Effects of Gas Rarefaction on Dynamic Characteristics of Micro Spiral-Grooved Thrust Bearing

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The effects of gas-rarefaction on dynamic characteristics of micro spiral-grooved-thrust-bearing are studied. The Reynolds equation is modified by the first order slip model, and the corresponding perturbation equations are then obtained on the basis of the linear small perturbation method. In the converted spiral-curve-coordinates system, the finite-volume-method (FVM) is employed to discrete the surface domain of micro bearing. The results show, compared with the continuum-flow model, that under the slip-flow regime, the decrease in the pressure and stiffness become obvious with the increasing of the compressibility number. Moreover, with the decrease of the relative gas-film-thickness, the deviations of dynamic coefficients between slip-flow-model and continuum-flow-model are increasing. [DOI: 10.1115/1.4006359]

1 Introduction

In power-MEMS, the rotor must spin at a super high speed to achieve high power density, and microbearings with low-friction are required for long life and low drag. Therefore, the gas-lubricated bearings are introduced in almost all the microsystems. In previous designs, micro gas thrust bearings were hydrostatic, in which the gas film supporting the axial load of the rotor was supplied through metering orifices from a pressure source external to the bearing [1–3]. Although this approach was shown to work well and had great operational and experimental flexibility, its high complexity in micro fabrication is still an issue, as hydrostatic thrust bearings need an external pressurized air source. However, the hydrodynamic thrust bearing can offer an alternative approach. In these bearings, the gas film pressure is generated within the bearing by the motion of the rotor relative to the stator. Huang et al. [4] proposed a design for a micro fabricated hydrodynamic thrust bearing and discussed its scaling issues. Chee Wei et al. [5] described the design, fabrication, and test of self-acting thrust bearing integrated into a high-speed micro turbine.

The previous investigation has revealed many important aspects of the micro thrust bearing in design and fabrication. However, it is noted that up to now almost all the studies on gas micro thrust bearing are limited to continuum flow analysis where the gas rarefaction effects are neglected. Actually, when the gas film thickness in microbearing is from 0.65 μm to 6.5 μm , the Knudsen number, which is defined as the ratio of the molecular mean free path to the microbearing gas film thickness, is within the range from 0.01 to 0.1 so that the continuum assumption of fluid mechanics is no longer a good approximation, and then the slip-flow assumption of fluid mechanics should be used to describe the hydrodynamic performance of the microbearing more accurately [6]. The objective of this paper is to investigate the dynamic characteristics of micro gas spiral-grooved thrust bearing with slip-flow assumption and analyze effects of gas rarefaction on dynamic characteristics of them.

2 Theoretical Analysis

The problem to be considered is a gas lubricated micro spiral groove thrust bearing, as shown in Fig. 1.

Expressed as $r = r_{\text{in}} \exp(\theta \tan \alpha)$, the relative motion between the grooved and flat mating surfaces causes a pumping action in the gas lubricant. The inner radius r_{in} and outer radius r_{out} denote the beginning and the end of the spiral groove, respectively. θ_r and θ_g represent the ridge and groove width, respectively. α is the spiral angle. The bearing is subjected to pressures p_{in} and p_{out} at its inner and outer boundary. As for the spiral groove geometry given in Fig. 1, h_r and h_g are the ridge clearance and groove depth, respectively. The origin of the Cartesian coordinates system is fixed in the steady equilibrium position of the shaft.

2.1 The Modified Reynolds Equation Based on the Slip-Flow Model. In the bearing gap, the gas flow is assumed to be isothermal, compressible, and laminar. As the Knudsen number (Kn) is within the range from 0.01 to 0.1, the time-dependent Reynolds' equation modified by the first order slip model [7] in polar coordinates is

$$\frac{1}{r} \frac{\partial}{\partial \theta} \left[ph^3 (1 + 6Kn') \frac{\partial p}{\partial \theta} \right] + \frac{\partial}{\partial r} \left[ph^3 (1 + 6Kn') r \frac{\partial p}{\partial r} \right] = 6\mu\omega r \frac{\partial}{\partial \theta} (ph) + 12\mu r \frac{\partial (ph)}{\partial t} \quad (1)$$

Here, p represents the hydrodynamic pressure, h is the fluid film thickness, and μ is the fluid viscosity. $Kn' = l'/h_{\text{min}}$, $l' = l(2 - \chi)/\chi$, where h_{min} is the minimum film thickness, l is the molecular mean free path, and χ is the momentum adjustment coefficient.

The finite volume method is employed to discrete the surface domain of the micro spiral groove bearing, and the Newton-Raphson method is used to solve those discretation formulas. The direct finite volume procedure is based on the papers by Miller and Greens [8,9], and some improvements about the coordinates system are made as follows.

Firstly, the polar coordinates $(r - \theta)$ are converted into the spiral coordinates $(\xi - \eta)$, that is,

$$\begin{cases} \xi = r \\ \eta = \theta - f(\xi) \end{cases} \quad (2)$$

where $f(\xi) = \ln(\xi/\xi_{\text{in}})/\tan \alpha$. This function is monotonically decreased in the range of $\xi_{\text{in}} \leq \xi \leq \xi_{\text{out}}$, and has the first order

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Fig. 1 Schematic of spiral groove bearing system (a) spiral groove geometry and (b) spiral groove bearing system

continuous partial derivatives. The spiral patterns before and after the coordinates conversion are shown in Fig. 2.

The purpose of altering the coordinates system is to make the radial mesh that discretizes the bearing surface domain coincide with the spiral pattern in the $\xi - \eta$ coordinates. In the polar coordinates, the finite volume Ω with the boundary Γ is divided by spiral grooves into different parts with nonradial shape. In the $\xi - \eta$ coordinates system, the radial shape can be better ensured. Therefore, the mass flow into the finite volume can be computed in the regular shape meshes, and the complexity of the computation is reduced.

In the $\xi - \eta$ coordinates system, the Reynolds equation is changed into

$$\vec{\nabla} \cdot \left[(1 + 6K_n') ph^3 \vec{\nabla} p - 6\mu \omega \xi p h \vec{i}_\eta \right] = 12\mu \frac{\partial(ph)}{\partial t} \quad (3)$$

Fig. 2 Coordinates transformation of spiral curve

where $\vec{\nabla} p = \left[\frac{\partial p}{\partial \xi} - f'(\xi) \frac{\partial p}{\partial \eta} \right] \sqrt{1 + \xi^2 [f'(\xi)]^2} \vec{i}_\xi + \left[-\frac{\partial p}{\partial \xi} \xi f'(\xi) + \frac{1}{\xi} \frac{\partial p}{\partial \eta} (1 + \xi^2 [f'(\xi)]^2) \right] \vec{i}_\eta$, and the boundary conditions in the spiral coordinates are

$$\begin{cases} p_{\xi=\xi_{out}} = p_{out} \\ p_{\xi=\xi_{in}} = p_{in} \\ p_{\eta=0} = p_{\eta=2\pi} \end{cases} \quad (4)$$

To accommodate these discontinuities (Fig. 3, type II), the technique of Kogure et al. [10] is implemented by introducing the

Fig. 3 Finite volume discretization in spiral coordinates. I: finite volume in the ridge area; II: finite volume separated by boundary; III: finite volume in the groove area.

weight factor ψ , which is defined to be the length of a finite volume face that is in a ridge divided by its overall length (see Fig. 3). In this case, the mass flow into the finite volume along the radial direction can be split and computed separately based on the discrete values of the film thicknesses in the ridge and groove at that finite volume, and the total mass flow through a finite volume is computed by adding the groove and ridge mass flows averaged by the appropriate weight factor, ψ and $1 - \psi$, respectively. More details about the discontinuities solution can be seen in the authors' paper [6].

2.2 Perturbed Reynolds Equations of Gas Films. In the practical micro systems, the rotor typically has certain axial runout and angular misalignment due to either manufacturing or assembly tolerances, thus producing the axial pulsation and wobbling motion during the dynamic operation. Although the relative motion between the rotor and the stator will generate hydrodynamic pressure, which can be further enhanced by a lift-off feature such as the spiral grooves, the rotor system stability will be inevitably affected by those factors.

A layout of the gas thrust bearing system is shown in Fig. 1(b). The origin of Cartesian coordinate is fixed in the steady equilibrium position of the bearing surface. The possible independent motions of the bearing are of three degrees of freedom, that is, axial movement z and angular wobbles α and β around the x and y axis, respectively. Based on the linear small perturbation method, three disturbed displacements can be expressed in the following form:

$$\begin{aligned} z(t) &= \Delta z e^{i\gamma t} \\ \alpha(t) &= \Delta \phi_x e^{i\gamma t} \\ \beta(t) &= \Delta \phi_y e^{i\gamma t} \end{aligned} \quad (5)$$

where Δz , $\Delta \phi_x$, and $\Delta \phi_y$ represent the axial perturbation and angular perturbations around the x and y axis at excitation frequency γ , respectively.

Assuming the rotating surface undergoes small axial motion ($z(t)$) and angular motions ($\alpha(t)$, $\beta(t)$) from the equilibrium position (h_0), the film thickness is

$$\begin{aligned} h &= h_0 + z(t) + \beta(t)r \cos \theta - \alpha(t)r \sin \theta \\ &= h_r + h_g + \Delta z e^{i\gamma t} + r \cos \theta \Delta \phi_y e^{i\gamma t} - r \sin \theta \Delta \phi_x e^{i\gamma t} \end{aligned} \quad (6)$$

Here, h_g is considered at the groove region only. According to the expression of Knudsen number $Kn = \lambda/h_0$, where h_0 represents the film thickness at the equilibrium position, Kn is not perturbed [11].

These small amplitude motions will cause the perturbations in the pressure field, which is given by

$$\begin{aligned} p &= p_0 + z(t)p_z + \alpha(t)p_{\phi_x} + \beta(t)p_{\phi_y} \\ &= p_0 + p_z \Delta z e^{i\gamma t} + p_{\phi_x} \Delta \phi_x e^{i\gamma t} + p_{\phi_y} \Delta \phi_y e^{i\gamma t} \end{aligned} \quad (7)$$

The dimensionless variables are given,

$$\begin{aligned} \bar{p}_0 &= p_0/p_a & \bar{\xi} &= \xi/\xi_{in} & \bar{h} &= h/h_g \\ \bar{t} &= t\omega & \gamma &= v\omega & \sigma &= 2\Lambda v \\ \Lambda &= \frac{6\mu\omega\xi_{in}^2}{p_a h_g^2} & \bar{p}_z &= \frac{p_z h_g}{p_a} \\ \bar{p}_{\phi_x} &= \frac{p_{\phi_x} h_g}{p_a \xi_{in}} & \bar{p}_{\phi_y} &= \frac{p_{\phi_y} h_g}{p_a \xi_{in}} \end{aligned} \quad (8)$$

Substituting the nondimensional perturbed film thickness and pressure into Eq. (1), and neglecting higher-order terms, it leads to the zeroth and first-order equations are then given as:

Table 1 Comparison With Muijderman's Results in Load Capacity (N)

λ	α	ρ	δ	Muijderman [3]	Present Analysis	Difference
0.4	12.4	1.16	0.32	100.48252	94.88441	5.57%
0.5	12.2	1.17	0.31	95.668	95.009	0.689%
0.6	11.7	1.19	0.31	86.711	86.836	0.144%
0.7	10.9	1.24	0.29	72.243	73.293	1.453%

Fig. 4 Comparison with the available numerical results of dimensionless load for spiral groove gas thrust bearings

Fig. 5 Comparison with the results of Malanoski and Pan [16] and Zirelback and Andres [14]. (a) Dimensionless axial stiffness versus frequency number for various compressibility numbers; (b) dimensionless axial damping versus frequency number for various compressibility numbers.

$$\begin{aligned} & \frac{1}{\xi} \frac{\partial}{\partial \eta} \left\{ \bar{p}_o \bar{h}_o^3 (1 + 6K_n') \left[\frac{\partial \bar{p}_o}{\partial \xi} - f'(\xi) \frac{\partial \bar{p}_o}{\partial \eta} \right] \sqrt{1 + \xi^2 [f'(\xi)]^2} \right\} \\ & + \frac{\partial}{\partial \xi} \left\{ \bar{p}_o \bar{h}_o^3 (1 + 6K_n') \left[\frac{1}{\xi} \frac{\partial \bar{p}_o}{\partial \eta} + \frac{\partial \bar{p}_o}{\partial \eta} \xi [f'(\xi)]^2 - \frac{\partial \bar{p}_o}{\partial \xi} \xi f'(\xi) \right] \right\} \\ & = \Lambda \xi \frac{\partial}{\partial \eta} (\bar{p}_o \bar{h}_o) \end{aligned} \quad (9)$$

$$\begin{aligned} & \frac{1}{\xi} \frac{\partial}{\partial \eta} \left(\bar{h}_o^3 \left[\frac{\partial \bar{p}_o \bar{p}_q}{\partial \xi} - f'(\xi) \frac{\partial \bar{p}_o \bar{p}_q}{\partial \eta} \right] \sqrt{1 + \xi^2 [f'(\xi)]^2} \right) \\ & + \frac{1}{\xi} \frac{\partial}{\partial \eta} \left(\frac{3}{2} \bar{h}_o^2 l_q \left[\frac{\partial \bar{p}_o^2}{\partial \xi} - f'(\xi) \frac{\partial \bar{p}_o^2}{\partial \eta} \right] \sqrt{1 + \xi^2 [f'(\xi)]^2} \right) \\ & + \frac{\partial}{\partial \xi} \left(\bar{h}_o^2 \xi \left[\frac{1}{\xi} \frac{\partial (\bar{p}_o \bar{p}_q)}{\partial \eta} + \frac{\partial (\bar{p}_o \bar{p}_q)}{\partial \eta} \xi [f'(\xi)]^2 - \frac{\partial (\bar{p}_o \bar{p}_q)}{\partial \xi} \xi f'(\xi) \right] \right) \\ & + \frac{\partial}{\partial \xi} \left(\frac{3}{2} \bar{h}_o^2 \xi l_q \left[-\frac{\partial (\bar{p}_o^2)}{\partial \xi} \xi f'(\xi) + \frac{1}{\xi} \frac{\partial (\bar{p}_o^2)}{\partial \eta} + \frac{\partial (\bar{p}_o^2)}{\partial \eta} \xi [f'(\xi)]^2 \right] \right) \\ & = \frac{\Lambda \xi}{1 + 6K_n'} \frac{\partial}{\partial \eta} (\bar{p}_o l_q + \bar{h}_o \bar{p}_q) + i \frac{\sigma \xi}{1 + 6K_n'} (\bar{p}_o l_q + \bar{h}_o \bar{p}_q) \end{aligned} \quad (10)$$

where the subscript q denotes z , φ_x , and φ_y , and $l_z = 1$, $l_{\varphi_x} = -\xi \sin \eta$, and $l_{\varphi_y} = \xi \cos \eta$. The first-order Eq. (13) contain the zeroth-order pressure $\bar{p}_o$; therefore, the equations for the zeroth-order pressure field (Eq. (12)) must be solved first for $\bar{p}_o$, then the first-order pressure field solution $\bar{p}_q$ can be obtain. For the sake of calculation, $\bar{p}_q$ is redefined as complex.

$$\bar{p}_q = \bar{p}_{qr} + i \bar{p}_{qi} (q = z, \varphi_x, \varphi_y) \quad (11)$$

By substitution Eq. (14) into Eq. (13), integration of the first-order pressure field ($\bar{p}_q$) on the surface of the micro spiral groove

(a)

(b)

Fig. 6 Circumferential pressure distribution at radius is 1.5 mm. (a) $\Lambda = 12$; (b) $\Lambda = 60$.

thrust bearing gives stiffness and damping coefficients, and the complex impedance can be written as

$$K_{qj} + i \gamma C_{qj} = - \int_0^{2\pi} \int_{\xi_{in}}^{\xi_{out}} \bar{p}_q \xi l_j d\xi d\eta \quad (12)$$

For the coefficients K and C , the subscript j denotes the direction of displacement perturbations while the first one stands for the direction of the corresponding hydrodynamic force.

The boundary conditions for the equilibrium state are

$$\begin{cases} \bar{p}_o(1, \theta) = \bar{p}_{in} \\ \bar{p}_o(\xi_{out}, \theta) = \bar{p}_{out} \\ \bar{p}_o(\xi, 0) = \bar{p}_o(\xi, 2\pi) \end{cases} \quad (13)$$

The boundary conditions for the small perturbation state are

$$\begin{cases} \bar{p}_{qj} = 0 \\ \bar{p}_{qj}(\xi, 0) = \bar{p}_{qj}(\xi, 2\pi) \end{cases} \quad (14)$$

3 Results and Discussion

3.1 Validation of the Present Work. Several examples in continuum flow regime are presented in this section for validating both the present model and computing method against the published work. It should be noted that the published results used for comparison did not incorporate rarefaction effects, so the

(a)

(b)

Fig. 7 Effects of gas rarefaction on dynamic coefficients with fixed compressibility number: (a) dimensionless stiffness coefficients; (b) dimensionless damping coefficients

discussions of all of the results in this section (including the works of the present authors) are not considered rarefaction effects.

The first example is to compare the present numerical solution of steady load capacity for spiral groove thrust bearing with the analytic solutions published by Mujinderman [3]. The operation conditions and structure parameters are as follows:

Groove number $G_n = 15$, $r_{out} = \xi_{out} = 15$ mm, $\mu = 18.1 \times 10^{-6}$ Pa · s, and $n = 36$ rpm.

The load capacity and the differences under different coefficients of the radius ratio $\lambda = r_{in}/r_{out} = \xi_{in}/\xi_{out}$, the width ratio of the ridge and groove $\rho = \theta_r/\theta_g$, the ratio of the film thickness and the depth of groove $\delta = h_r/h_g$, and the spiral groove angle α are presented in Table 1. It is shown that the analysis by the present finite volume method matches well with Mujinderman's results.

The second example shown in Fig. 4 is a comparison of steady load capacity with those previously published numerical calculations, i.e., James and Potter's FDM solution [12], Bonneau's FEM solution [13], Zirkelback and Andres's FEM solution [14], and the FEM solution of Liu et al. [15]. The parameters of the bearing in this example are listed as follow:

$G_n = 12$, $\alpha = 20$, $r_{in} = \xi_{in} = 0.0635$ m, $\lambda = 0.3848$, $\delta = 0.17$, $\rho = 0.25$, $\mu = 3.237 \times 10^{-5}$ N s/m², $n = 24,000$ rpm, $p_{in} = 3.2 \times 10^5$ Pa, and varied p_{out} . As we can see from Fig. 4, the present calculation shows reasonable agreement with the available results.

The third example is a comparison of dynamic axial stiffness and damping with the results by Zirkelback and Andres [14], Liu et al. [15] and Malanoski and Pan [16] for spiral groove gas thrust bearings, as shown in Fig. 5, validating the improved FVM model and the computer program presented in this paper, which will be

used for the further analysis of the micro gas spiral groove thrust bearing.

3.2 Characteristics of Micro Gas Spiral Groove Thrust Bearing.

A few representative results will be presented in this part to illustrate the effects of gas rarefaction on both pressure distribution and dynamic coefficients of micro gas spiral groove thrust bearing. The parameters are: $G_n = 5$, $\mu = 18.1 \times 10^{-6}$ N s/m², $r_{out} = \xi_{out} = 2$ mm, and $h_r = 3.05$ μ m.

Figures 6(a) and 6(b) display the circumferential pressure distribution at the radius of $r_c = \xi_c = 1.5$ mm with different compressibility number of $\Lambda = 12$ and $\Lambda = 60$. The pressure value increases along with compressibility number in both the continuum-flow model and slip-flow model. However, the pressure with the slip-flow model is lower than those with continuum-flow model. Figure 6 also indicates that in the ridge area with the lower clearance (Part A), the effects of gas rarefaction on slope of the pressure are obvious, whereas in the groove area with greater clearance (Part B), the slope of pressure is close in both continuum-flow and slip-flow model.

Figures 7(a) and 7(b) depicts the gas rarefaction effects on dynamic coefficients for different frequency numbers. It is observed that, due to axial symmetry, the angular stiffness and angular damping coefficients directly caused by the angular perturbations $\alpha(t)$ and $\beta(t)$ are equal ($K_{xx} = K_{yy}$, $C_{xx} = C_{yy}$), respectively, and the cross-coupled angular stiffness and angular damping coefficients created by the angular perturbations $\alpha(t)$ and

Fig. 8 Influence of dimensionless gas rarefaction on stiffness with different compressibility number: (a) dimensionless stiffness K_{xx} , K_{yy} ; (b) dimensionless stiffness K_{zz}

Fig. 9 Comparison of the results between dynamic coefficients considering and without considering micro scale effects: (a) dimensionless stiffness coefficients versus compressibility number; (b) dimensionless damping coefficients versus compressibility number

Fig. 10 Influence of dimensionless gas rarefaction on dynamic coefficients with different gas film thickness: (a) dimensionless stiffness; (b) dimensionless damping coefficients

$\beta(t)$ are of opposite sign ($K_{xy} = -K_{yx}$, $C_{xy} = -C_{yx}$). The angular stiffness and angular damping coefficients caused by the axial perturbation (K_{xz} , K_{yz} , C_{xz} , C_{yz}) are approximately equal to zero, as well as the axial stiffness and axial damping coefficients caused by angular perturbations (K_{zx} , K_{zy} , C_{zx} , C_{zy}). These results are consistent with Liu et al. [15]. Because the pressure profiles created by axial perturbation generate axial force, the cross-coupled dynamic coefficients caused by angular perturbation should be zero. The nonzero values might come from intrinsic numerical truncation and round-off errors. The axial stiffness coefficient caused by axial perturbation (K_{zz}) increases with the increasing of frequency number, and the angular stiffness caused by angular perturbations (K_{xx} , K_{yy}) shows the same trend. All of the damping coefficients decrease with the increasing of frequency number. Moreover, Fig. 7(a) shows that the stiffness with slip-flow model is lower than that continuum-flow model. It also can be seen in Fig. 7(b), the damping coefficients with slip-flow model are higher than those continuum-flow model.

Figure 8 presents the gas rarefaction effects on stiffness for different compressibility numbers. As shown, when the values of frequency number are less than 200, the stiffness increases quickly, for the frequency number larger than 200, this incensement slows down and gradually becomes flat. And the stiffness increases as the compressibility number increases from $\Lambda = 20$ to $\Lambda = 100$; the deviation of stiffness of two models under the condition of $\Lambda = 100$ is larger than that under the condition $\Lambda = 20$.

Figure 9 presents the gas rarefaction effects on stiffness and damping coefficients for different compressibility numbers. The slip flow assumption reduces the damping coefficients at lower compressibility number but increases the damping coefficients at larger compressibility number, which are shown to be similar trends to the results of Lee et al. in Ref. [11].

Figure 10 presents the numerical results of 18 dynamic coefficients against dimensionless gas film thickness δ under the condition of $h_g = 3.0 \mu\text{m}$. As shown in Fig. 10, the direct stiffness coefficients and damping coefficients decrease with increasing dimensionless gas film thickness δ . At higher δ , the dynamic coefficients in the slip-flow model are very close to those in the continuum-flow model. The deviations between the continuum flow model and slip flow model decrease with increasing δ , which means rarefaction effects have been weak when the gas film thickness is big. On the other hand, the cross-coupled stiffness and damping coefficients (K_{xz} , K_{yz} , K_{zx} , K_{zy} , C_{xz} , C_{yz} , C_{zx} , C_{zy}) are approximately equal to zero.

4 Conclusions

A numerical study about effects of gas rarefaction on dynamic characteristics of micro gas spiral-grooved thrust bearing is presented. From the results obtained in this paper, the following conclusions have been drawn:

- (1) The modified Reynolds equation based on slip-flow model and the improved finite volume method by coordinates transformation of spiral curve are validated for gas spiral-grooved thrust bearing calculation.
- (2) The effects of gas rarefaction on the pressure distribution and dynamic coefficients increase with the increasing of compressibility number.
- (3) With the increase of the gas film thickness, the deviation of the dynamic coefficients between slip-flow model and continuum-flow model is decreasing.

Therefore, the gas rarefaction effects on steady performances and dynamic coefficients must be taken into account in the micro systems operating with high compressibility number and small gas film thickness in order to analyze its dynamic characteristics more accurately.

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Nomenclature

- c , C = damping coefficients (N/(rad/s)), dimensionless damping coefficients
 G_n = groove number, (pairs)
 h , $\bar{h}$ = film thickness, (m); dimensionless film thickness
 h_o = equilibrium film thickness (m)
 h_r , h_g = steady film thickness at ridge area and groove area (m)
 $\bar{h}_r$ = dimensionless gas film thickness at ridge area
 $h_{\min}$ = the minimum gas film thickness (m)
 k , K = stiffness coefficients, (N/m, N/rad), dimensionless stiffness coefficients.
 Kn = Knudsen number
 l = molecular mean free path length (m)
 l' = modified molecular mean free path length by first order slip model (m)
 n = rotational speed, (rpm)
 p , $\bar{p}$ = pressure (Pa), dimensionless pressure
 p_a , p_{in} , p_{out} = ambient pressure, pressure at inner radius, and outer radius (Pa)
 p_z = axial perturbation pressure components (Pa)
 $\bar{p}_z$ = dimensionless axial perturbation pressure components
 p_{φ_x} , p_{φ_y} = angular perturbation pressure components (Pa)
 $\bar{p}_{\varphi_x}$, $\bar{p}_{\varphi_y}$ = dimensionless angular perturbation pressure components
 r_{in} , r_{out} = inner radius in cylindrical coordinates, outer radius in cylindrical coordinates (m)

r, θ, z = cylindrical coordinates
 $t, \bar{t}$ = time (s), dimensionless time
 x, y, z = rectangular coordinates
 $\Delta z, \Delta \varphi_x, \Delta \varphi_y$ = initial values of the perturbations
 $z(t), \alpha(t), \beta(t)$ = axial and angular perturbed displacement (m, rad)
 α = spiral angle (deg)
 γ, ν = perturbation frequency (rad/s), dimensional perturbation frequency
 δ = the ratio of the film height and the depth of the groove $\delta = h_r/h_g$
 θ_g, θ_r = groove width, ridge width (deg)
 λ = the radius ratio $\lambda = r_{in}/r_{out}$
 μ = fluid viscosity (N s/m²)
 ξ, η, z = spiral coordinates
 ξ_{in}, ξ_{out} = inner radius in spiral coordinates, outer radius in spiral coordinates (m)
 ρ = width ratio of the groove and ridge $\rho = \theta_r/\theta_g$
 σ = frequency number
 χ = momentum adjustment coefficient
 ω = rotational speed (rad/s)
 Λ = compressibility number
 Γ = finite volume boundary
 Ω = finite volume domain

Subscripts

min = the minimum variables
 o = variables in equilibrium position
 r, i = real part and imaginary part of the complex
 x, y = along x and y axis
 ξ, η = along x and y axis
 z, φ_x, φ_y = position of the perturbations

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