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Analytic derivation of pinhole collimation sensitivity for a general source model using spherical harmonics

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Abstract

Pinhole collimators are widely used for SPECT imaging of small organs and animals. There also has been renewed interest in using pinhole arrays for clinical cardiac SPECT imaging to achieve high sensitivity and complete data sampling. Overall sensitivity of a pinhole array is critical in determining a system's performance. Conventionally, a point source model has been used to evaluate the sensitivity and optimize the system design. This model is simple but far from realistic. This work addresses the use of more realistic source models to assess the sensitivity performance of pinhole collimation. We have derived an analytical formula for pinhole collimation sensitivity with a general source distribution model using spherical harmonics. As special cases of this general model, we provided the pinhole sensitivity formulae for line, disk and sphere sources. These results show that the point source model is just the zeroth-order approximation of the other source models. The point source model overestimates or underestimates the sensitivity relative to the more realistic model. The sphere source model yields the same sensitivity as a point source located at the center of the sphere when attenuation is not taken into account. In the presence of attenuation, the average path length of emitted gamma-rays is $3/4$ of the radius of the sphere source. The calculated sensitivities based on these formulae show good agreement with separate Monte Carlo simulations in simple cases. The general and special sensitivity formulae derived here can be useful for the design and optimization of SPECT systems that utilize pinhole collimators.

1. Introduction

Pinhole collimation was one of the first techniques used in gamma ray imaging (Anger and Rosenthal 1959, Mallard and Myers 1963, Paix 1967). Pinhole collimators are widely used for small animal single photon emission computed tomography (SPECT) imaging (Jaszczak et al. 1994, Weber et al. 1994, Meikle et al. 2005, Beekman and van der Have 2007). Recently, there has been renewed interest in using pinhole collimators for clinical cardiac SPECT imaging (Funk et al. 2006, Steele et al. 2008, Huang et al. 2009), motivated in part by a desire to achieve high sensitivity using multiple pinholes (Rogulski et al. 1993).

A number of pinhole studies have been reported in the literature. Some studies were focused on the characterization of the pinhole sensitivity, resolution and point spread function (PSF) (Smith and Jaszczak 1997, Metzler et al. 2001, Metzler et al. 2002, Accorsi and Metzler 2004, Bal and Acton 2006, Shokouhi et al. 2009). Other studies described methods for the design and optimization of pinhole or multi-pinhole parameters using Monte Carlo or analytical methods (Beekman and Vastenhouw 2004, Cao et al. 2005, Rentmeester et al. 2007). An alternative to the pinhole approach is to use the slit-slat (Metzler et al. 2006, Li et al. 2009, Chang et al. 2009) or skew-slit collimators (Huang and Zeng 2006, Zeng 2008).

The slit acts as a 1D pinhole which provides in the slit-slat and skew-slit collimators the essential feature to magnify or minify the projections.

The overall sensitivity of a pinhole array is critical in determining a SPECT system's performance. The sensitivity of a pinhole collimator is the fraction of gamma rays emitted from a radiation source that pass through the aperture of the collimator and is more appropriately called geometric efficiency (Cherry et al. 2003). Conventionally, a point source model has been used to evaluate the overall sensitivity and optimize the system design. This model is simple and ideal, but far from realistic. Thus we need more realistic source models to optimize pinhole designs and assess their sensitivity performance. Simulations designed to optimize the performance of pinhole collimation are computationally intensive because of the large number of parameters to be varied. A simple and straight forward method would be very useful. We have developed an analytical prediction for the pinhole collimation sensitivity with a general source model using spherical harmonics. As special cases of this general model, we have also provided the pinhole sensitivity predictions for line, disk and sphere sources.

2. Mathematical derivation

2.1. General expression

The basic pinhole collimation geometry is illustrated in figure 1. A spherical coordinate system is introduced to facilitate the derivation. A circular pinhole of diameter d is located at $\vec{r}_p = (r_p \sin \theta_p \cos \phi_p, r_p \sin \theta_p \sin \phi_p, r_p \cos \theta_p = b)$. The normal direction of the pinhole aperture is along the direction $\hat{m}$. The sensitivity formula for a point source located at $\vec{r} = (r \sin \theta \cos \phi, r \sin \theta \sin \phi, r \cos \theta)$ can be expressed as (Mallard and Myers 1963, Cherry et al. 2003),

$$g(\vec{r}_p, \vec{r}) \approx \frac{d^2 \cos^3 \beta}{16 h^2} = \frac{d^2 \hat{m} \cdot (\vec{r}_p - \vec{r})}{16 |\vec{r}_p - \vec{r}|^3}, \quad (1)$$

where $h = \hat{m} \cdot (\vec{r}_p - \vec{r})$ is the perpendicular distance between the point source and the pinhole aperture plane, and β is the incident angle with $\cos \beta = h / |\vec{r}_p - \vec{r}|$. Basically, $g(\vec{r}, \vec{r}_p)$ is the solid angle for the pinhole aperture $\pi d^2/4$ subtended at the point $\vec{r}$ normalized by 4π steradians. Equation (1) is not exact, it gives accurate estimation of the normalized solid angle only when $d/|\vec{r}_p - \vec{r}| \ll 1$, and this condition is satisfied in most pinhole applications. The exact solid angle for a pinhole aperture subtended by a point is given in (Jaffey 1954). When the pinhole is right above the point source, i.e. $\beta = 0$, the exact sensitivity is $(1 - h / \sqrt{h^2 + d^2/4})/2$.

For a more realistic source model, we assume that the normalized distribution of the source concentration (activity per unit volume) is denoted by $\rho(\vec{r})$, where $\vec{r}$ denotes the position inside the volume. Without loss of generality, we assume that the normal direction $\hat{m}$ is along the z -axis. Then the mean geometric efficiency for the whole volume is given by

$$G(\vec{r}_p) = \frac{d^2}{16} \iiint_{-\infty}^{+\infty} \frac{\hat{e}_z \cdot (\vec{r}_p - \vec{r})}{|\vec{r}_p - \vec{r}|^3} \rho(\vec{r}) d\vec{r}, \quad (2)$$

where $\hat{e}_z$ is the unit vector along z -axis.

The integrand in equation (2) can be expressed as the $\hat{e}_z$ -direction component of the gradient

of the scalar $\frac{1}{|\vec{r}_p - \vec{r}|}$, i.e. $\hat{e}_z \cdot (\vec{r}_p - \vec{r}) / |\vec{r}_p - \vec{r}|^3 = \hat{e}_z \cdot \nabla \frac{1}{|\vec{r}_p - \vec{r}|}$. As shown in Appendix A, by expressing the component gradient operator $\hat{e}_z \cdot \nabla$ in spherical coordinates and

expressing the scalar $\frac{1}{|\vec{r}_p - \vec{r}|}$ in terms of spherical harmonics, the integrand in equation (2) can be written as

$$\frac{\hat{e}_z \cdot (\vec{r}_p - \vec{r})}{|\vec{r}_p - \vec{r}|^3} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{(\ell+1-m)!}{(\ell+m)!} \frac{r^\ell}{r_p^{\ell+2}} P_{\ell+1}^m(\cos\theta_p) P_\ell^m(\cos\theta) e^{im(\varphi_p - \varphi)}, \quad (3)$$

where P_ℓ^m is an associated Legendre function which can be expressed in the form (Abramowitz and Stegun 1964)

$$P_\ell^m(x) = \frac{(-1)^m}{2^\ell \ell!} (1-x^2)^{m/2} \frac{d^{\ell+m}}{dx^{\ell+m}} (x^2-1)^\ell. \quad (4)$$

Putting equation (3) into equation (2), the mean geometric sensitivity for the general source model can be given by

$$G(\vec{r}_p) = \frac{d^2}{16} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{(\ell+1-m)!}{(\ell+m)!} \frac{T_{\ell m}}{r_p^{\ell+2}} P_{\ell+1}^m(\cos\theta_p) e^{im\varphi_p}, \quad (5)$$

where the coefficient $T_{\ell m}$ is

$$T_{\ell m} = \iiint_{-\infty}^{+\infty} \rho(\vec{r}) r^\ell P_\ell^m(\cos\theta) e^{-im\varphi} d\vec{r}. \quad (6)$$

Since the spherical harmonics form a complete set of orthonormal functions and form an orthonormal basis of the Hilbert space of square-integrable functions, the source distribution can thus be expanded as a linear combination of these functions:

$$\rho(\vec{r}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{2\ell+1}{4\pi} \frac{(\ell-m)!}{(\ell+m)!} \rho_{\ell m}(r) P_\ell^m(\cos\theta) e^{im\varphi}, \quad (7)$$

where $\rho_{\ell m}(r)$ can be written as

$$\rho_{\ell m}(r) = \iint_{s^2} \rho(\vec{r}) P_\ell^m(\cos\theta) e^{-im\varphi} d\Omega = \int_0^{2\pi} d\varphi \int_0^\pi \sin\theta \rho(r, \theta, \varphi) P_\ell^m(\cos\theta) e^{-im\varphi} d\theta. \quad (8)$$

Thus the coefficient $T_{\ell m}$ in equation (6) can be rewritten as $T_{\ell m} = \int_{-\infty}^{+\infty} \rho_{\ell m}(r) r^{\ell+2} dr$.

2.2. Attenuation effect

For the case of uniform attenuation, the integrand in equation (2) will have an additional attenuation factor $\exp[-\mu L(\vec{r}, \vec{r}_p)]$. Here μ is the linear attenuation coefficient of the source material and $L(\vec{r}, \vec{r}_p)$ is the length of the intersection path of the emitted gamma-ray with the source object. This length lies along a line from the source to the center of the pinhole, or in other words, along the $\vec{r}_p - \vec{r}$ direction. Hence the mean geometric sensitivity for the whole volume with uniform attenuation can be given by

$$G(\vec{r}_p) = \frac{d^2}{16} \iiint_{-\infty}^{+\infty} \frac{\hat{e}_z \cdot (\vec{r}_p - \vec{r})}{|\vec{r}_p - \vec{r}|^3} \rho(\vec{r}) \exp[-\mu L(\vec{r}, \vec{r}_p)] d\vec{r}. \quad (9)$$

Comparing equations (2) and (9), and assuming an attenuated source distribution $\rho_A(\vec{r})$ can be defined as

$$\rho_A(\vec{r}) = \rho(\vec{r}) \exp[-\mu L(\vec{r}, \vec{r}_p)], \quad (10)$$

then equations (5) and (6) can still be used to calculate the pinhole geometric efficiency for source distributions with attenuation by using $\rho_A(\vec{r})$ instead of $\rho(\vec{r})$. The geometric efficiency calculation for a pinhole with non-uniform attenuation is complex and is outside the scope of this paper. The special case for a uniform sphere source distribution is addressed and the pinhole is assumed at $\vec{r}_p = b\hat{e}_z$. The gamma-ray along the $\vec{r}_p - \vec{r}$ direction emitted from position $\vec{r}$ intersects the sphere boundary at one point. By calculating the length between point $\vec{r}$ and that intersection point, the intersection path length is given by

$$L(\vec{r}, b\hat{e}_z) = R \sqrt{1 - \frac{b^2 r^2 \sin^2 \theta}{R^2(b^2 + r^2 - 2br \cos \theta)}} + \frac{r^2 - br \cos \theta}{\sqrt{b^2 + r^2 - 2br \cos \theta}}. \quad (11)$$

3. Sensitivity for special source models

3.1. Point source model

A point source can be represented by a Dirac delta function, and the delta function in spherical coordinates can be given by[‡]

$$\delta(\vec{r}) = \frac{\delta(r)}{2\pi r^2}. \quad (12)$$

Putting equation (12) into equation (6), the factor T_{lm} for point source becomes $\delta_0 \delta_{m0}$. Thus the sensitivity formula (5) for a point source becomes

$$G(\vec{r}_p) = \frac{d^2}{16} \frac{P_1(\cos \theta_p)}{r_p^2} = \frac{d^2}{16} \frac{\cos \theta_p}{r_p^2} \quad (13)$$

which is the same as equation (1) for a point source located at $\vec{r} = 0$.

3.2. Line source models

The line source models are segmented line source models which have two endpoints. The specific line sources addressed here can have two different orientations, i.e., the line sources can lie in a plane parallel or perpendicular to the pinhole aperture plane. We refer to these two scenarios as parallel and perpendicular line source models.

[‡]In spherical coordinates, the three-dimensional delta function can be expressed as

$\delta(\vec{r} - \vec{r}') = \frac{1}{r^2} \delta(r - r') \delta(\varphi - \varphi') \delta(\cos \theta - \cos \theta')$ for $r' \neq 0$ (Jackson 1998), and $\delta(\vec{r}) = \frac{\delta(r)}{2\pi r^2}$ when $r' = 0$ (Barrett and Myers 2004). Since $r \in [0, +\infty)$, we have $\int_0^{+\infty} \delta(r) dr = \frac{1}{2} \int_{-\infty}^{+\infty} \delta(r) dr = \frac{1}{2}$. Others may use $\delta(\vec{r}) = \frac{\delta(r)}{4\pi r^2}$, where $\delta(r)$ should be considered as $\delta(r - 0^+)$, resulting in $\int_0^{+\infty} \delta(r - 0^+) dr = 1$.

3.2.1. Parallel Line source—The parallel line source is located on the xy plane which is parallel to the pinhole aperture plane whose normal direction is along the z -axis. Without loss of generality, let the line source lie on the x -axis with its length denoted by $2R$. The two endpoints of the source are located at $(\pm R, 0, 0)$. With the help of delta functions, the source density can be written as

$$\rho(\vec{r}) = \frac{1}{2Rr^2} \mathbf{1}_{[0,R]}(r) [\delta(\varphi) + \delta(\varphi - \pi)] \delta(\cos\theta). \quad (14)$$

Here, $\mathbf{1}_{[0,R]}(r)$ is an indicator function that is 1 in $[0, R]$ and 0 outside. Putting equation (14) into equation (6), $T_{\ell m}$ becomes

$$T_{\ell m} = \frac{R^\ell}{2(\ell+1)} P_\ell^m(0)(1 + e^{-im\pi}). \quad (15)$$

Hence the mean sensitivity for a parallel line source becomes

$$G_{\parallel}(\vec{r}_p) = \frac{d^2}{32} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{(\ell+1-m)!}{(\ell+1)(\ell+m)!} \frac{R^\ell}{r_p^{\ell+2}} P_\ell^m(0) P_{\ell+1}^m(\cos\theta_p) (1 + e^{-im\pi}) e^{im\varphi_p}. \quad (16)$$

Through Appendix B, equation (16) can be further simplified as

$$G_{\parallel}(\vec{r}_p) = \frac{d^2}{32} \frac{b}{R(b^2 + y_p^2)} \left(\frac{R - x_p}{\sqrt{R^2 - 2Rx_p + r_p^2}} + \frac{R + x_p}{\sqrt{R^2 + 2Rx_p + r_p^2}} \right). \quad (17)$$

When $\theta_p = 0$, equation (17) simply becomes

$$G_{\parallel}(\vec{r}_p) = \frac{d^2}{16} \frac{1}{b_{\text{eff}}^2} = \frac{d^2}{16} \frac{1}{b \sqrt{b^2 + R^2}}. \quad (18)$$

Here the effective distance $b_{\text{eff}} = \sqrt{b \cdot \sqrt{b^2 + R^2}}$ is the geometric mean of the two distances from the center of the pinhole to the center and one of the endpoints of the line source, respectively.

3.2.2. Perpendicular Line source—We assume the line source is located on the z -axis with endpoints at $\pm R\hat{e}_z$. Then the activity distribution can be written as

$$\rho(\vec{r}) = \frac{1}{2R} \frac{1}{2\pi r^2} \mathbf{1}_{[0,R]}(r) [\delta(\cos\theta + 1) + \delta(\cos\theta - 1)]. \quad (19)$$

One can easily verify that $\iiint_{-\infty}^{+\infty} \rho(\vec{r}) d\vec{r} = 1$. Putting equation (19) into equation (6), the factor $T_{\ell m}$ for the perpendicular line source becomes

$$T_{\ell m} = \frac{R^\ell}{\ell+1} \frac{1 + (-1)^\ell}{2} \delta_m. \quad (20)$$

And the sensitivity formula is

$$G_{\perp}(\vec{r}_p) = \frac{d^2}{32} \sum_{\ell=0}^{\infty} \frac{R^{\ell}}{r_p^{\ell+2}} [1 + (-1)^{\ell}] P_{\ell+1}(\cos\theta_p) = \frac{d^2}{16} \sum_{n=0}^{\infty} \frac{R^{2n}}{r_p^{2n+2}} P_{2n+1}(\cos\theta_p). \quad (21)$$

By splitting equation (21), the sensitivity formula can be rewritten as

$$\begin{aligned} G_{\perp}(\vec{r}_p) &= \frac{d^2}{16} \frac{1}{2R} \left[\sum_{\ell=0}^{\infty} \frac{R^{\ell+1}}{r_p^{\ell+2}} P_{\ell+1}(\cos\theta_p) - \sum_{\ell=0}^{\infty} \frac{(-R)^{\ell+1}}{r_p^{\ell+2}} P_{\ell+1}(\cos\theta_p) \right] \\ &= \frac{d^2}{16} \frac{1}{2R} \left(\frac{1}{|\vec{r}_p - R\hat{e}_z|} - \frac{1}{|\vec{r}_p + R\hat{e}_z|} \right) = \frac{d^2}{16} \frac{1}{2R} \left(\frac{1}{\sqrt{x_p^2 + y_p^2 + (b-R)^2}} - \frac{1}{\sqrt{x_p^2 + y_p^2 + (b+R)^2}} \right). \end{aligned} \quad (22)$$

When $\theta_p = 0$, equation (22) simply becomes

$$G_{\perp}(\vec{r}_p) = \frac{d^2}{16} \frac{1}{b_{\text{eff}}^2} = \frac{d^2}{16} \frac{1}{b^2 - R^2}. \quad (23)$$

Here the effective distance $b_{\text{eff}} = \sqrt{b^2 - R^2}$ is the geometric mean of the two distances from the center of the pinhole to the two endpoints of the line segment.

3.3. Disk source models

The disk sources addressed here also can have two different orientations, i.e., the disk sources can lie in a plane parallel or perpendicular to the pinhole aperture plane. We refer to these two scenarios as the parallel and perpendicular disk source models.

3.3.1. Parallel disk source—We assume a disk source of radius R located on the xy plane and parallel to the pinhole aperture plane. Again with the help of a delta function, the source density can be written as

$$\rho(\vec{r}) = \frac{1}{\pi R^2} \frac{1}{r} \mathbf{1}_{[0,R]}(r) \delta(\cos\theta). \quad (24)$$

One can easily calculate that $T_{\ell m} = 2R^{\ell} P_{\ell}(0) \delta_{m0} / (\ell + 2)$. So the sensitivity is

$$G_{\parallel}(\vec{r}_p) = \frac{d^2}{16} \sum_{\ell=0}^{\infty} \frac{2R^{\ell}}{r_p^{\ell+2}} \frac{\ell+1}{\ell+2} P_{\ell}(0) P_{\ell+1}(\cos\theta_p) = -\frac{d^2}{8R^2} \sum_{n=1}^{\infty} \frac{R^{2n}}{r_p^{2n}} P_{2n}(0) P_{2n-1}(\cos\theta_p) = -\frac{d^2}{8R^2} \sum_{n=1}^{\infty} \frac{(-1)^n (2n-1)}{(2n)} \frac{R^{2n}}{r_p^{2n}} P_{2n-1}(\cos\theta_p). \quad (25)$$

Here we used the fact that $P_{2n+1}(0) = 0$, $P_{2n}(0) = \frac{(-1)^n (2n-1)}{(2n)}$ and $(\ell+2) P_{\ell+2}(0) = -(\ell+1) P_{\ell}(0)$. When $R/r_p \ll 1$, the sensitivity formula can be approximated by

$$G_{\parallel}(\vec{r}_p) \approx \frac{d^2}{16} \frac{\cos\theta_p}{r_p^2} \left[1 - \frac{3}{8} \frac{R^2}{r_p^2} (5\cos^2\theta_p - 3) \right]. \quad (26)$$

When $\theta_p = 0$, using the fact that $P_{\ell}(1) = 1$, equation (25) is simplified as

$$G_{\parallel}(b) = -\frac{d^2}{16} \frac{2}{R^2} \sum_{n=1}^{\infty} P_{2n}(0) \frac{R^{2n}}{r_p^{2n}} = \frac{d^2}{16} \frac{2}{R^2} \left[1 - \sum_{n=0}^{\infty} \binom{-1/2}{n} \frac{R^{2n}}{r_p^{2n}} \right] = \frac{d^2}{16} \frac{2}{R^2} \left[1 - \frac{b}{\sqrt{b^2 + R^2}} \right] = \frac{d^2}{16} \frac{1}{b_{\text{eff}}^2}. \quad (27)$$

Here, $\binom{-1/2}{n}$ is a binomial coefficient, and one can easily verify that $\binom{-1/2}{n} = P_{2n}(0)$ (Graham et al. 1994). The effective distance b_{eff} is given by

$$b_{\text{eff}}^2 = \sqrt{R^2 + b^2} \frac{\sqrt{R^2 + b^2} + b}{2}. \quad (28)$$

Obviously, the effective distance b_{eff} is the geometric mean of $\sqrt{R^2 + b^2}$ and arithmetic mean $(\sqrt{R^2 + b^2} + b)/2$.

3.3.2. Perpendicular disk source—The perpendicular disk source is located on a plane perpendicular to the pinhole aperture plane and has a disk shape of radius R . Without loss of generality, the perpendicular disk source is assumed to be located at the xz plane. Then the distribution of a perpendicular disk shaped source is

$$\rho(\vec{r}) = \frac{1}{\pi R^2} \frac{1}{r \sin \theta} \mathbf{1}_{[0,R]}(r) [\delta(\varphi) + \delta(\varphi - \pi)]. \quad (29)$$

One can calculate that the $T_{\ell m}$ is

$$T_{\ell m} = \frac{1}{\pi} \frac{R^\ell}{\ell + 2} (1 + e^{-im\pi}) \int_{-1}^1 \frac{1}{\sqrt{1-x^2}} P_\ell^m(x) dx. \quad (30)$$

One can easily see that $T_{\ell, 2k+1} = 0$ when $m = 2k + 1$. We also have $T_{2n+1, 2k} = 0$ when $\ell = 2n + 1$ and $m = 2k$ since $P_{2n+1}^{2k}(x)$ is an odd function. The integration in equation (30) can be carried out using the identity in (Gradshteyn and Ryzhik 2007):

$$\int_{-1}^1 \frac{1}{\sqrt{1-x^2}} P_{2n}^{2k}(x) dx = \pi \frac{(2n+2k-1)(2n-1)}{(2n-2k)(2n)}. \quad (31)$$

Putting equations (30) and (31) into equation (5), the sensitivity formula for the perpendicular disk source becomes

$$G_\perp(\vec{r}_p) = \frac{d^2}{8} \sum_{n=0}^{\infty} \sum_{k=-n}^n \frac{(2n-2k+1)(2n-1)}{(2n+2k)(2n+2)} \frac{R^{2n}}{r_p^{2n+2}} P_{2n+1}^{2k}(\cos \theta_p) e^{i2k\varphi_p}. \quad (32)$$

Using the property of associated Legendre functions that $P_{2n+1}^{-2k} = \frac{(2n+1-2k)!}{(2n+1+2k)!} P_{2n+1}^{2k}$, we can rewrite equation (32) as

$$G_\perp(\vec{r}_p) = \frac{d^2}{8} \sum_{n=0}^{\infty} \frac{(2n+1)(2n-1)}{(2n)(2n+2)} \frac{R^{2n}}{r_p^{2n+2}} P_{2n+1}(\cos \theta_p) + \frac{d^2}{4} \sum_{n=1}^{\infty} \sum_{k=1}^n \frac{(2n-2k+1)(2n-1)}{(2n+2k)(2n+2)} \frac{R^{2n}}{r_p^{2n+2}} P_{2n+1}^{2k}(\cos \theta_p) \cos 2k\varphi_p. \quad (33)$$

When $R/R_p \ll 1$, equation (33) can be approximated by

$$G_\perp(\vec{r}_p) \approx \frac{d^2}{16} \frac{\cos \theta_p}{r_p^2} \left[1 + \frac{3}{16} \frac{R^2}{r_p^2} (5 \cos^2 \theta_p - 3 + 5 \sin^2 \theta_p \cos 2\phi_p) \right]. \quad (34)$$

When $\theta_p = 0$, equation (33) can be simplified as

$$G_{\perp}(b) = \frac{d^2}{8} \sum_{n=0}^{\infty} \frac{(2n+1)(2n-1)}{(2n)(2n+2)} \frac{R^{2n}}{b^{2n+2}}. \quad (35)$$

By comparing equation (35) with a hypergeometric function ${}_2F_1(a, b; c; z)$ whose series expansion is

$${}_2F_1(a, b; c; z) = 1 + \frac{ab}{c} \frac{z}{1!} + \frac{a(a+1)b(b+1)}{c(c+1)} \frac{z^2}{2!} + \dots, \quad (36)$$

we can easily see that

$$G_{\perp}(b) = \frac{d^2}{16} \frac{1}{b^2} {}_2F_1(3/2, 1/2; 2; R^2/b^2). \quad (37)$$

Here ${}_2F_1(3/2, 1/2; 2; R^2/b^2)$ can be considered as a correction factor. This factor equals 1 when $R = 0$, and equals 1.743 when $R = 0.9b$. For a disk shape, the effective distance is b divided by the square root of this correction factor.

3.4. Sphere source model

The distribution of a sphere shaped source of radius R is

$$\rho(\vec{r}) = \frac{3}{4\pi R^3} \mathbf{1}_{[0,R]}(r). \quad (38)$$

The factor T_{lm} for a sphere source in equation (6) becomes $\delta_{l0}\delta_{m0}$. The sensitivity formula for a sphere source is

$$G(\vec{r}_p) = \frac{d^2}{16} \frac{P_1(\cos\theta_p)}{r_p^2} = \frac{d^2}{16} \frac{\cos\theta_p}{r_p^2}. \quad (39)$$

The sensitivity formulae for a sphere and a point source are exactly the same when attenuation is not taken into account. The sphere source model is equivalent to a point source located at its center.

When attenuation is considered, the special case for a sphere source model with uniform attenuation and a pinhole located at $\vec{r}_p = b\hat{e}_z$ is treated here. For this special case, the equation (9) becomes

$$G(\vec{r}_p) = \frac{d^2}{16} \frac{3}{4\pi R^3} \iiint_{-\infty}^{+\infty} \frac{b - r\cos\theta}{(b^2 + r^2 - 2br\cos\theta)^{3/2}} \mathbf{1}_{[0,R]}(r) \exp[-\mu L(\vec{r}, \vec{r}_p)] d\vec{r}. \quad (40)$$

Here, $L(\vec{r}, \vec{r}_p)$ is given by equation (11). The sensitivity formula (40) is still arduous to calculate. By taking the Taylor series of $\exp[-\mu L(\vec{r}, \vec{r}_p)]$, an approximate analytic sensitivity formula for a sphere source with uniform attenuation can be given by

$$G(\vec{r}_p) \approx \frac{d^2}{16} \frac{1}{b^2} \left(1 - \frac{3\mu R}{4} + \frac{2\mu^2 R^2}{5} - \frac{\mu^3 R^3}{6} + \frac{2\mu^4 R^4}{35} \right). \quad (41)$$

Equation (41) can be rewritten into an exponential function as

$$G(\vec{r}_p) \approx \frac{d^2}{16} \frac{1}{b^2} \exp \left[-\frac{3\mu R}{4} \left(1 - \frac{19\mu R}{120} + \frac{7\mu^2 R^2}{720} \right) \right]. \quad (42)$$

When $\mu R \ll 1$, we can see from equation (42) that the average length of the intersection path of emitted gamma-rays with a sphere source is $3R/4$.

4. Simulations

To verify dependence on the shape of different source models, the predictions of the formulae derived above were compared to results from Monte Carlo (MC) simulations. Since the formulae only predict the geometric sensitivity, a simple code was developed to simulate total absorption in a pinhole assembly. The code first randomly picks a point inside the predetermined source volume, then a random ray (direction) was picked from an isotropic emission. The method of “picking a random point on the sphere,” (Press et al. 2007), was used to pick a ray from an isotropic emission. Intersections of the emitted photon with the pinhole aperture plane are calculated. A photon is considered to pass the collimator if its path is within the pinhole aperture. The average sensitivity of the the source model is then calculated as the fraction of passed to emitted photons. For the sphere source model with attenuation, the intersection path length for each ray was calculated by solving a quadratic equation numerically. The corresponding ray was then randomly picked based on transmittance. Penetration and scatter were not considered initially.

For the geometric sensitivity, we provide MC simulations for several simple cases. We assume the distance from the center of source to the pinhole aperture is $b = 12$ cm. The pinhole is assumed ideal and has a geometric diameter of 6 mm. We simulated the sensitivities for line, disk and sphere sources with sizes from 0 to $0.9b$ in $0.05b$ steps and calculated the sensitivities which are normalized to the sensitivity of a point source. We choose a relatively large sample size $N = 2 \times 10^8$ for the MC simulation since the error decreases asymptotically as $1/\sqrt{N}$. The results of MC simulations and analytic predictions for line, disk and sphere sources are given in figures 2, 3 and 4. The relative deviation between the simulations and analytic predictions are less than 1.6%. A sphere source with attenuation was also simulated. The linear attenuation coefficient is assumed to be 0.154 cm^{-1} which is the appropriate for water (with $^{99\text{m}}\text{Tc}$) at 140 keV. The results of MC simulation and analytic prediction for a sphere source with attenuation are given in figure 4. The relative deviation is less than 2%. The error is dependent on the product of the sphere source diameter and its linear attenuation coefficient. Thus, the geometric sensitivity prediction using equation (42) is applicable to a sphere source of water with diameter under 21.6 cm and is accurate to within 2%.

When taking penetration and attenuation effects of the edge pinhole aperture material into account, an effective pinhole diameter d_{eff} instead of d should be used to calculate the total sensitivities. The total sensitivity of the pinhole collimator is the sum of the geometric and penetrative sensitivities. The determination of the effective diameter of the pinhole collimator is a complex problem, and no attempt has been made here to give a rigorous solution. Metzler *et al* have derived an incident-angle-dependent effective diameter for point source model (Metzler et al. 2001). As an approximation, the effective diameter can be used to predict the total sensitivities for extended source models. As expected, the prediction becomes more accurate when the source volume becomes small and approaches a point source. The effective diameter d_{eff} as a function of incident angle β is given by

$$d_{\text{eff}}(\beta) = \left[d^2 + \left(1 - \frac{\cot^2 \beta}{\tan^2 \frac{\alpha}{2}} \right)^{1/2} \left(\frac{2d \sin \beta \tan \frac{\alpha}{2}}{\mu} + \frac{2 \sin^2 \beta \tan^2 \frac{\alpha}{2}}{\mu^2} - \frac{2 \cos^2 \beta}{\mu^2} \right) \right]^{1/2}, \quad (43)$$

where α is the acceptance angle of the pinhole collimator (Metzler et al. 2001).

To simulate penetration and scatter effects, more realistic pinhole simulations were performed using the GATE (Geant4 Application for Tomographic Emission) code (Jan et al. 2004). The pinhole insert was knife-edge shaped with a geometric diameter of 6 mm, and made of lead (^{82}Pb). The acceptance angle of the pinhole inserts was 90° . The detector crystal was made of NaI(Tl) with a thickness of 2 cm. Spherical source models of 30 kBq of ^{99m}Tc , with diameters of 0.2, 4, 8, 12 and 16 cm filled with both air and water were simulated separately. The simulated acquisition was 10 minutes in duration, and approximately 1.8×10^7 decays occur during the simulated acquisition. The low energy electromagnetic process package was used to model the physics, and perfect readout was assumed. The scattered events were cut by setting an energy window. The total sensitivities, defined by the count rate per unit source activity (cps/MBq), were calculated for the cases of 'in air' and 'in water'. Due to the limited number of events simulated, the Poisson fluctuations were between 1.8% and 2.6%. The linear attenuation coefficient of lead at 140 keV is assumed to be $\mu = 27.2 \text{ cm}^{-1}$ (Metzler et al. 2001); the calculated effective diameter of the pinhole insert was 6.38 mm.

The comparison of GATE simulations and analytic predictions of the total sensitivities for the sphere sources 'in air' and 'in water' are shown in figure 5. Our analytic predictions of the total sensitivities quantitatively agree with GATE simulations for the sphere sources with diameters smaller than 12 cm, and have slightly larger deviation for the sphere source of 16 cm diameter. The approximation of the effective diameters for large spheres becomes inaccurate. Each point inside the sphere source has a different incident angle, which makes the average effective diameter of the sphere smaller than that for a point source. As expected, the analytic prediction for a large sphere source overestimates the total sensitivity. In such case, the effective diameter for a large extended source model, or an approximation, needs to be derived.

5. Conclusion

We have derived an analytic prediction for the geometric sensitivity of pinhole collimation with a general source model. To our knowledge, this is the first closed-form pinhole collimator sensitivity formula. Since any physical source distribution can be expanded in terms of the spherical harmonics, the formula can be applied to any, including non-uniform, activity source distributions. These analytical predictions extend the zeroth-order sensitivity prediction of the point source model to higher orders of a general source model. The general formula is quite complex. For source distributions that have spherical symmetries (for example, the source can be better expressed in spherical coordinates, or can be expressed by simple harmonics), the formula is simplified. We have also provided pinhole sensitivity predictions for line, disk and sphere source models. Other source models, such as a ring, hemisphere, etc., can also be derived easily.

The analytic predictions of geometric sensitivity can also be applied to tilted pinholes for which calculations of geometric sensitivities for multi-pinhole systems are necessary. The discrete object model (where the object is approximated by a linear combination of N points) and MC simulations are other approaches which can be quite accurate. For optimization of pinhole systems, these approaches become problematic because many system parameters need to be varied. The analytic formula is a complementary approach that

provides additional mathematical insights. Because of the lack of an effective-diameter formula for general sources, the prediction of the total sensitivities including the penetrative sensitivity for a large source volume is often inaccurate. The effective diameter for general sources, instead of a point source, is needed to predict the total sensitivity accurately. The predictions of the analytic formulae for geometric and total sensitivities were in excellent agreement with simple MC and GATE simulations. Because the predictions are based on closed-form expressions, they can be useful for the design and optimization of SPECT systems that utilize pinhole collimators.

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Appendix A

Derivation of Equation (3)

The vector factor in the integrand of equation (2), viewed as a function of $\vec{r}$, can be written as the gradient of the scalar $1/|\vec{r}_p - \vec{r}|$ (Jackson 1998):

$$\frac{\vec{r}_p - \vec{r}}{|\vec{r}_p - \vec{r}|^3} = \nabla \frac{1}{|\vec{r}_p - \vec{r}|}. \quad (\text{A.1})$$

The gradient operator ∇ in the spherical coordinate system is written as (Arfken and Weber 2005),

$$\nabla = \hat{e}_r \frac{\partial}{\partial r} + \hat{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \hat{e}_\phi \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi}, \quad (\text{A.2})$$

where the unit vector $\hat{e}_r$, $\hat{e}_\theta$ and $\hat{e}_\phi$ respectively are

$$\hat{e}_r = \hat{e}_x \sin \theta \cos \phi + \hat{e}_y \sin \theta \sin \phi + \hat{e}_z \cos \theta, \quad \hat{e}_\theta = \hat{e}_x \cos \theta \cos \phi + \hat{e}_y \cos \theta \sin \phi - \hat{e}_z \sin \theta = \frac{\partial \hat{e}_r}{\partial \theta}, \quad \hat{e}_\phi = -\hat{e}_x \sin \phi + \hat{e}_y \cos \phi = \frac{1}{\sin \theta} \frac{\partial \hat{e}_r}{\partial \phi}.$$

Here, $\hat{e}_x$, $\hat{e}_y$ and $\hat{e}_z$ are unit vectors along the x , y and z directions respectively. We can easily obtain

$$\hat{e}_z \cdot \hat{e}_r = \cos \theta, \quad \hat{e}_z \cdot \hat{e}_\theta = -\sin \theta, \quad \hat{e}_z \cdot \hat{e}_\phi = 0. \quad (\text{A.3})$$

Using the equations (A.1), (A.2) and (A.3), The dot product factor in the integrand of equation (2) can then be written as,

$$\frac{\hat{e}_z \cdot (\vec{r}_p - \vec{r})}{|\vec{r}_p - \vec{r}|^3} = \left(\cos \theta \frac{\partial}{\partial r} - \sin \theta \frac{1}{r} \frac{\partial}{\partial \theta} \right) \frac{1}{|\vec{r}_p - \vec{r}|} = \left(\cos \theta \frac{\partial}{\partial r} + \sin^2 \theta \frac{1}{r} \frac{\partial}{\partial \cos \theta} \right) \frac{1}{|\vec{r}_p - \vec{r}|} \quad (\text{A.4})$$

The scalar $1/|\vec{r}_p - \vec{r}|$ can also be expressed in terms of products of two spherical harmonics with angular coordinates (θ, ϕ) and (θ_p, ϕ_p) (Jackson 1998):

$$\frac{1}{|\vec{r}_p - \vec{r}|} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{4\pi}{2\ell+1} \frac{r^\ell}{r_p^{\ell+1}} Y_{\ell m}^*(\theta, \varphi) Y_{\ell m}(\theta_p, \varphi_p). \quad (\text{A.5})$$

Here $Y_{\ell m}(\theta, \varphi)$ is the spherical harmonic functions of degree ℓ and order m . They are defined as

$$Y_{\ell}^m(\theta, \varphi) = \sqrt{\frac{(2\ell+1)(\ell-m)!}{4\pi(\ell+m)!}} P_{\ell}^m(\cos\theta) e^{im\varphi}, \quad (\text{A.6})$$

where P_{ℓ}^m is an associated Legendre function given by equation (4). Putting equation (A.6) into equation (A.5), we get,

$$\frac{1}{|\vec{r}_p - \vec{r}|} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{(\ell-m)!}{(\ell+m)!} \frac{r^\ell}{r_p^{\ell+1}} P_{\ell}^m(\cos\theta) P_{\ell}^m(\cos\theta_p) e^{im(\varphi_p - \varphi)}. \quad (\text{A.7})$$

The associated Legendre function P_{ℓ}^m has the recurrence formula (Abramowitz and Stegun 1964),

$$(1-x^2)P_{\ell}^{m'}(x) = (\ell+m)P_{\ell-1}^m(x) - \ell x P_{\ell}^m(x).$$

Using the above equation with $x = \cos \theta$, we can get

$$\left(\cos\theta \frac{\partial}{\partial r} + \sin^2\theta \frac{1}{r} \frac{\partial}{\partial \cos\theta} \right) r^{\ell} P_{\ell}^m(\cos\theta) = r^{\ell-1} (\ell+m) P_{\ell-1}^m(\cos\theta). \quad (\text{A.8})$$

Putting equations (A.7) and (A.8) into equation (A.4), we then get equation (3).

Appendix B

Derivation of Equation (17)

First, we assume that $G(\vec{r}_p)$ in equation (15) is also a function of R . We define $f(R, \vec{r}_p) = G(\vec{r}_p)$, then

$$\frac{d}{dR} [R \cdot f(R, \vec{r}_p)] = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{(\ell+1-m)!}{(\ell+m)!} \frac{R^{\ell}}{r_p^{\ell+2}} P_{\ell}^m(0) P_{\ell+1}^m(\cos\theta_p) (e^{im\varphi_p} + e^{im(\varphi_p - \pi)}). \quad (\text{B.1})$$

Using equation (3), the above equation can be simplified as

$$\frac{d}{dR} [R \cdot f(R, \vec{r}_p)] = \frac{d^2}{32} \left(\frac{b}{|\vec{r}_p - R\hat{e}_x|^3} + \frac{b}{|\vec{r}_p + R\hat{e}_x|^3} \right) = \frac{d^2}{32} \left[\frac{b}{(R^2 - 2Rx_p + r_p^2)^{3/2}} + \frac{b}{(R^2 + 2Rx_p + r_p^2)^{3/2}} \right]. \quad (\text{B.2})$$

By calculating the indefinite integral of equation (B.2), we get

$$f(R, \vec{r}_p) = \frac{d^2}{32} \frac{b}{R(b^2 + y_p^2)} \left(\frac{R - x_p}{\sqrt{R^2 - 2Rx_p + r_p^2}} + \frac{R + x_p}{\sqrt{R^2 + 2Rx_p + r_p^2}} \right) + C(\vec{r}_p), \quad (\text{B.3})$$

where $C(\vec{r}_p)$ is a function of variable $\vec{r}_p$ but a constant of variable R . By expanding equation (B.3) around $R = 0$ with respect to variable R and taking the limit as $R \rightarrow 0$ we have

$$\lim_{R \rightarrow 0} f(R, \vec{r}_p) = \frac{d^2}{16} \frac{b}{r_p^3} + C(\vec{r}_p). \quad (\text{B.4})$$

Since the left-hand side of equation (B.4) is just the sensitivity for a point source, we get $C(\vec{r}_p) = 0$. Thus equation (17) is proved.

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Figure 1.
Schematic diagram of pinhole collimation geometry.

Figure 2. Dependencies of sensitivity on the normalized object size R/b for line sources. Line-I/Line-II represents the line source parallel/perpendicular to the pinhole aperture.

Figure 3. Dependencies of sensitivity on the normalized object size R/b for disk sources. Disk-I/Disk-II represents the disk source parallel/perpendicular to the pinhole aperture.

Figure 4.
Dependence of sensitivity factor on the normalized object size R/b for sphere source with/without attenuation.

Figure 5.

Comparison between GATE simulations and analytic predictions of the total sensitivities for sphere sources 'in air' and 'in water'. The relative deviation are -0.9% , 0.1% , -1.0% , -1.9% and -7.3% for the case of 'in air'; and -0.6% , 2.1% , -2.6% , 3.3% and -5.8% for the case of 'in water'.