast x-ray imaging. As x-rays traverse a sample, x-rays undergo phase shifts in addition to x-ray attenuation. The amount of the phase shift along an exit ray is determined by the sample's electron densities. The amount of x-ray phase shift along an exit ray is $\phi = -\lambda r_e \int \rho_e ds$, where λ is the x-ray wavelength, ρ_e denotes the electron density, r_e the classical electron radius ($r_e = 2.818 \times 10^{-15}$ m,) and the integral is over the ray path [1, 2]. The need for phase retrieval, i.e. retrieving the x-ray phase shifts generated in a projection of a sample, arises from the efforts to provide quantitative tissue characterization for medical imaging applications. Take the phase contrast tomography as an example. A sample's tomogram reconstructed directly from angular phase contrast projections is actually a map of the sample's apparent linear attenuation coefficients. In fact, we found that the apparent linear attenuation coefficients are given by [3]:

$$\mu_{\text{apparent}} = \cos((\lambda R_2/4\pi M)\nabla_{3D}^2)\mu - 2\lambda r_e \sin((\lambda R_2/4\pi M)\nabla_{3D}^2)\rho_e,$$

where the three-dimensional Laplacian operator $\nabla_{3D}^2 \equiv (\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2)$, μ denotes the sample's actual linear attenuation coefficient, R_2 the sample-detector distance, and M the magnification factor. The operators $\sin((\lambda R_2/4\pi M)\nabla_{3D}^2)$ and $\cos((\lambda R_2/4\pi M)\nabla_{3D}^2)$ are the three-dimensional pseudodifferential operators as will be explained in details below. The quantitative information on the sample's attenuation coefficients and electron densities cannot be obtained from these apparent linear attenuation coefficients. This is because the differentials of the sample electron density can generate anomalously large or even negative apparent attenuation coefficients at tissue interfaces and boundaries. For example, the apparent attenuation coefficient values of small bronchi were found even lower than that of air in phase contrast tomography experiments [4]. These apparent artifacts may cause faulty pathological interpretation of sample structures. To overcome these problems, one has to perform phase retrievals for each of angular projections and thereby to extract the phase map and attenuation map for each of angular projections. In this way one can reconstruct the tomograms of sample electron densities and attenuation coefficients respectively [5–7]. Such phase retrieval based tomography is also called the quantitative phase contrast tomography.

Phase retrieval is needed not only for quantitative tomography, but a retrieved phase map per se may also be useful as well for tissue characterizations in medical imaging. For example, the phase map of a breast can be used to compute the volumetric breast density, i.e. the volume fraction of fibroglandular tissue in the breast. In fact, recent scientific studies have found that high breast density is associated with two- to fourfold increase in risk for breast cancer [8]. Recent new legislature and regulatory efforts in the U. S. require to provide patients the breast density information from their breast imaging exams. Current mammography-associated breast density measurement methods are based on the small differences in x-ray attenuations between the breast fibroglandular and adipose tissues. All these methods require conducting elaborated system calibrations. Rather than measuring tissue attenuation as a surrogate of the breast density, we recently proposed a phase-shifts-based method to measure breast density directly. We found the volumetric breast density (VBD), i.e. the fibroglandular-tissue's volume fractions of the breast, can be determined from the retrieved breast phase map as [9]: $VBD = \text{breast}(-\phi(\vec{r})/\tilde{\lambda} r_e - \rho_{e,ad}T_c)/(N_p T_c (\rho_{e,fg} - \rho_{e,ad}))$ where $\phi(\vec{r})$ denotes the breast phase map, T_c is the compressed breast thickness, $\tilde{\lambda}$ is the average x-ray wavelength, $\rho_{e,fg} = 3.448 \times 10^{23}/\text{cm}^3$ is the electron density of breast fibroglandular tissue, and $\rho_{e,ad} = 3.108 \times 10^{23}/\text{cm}^3$ is that of the adipose tissue.

In this presentation, we give a brief review of the general phase retrieval methods from medical imaging perspective, and compare the most commonly used phase retrieval method in literature to a general phase retrieval method that we developed in recent years. In a phase contrast projection the attenuation contrast and phase contrast are encoded mixed together. In order to retrieve a phase map from phase contrast projections, one should decode the projection images based on the x-ray transport equation. The general x-ray transport equation is originally a transfer-function-based equation derived in the Fourier domain [10, 11]. For sake of succinctness and clarity we rewrite it here in the image space domain:

$$I(\vec{r}_D) = \frac{I_{\text{in}}}{M^2} \cos\left(\frac{\lambda R_2}{4\pi M} \nabla^2\right) \left[A^2(\vec{r}) + \frac{\lambda R_2}{2\pi M} (\nabla \cdot (\phi(\vec{r}) \nabla A^2(\vec{r}))) \right] - 2 \sin\left(\frac{\lambda R_2}{4\pi M} \nabla^2\right) [A^2(\vec{r}) \phi(\vec{r})] \quad (1.1)$$

where $I(\vec{r}_D)$ denotes the projection x-ray intensity, I_{in} the entrance x-ray intensity, $A^2(\vec{r})$ and $\phi(\vec{r})$ are the attenuation map and phase-shift map respectively. In eq. (1.1) R_2 is the object-

detector distance, M the magnification factor employed in the projection. and $\vec{r}_D$ denotes the position vector in the detector plane and $\vec{r}_D = M\vec{r}$. Note that in this equation $\sin(\lambda R_2 \nabla^2 / 4\pi M)$ and $\cos(\lambda R_2 \nabla^2 / 4\pi M)$ are the two-dimensional pseudodifferential operators, where the two-dimensional transverse Laplacian operator $\nabla^2 \equiv (\partial^2 / \partial x^2 + \partial^2 / \partial y^2)$. In mathematics the action on a function $g(\vec{r}_D)$ of a two-dimensional pseudodifferential operator, such as $\sin(c\nabla^2)$, is defined as: $\sin(c\nabla^2) g(\vec{r}_D) \equiv \iint \exp(i2\pi(\vec{q} - \vec{r}_D) \cdot \vec{f}) \cdot \sin(-4\pi^2 c f^2) g(\vec{q}) d^2 \vec{q} d^2 f$. The three-dimensional pseudodifferential operators described earlier are defined similarly. This general transport equation forms the basis from which phase retrieval methods are developed.

From medical imaging perspective, we noted two special features that are important in developing phase retrieval methods for such applications. First, the x-ray transport equation (eq. (1.1)) gets simplified for the medical imaging. In fact, medical imaging requires only moderate spatial resolution σ of tens micrometers or coarser. Combined with relatively short object-detector distances limited by the sizes of hospital rooms, so $\pi \lambda R_2 / 4M\sigma^2 \ll 1$, so the general x-ray transport equation (eq. (1.1)) is reduced to:

$$I(\vec{r}_D) = \frac{I_{in}}{M^2} \{A^2(\vec{r}) - \frac{\lambda R_2}{2\pi M} (\nabla \cdot (A^2(\vec{r}) \nabla \varphi(\vec{r})))\} \quad (1.2)$$

This simplified transport equation is just the so-called Transport of Intensity Equation (TIE) in literature [12, 13]. Hence the task of phase retrieval for medical imaging is to extract the phase map $\varphi(\vec{r})$ from phase contrast projections by using eq. (1.2). Obviously the projection intensity $I(\vec{r}_D)$ is determined not only by attenuation-map $A^2(\vec{r})$ in the projection, but also by the encoded phase contrast, that is, by the transverse Laplacian and gradient differentials of the phase-shift map $\varphi(\vec{r})$. Since the phase contrast and attenuation contrast are entangled together in a phase contrast image, as is shown by eq. (1.2), hence a single phase-contrast image is not enough for retrieving the phase map $\varphi(\vec{r})$ from it. In general several projection images, at least two projections, are needed for retrieving $\varphi(\vec{r})$ from them.

Second, since medical images are always contaminated with substantial quantum noise because of the imperative radiation dose limits, hence medical imaging places a stringent requirement on the robustness of a phase retrieval method. As the public and medical community becomes ever vigilant about the radiation risks involved in x-ray imaging such as mammography and computed tomography, it is important for us to develop highly robust phase retrieval methods for medical imaging applications.

In this presentation we compare the robustness of two phase-retrieval methods that are developed based on the simplified x-ray transport equation (eq. (1.2)). We will first introduce the phase retrieval method that is based on the direct inversion of eq. (1.2), which is called the TIE-inversion method. This method is the most commonly used method for phase retrieval in the literature [13, 14]. We will then discuss a recently developed phase retrieval method, which is called the Attenuation Partition based (AP-based) method [15, 16]. In order to examine the robustness of these two methods, we applied these two phase-retrieval methods to two experimental cases, and compare their performances in the robustness against the noise in projection acquisitions.

2 Two general phase retrieval methods

The mostly used phase retrieval method in the literature is the TIE-inversion method [13, 14]. In this method two acquired projection images of an object are used for phase retrieval: one is a contact radiograph $A^2(\vec{r})$, the other is a phase contrast projection image $I(\vec{r}_D)$ of the object. One then is able to retrieve the phase-shift map $\varphi(\vec{r})$ by directly inverting eq. (1.2) to solve for $\varphi(\vec{r})$ as follows [14]:

$$\varphi(\vec{r}) = - (2\pi M / \lambda R_2) \nabla^{-2} \left\{ \nabla \cdot \left[\frac{1}{A^2} \nabla \left(\nabla^{-2} \left(\frac{M^2 I}{I_{in}} - A^2 \right) \right) \right] \right\}. \quad (2.1)$$

In above equation the pseudodifferential operator ∇^{-2} denotes the inverse of the Laplacian operator $\nabla^2 \equiv (\partial^2 / \partial x^2 + \partial^2 / \partial y^2)$. The TIE-inversion method is effective and fast for phase retrievals in the cases with strong phase contrast effects and low noise levels. However, the Achilles heel of the TIE-inversion method lies at the inverse Laplacian operator involved in the TIE-inversion (eq. (2.1)). The inverse Laplacian operator ∇^{-2} embeds a zero-frequency singularity into eq. (2.1). As we show below, the singularity may grossly amplify the noise in the acquired projections and make the phase retrieval instable. To mend the instability problem, one can try to utilize the regularization techniques to tame the severity of the singularity. A common regularization technique is the so-called Tikhonov regularization [15]. With this regularization scheme one replaces the inverse Laplacian operator ∇^{-2} by a pseudodifferential operator $\nabla^2 / [(\nabla^2)^2 + \alpha^2]$, where α is the regularization parameter [15]. In essence Tikhonov regularization seeks the minimum-norm, least squares solution of eq. (2.1). The regularization parameter α was selected by comparing the quality of retrieved phase maps with different α -values. However, this mathematical procedure just limits the overall magnitude of phase errors caused by the noise, but the retrieved phase map could still suffer from loss of details, as is demonstrated for experimental cases shown in section 3 below.

In order to get rid of the singular pseudodifferential operator ∇^{-2} involved in the TIE-inversion method, we recently developed a novel phase retrieval method: the attenuation partition (AP) based method, or the AP-based method for short [16, 17]. Our idea is to utilize the correlation between the x-ray phase shift and attenuation to eradicate the singularity involved in phase retrieval. In fact, we partition the x-ray attenuation into two factors:

$$A^2(\vec{r}) = A_{KN}^2(\vec{r}) \cdot A_{pe,coh}^2(\vec{r}) \quad (2.2)$$

where A_{KN}^2 denotes the Compton scattering-generated attenuation, and $A_{pe,coh}^2$ is the attenuation generated by photoelectric absorption and coherent scattering. We noted that both $A_{KN}^2(\vec{r})$ and $\varphi(\vec{r})$ are all determined by the tissue's electron density:

$$A_{KN}^2(\vec{r}) = \exp(-\sigma_{KN} \rho_{e,p}(\vec{r})), \quad \varphi(\vec{r}) = -\lambda r_e \rho_{e,p}(\vec{r}), \quad (2.3)$$

where $\rho_{e,p}(\vec{r}) = \int \rho_e(\vec{r}, z) dz$ is the projected electron density along the ray path, and σ_{KN} denotes the Compton scattering cross-section, which is given by the Klein-Nishina total cross-section [16, 17]:

$$\sigma_{KN}(E) = 2\pi r_e^2 \left\{ \frac{1+\eta}{\eta^2} \left[\frac{2(1+\eta)}{1+2\eta} - \frac{1}{\eta} \log(1+2\eta) \right] + \frac{1}{2\eta} \log(1+2\eta) - \frac{1+3\eta}{(1+2\eta)^2} \right\} \quad (2.4)$$

where $\eta \equiv E/511$ keV. The extent of $\varphi - A^2$ -correlation is really depends on the sample elemental compositions and x-ray energy. For samples made of light elements with atomic numbers $Z < 10$, such as soft tissues, the $\varphi - A^2$ -correlation can be very strong. As E reaches 60 keV or higher, the Compton scattering dominates the attenuation in soft tissues, then x-ray phase shift and attenuation are all determined by soft tissues' electron density distributions, thereby φ and A^2 are fully correlated and we call this full-correlation as the phase-attenuation duality [18, 19]. Moreover, we found that the phase-attenuation duality greatly simplified the phase retrieval such that the phase map $\varphi(\vec{r})$ can be retrieved from just

a single phase contrast projection image $I(\vec{r}_D)$ by using the so-called duality transform $\mathfrak{D}(I(\vec{r}_D))$:

$$\varphi(\vec{r}) = \frac{\lambda r_e}{\sigma_{KN}} \ln \mathfrak{D}(I(\vec{r}_D)). \quad (2.5)$$

where the duality-transform $\mathfrak{D}(I(\vec{r}_D))$ is defined as

$$\mathfrak{D}(I(\vec{r}_D)) \equiv (1 - (\lambda^2 R_2 r_e / 2\pi M \sigma_{KN}) \nabla^2)^{-1} \cdot \left(\frac{M^2 I(\vec{r}_D)}{I_{IN}} \right). \quad (2.6)$$

Although the phase-attenuation duality does not hold in some cases such as imaging with low energy x-rays or imaging bones and calcifications that contain substantial amount of heavy elements, we decided to use eqs. (2.5)–(2.6) for obtaining a coarse estimates of the real phase maps in these general cases. We further seek to correct the estimation errors through an iterative algorithm, and its flow chart is shown in figure 1 [16]. In this flow chart I is the acquired phase contrast projection of the sample, A^2 the sample's attenuation map measured from sample contact radiograph, and I_{KN} denotes the computed phase contrast projection assuming Compton scattering as the sole attenuation mechanism.

In figure 1 two transforms are implemented in the iterations, one is the duality transform $\mathfrak{D}$ as is defined in eq. (2.6), the other is the Fresnel diffraction transform $\mathfrak{F}_r$ implementing x-ray Fresnel diffraction. According to this flow chart, one first applies the duality transform $\mathfrak{D}$ to the acquired phase contrast projection I to obtain an estimate of A_{KN}^2 and the phase map φ of the sample. Comparing A_{KN}^2 to the measured A^2 , one then computes the error $\delta A \equiv A_{KN} - A$. Computing the Fresnel diffraction of the fictitious object with a transmission function $\delta A \cdot \exp(i\varphi)$, one obtains the I_{KN} -correction $\delta I \equiv \{\mathfrak{F}_r(\delta A \cdot \exp(i\varphi))\}^2$, and a new I_{KN} -estimate determined as $I_{KN} = (\sqrt{I} + \sqrt{\delta I})^2$. With this updated estimate of I_{KN} one can start a new round of the iteration. The iteration converges when the I_{KN} does not change substantially with further iterations. Interested readers can find the detailed mathematical proofs of the algorithm in [16]. One significant advantage of this AP-based phase retrieval method is that the duality transform $\mathfrak{D} \equiv (1 - (\lambda^2 R_2 r_e / 2\pi M \sigma_{KN}) \nabla^2)^{-1}$ and Fresnel diffraction transform $\mathfrak{F}_r$ used in the iterations are all singularity-free.

Therefore, the AP-based method is stable and robust against the noise in acquired projections, as is demonstrated below with two experimental cases.

3 Comparison of robustness of the phase retrieval methods

In order to test the robustness of these two phase retrieval methods, namely the TIE-based method and the AP-based method, we applied the two methods to two experimental studies. The first case in testing is to retrieve the phase map of a breast lumpectomy specimen imaged with a microfocus tube operating at 40 kVp with an estimated average x-ray photon energy of 14.6 keV [16, 20]. Figure 2(a) is the specimen's contact radiograph acquired with a source-image receptor-distance (SID) of 66 cm. Figure 2(b) is the specimen's phase contrast projection acquired with SID = 1.85 m and $R_2 = 1.19$ m. Compared to figure 2(a), figure 2(b) demonstrates the dark-bright fringes at enhanced tissue interfaces and boundaries. We first employed the TIE-inversion method (eq. (2.1)) to retrieve the specimen's phase map using its radiograph and phase contrast image.

But the phase retrieval failed completely as specimen details would be hardly recognizable in the retrieved phase map. The failure can be ascribed to the intrinsic instability of the TIE-inversion method in presence of substantial noise in the two specimen projection images. As is pointed out in section 2, the inverse Laplacian operator ∇^{-2} plugs in the zero-frequency singularity into the TIE-inversion method. The singularity amplifies the noise in the projections and causes the failure in the phase retrieval.

Figure 3 is the specimen's phase map retrieved using the TIE-inversion method and Tikhonov regularization with an "optimal" $\alpha = 50\Delta u^2$, where Δu is the frequency sampling-step adopted in the retrieval. In figure 3 the tissue components of the lumpectomy specimen are hardly visible as breast tissue contrast is extremely poor across. The retrieved phase-shift values, as is shown in the gray-scale bar on right side in figure 3, range from $-79,000$ to $26,000$ radians. These retrieved phase values are grossly erroneous, since the actual phase-shift in the specimen ranges from -1400 to -1800 radians, as is estimated from the breast tissue compositions, thickness profile and average x-ray energy. Moreover, the phase-shifts outside the specimen should be close to zero as the physics dictates, but the outside phase values nonsensically swing from $-20,000$ to $+70,000$ radians in figure 3. As a comparison, we applied the AP-based method that we developed for the phase retrieval as well. Following the iteration flow chart in figure 1, we succeeded in retrieving the specimen's phase map with ten iteration steps. Figure 4 is the retrieved specimen's phase map with the AP-based method. In a stark contrast to figure 3, this time the different breast tissue components are depicted with good contrast. The retrieved phase-shift values are shown in the side gray-scale bar in figure 4. These phase values are ranging from -1400 to -1800 radians, are consistent with the estimate based on the tissue composition, thickness profile and average x-ray energy. The phase values along the guide-wire track in the specimen are about -3600 radians because of the metal wire in the track. Outside the specimen, the retrieved phase-shifts are only -1 to -2 radians, as is expected. These results demonstrate that the AP-based method is obviously superior to the TIE-inversion method in performing the phase retrieval in this study.

The second case in testing is to retrieve the phase map of a piece of air bubble wrap [21]. Shown in figure 5(a) is the wrap's contact radiograph acquired by the same tube operating at 40 kVp. In this acquisition the SID was set to 1.78 m. Since the thin polyethylene films and the air bubbles generate little differences in x-ray attenuation, so the low-contrast bubble rims are poorly depicted as they are masked by the noise in figure 5(a). On the other hand, figure 5(b) is a phase contrast projection of the bubble wrap. This phase contrast image was acquired with a sample-detector distance $R^2 = 1.15$ m and otherwise the same technique settings as that used for the radiograph. As is shown in figure 5(b), the phase contrast depicts not only the bubble rims, but also the folds inside the bubble. In order to retrieve the wrap's phase-shift map from the wrap's radiograph and phase contrast image, we first employed the TIE-inversion method and Tikhonov regularization. Figure 6(a) is the phase map retrieved by using the TIE-method and Tikhonov regularization. The optimal regularization parameter adopted is $\alpha = 200\Delta u^2$, as was determined from the trials with different α -values.

Apparently the quality of the phase map retrieved with TIE-inversion method is very poor, as the bubble rims get almost lost in the extremely cluttered background in figure 6(a). As a comparison, we applied the AP-based method again in this case. Figure 6(b) is the wrap's phase map retrieved using the AP-based method. In stark contrast to figure 6(a), the bubble rims are prominently depicted in the AP-retrieved phase map, as is shown in figure 6(b). In order to gauge the accuracies of the retrieved phase values, we measured the thicknesses of the flat bases between bubbles, and found it of 0.055 mm thick. Note that the molecular formula of the low-density polyethylene film is $(C_2H_4)_n$ and its mass density is 0.925 g/cm^3 , we found its electron density as $3.18 \times 10^{23}/\text{cm}^3$. Using the values of the base-thickness, wrap's electron density and average x-ray energy, we found that the estimated phase-shifts

at the flat bases between bubbles is -4.2 radian. Figure 7(a) shows the profile of the retrieved phase values obtained with the TIE-inversion method and Tikhonov regularization. The profile is plotted along the dash line marked on figure 6(a). Note that all the marked lines in figures 5(a), (b) and figures 6(a), (b) correspond to the same line drawn on the bubble wrap. According to this phase profile, the retrieved phase values at the flat bases fluctuate violently from -24.7 to $+87.5$ radians with an average of $+27.3$ radians, but x-ray phase shifts should be of negative values. Hence the retrieved average phase-shift of the flat bases is in gross discrepancy with the thickness-measurement-based estimate of -4.2 radians. A profile of retrieved phase values plotted from figure 6(b), namely the phase map retrieved using the AP-based method, is shown in figure 7(b). Note that the three sharp negative peaks shown in figure 7(b) reflect the large phase-shifts incurred at the three rims along the marked line. According to this phase profile, the average phase shift at the flat bases between bubbles is -1.05 radians. This value is reasonably close to the thickness-based phase estimate, and the errors may be ascribed to the the background nonuniformity in the acquired projection images.

4 Discussion and conclusions

The striking differences in performance of the two phase retrieval methods, namely the TIE-inversion method vs. the AP-based method as described above, underline the importance of handling the singularity associated with the direct inversion of the x-ray transport equation eq. (1.2). This transport equation dictates that the phase contrast projection is insensitive to the slow variations of x-ray phase shifts. Hence the TIE-inversion method is ineffective to recover slow phase variation. From mathematical viewpoint, the TIE-inversion formula (eq. (2.1)) is ill-posed, since it involves the inverse Laplacian operator ∇^{-2} that contains a zero-frequency singularity. As one tries to retrieve the slowly varying phase shifts with this method, the zero-frequency singularity amplifies the noise embedded in the acquired projections, and ruined the phase retrieval completely. The Tikhonov regularization as described above just limits the overall magnitude of the phase errors caused by the noise, but the intrinsic tissue contrast could still get lost in the retrieval, as is demonstrated in the two experimental studies discussed above. The noise problem is especially challenging in medical imaging where acquired images are generally with substantial quantum noise resulted from the imperative limits on allowed radiation doses. Hence the TIE-inversion method are ineffective at least, and may even be infeasible for medical imaging applications. On the other hand, the AP-based method eradicates the singularity problem by utilizing the correlations between x-ray attenuation and x-ray phase shifts. With such a correlation the resulted duality-transform operator $(1 - (\lambda^2 R_2 r_d / 2\pi M \sigma_{KN}) \nabla^2)^{-1}$ becomes singularity-free. That is, the AP-based method utilizes an object's own x-ray attenuation gets rid of the zero-frequency singularity and consequent noise amplification. Hence this physics-based regularization scheme of the AP-based method is expected to be more effective than the mathematical regularization. The greatly improved quality of the phase maps retrieved with the AP-based method, as is demonstrated in above two cases, validates this expectation.

Note that both the TIE-inversion method and AP-based method are the general phase retrieval methods that are applicable in principle for any general objects imaged with x-rays of any energies in medical imaging, and both require at least two projection images of an object for retrieving the object's phase map. We point out that there are two single-projection based phase retrieval methods that are applicable only for certain special imaging tasks. One of the single-projection based retrieval methods works specifically for a single-material sample, as long as the linear attenuation coefficient and refractive index of this material are known in advance [22]. In such special cases the phase map of a single-material sample can be retrieved from just a single phase contrast projection of the sample. While this phase retrieval method is very robust, the stringent requirement on the material

homogeneity and prior knowledge of material attenuation coefficients and refractive index limits its applicability. The other single-projection based retrieval methods the phase-attenuation-duality-based method (PAD-based method) that is applicable for any inhomogeneous samples made of light elements ($Z < 10$) and being imaged with high energy x-rays [18]. Imaging light-elemental samples such as soft tissue organs (e.g., breast, liver and brain etc.), one can utilize the phase-attenuation duality (PAD) as discussed in section 2 for phase retrieval. In this PAD-based method one can retrieve a sample's phase map using just a single phase contrast projection of the sample, without requiring any prior information on the tissue linear attenuation coefficient and refractive index at all. In this method the phase retrieval is accomplished by the duality transform as defined eqs. (2.5)–(2.6). Both the single-projection based phase retrieval methods utilize an object's own x-ray attenuation to get rid of the singularity, hence both the methods are very robust against noise and can be used for special medical imaging tasks as specified above. Moreover, these two single-projection based phase retrieval methods may greatly facilitate the quantitative phase contrast tomography as only a single projection is required for a given angular view [6, 23–25]. Several reports on quantitative tomography using these special phase retrieval methods can be found in this proceedings as well.

In summary, in this presentation a brief review of general phase retrieval methods is given from medical imaging perspective. While the x-ray transport equation gets simplified for medical imaging applications, the requirement on phase retrieval robustness is demanding. We compared the most commonly used phase retrieval method, namely the TIE-inversion method, to the attenuation partition based (AP-based) method. We showed that the TIE-inversion method, regardless if being assisted with the Tikhonov regularization, failed to retrieve the phase maps in two experimental studies. The failure exposes this method's weakness as being unstable against the noise. In contrast, the sample phase maps are retrieved successfully by using the AP-based method. The stark performance differences of the two methods are rooted in their different techniques dealing with the singularity problem. This comparison shows that the robust AP-based phase retrieval method will be superior to the TIE-inversion method for medical imaging applications where radiation doses are stringently limited.

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Figure 1.
The iteration flow chart of the AP-based method.

(a)

(b)

Figure 2.

(a) Contact radiograph of a lumpectomy specimen (© IEEE 2008) acquired at 40 kVp using a x-ray tube. (b) Phase contrast projection of a lumpectomy specimen (© IEEE 2008) acquired at 40 kVp using a x-ray tube.

Figure 3.
Phase map of the lumpectomy specimen retrieved by using TIE-inversion method and Tikhonov regularization.

Figure 4.
Phase map of the lumpectomy specimen retrieved by using the AP-based method.

(a)

(b)

Figure 5.

(a) Contact radiograph of a piece of air bubble wrap. (b) Phase contrast projection of the bubble wrap. The two were all acquired with a x-ray tube at 40 kVp.

(a)

(b)

Figure 6.

(a) Phase map of the bubble wrap retrieved using TIE-inversion method and Tikhonov regularization. (b) Phase map of the bubble wrap retrieved using the AP-based method.

Figure 7. (a) Profile of the phase values retrieved using the TIE-inversion method. (b) Profile of the phase values retrieved using the AP-based method.