H_∞ Estimation for Stochastic Time Delays in Networked Control Systems by Partly Unknown Transition Probabilities of Markovian Chains

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This paper is concerned with the problem of H_∞ estimation for networked control systems. Time delays and packet dropouts are considered simultaneously. The occurrence probability of each time delay is considered. The packet dropouts have the Bernoulli distributions. The system is modeled as Markovian jump linear systems with partly unknown transition probability. State observer is designed to estimate the practical state with H_∞ feature. The estimation problem is cast into a set of linear matrix inequalities. An example is provided to illustrate the effectiveness and applicability of the proposed method. [DOI: 10.1115/1.4007973]

Keywords: H_∞ estimation, network induced dropouts, linear matrix inequality, Markovian jump linear systems

1 Introduction

Network-induced delay and packet-dropout are two main problems in networked control systems (NCS), and have attracted much research interest [1,2].

The problem of time delay was considered in Refs. [3–6]. In these papers, time delays were bounded and dealt with by stochastic parameters. The determined methods were used to deal with stability analysis and controller design [3,4]. The stochastic methods which adopted Markovian chains and Bernoulli distribution were used in Refs. [5,6]. Time delays were known variables which were constant or stochastic values determined by certain distribution in these two methods, then controller was designed to provide stochastic stability and the desired performance for the closed-loop networked control systems. Different with the above methods, communication protocol was designed to decrease the influence of time delays [7].

The problem of packet dropout was considered in Refs. [8–11]. The problem of estimation and filtering were investigated in Refs. [12–16]. Considering the case in which time delays and dropouts exit simultaneously. Determined method was proposed in Refs. [17,18]. In Ref. [17], the problem of network-induced data dropouts and variable delays with NCS was investigated. The deterministic method was proposed to deal with time delays and dropouts with upper bound and lower bound. Sufficient conditions for Lyapunov stability are derived with uncertainty of drops and

delays. In Ref. [18], a new switched linear system model was proposed to describe the NCS with both network-induced delay and packet-dropout. A quantitative relation was constructed between the packet-dropout rate and the stability of the NCS. Design procedures for the state feedback stabilizing controllers are also presented by using the augmenting technique. The number of the packet dropouts was supposed to be bounded. The dropouts were considered in determined way. Stochastic method including Markovian chain was proposed in Refs. [5,19]. In paper [19], the stability and stabilization problems of a class of continuous-time and discrete-time Markovian jump linear system with partly unknown transition probabilities are investigated. In contrast with the uncertain transition probabilities studied recently, the concept of partly unknown transition probabilities proposed in this paper does not require any knowledge of the unknown elements. Reference [20] was concerned with the stabilization problem for a networked control system with Markovian characterization. Reference [5] considered the case that the random communication delays existed both in the system state and in the mode signal. It was modeled as a Markov chain. The resulting closed-loop system is modeled as a Markovian jump linear system with two jumping parameters, and a necessary and sufficient condition on the existence of stabilizing controllers is established.

In most existing work, time delays and dropouts were assumed to be up or lower bounded. Certain stochastic variables were introduced to represent all the values between the two bound. Determined and stochastic methods were presented to deal with the problem. While this treatment can simplify the networked control problems, it may also cause significant conservativeness as large time delays often occur with a low probability.

In order to reduce the conservativeness, the probability of each time delay was taken into account in stabilization design [21]. However, disturbance and packet dropouts were not considered in this paper. The determined method was used to model the system. So far, time delays and packet dropouts are not considered simultaneously by the model method with partly known transition probability Markovian chain. The other problem was that every occurrence probability of time delay and packet dropout was not considered. The packet dropouts can be considered as a kind of infinite delay. Yue, and co-workers have already deal with this issue with a delay system formulation including paper [22–26]. In Ref. [22], the descriptor systems with time-varying discrete and distributed delays is investigated. In Ref. [24], the filtering problem is studied for descriptor systems. In Ref. [23], the exponential stability for stochastic systems is investigated with time-varying delays, nonlinearities and Markovian jump parameters. In Ref. [25], the problem of stabilization of uncertain systems with unknown input delay is studied. In Ref. [26], the feedback design is extended to a class of nonlinear time-delay systems.

There are a fair amount of work in this area in recent years [20,27–46]. Wang and his co-workers have studied similar problems under the missing measurement formulation in Refs. [47–50]. In Ref. [47], the problem of robust filtering is investigated for a class of discrete time-varying Markovian jump systems. The randomly occurring nonlinearities and sensor saturation are considered in this system. In Ref. [48], the distributed state estimation problem is investigated for a class of sensor networks. The system is described by uncertain discrete-time dynamical systems with Markovian jumping parameters and distributed time delays. Reference [49] is concerned with the mixed H_2/H_∞ control problem over a finite horizon for a class of nonlinear Markovian jump systems with both stochastic nonlinearities and probabilistic sensor failures. In Ref. [50], the globally exponential stabilization problem is investigated for a general class of stochastic systems with both Markovian jumping parameters and mixed time-delays. The mixed mode dependent time-delays consist of both discrete and distributed delays. A more updated literature review of partly known probabilities in the Markov process can be states as [19,51–54]. In Ref. [19], the problem of stability and stabilization of Markovian jump linear systems

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with partly unknown transition probabilities is investigated. In Ref. [51], it extended the known results to this system with time delays. In Ref. [52], the problem of filtering for Markovian jump linear systems with partly unknown transition probabilities is investigated. In Ref. [53], it extended the partly unknown transition probabilities of Markovian jump linear systems to the defective statistics of modes transitions. In Ref. [54], the H_∞ filter for Markov jumping linear systems with non-accessible mode information is studied.

In this paper, the problem of H_∞ estimation is investigated for NCS with delays and dropouts. Partly known transition probability Markovian chain is used to model the system. Stability analysis and filtering design are developed based the occurrence probabilities of time delays and packet dropouts. State observer is designed to estimate the practical state with H_∞ feature.

2 Problem Statement

Consider the following NCS:

$$\begin{cases} x(k+1) = A_s x(k) + B_{2s} w(k) \\ y(k) = C_s x(k) + D_{2s} w(k) \\ z(k) = M_s x(k) \end{cases} \quad (1)$$

where $x(k) \in R^n$ is the state vector, $w(k) \in L_2$ is the exogenous disturbance signal, $y(k) \in R^q$ is the controller output, $z(k)$ is the state which need to be estimated. $r \in \{1, 2, \dots, M\}$ governs the switching among the different system modes. A_r, B_{2r}, C_r, D_{2r} are some constant matrices of appropriate dimensions.

The values of the network induced delay have a certain probability distribution. Let $T_m, m \in \{1, 2, \dots, p\}$ denotes each time delay. Define

$$\lambda\{T_m = \tau_i\} = \begin{cases} 1, T_m = \tau_i \\ 0, T_m \neq \tau_i \end{cases} \quad (2)$$

$$E[\lambda\{T_m = \tau_i\}] = \text{prob}\{T_m = \tau_i\} = \beta_i, \quad i = 1, \dots, p$$

The state $x(k)$ in sensor nodes becomes $x(k - \tau_i)$ when it is transferred to controller nodes in the influence of time-delays. Consider the influence of time delays and the system. The NCS can be expressed as follows:

$$\begin{cases} x(k+1) = A_s x(k) + \sum_{i=1}^p \lambda\{T_m = \tau_i\} B_{1s} x(k - \tau_i) + B_{2s} w(k) \\ y(k) = C_s x(k) + \sum_{i=1}^p \lambda\{T_m = \tau_i\} D_{1s} x(k - \tau_i) + D_{2s} w(k) \\ z(k) = M_s x(k) \quad x(k) = \varphi(k), k = -\tau_g, g = 1, \dots, p \end{cases} \quad (3)$$

$r \in \{1, 2, \dots, N\}$ is the M modes. Their relationship is decided by Markovian chain. Furthermore, the Markov process transition rates matrix Λ is defined by

$$\Lambda = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \dots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \dots & \sigma_{2N} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \dots & \sigma_{3N} \\ \dots & \dots & \dots & \dots & \dots \\ \sigma_{N1} & \sigma_{N2} & \sigma_{N3} & \dots & \sigma_{NN} \end{pmatrix} \quad (4)$$

The mode transition probabilities can be expressed as $\text{prob}\{r_{k+1} = v | r_k = s\} = \sigma_{sv}$.

$$\text{where } \sigma_{sv} > 0, \quad s, v \in \{1, 2, \dots, N\}, \quad \sum_{v=1}^N \sigma_{sv} = 1$$

In addition, the transition rates or probabilities of the jumping process in this paper are considered to be partly accessed, and some elements are unknown.

$$\Lambda = \begin{pmatrix} ! & \sigma_{12} & \sigma_{13} & \dots & ! \\ \sigma_{21} & \sigma_{22} & ! & \dots & \sigma_{2N} \\ \sigma_{31} & ! & \sigma_{33} & \dots & \sigma_{3N} \\ \dots & \dots & \dots & \dots & \dots \\ \sigma_{N1} & \sigma_{N2} & ! & \dots & \sigma_{NN} \end{pmatrix} \quad (5)$$

! stands for the unknown elements.

$$s \in r, \quad r = r_k^s + r_{uk}^s, \quad r_k^s = \{v, \sigma_{sv} \text{ is known}\}, \quad r_{uk}^s = \{v, \sigma_{sv} \text{ is unknown}\} \quad (6)$$

If r_k^s is not empty, it can be expressed as $r_k^s = (\eta_1^s, \dots, \eta_l^s)$, $1 \leq l \leq N$. η_l^s stands for the l th known element in the s th row of matrix Λ . We defined $\sigma_k^s = \sum_{v \in r_k^s} \sigma_{sv}$ throughout the paper.

Considering the linear stochastic discrete-time system in Eq. (4), we want to find the estimate $\hat{z}(k)$ of $z(k)$ such that the H_∞ -norm of the filtering error dynamics is minimized. Therefore, a filter needs to be designed.

Now, consider the following filter:

$$\begin{cases} \hat{x}(k+1) = a_{fs} \hat{x}(k) + b_{fs} \psi(k) + c_{fs} y(k) \\ \hat{z}(k) = M_{fs} \hat{x}(k) \\ \psi(k) = \sum_{i=1}^p \lambda\{T_m = \tau_i\} x(k - \tau_i) \end{cases} \quad (7)$$

The augment system can be written as

$$\begin{aligned} \tilde{z}(k) &= z(k) - \hat{z}(k) \begin{pmatrix} x(k+1) \\ x(k+1) \end{pmatrix} \\ &= \begin{pmatrix} A_s & 0 \\ c_{fs} C_s & a_{fs} \end{pmatrix} \begin{pmatrix} x(k) \\ x(k) \end{pmatrix} \\ &\quad + \begin{pmatrix} B_{1s} \\ b_{fs} + c_{fs} D_{1s} \end{pmatrix} \psi(k) + \begin{pmatrix} B_{2s} \\ c_{fs} D_{2s} \end{pmatrix} w(k) \\ \tilde{z}(k) &= (M_s \quad -M_{fs}) \begin{pmatrix} x(k) \\ \hat{x}(k) \end{pmatrix} \end{aligned} \quad (8)$$

The following definition is adopted in order to get the result.

Definition 1. System is said to be stochastically stable if for $u(k) = 0, w(k) = 0$ and every initial condition $x(0) \in R^n$, and $r(0) \in \{1, 2, \dots, M\}$, the following holds:

$$E \left\{ \sum_{k=0}^{\infty} \|x(k)\|^2 \middle| x(0), r(0) \right\} < \infty$$

Definition 2. Given the disturbance input $w(k) \in L_2, w(k) \neq 0$ in Eq. (4), a filter in the form of Eq. (8) is designed such that the filtering error system (9) is stochastically stable and satisfies: $\sum_{k=0}^{\infty} E \left\| \tilde{z}(k) \right\|^2 < \gamma^2 \sum_{k=0}^{\infty} E \|w(k)\|^2$, then system (9) has the γ feature of filtering.

Lemma 1. The unforced system is globally mean-square asymptotical stability (GUAS) if, and only if, there exists a set of symmetric and positive definite matrices $P_i, i \in \{1, 2, \dots, N\}$ satisfying [27]

$$A_i^T \zeta^i A_i - P_i < 0 \quad (9)$$

where $\zeta^i = \sum_{j \in I} \sigma_{ij} P_j$.

Lemma 2. When $w(k) = 0$, considering the system (1) with partly unknown transition probabilities. The corresponding system is stochastically stable if there exists $P_r > 0, r \in \{1, 2, \dots, N\}$, such that

$$\begin{aligned} A_r^T P_r^s A_r - \sigma_k^s P_r &< 0, \quad A_r^T P_v A_r - P_r < 0, \quad \forall v \in r_{uk}^s, \\ P_k^s &= \sum_{v \in r_k^s} \sigma_{sv} P_v \end{aligned} \quad (10)$$

Proof. Based on Lemma 1, we know that the system (1) is stochastically stable if (10) holds.

Due to $\sum_{v \in r} \sigma_{sv} = 1$, the left hand side of Eq. (10) can be written in the form of

$$\begin{aligned} A_r^T \left(\sum_{v \in r} \sigma_{sv} P_v \right) A_r - \left(\sum_{v \in r} \sigma_{sv} \right) P_r &= A_r^T \left(\sum_{v \in r_k^s} \sigma_{sv} P_v \right) A_r - \left(\sum_{v \in r_k^s} \sigma_{sv} \right) P_r \\ &+ A_r^T \left(\sum_{v \in r_{uk}^s} \sigma_{sv} P_v \right) A_r - \left(\sum_{v \in r_{uk}^s} \sigma_{sv} \right) P_r = A_r^T P_k^s A_r - \sigma_k^s P_r \\ &+ \sum_{v \in r_{uk}^s} \sigma_{sv} (A_r^T P_v A_r - P_r) \end{aligned} \quad (11)$$

Since one always has $\sigma_{sv} \geq 0, \forall v \in r$, it is straightforward Lemma 2 can be satisfied if Eq. (11) holds. Obviously, no knowledge on $\sigma_{sv}, \forall v \in r_{uk}^s$ is needed in Eq. (11). we can hereby conclude that the system (1) is stochastically stable against the partly unknown transition probabilities (5). This completes the proof.

3 Main Results

Theorem 1. The system in Eq. (8) is GUAS if there are symmetric matrices, $P_s > 0, P_v > 0, N, M_v, M_s, S_v, s \in \{1, 2, \dots, N\}, v \in \{1, 2, \dots, N\}$, the disturbance rejection index γ and filters with parameters a_{fs}, b_{fs}, c_{fs} such that the following linear matrix inequalities (LMI) hold:

$$\begin{pmatrix} N_{11} & -\left(\sum_{v \in r_{uk}^s} P_v\right) \\ * & N_{21} \end{pmatrix} \geq 0, \dots, \begin{pmatrix} N_{1p} & -\left(\sum_{v \in r_{uk}^s} P_v\right) \\ * & N_{2p} \end{pmatrix} \geq 0, \quad (12)$$

$$\begin{pmatrix} N_{1w} & -\left(\sum_{v \in r_{uk}^s} P_v\right) \\ * & N_{2w} \end{pmatrix} \geq 0$$

$$\begin{pmatrix} N_{11} & -\tau_1 \left(\sum_{v \in r_{uk}^s} M_v\right) \\ * & N_{21} \end{pmatrix} \geq 0, \dots, \begin{pmatrix} N_{1p} & -\tau_p \left(\sum_{v \in r_{uk}^s} M_v\right) \\ * & N_{2p} \end{pmatrix} \geq 0, \quad (13)$$

$$\begin{pmatrix} N_{1w} & -\sum_{i=1}^p \tau_i \left(\sum_{v \in r_{uk}^s} \sigma_{sv} M_v\right) \\ * & N_{2w} \end{pmatrix} \geq 0$$

$$\begin{pmatrix} \phi & \Phi_1^T & \Phi_2^T & \Phi_2^T & \Phi_1^T & \Phi_2^T & \Phi_2^T \\ * & -\Gamma_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\Gamma_1 & 0 & 0 & 0 & 0 \\ * & * & * & -\Gamma_2 & 0 & 0 & 0 \\ * & * & * & * & -\Gamma_3 & 0 & 0 \\ * & * & * & * & * & \Gamma_3 & 0 \\ * & * & * & * & * & * & \Gamma_4 \end{pmatrix} < 0 \quad (14)$$

Where

$$\begin{aligned} \phi &= \phi_{112} + \phi_{114} + \phi_{115} + \phi_{116} + \psi_1 + \phi_{1212} + \psi_2 + \phi_{1222} \\ &+ \phi_{312} + \phi_{314} + \phi_{315} + \phi_{316} + \psi_3 + \phi_{3212} + \psi_3 + \phi_{3222} \\ &+ \phi_{331} + \phi_{332} \end{aligned} \quad (15)$$

$$\Phi_1^T = \begin{pmatrix} 0 & 0 & \sqrt{\beta_1} A_s^T & \dots & \sqrt{\beta_p} A_s^T & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \sqrt{\beta_1} B_{1s}^T & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & \dots \\ 0 & 0 & 0 & 0 & \sqrt{\beta_p} B_{1s}^T & 0 \\ 0 & 0 & \sqrt{\beta_1} B_{2s}^T & \dots & \sqrt{\beta_p} B_{2s}^T & 0 \end{pmatrix} \quad (16)$$

$$\Gamma_1 = \left(\sum_{v \in r_k^s} \sigma_{sv} P_v \right) \quad (17)$$

$$\phi_{112} = \text{diag} \left(-\left(\sum_{v \in r_k^s} \sigma_{sv} P_s \right) 0 \ 0 \ \dots \ 0 \ 0 \right) \quad (18)$$

$$\phi_{114} = \text{diag} \left(-\left(\sum_{v \in r_{uk}^s} \sigma_{sv} P_s \right) 0 \ 0 \ \dots \ 0 \ 0 \right) \quad (19)$$

$$\begin{aligned} \phi_{115} &= \sum_{v \in r_{uk}^s} \sigma_{sv} \text{diag} (A_s^T \ 0 \ \beta_1 B_{1s}^T \ \dots \ \beta_p B_{1s}^T \ 0) \\ &\times P_v \text{diag} (A_s \ 0 \ B_{1s} \beta_1 \ \dots \ B_{1s} \beta_p \ 0) \end{aligned} \quad (20)$$

$$\begin{aligned} \phi_{116} &= \text{diag} (A_s^T + (N_{11}) + \dots N_{1p} + N_{1w} A_s 0 \beta_1 B_{1s}^T (N_{21} + N_{1w}) \\ &\times \beta_1 B_{1s} \dots \beta_p B_{1s}^T (N_{2p} + N_{1w}) \beta_p B_{1s} B_{2s}^T (p+1) N_{2w} B_{2s}) \end{aligned} \quad (21)$$

$$\Phi_2^T = \begin{pmatrix} 0 & 0 & \sqrt{\beta_1} C_s^T c_{fs}^T & \dots & \sqrt{\beta_p} C_s^T c_{fs}^T & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \sqrt{\beta_1} b_{fs}^T & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & \dots \\ 0 & 0 & 0 & 0 & \sqrt{\beta_p} b_{fs}^T & 0 \\ 0 & 0 & \sqrt{\beta_1} c_{fs} D_{2s} & \dots & \sqrt{\beta_p} c_{fs} D_{2s} & D_{2s}^T c_{fs}^T \end{pmatrix} \quad (22)$$

$$\psi_1 = \text{diag} \left(0 \ -\left(\sum_{v \in r_k^s} \sigma_{sv} P_s \right) 0 \ \dots \ 0 \ 0 \right) \quad (23)$$

$$\phi_{1212} = \begin{pmatrix} C_s^T c_{fs}^T (N_{11} + \dots N_{1p}) c_{fs} C_s & 0 & 0 & \dots & 0 & 0 \\ * & a_{fs}^T (N_{11} + \dots N_{1p}) a_{fs} + a_{fs}^T \sum_{v \in r_k^s} \sigma_{sv} P_v a_{fs} - \sum_{v \in r_k^s} \sigma_{sv} P_s & \sum_{v \in r_k^s} \sigma_{sv} P_v b_{fs} \beta_1 & \dots & \sum_{v \in r_k^s} \sigma_{sv} P_v b_{fs} \beta_p & 0 \\ * & * & 2\beta_1 D_{1s}^T c_{fs}^T N_{21} \beta_1 c_{fs} D_{1s} & 0 & 0 & 0 \\ * & * & * & \dots & \dots & \dots \\ * & * & * & * & 2\beta_p D_{1s}^T c_{fs}^T N_{2p} \beta_p c_{fs} D_{1s} & 0 \\ * & * & * & * & * & 0 \end{pmatrix} \quad (24)$$

$$\Gamma_2 = \left(\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \right) \quad (25)$$

$$\psi_2 = \text{diag} \left(0 \quad - \left(\sum_{v \in I_{uk}^s} \sigma_{sv} P_s \right) 0 \dots 0 \quad 0 \right) \quad (26)$$

$$\phi_{1222} = \begin{pmatrix} C_s^T c_{fs}^T (N_{11} + \dots N_{1p}) c_{fs} C_s & 0 & 0 & \dots & 0 & 0 \\ * & a_{fs}^T (N_{11} + \dots N_{1p}) a_{fs} + a_{fs}^T \sum_{v \in I_{uk}^s} \sigma_{sv} P_v a_{fs} - \sum_{v \in I_{uk}^s} \sigma_{sv} P_s & \sum_{v \in I_{uk}^s} P_v b_{fs} \beta_1 & \dots & \sum_{v \in I_{uk}^s} P_v b_{fs} \beta_p & 0 \\ * & * & 2\beta_1 D_{1s}^T c_{fs}^T N_{21} \beta_1 c_{fs} D_{1s} + 0 & 0 & 0 & 0 \\ * & * & * & \dots & \dots & \dots \\ * & * & * & * & 2\beta_p D_{1s}^T c_{fs}^T N_{2p} \beta_p c_{fs} D_{1s} & 0 \\ * & * & * & * & * & 0 \end{pmatrix} \quad (27)$$

$$\phi_{21} = \text{diag} \left(\left(\sum_{v \in I_k^s} \sigma_{sv} S_v \right) \quad \left(\sum_{v \in I_k^s} \sigma_{sv} S_v \right) \quad - \sum_{v \in I_k^s} \sigma_{sv} S_v \quad \dots \quad - \sum_{v \in I_k^s} \sigma_{sv} S_v \quad 0 \right) \quad (28)$$

$$\phi_{22} = \sum_{v \in I_{uk}^s} \sigma_{sv} \text{diag} (S_v \quad S_v \quad -N \quad \dots \quad -N \quad I) \quad (29)$$

$$\Gamma_3 = \sum_{i=1}^p \tau_i \left(\sum_{v \in I_k^s} \sigma_{sv} M_v \right) \quad (30)$$

$$\phi_{312} = \text{diag} \left(- \sum_{i=1}^p \tau_i \left(\sum_{v \in I_k^s} \sigma_{sv} M_s \right) 0 \quad 0 \dots 0 \quad 0 \right) \quad (31)$$

$$\phi_{314} = \text{diag} \left(- \sum_{i=1}^p \tau_i \left(\sum_{v \in I_{uk}^s} M_s \right) 0 \quad 0 \dots 0 \quad 0 \right) \quad (32)$$

$$\begin{aligned} \phi_{315} &= \sum_{v \in I_{uk}^s} \sigma_{sv} \text{diag} \left(\sqrt{\sum_{i=1}^p \tau_i A_s^T} \quad 0 \quad \sqrt{\tau_1} \beta_1 B_{1s}^T \quad \dots \quad \sqrt{\tau_p} \beta_p B_{1s}^T \quad 0 \right) \\ &M_v \text{diag} \left(\sqrt{\sum_{i=1}^p \tau_i A_s} \quad 0 \quad \sqrt{\tau_1} B_{1s} \beta_1 \quad \dots \quad \sqrt{\tau_p} B_{1s} \beta_p \quad 0 \right) \end{aligned} \quad (33)$$

$$\begin{aligned} \phi_{316} &= \text{diag} \left(\sqrt{\sum_{i=1}^p \tau_i A_s^T} \quad 0 \quad \sqrt{\tau_1} \beta_1 B_{1s}^T \quad \dots \quad \sqrt{\tau_p} \beta_p B_{1s}^T \quad \sqrt{\sum_{i=1}^p \tau_i (p+1) B_{2s}^T} \right) \\ &\text{diag} (N_{11} + \dots N_{1p} + N_{1w} \quad 0 \quad N_{21} + N_{1w} \quad \dots \quad N_{2p} + N_{1w} \quad (p+1) N_{2w}) \\ &\text{diag} \left(\sqrt{\sum_{i=1}^p \tau_i A_s} \quad 0 \quad \sqrt{\tau_1} \beta_1 B_{1s} \quad \dots \quad \sqrt{\tau_p} \beta_p B_{1s} \quad \sqrt{\sum_{i=1}^p \tau_i (p+1) B_{2s}} \right) \end{aligned} \quad (34)$$

$$\Gamma_3 = \sum_{i=1}^p \tau_i \left(\sum_{v \in I_k^s} \sigma_{sv} M_v \right) \quad (35)$$

$$\psi_3 = \text{diag} \left(0 \quad - \left(\sum_{v \in I_k^s} \sigma_{sv} M_s \right) 0 \dots 0 \quad 0 \right) \quad (36)$$

Where

$$\phi_4 = \text{diag} (\Xi_1 \quad \Xi_2) \quad (37)$$

$$\begin{aligned}\Xi_1 &= \begin{pmatrix} M_s^T M_s & -M_s^T M_{fs} \\ * & M_{fs}^T M_{fs} \end{pmatrix} \\ \Xi_2 &= \text{diag}(0 \quad \dots \quad 0 \quad -\gamma^2)\end{aligned}\quad (38)$$

ϕ_{3212} can be written as

$$\phi_{3212} = \begin{pmatrix} C_s^T c_{fs}^T (N_{11} + \dots N_{1p}) c_{fs} C_s & 0 & 0 & \dots & 0 & 0 \\ * & a_{fs}^T (N_{11} + \dots N_{1p}) a_{fs} + a_{fs}^T \sum_{i=1}^p \tau_i \sum_{v \in I_k^p} \sigma_{sv} M_v a_{fs} - \sum_{i=1}^p \tau_i \sum_{v \in I_k^p} \sigma_{sv} M_s & \tau_1 \sum_{v \in I_k^p} \sigma_{sv} P_v b_{fs} \beta_1 & \dots & \tau_p \sum_{v \in I_k^p} \sigma_{sv} M_v b_{fs} \beta_p & 0 \\ * & * & 2\tau_1 \beta_1 D_{1s}^T c_{fs}^T N_{21} \beta_1 c_{fs} D_{1s} & 0 & 0 & 0 \\ * & * & * & \dots & \dots & \dots \\ * & * & * & * & 2\tau_p \beta_p D_{1s}^T c_{fs}^T N_{2p} \beta_p c_{fs} D_{1s} & 0 \\ * & * & * & * & * & 0 \end{pmatrix}\quad (39)$$

Where

$$\Gamma_4 = \sum_{i=1}^p \tau_i \left(\sum_{v \in I_{uk}^p} \sigma_{sv} M_v \right) \quad (40)$$

$$\psi_4 = \text{diag} \left(0 \quad - \left(\sum_{v \in I_{uk}^p} \sigma_{sv} P_s \right) 0 \dots 0 \right) \quad (41)$$

$$\phi_{1212} = \begin{pmatrix} C_s^T c_{fs}^T (N_{11} + \dots N_{1p}) c_{fs} C_s & 0 & 0 & \dots & 0 & 0 \\ * & a_{fs}^T (N_{11} + \dots N_{1p}) a_{fs} + a_{fs}^T \sum_{i=1}^p \tau_i \sum_{v \in I_{uk}^p} \sigma_{sv} M_v a_{fs} - \sum_{i=1}^p \tau_i \sum_{v \in I_{uk}^p} \sigma_{sv} M_s & \tau_1 M_v b_{fs} \beta_1 & \dots & \tau_p M_v b_{fs} \beta_p & 0 \\ * & * & 2\tau_1 \beta_1 D_{1s}^T c_{fs}^T N_{21} \beta_1 c_{fs} D_{1s} & 0 & 0 & 0 \\ * & * & * & \dots & \dots & \dots \\ * & * & * & * & 2\tau_p \beta_p D_{1s}^T c_{fs}^T N_{2p} \beta_p c_{fs} D_{1s} & 0 \\ * & * & * & * & * & 0 \end{pmatrix}\quad (42)$$

$$\phi_{331} = \begin{pmatrix} 0 & - \sum_{i=1}^p \tau_i C_s^T c_{fs}^T \sum_{v \in I_k^p} \sigma_{sv} P_v & 0 & 0 & 0 & 0 \\ * & - \sum_{i=1}^p \tau_i \sum_{v \in I_k^p} \sigma_{sv} P_v a_{fs} & - \tau_1 \sum_{v \in I_k^p} \sigma_{sv} P_v [b_{fs} \beta_1 + c_{fs} D_{1s} \beta_1] & \dots & - \tau_p \sum_{v \in I_k^p} \sigma_{sv} P_v [b_{fs} \beta_p + c_{fs} D_{1s} \beta_p] & \sum_{i=1}^p \tau_i \sum_{v \in I_k^p} \sigma_{sv} P_v c_{fs} D_{2s} \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & \dots & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{pmatrix}\quad (43)$$

$$\phi_{332} = \begin{pmatrix} 0 & - \sum_{i=1}^p \tau_i C_s^T c_{fs}^T \sum_{v \in I_{uk}^p} P_v & 0 & 0 & 0 & 0 \\ * & - \sum_{i=1}^p \tau_i \sum_{v \in I_{uk}^p} P_v a_{fs} & - \tau_1 \sum_{v \in I_{uk}^p} P_v [b_{fs} \beta_1 + c_{fs} D_{1s} \beta_1] & \dots & - \tau_p \sum_{v \in I_{uk}^p} P_v [b_{fs} \beta_p + c_{fs} D_{1s} \beta_p] & \sum_{i=1}^p \tau_i \sum_{v \in I_{uk}^p} P_v c_{fs} D_{2s} \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & \dots & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{pmatrix}\quad (44)$$

Proof. Define Lyapunov function as

$$V(k) = \zeta^T(k) P_s \zeta(k) \quad (45)$$

$$V(x(k), k) = V_1 + V_2 + V_3, \quad V_1 = \zeta^T(k) P_s \zeta(k), \quad V_2 = \sum_{i=1}^p \sum_{i=k-\tau_i}^{k-1} \zeta^T(i) \left(\sum_{s \in I} \sigma_{sv} S_v \right) \zeta(i), \quad V_3 = \sum_{j=1}^q \sum_{i=-\tau_j}^{-1} \sum_{m=k+i}^{k-1} \delta^T(m) \left(\sum_{s \in I} \sigma_{sv} M_v \right) \delta(m) \quad (46)$$

$$\delta(m) = \zeta(m+1) - \zeta(m)$$

P_v are the different symmetric matrices determined by transition matrices.
Define

$$\zeta^T(k) = \begin{pmatrix} x^T(k) & \hat{x}^T(k) \end{pmatrix} \quad (47)$$

The forward difference of Lyapunov function can be written as

$$E\{\Delta V_1(k)\} = E\{\Delta V_{11}(k) + \Delta V_{12}(k)\}, \quad E\{\Delta V_{11}(k)\} = E\{x^T(k+1) \sum_{v \in I_k^s} \sigma_{sv} P_v x(k+1) - x^T(k) \sum_{v \in I_k^s} \sigma_{sv} P_v x(k)\} \quad (48)$$

$$E\{\Delta V_{12}(k)\} = E\{\hat{x}^T(k+1) \sum_{v \in I_k^s} \sigma_{sv} P_v \hat{x}(k+1) - \hat{x}^T(k) \sum_{v \in I_k^s} \sigma_{sv} P_v \hat{x}(k)\}$$

ΔV_{11} can be expressed as

$$\begin{aligned} \Delta V_{11} = E[\Delta V_{11}(x(k))] &= x^T(k) A_s^T \left(\sum_{v \in I_k^s} \sigma_{sv} P_v \right) A_s x(k) + 2x^T(k) A_s^T \left(\sum_{v \in I_k^s} \sigma_{sv} P_v \right) \sum_{i=1}^p \beta_i B_{1s} x(k - \tau_i) \\ &+ 2x^T(k) A_s^T \left(\sum_{v \in I_k^s} \sigma_{sv} P_v \right) B_{2s} w(k) + 2 \sum_{i=1}^p \beta_i x^T(k - \tau_i) B_{1s}^T \left(\sum_{v \in I_k^s} \sigma_{sv} P_v \right) B_{2s} w(k) \sum_{i=1}^p \beta_i x^T(k - \tau_i) \\ &\times B_{1s}^T \left(\sum_{v \in I_k^s} \sigma_{sv} P_v \right) \beta_i B_{1s} x(k - \tau_i) + w^T(k) B_{2s}^T \left(\sum_{v \in I_k^s} \sigma_{sv} P_v \right) B_{2s} w(k) - x^T(k) \left(\sum_{v \in I_k^s} \sigma_{sv} P_v \right) x(k) \end{aligned} \quad (49)$$

Define

$$\Pi_w^T = \begin{pmatrix} x^T(k) & \hat{x}^T(k) & x^T(k - \tau_1) & \dots & x^T(k - \tau_p) & \omega^T(k) \end{pmatrix} \quad (50)$$

ΔV_{11} can be written as

$$\Delta V_{11} = \Pi_w^T (\phi_{111} + \phi_{112} + \phi_{113} + \phi_{114}) \Pi_w \quad (51)$$

Where

$$\phi_{311} = \begin{pmatrix} A_s^T \left(\sum_{v \in I_k^s} \sigma_{sv} P_v \right) A_s & 0 & A_s^T \left(\sum_{v \in I_k^s} \sigma_{sv} P_v \right) \beta_1 B_{1s} & \dots & A_s^T \left(\sum_{v \in I_k^s} \sigma_{sv} P_v \right) \beta_p B_{1s} & A_s^T \left(\sum_{v \in I_k^s} \sigma_{sv} P_v \right) B_{2s} \\ * & 0 & 0 & \dots & 0 & 0 \\ * & * & \beta_1 B_{1s}^T \left(\sum_{v \in I_k^s} \sigma_{sv} P_v \right) B_{1s} \beta_1 & 0 & 0 & \beta_1 B_{1s}^T \left(\sum_{v \in I_k^s} \sigma_{sv} P_v \right) B_{2s} \\ * & * & * & \dots & 0 & \dots \\ * & * & * & * & \beta_p B_{1s}^T \left(\sum_{v \in I_k^s} \sigma_{sv} P_v \right) B_{1s} \beta_p & \beta_p B_{1s}^T \left(\sum_{v \in I_k^s} \sigma_{sv} P_v \right) B_{2s} \\ * & * & * & * & * & B_{2s}^T \left(\sum_{v \in I_k^s} \sigma_{sv} P_v \right) B_{2s} \end{pmatrix} \quad (52)$$

$$\phi_{113} = \begin{pmatrix} A_s^T \left(\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \right) A_s & 0 & A_s^T \left(\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \right) \beta_1 B_{1s} & \dots & A_s^T \left(\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \right) \beta_p B_{1s} & A_s^T \left(\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \right) B_{2s} \\ * & 0 & 0 & \dots & 0 & 0 \\ * & * & \beta_1 B_{1s}^T \left(\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \right) B_{1s} \beta_1 & 0 & 0 & \beta_1 B_{1s}^T \left(\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \right) B_{2s} \\ * & * & * & \dots & 0 & \dots \\ * & * & * & * & \beta_p B_{1s}^T \left(\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \right) B_{1s} \beta_p & \beta_p B_{1s}^T \left(\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \right) B_{2s} \\ * & * & * & * & * & B_{2s}^T \left(\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \right) B_{2s} \end{pmatrix} \quad (53)$$

ϕ_{112}, ϕ_{114} are the same as the terms in Theorem 1.
 ϕ_{111} can be written as

$$\phi_{111} = \Phi_1^T \Gamma_1 \Phi_1 \quad (54)$$

Where
 Φ_1^T, Γ_1 are the same as the terms in Theorem 1.
Define

$$\begin{pmatrix} N_{11} & -\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \\ * & N_{21} \end{pmatrix} \geq 0 \quad (55)$$

we can get

$$\begin{pmatrix} x^T(k) A_s^T & x^T(k - \tau_1) \beta_1 B_{1s}^T \end{pmatrix} \begin{pmatrix} N_{11} & -\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \\ * & N_{21} \end{pmatrix} \begin{pmatrix} A_s x(k) \\ \beta_1 B_{1s} x(k - \tau_1) \end{pmatrix} \geq 0 \quad (56)$$

It can be expressed as

$$2x^T(k) A_s^T \left(\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \right) \beta_1 B_{1s} x(k - \tau_1) \leq x^T(k) A_s^T N_{11} A_s x(k) + x^T(k - \tau_1) \beta_1 B_{1s}^T N_{21} \beta_1 B_{1s} x(k - \tau_1) \quad (57)$$

Define

$$\begin{pmatrix} N_{1p} & -\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \\ * & N_{2p} \end{pmatrix} \geq 0 \quad (58)$$

We can obtain

$$2x^T(k) A_s^T \left(\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \right) \beta_p B_{1s} x(k - \tau_p) \leq x^T(k) A_s^T N_{1p} A_s x(k) + x^T(k - \tau_p) \beta_p B_{1s}^T N_{2p} \beta_p B_{1s} x(k - \tau_p) \quad (59)$$

According to Eqs. (55) and (57)

$$\begin{aligned} & 2x^T(k) A_s^T \left(\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \right) \beta_1 B_{1s} x(k - \tau_1) + 2x^T(k) A_s^T \left(\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \right) \beta_p B_{1s} x(k - \tau_p) \leq \Pi_w^T \text{diag} \\ & \times (A_s^T N_{11} + \dots N_{1p} A_s \quad 0 \quad \beta_1 B_{1s}^T N_{21} \beta_1 B_{1s} \quad \dots \quad \beta_p B_{1s}^T N_{2p} \beta_p B_{1s} \quad 0) \Pi_w \end{aligned} \quad (60)$$

Suppose

$$\begin{pmatrix} N_{1w} & -\left(\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \right) \\ * & N_{2w} \end{pmatrix} \geq 0 \quad (61)$$

From

$$\begin{pmatrix} x^T(k)A_s^T & w^T(k)B_{2s}^T \end{pmatrix} \begin{pmatrix} N_{1w} & -\left(\sum_{v \in P_{uk}^s} P_v\right) \\ * & N_{2w} \end{pmatrix} \begin{pmatrix} A_s x(k) \\ B_{2s} w(k) \end{pmatrix} \geq 0 \quad (62)$$

We can get

$$2x^T(k)A_s^T \sum_{v \in P_{uk}^s} \sigma_{sv} P_v B_{2s} w(k) \leq x^T(k)A_s^T N_{1w} A_s x(k) + w^T(k)B_{2s}^T N_{2w} B_{2s} w(k) \quad (63)$$

With the similar computation

$$\begin{pmatrix} x^T(k - \tau_1)\beta_1 B_{1s}^T & w^T(k)B_{2s}^T \end{pmatrix} \begin{pmatrix} N_{1w} & -\left(\sum_{v \in P_{uk}^s} P_v\right) \\ * & N_{2w} \end{pmatrix} \begin{pmatrix} \beta_1 B_{1s} x(k - \tau_1) \\ B_{2s} w(k) \end{pmatrix} \geq 0$$

$$\begin{pmatrix} x^T(k - \tau_p)\beta_p B_{1s}^T & w^T(k)B_{2s}^T \end{pmatrix} \begin{pmatrix} N_{1w} & -\left(\sum_{v \in P_{uk}^s} P_v\right) \\ * & N_{2w} \end{pmatrix} \begin{pmatrix} \beta_p B_{1s} x(k - \tau_p) \\ B_{2s} w(k) \end{pmatrix} \geq 0 \quad (64)$$

Then

$$\phi_{113} \leq \phi_{115} + \phi_{116} \quad (65)$$

Where ϕ_{115}, ϕ_{116} can be written as the terms in Theorem 1.

ΔV_{12} can be written in the form of

$$\begin{aligned} \Delta V_{12} = & E\{x^T(k+1) \sum_{v \in l} \sigma_{sv} P_v \hat{x}(k+1) - x^T(k) \sum_{v \in l} \sigma_{sv} P_s \hat{x}(k)\} = \{x^T(k)a_{fs}^T + \sum_{i=1}^p x^T(k - \tau_i)\beta_i b_{fs}^T + [x^T(k)C_s^T \\ & + \sum_{i=1}^p x^T(k - \tau_i)\beta_i D_{1s}^T + \omega^T(k)D_{2s}^T]c_{fs}^T\} \sum_{v \in l} \sigma_{sv} P_v \{a_{fs} \hat{x}(k) + \sum_{i=1}^p b_{fs} \beta_i x(k - \tau_i) + c_{fs}[C_s x(k) + D_{1s} \beta_i \sum_{i=1}^p x(k - \tau_i) \\ & + D_{2s} \omega(k)]\} - x^T(k) \sum_{v \in l} \sigma_{sv} P_s \hat{x}(k) = \Delta V_{121} + \Delta V_{122} + \Delta V_{123} \end{aligned} \quad (66)$$

ΔV_{121} can be written as

$$\begin{aligned} = & \{x^T(k)a_{fs}^T \sum_{v \in l} \sigma_{sv} P_v a_{fs} \hat{x}(k) + x^T(k)a_{fs}^T \sum_{v \in l} \sigma_{sv} P_v \sum_{i=1}^p b_{fs} \beta_i x(k - \tau_i) + x^T(k)a_{fs}^T \sum_{v \in l} \sigma_{sv} P_v c_{fs} C_s x(k) + x^T(k)a_{fs}^T \sum_{v \in l} \sigma_{sv} P_v c_{fs} D_{1s} \beta_i \\ & \times \sum_{i=1}^p x(k - \tau_i) + x^T(k)a_{fs}^T \sum_{v \in l} \sigma_{sv} P_v c_{fs} D_{2s} \omega(k) - x^T(k) \sum_{v \in l} \sigma_{sv} P_s \hat{x}(k)\} \end{aligned} \quad (67)$$

ΔV_{122} can be written as

$$\begin{aligned} \{ & \sum_{i=1}^p x^T(k - \tau_i)\beta_i b_{fs}^T \sum_{v \in l} \sigma_{sv} P_v a_{fs} \hat{x}(k) + \sum_{i=1}^p x^T(k - \tau_i)\beta_i b_{fs}^T \sum_{v \in l} \sigma_{sv} P_v \sum_{i=1}^p b_{fs} \beta_i x(k - \tau_i) \\ & + \sum_{i=1}^p x^T(k - \tau_i)\beta_i b_{fs}^T \sum_{v \in l} \sigma_{sv} P_v c_{fs} C_s x(k) + \sum_{i=1}^p x^T(k - \tau_i)\beta_i b_{fs}^T \sum_{v \in l} \sigma_{sv} P_v c_{fs} D_{1s} \beta_i \sum_{i=1}^p x(k - \tau_i) \\ & + \sum_{i=1}^p x^T(k - \tau_i)\beta_i b_{fs}^T \sum_{v \in l} \sigma_{sv} P_v c_{fs} D_{2s} \omega(k)\} \end{aligned} \quad (68)$$

ΔV_{123} can be expressed as

$$\begin{aligned} x^T(k)C_s^T c_{fs}^T \sum_{v \in l} \sigma_{sv} P_v \{a_{fs} \hat{x}(k) + \sum_{i=1}^p b_{fs} \beta_i x(k - \tau_i) + c_{fs}[C_s x(k) + D_{1s} \beta_i \sum_{i=1}^p x(k - \tau_i) + D_{2s} \omega(k)]\} \\ + \omega^T(k)D_{2s}^T c_{fs}^T \sum_{v \in l} \sigma_{sv} P_v c_{fs} D_{2s} \omega(k) \end{aligned} \quad (69)$$

ΔV_{12} can be written as

$$\Delta V_{12} = \Pi^T \phi_{121} \Pi + \Pi^T \phi_{122} \Pi \quad (70)$$

Where

$$\phi_{121} = \begin{pmatrix} C_s^T c_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} C_s & C_s^T c_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v a_{fs} & C_s^T c_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v (b_{fs}(\beta_1 + c_{fs} D_{1s} \beta_1)) & \dots & C_s^T c_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v b_{fs} \beta_p + c_{fs} D_{1s} \beta_p & C_s^T c_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{2s} \\ * & a_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v a_{fs} - \sum_{v \in I_k^s} \sigma_{sv} P_s & \sum_{v \in I_k^s} \sigma_{sv} P_v b_{fs} \beta_1 + a_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{1s} \beta_1 & \dots & \sum_{v \in I_k^s} \sigma_{sv} P_v b_{fs} \beta_p + a_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{1s} \beta_p & a_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{2s} \\ * & * & \beta_1 b_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v b_{fs} \beta_1 + \beta_1 b_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{1s} \beta_1 & \dots & 0 & \beta_1 b_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{2s} \\ * & * & * & \dots & 0 & \dots \\ * & * & * & * & \beta_p b_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v b_{fs} \beta_p + \beta_p b_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{1s} \beta_p & \beta_p b_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{2s} \\ * & * & * & * & * & D_{2s}^T c_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{2s} \end{pmatrix} \quad (71)$$

$$\phi_{122} = \begin{pmatrix} C_s^T c_{fs}^T \sum_{v \in I_{uk}^s} \sigma_{sv} P_v c_{fs} C_s & C_s^T c_{fs}^T \sum_{v \in I_{uk}^s} \sigma_{sv} P_v a_{fs} & C_s^T c_{fs}^T \sum_{v \in I_{uk}^s} \sigma_{sv} P_v (b_{fs}(\beta_1 + c_{fs} D_{1s} \beta_1)) & \dots & C_s^T c_{fs}^T \sum_{v \in I_{uk}^s} \sigma_{sv} P_v b_{fs} \beta_p + c_{fs} D_{1s} \beta_p & C_s^T c_{fs}^T \sum_{v \in I_{uk}^s} \sigma_{sv} P_v c_{fs} D_{2s} \\ * & a_{fs}^T \sum_{v \in I_{uk}^s} \sigma_{sv} P_v a_{fs} - \sum_{v \in I_{uk}^s} \sigma_{sv} P_s & \sum_{v \in I_{uk}^s} \sigma_{sv} P_v b_{fs} \beta_1 + a_{fs}^T \sum_{v \in I_{uk}^s} \sigma_{sv} P_v c_{fs} D_{1s} \beta_1 & \dots & \sum_{v \in I_{uk}^s} \sigma_{sv} P_v b_{fs} \beta_p + a_{fs}^T \sum_{v \in I_{uk}^s} \sigma_{sv} P_v c_{fs} D_{1s} \beta_p & a_{fs}^T \sum_{v \in I_{uk}^s} \sigma_{sv} P_v c_{fs} D_{2s} \\ * & * & \beta_1 b_{fs}^T \sum_{v \in I_{uk}^s} \sigma_{sv} P_v b_{fs} \beta_1 + \beta_1 b_{fs}^T \sum_{v \in I_{uk}^s} \sigma_{sv} P_v c_{fs} D_{1s} \beta_1 & \dots & 0 & \beta_1 b_{fs}^T \sum_{v \in I_{uk}^s} \sigma_{sv} P_v c_{fs} D_{2s} \\ * & * & * & \dots & 0 & \dots \\ * & * & * & * & \beta_p b_{fs}^T \sum_{v \in I_{uk}^s} \sigma_{sv} P_v b_{fs} \beta_p + \beta_p b_{fs}^T \sum_{v \in I_{uk}^s} \sigma_{sv} P_v c_{fs} D_{1s} \beta_p & \beta_p b_{fs}^T \sum_{v \in I_{uk}^s} \sigma_{sv} P_v c_{fs} D_{2s} \\ * & * & * & * & * & D_{2s}^T c_{fs}^T \sum_{v \in I_{uk}^s} \sigma_{sv} P_v c_{fs} D_{2s} \end{pmatrix} \quad (72)$$

ϕ_{121} can be expressed as

$$\phi_{121} = \phi_{1211} + \phi_{1212} \quad (73)$$

$$\phi_{1211} = \begin{pmatrix} C_s^T c_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} C_s & C_s^T c_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v a_{fs} & C_s^T c_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v b_{fs} \beta_1 & \dots & C_s^T c_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v b_{fs} \beta_p & C_s^T c_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{2s} \\ 0 & 0 & 0 & \dots & 0 & 0 \\ * & 0 & \beta_1 b_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v b_{fs} & \dots & 0 & \beta_1 b_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{2s} \\ * & 0 & * & \dots & 0 & \dots \\ * & 0 & * & * & \beta_p b_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v b_{fs} & \beta_p b_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{2s} \\ * & * & * & * & * & D_{2s}^T c_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{2s} \end{pmatrix} \quad (74)$$

ϕ_{1211} can be written as

$$\phi_{1211} = \Phi_2^T \Gamma_1 \Phi_2 + \psi_1 \quad (75)$$

ϕ_{1212} can be written as

$$\phi_{1212} = \begin{pmatrix} 0 & \dots & C_s^T c_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{1s} \beta_1 & \dots & C_s^T c_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{1s} \beta_p & 0 \\ * & a_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v a_{fs} - \sum_{v \in I_k^s} \sigma_{sv} P_s & \sum_{v \in I_k^s} \sigma_{sv} P_v b_{fs} \beta_1 + a_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{1s} \beta_1 & \dots & \sum_{v \in I_k^s} \sigma_{sv} P_v b_{fs} \beta_p + a_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{1s} \beta_p & 0 \\ * & * & \beta_1 b_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{1s} & 0 & 0 & 0 \\ * & * & * & \dots & \dots & \dots \\ * & * & * & * & \beta_p b_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{1s} & 0 \\ * & * & * & * & * & 0 \end{pmatrix} \quad (76)$$

According to $2ab \leq a^2 + b^2$, we can get

$$\begin{aligned}\beta_1 b_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{1s} &\leq \frac{1}{2} [\beta_1 b_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v b_{fs} + \beta_1 D_{1s}^T c_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{1s}] \\ \beta_p b_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{1s} &\leq \frac{1}{2} [\beta_p b_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v b_{fs} + \beta_p D_{1s}^T c_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v c_{fs} D_{1s}]\end{aligned}\quad (77)$$

Suppose that

$$\begin{pmatrix} N_{11} & -\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \\ * & N_{21} \end{pmatrix} \geq 0 \quad (78)$$

According to

$$\begin{pmatrix} x^T(k) C_s^T c_{fs}^T & x^T(k - \tau_1) \beta_1 D_{1s}^T c_{fs}^T \end{pmatrix} \begin{pmatrix} N_{11} & -\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \\ * & N_{21} \end{pmatrix} \begin{pmatrix} c_{fs} C_s x(k) \\ c_{fs} D_{1s} \beta_1 x(k - \tau_1) \end{pmatrix} \geq 0 \quad (79)$$

$$2x^T(k) C_s^T c_{fs}^T \left(\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \right) c_{fs} D_{1s} \beta_1 x(k - \tau_1) \leq x^T(k) C_s^T c_{fs}^T, N_{11} c_{fs} C_s x(k) + x^T(k - \tau_1) \beta_1 D_{1s}^T c_{fs}^T N_{21} \beta_1 c_{fs} D_{1s} x(k - \tau_1) \quad (80)$$

$$\begin{pmatrix} N_{1p} & -\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \\ * & N_{2p} \end{pmatrix} \geq 0 \quad (81)$$

$$\begin{pmatrix} x^T(k) C_s^T c_{fs}^T & x^T(k - \tau_p) \beta_p D_{1s}^T c_{fs}^T \end{pmatrix} \begin{pmatrix} N_{1p} & -\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \\ * & N_{2p} \end{pmatrix} \begin{pmatrix} c_{fs} C_s x(k) \\ c_{fs} D_{1s} \beta_p x(k - \tau_p) \end{pmatrix} \geq 0 \quad (82)$$

$$2x^T(k) C_s^T c_{fs}^T \left(\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \right) c_{fs} D_{1s} \beta_p x(k - \tau_p) \leq x^T(k) C_s^T c_{fs}^T, N_{1p} c_{fs} C_s x(k) + x^T(k - \tau_p) \beta_p D_{1s}^T c_{fs}^T N_{2p} \beta_p c_{fs} D_{1s} x(k - \tau_p) \quad (83)$$

Suppose

$$\begin{pmatrix} N_{1p} & -\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \\ * & N_{2p} \end{pmatrix} \geq 0 \quad (84)$$

$$\begin{pmatrix} \hat{x}^T(k) a_{fs}^T & x^T(k - \tau_1) \beta_1 D_{1s}^T c_{fs}^T \end{pmatrix} \begin{pmatrix} N_{11} & -\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \\ * & N_{21} \end{pmatrix} \begin{pmatrix} a_{fs} \hat{x}(k) \\ c_{fs} D_{1s} \beta_1 x(k - \tau_1) \end{pmatrix} \geq 0 \quad (85)$$

$$\begin{pmatrix} \hat{x}^T(k) a_{fs}^T & x^T(k - \tau_p) \beta_p D_{1s}^T c_{fs}^T \end{pmatrix} \begin{pmatrix} N_{1p} & -\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \\ * & N_{2p} \end{pmatrix} \begin{pmatrix} a_{fs} \hat{x}(k) \\ c_{fs} D_{1s} \beta_p x(k - \tau_p) \end{pmatrix} \geq 0$$

$$2\hat{x}^T(k) a_{fs}^T \left(\sum_{v \in I_{uk}^s} \sigma_{sv} P_v \right) c_{fs} D_{1s} \beta_p x(k - \tau_p) \leq \hat{x}^T(k) a_{fs}^T, N_{1p} a_{fs} \hat{x}(k) + x^T(k - \tau_p) \beta_p D_{1s}^T c_{fs}^T N_{2p} \beta_p c_{fs} D_{1s} x(k - \tau_p) \quad (86)$$

$$\begin{pmatrix} C_s^T c_{fs}^T (N_{11} + \dots N_{1p}) c_{fs} C_s & \dots & \dots & \dots & 0 \\ * & a_{fs}^T (N_{11} + \dots N_{1p}) a_{fs} + a_{fs}^T \sum_{v \in I_k^s} \sigma_{sv} P_v a_{fs} - \sum_{v \in I_k^s} \sigma_{sv} P_s & \sum_{v \in I_k^s} \sigma_{sv} P_v b_{fs} \beta_1 & \dots & \sum_{v \in I_k^s} \sigma_{sv} P_v b_{fs} \beta_p & 0 \\ * & * & 2\beta_1 D_{1s}^T c_{fs}^T N_{21} \beta_1 c_{fs} D_{1s} & 0 & 0 & 0 \\ * & * & * & \dots & \dots & \dots \\ * & * & * & * & 2\beta_p D_{1s}^T c_{fs}^T N_{2p} \beta_p c_{fs} D_{1s} & 0 \\ * & * & * & * & * & 0 \end{pmatrix} \quad (87)$$

ϕ_{1221} can be written as

$$\phi_{1221} = \Phi_3^T \Gamma_2 \Phi_3 + \psi_2 \quad (88)$$

Where $\phi_{1222}, \Phi_3^T, \Gamma_2, \psi_2$ are the same as the terms in Theorem 1.

The forward difference of V_2 can be written in the form of

$$\begin{aligned} \Delta V_2 &\leq \left[\sum_{i=1}^p x^T(k) \left(\sum_{v \in I_k^p} \sigma_{sv} S_v \right) x(k) + \sum_{i=1}^p x^T(k) \left(\sum_{v \in I_{uk}^p} \sigma_{sv} S_v \right) x(k) \right] - \left[\sum_{i=1}^p x^T(k - \tau_i) \left(\sum_{v \in I_k^p} \sigma_{sv} S_v \right) x(k - \tau_i) + \sum_{i=1}^p x^T(k - \tau_i) \right. \\ &\quad \times \left. \left(\sum_{v \in I_{uk}^p} \sigma_{sv} S_v \right) x(k - \tau_i) \right] + \left[\sum_{i=1}^p \hat{x}^T(k) \left(\sum_{v \in I_k^p} \sigma_{sv} S_v \right) \hat{x}(k) + \sum_{i=1}^p \hat{x}^T(k) \left(\sum_{v \in I_{uk}^p} \sigma_{sv} S_v \right) \hat{x}(k) \right] - = \prod_w^T \phi_{2w} \prod_w \\ &= \prod_w^T (\phi_{21} + \phi_{22}) \prod_w \end{aligned} \quad (89)$$

Where ϕ_{21}, ϕ_{22} are the same as the terms in Theorem 1.

The forward differential of V_3 can be expressed in the form of

$$\Delta V_3 = \Delta V_{31} + \Delta V_{32}$$

$\Delta V_{31}, \Delta V_{32}$ can be written as $\Delta V_{31} = \Delta V_{311} + \Delta V_{312}, \Delta V_{32} = \Delta V_{321} + \Delta V_{322}$.

$$\Delta V_3 = \sum_{i=1}^p \sum_{i=-\tau_i}^{-1} \left\{ \delta^T(k) \left(\sum_{v \in I} \sigma_{sv} M_v \right) \delta(k) - \delta^T(k+i) \left(\sum_{v \in I} \sigma_{sv} M_v \right) \delta(k+i) \right\} \quad (90)$$

Where

$$\Delta V_{311} = \sum_{i=1}^p \tau_i x^T(k+1) \left(\sum_{v \in I} \sigma_{sv} M_v \right) x(k+1) - \sum_{i=1}^p \tau_i [x^T(k) \left(\sum_{v \in I} \sigma_{sv} M_v \right) x(k)] \quad (91)$$

$$\begin{aligned} \Delta V_{312} &= - \sum_{i=1}^p \tau_i [x^T(k+1) \left(\sum_{v \in I} \sigma_{sv} M_v \right) x(k) - \sum_{i=1}^p \tau_i [x^T(k) \left(\sum_{v \in I} \sigma_{sv} M_v \right) x(k+1) \\ &\quad - \sum_{i=1}^p [x(k) - x(k - \tau_i)]^T \left(\sum_{v \in I} \sigma_{sv} M_v \right) [x(k) - x(k - \tau_i)]] \end{aligned} \quad (92)$$

$$\Delta V_{321} = \sum_{i=1}^p \tau_i \hat{x}^T(k+1) \left(\sum_{v \in I} \sigma_{sv} M_v \right) \hat{x}(k+1) - \sum_{i=1}^p \tau_i \hat{x}^T(k) \left(\sum_{v \in I} \sigma_{sv} M_v \right) \hat{x}(k) \quad (93)$$

$$\begin{aligned} \Delta V_{322} &= - \sum_{i=1}^p \tau_i [\hat{x}^T(k+1) \left(\sum_{v \in I} \sigma_{sv} M_v \right) \hat{x}(k) - \sum_{i=1}^p \tau_i [\hat{x}^T(k) \left(\sum_{v \in I} \sigma_{sv} M_v \right) \hat{x}(k+1) \\ &\quad - \sum_{i=1}^p [\hat{x}(k) - \hat{x}(k - \tau_i)]^T \left(\sum_{v \in I} \sigma_{sv} M_v \right) [\hat{x}(k) - \hat{x}(k - \tau_i)]] \leq - \sum_{i=1}^p \tau_i [\hat{x}^T(k+1) \left(\sum_{v \in I} \sigma_{sv} M_v \right) x(k) \\ &\quad - \sum_{i=1}^p \tau_i [x^T(k) \left(\sum_{v \in I} \sigma_{sv} M_v \right) \hat{x}(k+1)] \end{aligned} \quad (94)$$

ΔV_{31} can be written as

$$\Delta V_{31} = \Pi^T (\phi_{311} + \phi_{312} + \phi_{313} + \phi_{314}) \Pi \quad (95)$$

Where

$$\phi_{311} = \begin{pmatrix} \sum_{i=1}^p \tau_i A_s^T \left(\sum_{v \in I_k^p} \sigma_{sv} M_v \right) A_s & 0 & \sum_{i=1}^p \tau_i A_s^T \left(\sum_{v \in I_k^p} \sigma_{sv} M_v \right) \beta_1 B_{1s} & \dots & \sum_{i=1}^p \tau_i A_s^T \left(\sum_{v \in I_k^p} \sigma_{sv} M_v \right) \beta_p B_{1s} \\ * & 0 & 0 & \dots & 0 \\ * & * & \tau_1 \beta_1 B_{1s}^T \left(\sum_{v \in I_k^p} \sigma_{sv} M_v \right) B_{1s} \beta_1 & 0 & 0 \\ * & * & * & \dots & 0 \\ * & * & * & * & \tau_p \beta_p B_{1s}^T \left(\sum_{v \in I_k^p} \sigma_{sv} M_v \right) B_{1s} \beta_p \end{pmatrix} \quad (96)$$

$$\phi_{313} = \begin{pmatrix} \sum_{i=1}^p \tau_i A_s^T \left(\sum_{v \in I_{uk}^p} \sigma_{sv} M_v \right) A_s & 0 & \sum_{i=1}^p \tau_i A_s^T \left(\sum_{v \in I_{uk}^p} \sigma_{sv} M_v \right) \beta_1 B_{1s} & \dots & \sum_{i=1}^p \tau_i A_s^T \left(\sum_{v \in I_{uk}^p} \sigma_{sv} M_v \right) \beta_p B_{1s} \\ * & 0 & 0 & \dots & 0 \\ * & * & \tau_1 \beta_1 B_{1s}^T \left(\sum_{v \in I_{uk}^p} \sigma_{sv} M_v \right) B_{1s} \beta_1 & 0 & 0 \\ * & * & * & \dots & 0 \\ * & * & * & * & \tau_p \beta_p B_{1s}^T \left(\sum_{v \in I_{uk}^p} \sigma_{sv} M_v \right) B_{1s} \beta_p \end{pmatrix} \quad (97)$$

ϕ_{312}, ϕ_{314} are the same as the terms in Theorem 1
 ΔV_{32} can be written in the form of

$$\begin{aligned} \Delta V_{32} = & \sum_{i=1}^p \tau_i E \left\{ \hat{x}^T(k+1) \sum_{v \in I} \sigma_{sv} M_v \hat{x}(k+1) - \hat{x}^T(k) \sum_{v \in I} \sigma_{sv} M_s \hat{x}(k) \right\} = \left\{ \hat{x}^T(k) a_{fs}^T + \sum_{i=1}^p x^T(k - \tau_i) \beta_i b_{fs}^T \right. \\ & + \left[x^T(k) C_s^T + \sum_{i=1}^p x^T(k - \tau_i) \beta_i D_{1s}^T \right] c_{fs}^T \left\{ \sum_{v \in I} \sigma_{sv} P_v \left\{ a_{fs} \hat{x}(k) + \sum_{i=1}^p b_{fs} \beta_i x(k - \tau_i) + c_{fs} \left[C_s x(k) + D_{1s} \beta_i \sum_{i=1}^p x(k - \tau_i) \right] \right\} \right. \\ & \left. \left. - \hat{x}^T(k) \sum_{v \in I} \sigma_{sv} P_s \hat{x}(k) \right\} \right\} \quad (98) \end{aligned}$$

ΔV_{32} can be written as

$$\Delta V_{321} = \Pi^T \phi_{321} \Pi + \Pi^T \phi_{322} \Pi \quad (99)$$

Where

$$\begin{aligned} \phi_{321} = & \begin{pmatrix} \sum_{i=1}^p \tau_i C_s^T c_{fs}^T \sum_{v \in I_k^p} \sigma_{sv} M_v c_{fs} C_s & C_s^T c_{fs}^T \sum_{v \in I_k^p} \sigma_{sv} M_v a_{fs} & \tau_1 [C_s^T c_{fs}^T \sum_{v \in I_k^p} \sigma_{sv} P_v (b_{fs} \beta_1 + c_{fs} D_{1s} \beta_1)] & \dots & \tau_p \left[C_s^T c_{fs}^T \sum_{v \in I_k^p} \sigma_{sv} M_v (b_{fs} \beta_p + c_{fs} D_{1s} \beta_p) \right] \\ * & \sum_{i=1}^p \tau_i \left[a_{fs}^T \sum_{v \in I_k^p} \sigma_{sv} M_v a_{fs} - \sum_{v \in I_k^p} \sigma_{sv} M_s \right] & \tau_1 \left[\sum_{v \in I_k^p} \sigma_{sv} P_v b_{fs} \beta_1 + a_{fs}^T \sum_{v \in I_k^p} \sigma_{sv} M_v c_{fs} D_{1s} \beta_1 \right] & \dots & \tau_p \left[\sum_{v \in I_k^p} \sigma_{sv} M_v b_{fs} \beta_p + a_{fs}^T \sum_{v \in I_k^p} \sigma_{sv} M_v c_{fs} D_{1s} \beta_p \right] \\ * & * & \tau_1 \left[\beta_1 b_{fs}^T \sum_{v \in I_k^p} \sigma_{sv} P_v b_{fs} \beta_1 + \beta_1 b_{fs}^T \sum_{v \in I_k^p} \sigma_{sv} M_v c_{fs} D_{1s} \beta_1 \right] & \dots & 0 \\ * & * & * & \dots & 0 \\ * & * & * & * & \tau_p [\beta_p b_{fs}^T \sum_{v \in I_k^p} \sigma_{sv} M_v b_{fs} \beta_p + \beta_p b_{fs}^T \sum_{v \in I_k^p} \sigma_{sv} M_v c_{fs} D_{1s} \beta_p] \end{pmatrix} \quad (100) \\ \phi_{322} = & \begin{pmatrix} \sum_{i=1}^p \tau_i C_s^T c_{fs}^T \sum_{v \in I_{at}^p} \sigma_{sv} M_v c_{fs} C_s & C_s^T c_{fs}^T \sum_{v \in I_{at}^p} \sigma_{sv} M_v a_{fs} & \tau_1 \left[C_s^T c_{fs}^T \sum_{v \in I_{at}^p} \sigma_{sv} P_v (b_{fs} \beta_1 + c_{fs} D_{1s} \beta_1) \right] & \dots & \tau_p \left[C_s^T c_{fs}^T \sum_{v \in I_{at}^p} \sigma_{sv} M_v (b_{fs} \beta_p + c_{fs} D_{1s} \beta_p) \right] \\ * & \sum_{i=1}^p \tau_i \left[a_{fs}^T \sum_{v \in I_{at}^p} \sigma_{sv} M_v a_{fs} - \sum_{v \in I_{at}^p} \sigma_{sv} M_s \right] & \tau_1 \left[\sum_{v \in I_{at}^p} \sigma_{sv} P_v b_{fs} \beta_1 + a_{fs}^T \sum_{v \in I_{at}^p} \sigma_{sv} M_v c_{fs} D_{1s} \beta_1 \right] & \dots & \tau_p \left[\sum_{v \in I_{at}^p} \sigma_{sv} M_v b_{fs} \beta_p + a_{fs}^T \sum_{v \in I_{at}^p} \sigma_{sv} M_v c_{fs} D_{1s} \beta_p \right] \\ * & * & \tau_1 \left[\beta_1 b_{fs}^T \sum_{v \in I_{at}^p} \sigma_{sv} P_v b_{fs} \beta_1 + \beta_1 b_{fs}^T \sum_{v \in I_{at}^p} \sigma_{sv} M_v c_{fs} D_{1s} \beta_1 \right] & \dots & 0 \\ * & * & * & \dots & 0 \\ * & * & * & * & \tau_p [\beta_p b_{fs}^T \sum_{v \in I_{at}^p} \sigma_{sv} M_v b_{fs} \beta_p + \beta_p b_{fs}^T \sum_{v \in I_{at}^p} \sigma_{sv} M_v c_{fs} D_{1s} \beta_p] \end{pmatrix} \quad (101) \end{aligned}$$

$$\begin{aligned} \Delta V_{322} \leq & - \sum_{i=1}^p \tau_i \left[\hat{x}^T(k+1) \left(\sum_{v \in I} \sigma_{sv} M_v \right) \hat{x}(k) - \sum_{i=1}^p \tau_i \left[x^T(k) \left(\sum_{v \in I} \sigma_{sv} M_v \right) \hat{x}(k+1) \right. \right. \\ & \left. \left. - \sum_{i=1}^p \tau_i \left\{ \hat{x}^T(k) a_{fs}^T + \sum_{i=1}^p x^T(k - \tau_i) \beta_i b_{fs}^T + \left[x^T(k) C_s^T + \sum_{i=1}^p x^T(k - \tau_i) \beta_i D_{1s}^T + \omega^T(k) D_{2s}^T \right] c_{fs}^T \right\} \right. \right. \\ & \times \sum_{v \in I} \sigma_{sv} P_v \hat{x}(k) - \sum_{i=1}^p \tau_i \hat{x}^T(k) \sum_{v \in I} \sigma_{sv} P_v \left\{ a_{fs} \hat{x}(k) + \sum_{i=1}^p b_{fs} \beta_i x(k - \tau_i) + c_{fs} [C_s x(k) \right. \\ & \left. \left. + D_{1s} \beta_i \sum_{i=1}^p x(k - \tau_i) + D_{2s} \omega(k)] \right\} \right] = \Pi^T \phi_{323} \Pi + \Pi^T \phi_{324} \Pi \quad (102) \end{aligned}$$

ϕ_{331}, ϕ_{332} are the same as the terms in Theorem 1

$$\begin{aligned}\Omega &= \phi_{11} + \phi_{12} + \phi_{21} + \phi_{22} + \phi_{31} + \phi_{32} + \phi_{33} = \phi_{111} + \phi_{112} + \phi_{114} + \phi_{115} + \phi_{116} + \phi_{1211} + \phi_{1212} + \phi_{1221} \\ &+ \phi_{1222} + \phi_{311} + \phi_{312} + \phi_{314} + \phi_{315} + \phi_{316} + \phi_{3211} + \phi_{3212} + \phi_{3221} + \phi_{3222} + \phi_{331} + \phi_{332} = \Phi_1^T \Gamma_1 \Phi_1 \\ &+ \phi_{112} + \phi_{114} + \phi_{115} + \phi_{116} + \Phi_2^T \Gamma_1 \Phi_2 + \psi_1 + \phi_{1212} + \Phi_2^T \Gamma_2 \Phi_2 + \psi_2 + \phi_{1222} + \Phi_1^T \Gamma_3 \Phi_1 + \phi_{312} \\ &+ \phi_{314} + \phi_{315} + \phi_{316} + \Phi_2^T \Gamma_4 \Phi_2 + \psi_3 + \phi_{3212} + \Phi_2^T \Gamma_4 \Phi_2 + \psi_3 + \phi_{3222} + \phi_{331} + \phi_{332} < 0\end{aligned}\quad (103)$$

$\tilde{z}(k)$ can be written as

$$\tilde{z}(k) = \begin{pmatrix} M_s & -M_{fs} \end{pmatrix} \begin{pmatrix} x(k) \\ \hat{x}(k) \end{pmatrix} \quad (104)$$

$$[M_s x(k) - M_{fs} \hat{x}(k)]^T [M_s x(k) - M_{fs} \hat{x}(k)] - \gamma^2 \omega^T(k) \omega(k) < 0 \quad (105)$$

$$x^T(k) M_s^T M_s x(k) - 2x^T(k) M_s^T M_{fs} \hat{x}(k) + \hat{x}^T(k) M_{fs}^T M_{fs} \hat{x}(k) - \gamma^2 \omega^T(k) \omega(k) < 0 \quad (106)$$

$$\begin{aligned}\text{diag}(\Xi_1 \quad \Xi_2) \\ \Xi_1 = \begin{pmatrix} M_s^T M_s & -M_s^T M_{fs} \\ * & M_{fs}^T M_{fs} \end{pmatrix} \quad \Xi_2 = \text{diag}(0 \quad \dots \quad 0 \quad -\gamma^2)\end{aligned}\quad (107)$$

$$-\gamma^2 < \varepsilon I$$

$$\Xi = \Omega + \text{diag}(\Xi_1 \quad \Xi_2) < 0 \quad (108)$$

$$\tilde{z}(k) < \gamma \omega(k)$$

$$\begin{aligned}\Phi_1^T \Gamma_1 \Phi_1 + \phi_{112} + \phi_{114} + \phi_{115} + \phi_{116} + \Phi_2^T \Gamma_1 \Phi_2 + \psi_1 + \phi_{1212} + \\ \Phi_2^T \Gamma_2 \Phi_2 + \psi_2 + \phi_{1222} + \Phi_1^T \Gamma_3 \Phi_1 + \phi_{312} + \phi_{314} + \phi_{315} + \phi_{316} + \\ \Phi_2^T \Gamma_3 \Phi_2 + \psi_3 + \phi_{3212} + \Phi_2^T \Gamma_4 \Phi_2 + \psi_3 + \phi_{3222} + \phi_{331} + \phi_{332} < 0\end{aligned}\quad (109)$$

It can be expressed as

$$\zeta^T(k) \Xi \zeta(k) < 0 \quad (110)$$

If $\Xi < 0$, that is $\Delta V < 0$, then system (8) is GUAS.

Remark 1. $\Xi < 0$ is not a LMI. Iterative methods are proposed to deal with it. The method can be stated as follows:

Theorem 2. System (8) is GUAS, if there exist a filter The filter can be designed as follows

Define

$$\Xi = \varphi(a_{fs}, b_{fs}, c_{fs}, P_s, P_v, N, M_v, M_s, S_v, N_{11}, \dots, N_{1p}, N_{1w}, N_{21}, \dots, N_{2p}, N_{2w}) \quad (111)$$

Λ is the maximum eigenvalue of the matrix

First Given $k = 0, (P_s, P_v, M_s, M_v) = (P_s^{(0)}, P_v^{(0)}, M_s^{(0)}, M_v^{(0)}) > 0$

Then $k = k + 1$ The solution to the $\text{Min}_{a_{fs}, b_{fs}, c_{fs}} \Lambda(\Xi = \varphi(a_{fs}, b_{fs}, c_{fs}, P_s^{(k-1)}, P_v^{(k-1)}, M_s^{(k-1)}, M_v^{(k-1)}))$

Can be written as $(a_{fs}^{(k)}, b_{fs}^{(k)}, c_{fs}^{(k)}) = a_{fs}, b_{fs}, c_{fs}$

Then according to $\text{Min}_{P_s, P_v, M_s, M_v} \Lambda(\Xi = \varphi(a_{fs}^{(k)}, b_{fs}^{(k)}, c_{fs}^{(k)}, P_s, P_v, M_s, M_v))$ we can get $P_s^{(k)}, P_v^{(k)}, M_s^{(k)}, M_v^{(k)} = P_s, P_v, M_s, M_v$

If $\Xi = \varphi(a_{fs}^{(k)}, b_{fs}^{(k)}, c_{fs}^{(k)}, P_s^{(k-1)}, P_v^{(k)}, M_s^{(k)}, M_v^{(k)}) < 0$ or $\Xi = \varphi(a_{fs}^{(k-1)}, b_{fs}^{(k-1)}, c_{fs}^{(k-1)}, P_s^{(k-1)}, P_v^{(k-1)}, M_s^{(k-1)}, M_v^{(k-1)}) < 0$

The filters a_{fs}, b_{fs}, c_{fs} , can be derived from this method. Then the minimum γ disturbance rejection feature can be gained consequently.

In this section, we use an example to illustrate the result proposed in this paper. The controlled plant is written in the form of four different Markovian chains.

$$\begin{aligned}
1. & \begin{cases} x(k+1) = \begin{pmatrix} 2 & 0 \\ 0.7 & 1.1 \end{pmatrix} x(k) + \begin{pmatrix} 1 \\ 2 \end{pmatrix} \left[\sum_{i=1}^4 \lambda \{T_m = \tau_i\} x(k - \tau_i) \right] + \begin{pmatrix} 1 \\ 2 \end{pmatrix} w(k) \\ z(k) = [0.5 & -1] x(k) \end{cases} \\
2. & \begin{cases} x(k+1) = \begin{pmatrix} 0.3 & 0 \\ 2.4 & 3.9 \end{pmatrix} x(k) + \begin{pmatrix} -1.1 \\ 0.8 \end{pmatrix} \left[\sum_{i=1}^4 \lambda \{T_m = \tau_i\} x(k - \tau_i) \right] + \begin{pmatrix} -1.1 \\ 0.8 \end{pmatrix} w(k) \\ z(k) = [5 & -2.3] x(k) \end{cases} \\
3. & \begin{cases} x(k+1) = \begin{pmatrix} 2.1 & 0.8 \\ 0.7 & -1.1 \end{pmatrix} x(k) + \begin{pmatrix} 1 \\ 0.2 \end{pmatrix} \left[\sum_{i=1}^4 \lambda \{T_m = \tau_i\} x(k - \tau_i) \right] + \begin{pmatrix} -1.5 \\ 2.3 \end{pmatrix} w(k) \\ z(k) = [-1.3 & 2.5] x(k) \end{cases} \\
4. & \begin{cases} x(k+1) = \begin{pmatrix} 0.3 & 0 \\ -0.2 & 11.4 \end{pmatrix} x(k) + \begin{pmatrix} 3.8 \\ -2.5 \end{pmatrix} \left[\sum_{i=1}^4 \lambda \{T_m = \tau_i\} x(k - \tau_i) \right] + \begin{pmatrix} 3.8 \\ -2.5 \end{pmatrix} w(k) \\ z(k) = [3.2 & 1.7] x(k) \end{cases}
\end{aligned}$$

The state transition matrix with partly unknown transition probabilities is supposed to be

$$\begin{pmatrix} ! & 0.2 & ! & 0.3 \\ 0 & ! & 0.2 & ! \\ 0.4 & ! & 0.3 & ! \\ ! & 0.2 & 0.4 & ! \end{pmatrix}$$

The networked induced delay and its probabilities are

$\text{Prob}(\tau_1 = 1) = \beta_1 = 0.2$, $\text{Prob}(\tau_2 = 2) = \beta_2 = 0.4$, $\text{Prob}(\tau_3 = 3) = \beta_3 = 0.1$, $\text{Prob}(\tau_4 = 4) = \beta_4 = 0.3$.

The initial state of this system is $[1, -0.5]^T$, $[0.6, -0.8]^T$, $[1.5, -0.9]^T$, $[-0.7, 0.5]^T$ the disturbance signal is a Gauss white noise.

$$\begin{aligned}
a_{f1} &= \begin{pmatrix} 12.81 & 9.25 \\ 5.23 & -2.35 \end{pmatrix}, b_{f1} = \begin{pmatrix} 6.29 \\ -1.47 \end{pmatrix}, c_{f1} = \begin{pmatrix} -1.83 & 7.71 \\ 123.96 & 0.75 \end{pmatrix}; \\
a_{f2} &= \begin{pmatrix} 3.91 & 9.81 \\ 10.37 & -3.61 \end{pmatrix}, b_{f2} = \begin{pmatrix} 1.09 \\ 8.28 \end{pmatrix}, c_{f2} = \begin{pmatrix} 0.79 & 5.19 \\ a_{21} & -8.01 \end{pmatrix}; \\
a_{f3} &= \begin{pmatrix} 2.38 & a_{12} \\ -2.03 & 72.11 \end{pmatrix}, b_{f3} = \begin{pmatrix} 78.81 \\ 2.72 \end{pmatrix}, c_{f3} = \begin{pmatrix} -1.92 & -0.03 \\ 23.71 & -5.50 \end{pmatrix}; \\
a_{f4} &= \begin{pmatrix} 9.15 & 5.49 \\ -2.13 & 11.26 \end{pmatrix}, b_{f4} = \begin{pmatrix} 4.64 \\ 1.05 \end{pmatrix}, c_{f4} = \begin{pmatrix} 1.38 & 8.53 \\ -0.29 & 4.76 \end{pmatrix}
\end{aligned}$$

The error between practical state value and its estimation of two modes can be expressed in Figs. 1 and 2 by this filter.

The error between practical state value and its estimation of modes 3 and 4 can be gotten

4 Conclusion

The problem of H_∞ filtering is investigated which time delays and packet dropouts are considered simultaneously in NCS. Unlike with upper bounded or lower bounded to stand for all the

Fig. 1 The error between $x(k)$ and estimation of mode 1

Fig. 2 The error between $x(k)$ and estimation of mode 2

time delays, the occurrence probability of each time delay is considered. The packet dropouts have the Bernoulli distributions. Markovian jump linear systems with partly unknown transition probability are adopted to model the system. A filter is designed to estimate the practical state with H_∞ feature. The estimation problem is cast into a set of linear matrix inequalities. An example is provided to illustrate the effectiveness and applicability of the proposed method. Further study will be focused on H_∞ control for stochastic time delays and dropouts in networked control systems by partly unknown transition probabilities of Markovian chains.

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References

- [1] Walsh, G. C., and Ye, H., 2001, "Scheduling of Networked Control Systems," *IEEE Control Syst.*, **21**(1), pp. 57–65.
- [2] Antsaklis, P., and Baillieul, J., 2004, "Special Issue on Networked Control Systems," *IEEE Trans. Autom. Control*, **49**(9), pp. 1421–1423.
- [3] Huang, D., and Nguang, S. K., 2008, "State Feedback Control of Uncertain Networked Control Systems With Random Time Delays," *IEEE Trans. Autom. Control*, **53**(3), pp. 829–834.
- [4] Hu, S., and Zhu, Q., 2003, "Stochastic Optimal Control and Analysis of Stability of Networked Control Systems With Long Delay," *Automatica*, **39**(11), pp. 1877–1884.
- [5] Liu, M., Ho, D. W. C., and Niu, Y., 2009, "Stabilization of Markovian Jump Linear System Over Networks With Random Communication Delay," *Automatica*, **45**(2), pp. 416–421.
- [6] Gao, H., Wu, J., and Shi, P., 2009, "Robust Sampled-Data H_∞ Control With Stochastic Sampling," *Automatica*, **45**(7), pp. 1729–1736.
- [7] Dačić, D. B., and Nešić, D., 2007, "Quadratic Stabilization of Linear Networked Control Systems via Simultaneous Protocol and Controller Design," *Automatica*, **43**(7), pp. 1145–1155.
- [8] Hu, S., and Yan, W.-Y., 2007, "Stability Robustness of Networked Control Systems With Respect to Packet Loss," *Automatica*, **43**(7), pp. 1243–1248.
- [9] Xiong, J., and Lam, J., 2007, "Stabilization of Linear Systems Over Networks With Bounded Packet Loss," *Automatica*, **43**(1), pp. 80–87.
- [10] Ling, Q., and Lemmon, M. D., 2004, "Power Spectral Analysis of Networked Control Systems With Data Dropouts," *IEEE Trans. Autom. Control*, **49**(6), pp. 955–959.
- [11] Zhang, W.-A., and Yu, L., 2007, "Output Feedback Stabilization of Networked Control Systems With Packet Dropouts," *IEEE Trans. Autom. Control*, **52**(9), pp. 1705–1710.
- [12] Sahebsara, M., Chen, T., and Shah, S. L., 2008, "Optimal H_∞ Filtering in Networked Control Systems With Multiple Packet Dropouts," *Syst. Control Lett.*, **57**(9), pp. 696–702.
- [13] Malyavej, V., and Savkin, A. V., 2005, "The Problem of Optimal Robust Kalman State Estimation via Limited Capacity Digital Communication Channels," *Syst. Control Lett.*, **54**(3), pp. 283–292.
- [14] Lee, K. H., and Huang, B., 2008, "Robust H_2 Optimal Filtering for Continuous-Time Stochastic Systems With Polytopic Parameter Uncertainty," *Automatica*, **44**(10), pp. 2686–2690.
- [15] Gao, H., Meng, X., and Chen, T., 2008, "A Parameter-Dependent Approach to Robust H_∞ Filtering for Time-Delay Systems," *IEEE Trans. Autom. Control*, **53**(10), pp. 2420–2425.
- [16] Zhang, J., Xia, Y., and Shi, P., 2009, "Parameter-Dependent Robust H_∞ Filtering for Uncertain Discrete-Time Systems," *Automatica*, **45**(2), pp. 560–565.
- [17] Garcia-Rivera, M., and Barreiro, A., 2007, "Analysis of Networked Control Systems With Drops and Variable Delays," *Automatica*, **43**(12), pp. 2054–2059.
- [18] Zhang, W.-A., and Yu, L., 2008, "Modelling and Control of Networked Control Systems With Both Network-Induced Delay and Packet-Dropout," *Automatica*, **44**(12), pp. 3206–3210.
- [19] Zhang, L., and Boukas, E., 2009, "Stability and Stabilization of Markovian Jump Linear Systems With Partly Unknown Transition Probabilities," *Automatica*, **45**(2), pp. 463–468.
- [20] Dong, H., Wang, Z., and Gao, H., 2010, "Observer-Based H_∞ Control for Systems With Repeated Scalar Nonlinearities and Multiple Packet Losses," *Int. J. Robust Nonlinear Control*, **20**(12), pp. 1363–1378.
- [21] Gao, H., Meng, X., and Chen, T., 2008, "Stabilization of Networked Control Systems With a New Delay Characterization," *IEEE Trans. Autom. Control*, **53**(9), pp. 2142–2148.
- [22] Yue, D., and Han, Q.-L., 2005, "Delay-Dependent Robust H_∞ Controller Design for Uncertain Descriptor Systems With Time-Varying Discrete and Distributed Delays," *IEE Proc.: Control Theory Appl.*, **152**(6), pp. 628–638.
- [23] Yue, D., and Han, Q.-L., 2005, "Delay-Dependent Exponential Stability of Stochastic Systems With Time-Varying Delay, Nonlinearity, and Markovian Switching," *IEEE Trans. Autom. Control*, **50**(2), pp. 217–222.
- [24] Yue, D., and Han, Q.-L., 2004, "Robust H_∞ Filter Design of Uncertain Descriptor Systems With Discrete and Distributed Delays," *IEEE Trans. Signal Process.*, **52**(11), pp. 3200–3212.
- [25] Yue, D., 2004, "Robust Stabilization of Uncertain Systems With Unknown Input Delay," *Automatica*, **40**(2), pp. 331–336.
- [26] Yue, D., and Lam, J., 2004, "Reliable Memory Feedback Design for a Class of Non-Linear Time-Delay Systems," *Int. J. Robust Nonlinear Control*, **14**(1), pp. 39–60.
- [27] Dong, H., Wang, Z., Ho, D. W. C., and Gao, H., 2010, "Robust H_∞ Fuzzy Output-Feedback Control With Multiple Probabilistic Delays and Multiple Missing Measurements," *IEEE Trans. Fuzzy Syst.*, **18**(4), pp. 712–725.
- [28] Wang, Z., Ho, D. W. C., Dong, H., and Gao, H., 2010, "Robust $calH_\infty$ Finite-Horizon Control for a Class of Stochastic Nonlinear Time-Varying Systems Subject to Sensor and Actuator Saturations," *IEEE Trans. Autom. Control*, **55**(7), pp. 1716–1722.
- [29] Wang, Y., Wang, Z., and Liang, J., 2010, "On Robust Stability of Stochastic Genetic Regulatory Networks With Time Delays: A Delay Fractioning Approach," *IEEE Trans. Syst., Man, Cybern., Part B: Cybern.*, **40**(3), pp. 729–740.
- [30] Dong, H., Wang, Z., Ho, D. W. C., and Gao, H., 2010, "Variance-Constrained $calH_\infty$ Filtering for a Class of Nonlinear Time-Varying Systems With Multiple Missing Measurements: The Finite-Horizon Case," *IEEE Trans. Signal Process.*, **58**(5), pp. 2534–2543.
- [31] Shen, B., Wang, Z., and Hung, Y. S., 2010, "Distributed H_∞ -Consensus Filtering in Sensor Networks With Multiple Missing Measurements: The Finite-Horizon Case," *Automatica*, **46**(10), pp. 1682–1688.
- [32] Dong, H., Wang, Z., and Gao, H., 2010, "Robust H_∞ Filtering for a Class of Nonlinear Networked Systems With Multiple Stochastic Communication Delays and Packet Dropouts," *IEEE Trans. Signal Process.*, **58**(4), pp. 1957–1966.
- [33] Wang, Z., Liu, Y., Wei, G., and Liu, X., 2010, "A Note on Control of a Class of Discrete-Time Stochastic Systems With Distributed Delays and Nonlinear Disturbances," *Automatica*, **46**(3), pp. 543–548.
- [34] Wang, Z., Wang, Y., and Liu, Y., 2010, "Global Synchronization for Discrete-Time Stochastic Complex Networks With Randomly Occurred Nonlinearities and Mixed Time Delays," *IEEE Trans. Neural Networks*, **21**(1), pp. 11–25.
- [35] Yang, H., Shi, P., Zhang, J., and Qiu, J., 2011, "Robust H_∞ Filtering for a Class of Markovian Jump Systems With Time-Varying Delays Based on Delta Operator Approach," *Asian J. Control*, **13**(3), pp. 398–407.
- [36] Yang, H., Xia, Y., and Shi, P., 2011, "Stabilization of Networked Control Systems With Nonuniform Random Sampling Periods," *Int. J. Robust Nonlinear Control*, **21**(5), pp. 501–526.
- [37] Jiang, B., Shi, P., and Mao, Z., 2011, "Sliding Mode Observer-Based Fault Estimation for Nonlinear Networked Control Systems," *Circuits, Syst., Signal Process.*, **30**(1), pp. 1–16.
- [38] Yin, Y., Shi, P., and Liu, F., 2011, "Gain-Scheduled PI Tracking Control on Stochastic Nonlinear Systems With Partially Known Transition Probabilities," *J. Franklin Inst.*, **348**(4), pp. 685–702.
- [39] Luan, X., Liu, F., and Shi, P., 2011, "Finite-Time Stabilization of Stochastic Systems With Partially Known Transition Probabilities," *ASME J. Dyn. Sys., Meas., Control*, **133**(1), p. 014504.
- [40] Luan, X., Liu, F., and Shi, P., 2010, "Finite-Time Filtering for Non-Linear Stochastic Systems With Partially Known Transition Jump Rates," *IET Control Theory Appl.*, **4**(5), pp. 735–745.
- [41] Zhang, L., Boukas, K., and Shi, P., 2009, " H_∞ Model Reduction for Discrete-Time Markov Jump Linear Systems With Partially Known Transition Probabilities," *Int. J. Control*, **82**(2), pp. 343–351.
- [42] Nakura, G., 2010, "Stochastic Optimal Tracking With Preview by State Feedback for Linear Discrete-Time Markovian Jump Systems," *Int. J. Innovative Comput., Inf. Control*, **6**(1), pp. 15–27.
- [43] Ding, Q., and Zhong, M., 2010, "On Designing H_∞ Fault Detection Filter for Markovian Jump Linear Systems With Polytopic Uncertainties," *Int. J. Innovative Comput., Inf. Control*, **6**(3(A)), pp. 995–1004.
- [44] Xia, Y., Zhu, Z., and Mahmoud, M. S., 2009, " H_2 Control for Networked Control Systems With Markovian Data Losses and Delays," *ICIC Express Lett.*, **3**(3(A)), pp. 271–276.
- [45] Huang, C., Bai, Y., and Liu, X., 2010, "Robust H -infinity Output Feedback Control for a Class of Networked Cascade Control Systems With Uncertain Delays," *ICIC Express Lett.*, **4**(1), pp. 231–238.
- [46] Vesely, V., and Quang, T. N., 2010, "Robust Output Networked Control System Design," *ICIC Express Lett.*, **4**(4), pp. 1399–1404.
- [47] Dong, H., Wang, Z., Ho, D. W. C., and Gao, H., 2011, "Robust $calH_\infty$ Filtering for Markovian Jump Systems With Randomly Occurring Nonlinearities and Sensor Saturation: The Finite-Horizon Case," *IEEE Trans. Signal Process.*, **59**(7), pp. 3048–3057.
- [48] Liang, J., Wang, Z., and Liu, X., 2012, "Distributed State Estimation for Uncertain Markov-Type Sensor Networks With Mode-Dependent Distributed Delays," *Int. J. Robust Nonlinear Control*, **22**(3), pp. 331–346.
- [49] Ma, L., Wang, Z., Bo, Y., and Guo, Z., 2011, "Finite-Horizon $\mathcal{H}_2/\mathcal{H}_\infty$ Control for a Class of Nonlinear Markovian Jump Systems With Probabilistic Sensor Failures," *Int. J. Control*, **84**(11), pp. 1847–1857.
- [50] Wang, Z., Liu, Y., and Liu, X., 2010, "Exponential Stabilization of a Class of Stochastic System With Markovian Jump Parameters and Mode-Dependent Mixed Time-Delays," *IEEE Trans. Autom. Control*, **55**(7), pp. 1656–1662.

- [51] Zhang, L. X., Boukas, E. K., and Lam, J., 2008, "Analysis and Synthesis of Markov Jump Linear Systems With Time-Varying Delays and Partially Known Transition Probabilities," *IEEE Trans. Autom. Control*, **53**(10), pp. 2458–2464.
- [52] Zhang, L. X., and Boukas, E. K., 2009, "Mode-Dependent H_∞ Filtering for Discrete-Time Markovian Jump Linear Systems With Partly Unknown Transition Probabilities," *Automatica*, **45**(6), pp. 1462–1467.
- [53] Gao, H., You, J., Shi, P., Zhang, L., and Zhao, Y., 2011, "Stabilization of Continuous-Time Markov Jump Linear Systems With Defective Statistics of Modes Transitions," The 18th *IFAC* World Congress, Milano, Italy, Aug. 28–Sept. 2.
- [54] Liu, H. P., Ho, D. W. C., and Sun, F. C., 2008, "Design of H_∞ Filter for Markov Jumping Linear Systems With Non-Accessible Mode Information," *Automatica*, **44**(10), pp. 2655–2660.