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Crack Identification of Cantilever Plates Based on a Kriging Surrogate Model

This work presents an effective method to identify the tip locations of an internal crack in cantilever plates based on a Kriging surrogate model. Samples of varying crack parameters (tip locations) and their corresponding root mean square (RMS) of random responses are used to construct the initial Kriging surrogate model. Moreover, the pseudo excitation method (PEM) is employed to speed up the spectral analysis. For identifying crack parameters based on the constructed Kriging model, a robust stochastic particle swarm optimization (SPSO) algorithm is adopted for enhancing the global searching ability. To improve the accuracy of the surrogate model without using extensive samples, a small number of samples are first used. Then an optimal point-adding process is carried out to reduce computational cost. Numerical studies of a cantilever plate with an internal crack are performed. The effectiveness and efficiency of this method are demonstrated by the identified results. The effect of initial sampling size on the precision of the identified results is also investigated. [DOI: 10.1115/1.4023813]

Keywords: crack identification, Kriging model, cantilever plate, pseudo excitation method

Introduction

Cantilever plates appear in a wide range of structural systems in civil, ocean, and aeronautical engineering, which may be subjected to known and unknown cyclic loads. Consequently, fatigue cracks may be initiated. A crack can lead to reduced performance and shorter lifetime, and, even worse, may induce catastrophic failure of structures, which should be considered seriously in both theoretical and experimental research.

A significant number of methods for health monitoring and crack identification have been proposed during recent years [1,2]. In the early stages of the crack detection development, the frequency contour plot method was one of the most popular techniques to identify a single crack in a beam by using the first few natural frequencies. Gudmundson [3] discussed the effect of geometrical imperfection on the eigenvalue by means of perturbation analysis. Liang et al. [4] proposed a massless rotational spring model for a crack based on the Euler–Bernoulli beam theory, in which the location and size of a crack could be identified via finding the intersection point of a few frequency contour lines. Ostachowicz and Krawczuk [5] studied the forced vibration of a beam and the effects of the crack locations and sizes on the vibration behavior of the structure. Lele and Maiti [6] extended the frequency contour plot method in beams based on the Timoshenko beam theory. However, the frequency contour plot method suffers from the drawback that the curves of the frequency contour plot might not intersect because of inaccuracies in the model with respect to the measured results on practical structures.

Many efforts have been devoted to crack identification in beams. However, there are few works on crack identification for plates. In the work by Krawczuk et al. [7], special spectral elements were used to analyze waves in cracked plates. Horibe and Watanabe [8] applied a genetic algorithm to determine the crack location using finite element models. Hadjileontiadis and Douka [9] presented an effective method for detecting cracks in plate structures based on fractal dimension analysis. The location and

length of the crack were identified as the abrupt changes in the spatial variation of the 2D fractal dimension signal for plates with cracks of varying lengths at different locations. In the paper by Lam and Yin [10], cracks were modeled as a set of linear springs with varying stiffness and a two-stage method for multicrack detection was proposed using statistical analysis. One shortcoming of these methods is that they are only suitable for cracks that are parallel to the plate edge, which of course is a limitation in real world situations. In order to overcome this shortcoming, Bagheri et al. [11] investigated the use of curvelet transform to identify the location of damage. More recently, Moore et al. [12] used a Markov–Chain Monte Carlo implementation of Bayes' rule to estimate the crack parameters such as location, orientation, and size. Although this method gave a good prediction with acceptable accuracy, it required a large amount of samples and hence computation time.

In essence, crack identification is an inverse problem. Optimization methods are commonly employed to estimate the most suitable crack parameters by minimizing the objective function related to the output discrepancies such as natural frequencies, model shapes, and dynamical responses. It is desirable to construct a simple relationship between the variables and the objective or constraint functions to avoid the repeated analysis of computationally expensive FE models during the optimization process. Recently, surrogate models based on neural networks (NNs) [13,14] and the polynomial response surface (PRS) method [15,16] were investigated by many researchers. Atalla and Inman [13] employed a neural network to estimate the updated model parameters. Real and imaginary components of the frequency response are integrated over selected frequency intervals and used to provide information about the system to the network. Lee et al. [14] used mode shape differences or mode shape ratios between before and after damage as the input to the NN to reduce the effect of the modeling errors in the baseline FE model, from which the training patterns are to be generated. While the NNs-based method is successful to some extent, the required number of training samples would exponentially increase, resulting in considerable computational efforts.

In the thesis by Cundy [17], a four-step process based on PRS was developed for simple physical systems and damage identification was performed successfully, given the limited amount of

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“training” data used. Faravelli and Casciati [18] and Casciati [19] employed acceleration time histories collected under different loading conditions for identifying the presence of damage and locating cracks by a comparison of the sum of the squared errors (SSE) histograms. Moreover, a derivative of SSE, defined as ADM, was proposed and used for detecting distributed cracks in an actual masonry [20]. Huang et al. [21] stated that PRS method not only tended to show severe oscillations, but also required too many support points, which might result in limited accuracy when the response data to be modeled had multiple local extrema.

Compared with NNs and PRS, Kriging surrogate models might offer a good alternative. It provides explicit functions to represent the relationships between the inputs and outputs of a linear or non-linear system for response estimation and parameter identification. The use of Kriging models often requires only a small number of samples and can reduce significantly the computing time in obtaining the optimal parameters. Therefore, Kriging models have drawn much attention and have already been increasingly used in industrial design [22]. However, to date, little research has been carried out in applying Kriging models to crack identification in plates. Bilicz et al. [23] provides a methodology for the characterization of a 3D defect embedded in a conductive nonmagnetic plate from the measurement of the impedance variations of an air-cored pancake coil at eddy current frequencies based on Kriging interpolation using the Expected Improvement algorithm. However, the boundary of the defect model must be parallel or perpendicular to the plate, which somewhat diminishes its utility. Other adaptive metamodels (e.g., radial basis functions, RBF) for crack characterization also can be found in Refs. [24,25].

The objective of this work is to apply a Kriging model to single arbitrary crack identification for plate-type structures. Firstly a number of FE models with varying tip locations are constructed in ANSYS code to solve dynamical responses. To speed up the spectral analysis, PEM is used to guarantee accuracy and reduce computational cost. The initial Kriging model is then constructed by the samples of crack parameters and their corresponding RMS values at specified points. To estimate the actual crack parameters, the SPSO algorithm is employed, which is demonstrated to be effective, accurate, and robust for searching for a global optimal solution. After that, the current optimal solution will be used in FE analysis and inserted into the initial sample set to update the initial Kriging model, until the surrogate model is sufficiently accurate and the optimization process converges. Finally, the effectiveness and efficiency of the presented method is demonstrated by numerical studies on a cantilever plate with an internal crack in the presence of noise. The identified results show that the proposed method holds considerable promise for practical engineering problems.

Finite Element Model of A Cracked Plate

A model of an elastic, rectangular plate with an internal crack at an arbitrary position is shown in Fig. 1. The plane dimensions of the plate are a and b in the x and y directions, respectively. x_i and y_i ($i = 1, 2$) represent the true coordinates of the two crack tips in a Cartesian coordinate system. For simplicity, the nondimensional locations are used in the form of $\xi_i = x_i/a$ and $\eta_i = y_i/b$ ($i = 1, 2$). Hence, the values of crack parameter are limited in the interval (0,1).

In previous works [9,10], a crack was modeled as massless line springs with varying stiffness along the crack, which divided the plate into two segments. Then a hypothetical displacement function and the governing differential equations of the plate segments was established to estimate the crack parameters such as location and size. However, this process is likely to be awkward when the crack slants to the plate edge because of the difficulties in describing the displacement field using the displacement function. As such, finite element techniques may be a promising choice for the construction of a plate model with an arbitrary crack. In the current study, the finite element models are established with a mesh

Fig. 1 Schematic of a plate with an arbitrary internal crack

of isoparametric elements that contain a singularity. As shown in Fig. 2, the elements away from the crack tips are eight-node quadrilateral, and the elements surrounding the tips are triangular with three nodes. It should be mentioned that it is assumed that the crack remains always open without changing the stiffness and mass properties of the plate during the motion, and the contacting nonlinearity and the crack propagation process are beyond the scope of our paper.

In order to assess the accuracy of the cracked plate model, a comparative study on a completely free square plate with an internal crack is carried out. Table 1 displays the first five nondimensional frequency parameters $\omega a^2 \sqrt{\rho h/D}$ calculated by using the finite element method and Ritz method [26] in six different cases. ω is natural frequencies of the plate, h is the thickness of the plate, ρ is the mass density of the plate; $D = Eh^3/12(1-\nu^2)$ is the flexural rigidity, E is the modulus of elasticity, and ν is Poisson's ratio.

Spectral Analysis Based on Pseudo Excitation Method

Natural frequencies as an overall global characteristic of a mechanical system are not very effective for crack identification in plates. Location-based characteristics should be considered to locate the crack tips. For this reason, vibration amplitudes and impulse responses are widely used for damage detection. However, practical engineering problems are very different from simulated cases due to the presence of modeling errors and measurement noise. As observed by the investigators of Refs. [27,28], these outputs are somewhat sensitive to environmental

Fig. 2 Mixed mesh based on FE method

Table 1 Frequency parameters $\omega a^2 \sqrt{\rho h/D}$ for completely free square plate with an internal crack

Tip locations (ξ_1, η_1) (ξ_2, η_2)	Mode				
	1	2	3	4	5
(0.4,0.5) (0.6,0.5)	{13.47} 13.48	{19.40} 19.40	{23.73} 23.74	{34.77} 34.77	{34.79} 34.81
(0.4,0.75) (0.6,0.75)	{13.47} 13.48	{19.55} 19.56	{24.06} 24.08	{34.75} 34.76	{34.80} 34.81
(0.4134,0.4500) (0.5866,0.5500)	{13.40} 13.41	{19.55} 19.54	{23.73} 23.73	{34.79} 34.79	{34.80} 34.81
(0.2402,0.3500) (0.7598,0.6500)	{12.54} 12.54	{18.66} 18.67	{21.29} 21.31	{33.31} 33.32	{34.74} 34.76
(0.4293,0.4293) (0.5707,0.5707)	{13.38} 13.40	{19.60} 19.60	{23.73} 23.74	{34.80} 34.82	{34.80} 34.83
(0.2879,0.2879) (0.7121,0.7121)	{12.35} 12.36	{19.56} 19.57	{20.86} 20.88	{33.89} 33.90	{34.78} 34.80

Note: { } results from Huang et al. [26] by the Ritz method.

changes and noise interference, which may lead to unreliable results of crack localization. Considering this, some researchers used stochastic response analysis for damage detection and system identification [29–31]. Liberatore and Carman [32] suggested that the RMS values of the power spectral density (PSD) could be adopted to identify damage in beams and give satisfactory predictions even for noisy cases. Inspired by their work, the RMS values at specified points are used in this work due to its strong capability in describing local features and against noise.

After the cracked plate models with varying tip locations are established, excitation and measurement points are located as shown in Fig. 3. The plate is excited by a simulated white-noise force spectrum with a constant amplitude. With four measurement points and four excitations points, 16 acceleration spectra are calculated for each damaged case.

To speed up the spectral analysis, PEM, known as the fast complete quadratic combination (fast-CQC method), is used to reduce the computational cost, whose basic concept was proposed by Lin [33]. Here, a brief description of the fundamental theory for stationary single excitation problems is given. For a linear system subjected to a zero-mean-valued stationary single excitation whose PSD matrix can be decomposed into $S_{ff}(\omega) = \gamma^* \gamma^T$, a response

vector $\mathbf{y} = \mathbf{H} \gamma e^{i\omega t}$ can be generated by the pseudo harmonic excitation $\mathbf{f} = \gamma e^{i\omega t}$. A structure subjected to a single random excitation is considered, whose equation of motion is in the form of

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{C}\dot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{A}\mathbf{u}(t) \quad (1)$$

where $\mathbf{u}(t)$ is a random process; $\mathbf{A}$ is a given constant, $\mathbf{y}$ is the displacement vector, $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ denote mass, damping, and stiffness matrices, respectively. The essence of PEM is to replace $\mathbf{u}(t)$ with the pseudo harmonic excitation $\mathbf{f}$

$$\mathbf{f} = \mathbf{E} \sqrt{S_{ff}(\omega)} e^{i\omega t}$$

where $\mathbf{E}$ is defined as a column vector consisted of 0 and 1 corresponding to the excitation node. Thus, the random spectral analysis can be replaced with a harmonic analysis whose stationary solution is

$$\mathbf{y}(t) = \mathbf{y}_\omega(\omega) e^{i\omega t} \quad (2)$$

Using its first n_q modes for mode-superposition, $\mathbf{y}_\omega$ can be written as

$$\mathbf{y}_\omega = \sum_{j=1}^{n_q} \mathbf{A} \boldsymbol{\varphi}_j^T \mathbf{h}_j \boldsymbol{\varphi}_j \gamma \quad (3)$$

where $\boldsymbol{\varphi}_j$ is the j th mode; $\mathbf{h}_j$ is a vector given by the j th column in matrix of frequency response function $\mathbf{H}$. The acceleration PSD matrix at each frequency point would be

$$\mathbf{S}_{\ddot{\mathbf{y}}\ddot{\mathbf{y}}}(\omega) = \omega^4 \mathbf{y}_\omega^* \mathbf{y}_\omega^T \quad (4)$$

The area under the acceleration PSD curve is the mean square (MS) value. Its square root (RMS) will be used to identify the crack parameters.

Construction of the Kriging Surrogate Model

Varying tip locations can result in different FE meshes. To calculate the corresponding RMS values based on FE models, remeshing should be made at every iterative step of optimization. This is awkward for complex problems that need a great deal of iterative steps. Therefore it is desirable to construct a simple relationship between crack parameters and the associated RMS values before optimization for avoiding the expensive FE analysis.

The Kriging surrogate model is a statistics-based interpolation method [34,35]. It can be postulated as a combination of polynomials and stochastic processing. When n samples of crack parameters and the corresponding RMS values are given

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}^1 \\ \mathbf{x}^2 \\ \mathbf{x}^3 \\ \vdots \\ \mathbf{x}^n \end{bmatrix} = \begin{bmatrix} \xi_1^1 & \xi_2^1 & \eta_1^1 & \eta_2^1 \\ \xi_1^2 & \xi_2^2 & \eta_1^2 & \eta_2^2 \\ \xi_1^3 & \xi_2^3 & \eta_1^3 & \eta_2^3 \\ \vdots & \vdots & \vdots & \vdots \\ \xi_1^n & \xi_2^n & \eta_1^n & \eta_2^n \end{bmatrix} \quad (5)$$

$$\mathbf{RMS} = \begin{bmatrix} \mathbf{rms}^1 \\ \mathbf{rms}^2 \\ \mathbf{rms}^3 \\ \vdots \\ \mathbf{rms}^n \end{bmatrix} = \begin{bmatrix} \mathbf{rms}_1(\mathbf{x}^1) & \mathbf{rms}_2(\mathbf{x}^1) & \mathbf{rms}_3(\mathbf{x}^1) & \dots & \mathbf{rms}_q(\mathbf{x}^1) \\ \mathbf{rms}_1(\mathbf{x}^2) & \mathbf{rms}_2(\mathbf{x}^2) & \mathbf{rms}_3(\mathbf{x}^2) & \dots & \mathbf{rms}_q(\mathbf{x}^2) \\ \mathbf{rms}_1(\mathbf{x}^3) & \mathbf{rms}_2(\mathbf{x}^3) & \mathbf{rms}_3(\mathbf{x}^3) & \dots & \mathbf{rms}_q(\mathbf{x}^3) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{rms}_1(\mathbf{x}^n) & \mathbf{rms}_2(\mathbf{x}^n) & \mathbf{rms}_3(\mathbf{x}^n) & \dots & \mathbf{rms}_q(\mathbf{x}^n) \end{bmatrix} \quad (6)$$

their relationship can be written as a model function

Fig. 3 Position of excitations (solid dot) and measurements (hollow dot)

$$\text{rms}_l(\mathbf{x}^i) = \mathbf{f}^T(\mathbf{x}^i)\boldsymbol{\beta}_l + z_l(\mathbf{x}^i) \quad i = 1, 2, \dots, n \quad l = 1, 2, \dots, q \quad (7)$$

where $\mathbf{x}^i = \{\xi_1^i, \xi_2^i, \eta_1^i, \eta_2^i\}^T$ is the i th set of sampling crack parameters expressed as a four-dimension variable vector; $\text{rms}_l(\mathbf{x}^i)$ is the l th component of the output RMS vector at the assigned measurement points on the cracked plate, $\mathbf{f}(\mathbf{x}^i)$ is a vector of a linear combination of p chosen functions, $\boldsymbol{\beta}_l$ is a $p \times 1$ vector given by the l th column in matrix of regression coefficients, q is the number of dimensions of the predicted RMS vector, and $z_l(\mathbf{x}^i)$ denotes a model of Gaussian and stationary stochastic process with a mean of zero and a variance of σ_l^2 . The covariance matrix between two given samples $\mathbf{x}^i$ and $\mathbf{x}^j$ is expressed by

$$\text{Cov}[z_l(\mathbf{x}^i), z_l(\mathbf{x}^j)] = \sigma_l^2 \mathbf{R}(\theta, \mathbf{x}^i, \mathbf{x}^j) \quad i, j = 1, 2, \dots, n \quad l = 1, 2, \dots, q \quad (8)$$

The above spatial $n \times n$ correlation function matrix $\mathbf{R}$ can be formed by Gaussian correlation function with only a single correlation parameter θ , given by

$$R_{i,j}(\theta, \mathbf{x}^i, \mathbf{x}^j) = \exp(-\theta \|\mathbf{x}^i - \mathbf{x}^j\|) \quad i, j = 1, 2, \dots, n \quad (9)$$

The RMS values at an untested sample $\mathbf{x}^*$ can be approximately estimated as a linear combination of the existing RMS values

$$\text{rms}_l^*(\mathbf{x}^*) = \mathbf{c}^T \mathbf{y} \text{rms}_l \quad l = 1, 2, \dots, q \quad (10)$$

where $\mathbf{c}$ is a $n \times 1$ coefficient vector and $\mathbf{y} \text{rms}_l$ is a $n \times 1$ vector given by the l th column in sample matrix $\mathbf{RMS}$. The vector of correlations between initial samples $\mathbf{x}^i$ and the new sample $\mathbf{x}^*$ can be written as

$$\mathbf{r}(\mathbf{x}) = [R(\theta, \mathbf{x}^1, \mathbf{x}^*), R(\theta, \mathbf{x}^2, \mathbf{x}^*), \dots, R(\theta, \mathbf{x}^n, \mathbf{x}^*)]^T \quad (11)$$

The predicted error by using Eq. (10) is

$$\begin{aligned} \text{rms}_l^*(\mathbf{x}^*) - \text{rms}_l(\mathbf{x}^*) &= \mathbf{c}^T \mathbf{y} \text{rms}_l - \text{rms}_l(\mathbf{x}^*) \\ &= \mathbf{c}^T (\mathbf{F} \boldsymbol{\beta}_l - \mathbf{z}_l) - (\mathbf{f}^T(\mathbf{x}^*) \boldsymbol{\beta}_l + z_l(\mathbf{x}^*)) \\ &= \mathbf{c}^T \mathbf{z}_l - z_l(\mathbf{x}^*) + (\mathbf{F}^T \mathbf{c} - \mathbf{f}^T(\mathbf{x}^*))^T \boldsymbol{\beta}_l \end{aligned} \quad (12)$$

in which

$$\mathbf{F} = \begin{bmatrix} f_1(\mathbf{x}^1) & \dots & f_p(\mathbf{x}^1) \\ \vdots & \ddots & \vdots \\ f_1(\mathbf{x}^n) & \dots & f_p(\mathbf{x}^n) \end{bmatrix} \quad (13)$$

$$\mathbf{z}_l = [z_l(\mathbf{x}^1), z_l(\mathbf{x}^2), \dots, z_l(\mathbf{x}^n)]^T \quad (14)$$

To make the Kriging predictor unbiased for $\mathbf{x}^*$, it is required that

$$\mathbf{F}^T \mathbf{c} - \mathbf{f}(\mathbf{x}^*) = \mathbf{0} \quad (15)$$

The mean squared error (MSE) of the predictor can be derived by using Eq. (8) and Eq. (11)

$$\begin{aligned} \varepsilon_l(\mathbf{x}^*) &= E[(\text{rms}_l^*(\mathbf{x}^*) - \text{rms}_l(\mathbf{x}^*))^2] \\ &= E[(\mathbf{c}^T \mathbf{z}_l - z_l(\mathbf{x}^*))^2] = \sigma_l^2 (1 + \mathbf{c}^T \mathbf{z}_l \mathbf{z}_l^T \mathbf{c} - 2\mathbf{c}^T \mathbf{z}_l z_l(\mathbf{x}^*)) \\ &= \sigma_l^2 (1 + \mathbf{c}^T \mathbf{R} \mathbf{c} - 2\mathbf{c}^T \mathbf{r}) \end{aligned} \quad (16)$$

Introducing Lagrange multiplier for minimizing the MSE with the constraint of Eq. (15)

$$L(\mathbf{c}, \boldsymbol{\lambda}) = \sigma_l^2 (1 + \mathbf{c}^T \mathbf{R} \mathbf{c} - 2\mathbf{c}^T \mathbf{r}) - \boldsymbol{\lambda}^T (\mathbf{F}^T \mathbf{c} - \mathbf{f}(\mathbf{x}^*)) \quad (17)$$

The gradient of the above Lagrange function with respect to $\mathbf{c}$ is

$$L'_c(\mathbf{c}, \boldsymbol{\lambda}) = 2\sigma_l^2 (\mathbf{R} \mathbf{c} - \mathbf{r}) - \mathbf{F} \boldsymbol{\lambda} \quad (18)$$

The coefficient vector $\mathbf{c}$ can be obtained from the first order necessary conditions for optimality

$$\mathbf{c} = \mathbf{R}^{-1} (\mathbf{r} - \mathbf{F} (\mathbf{F}^T \mathbf{R}^{-1} \mathbf{F})^{-1} (\mathbf{F}^T \mathbf{R}^{-1} \mathbf{r} - \mathbf{f}(\mathbf{x}^*))) \quad (19)$$

Substituting the above equation into Eq. (10) gives

$$\text{rms}_l^*(\mathbf{x}^*) = \mathbf{R}^{-T} (\mathbf{r} - \mathbf{F} (\mathbf{F}^T \mathbf{R}^{-1} \mathbf{F})^{-1} (\mathbf{F}^T \mathbf{R}^{-1} \mathbf{r} - \mathbf{f}(\mathbf{x}^*)))^T \mathbf{y} \text{rms}_l \quad l = 1, 2, \dots, q \quad (20)$$

Thus, the relation between crack parameters and the corresponding RMS values has been deduced in place of a full dynamic model. This surrogate model as a black box can be used to estimate the unknown information and tendency around the samples, and reflect global and local statistical properties, which can also be obtained via maximum likelihood estimation [21,22,36]. When the initial surrogate model is constructed, its quality can be assessed from the accuracy of predictions. The squared multiple correlations (SMC) and the empirical integrated squared error (EISE) criterion [16] are used in this work.

$$\text{SMC} = 1 - \frac{\sum_{l=1}^q [\text{rms}_l^*(\mathbf{x}^*) - \text{rms}_l^{(\text{FE})}(\mathbf{x}^*)]^2}{\sum_{l=1}^q [\text{rms}_l^{(\text{FE})}(\mathbf{x}^*) - \text{rms}^{(\text{Average})}(\mathbf{x}^*)]^2} \quad (21)$$

$$\text{EISE} = \frac{1}{q} \sum_{l=1}^q [\text{rms}_l^*(\mathbf{x}^*) - \text{rms}_l^{(\text{FE})}(\mathbf{x}^*)]^2 \quad (22)$$

where rms_l^* and $\text{rms}_l^{(\text{FE})}$ are the l th component of the RMS vector of surrogate model and the true value calculated via FE analysis, respectively; $\text{rms}^{(\text{Average})}$ is the mean of all true values.

Crack Identification Based on Kriging Model

The initial Kriging model is established by using samples of varying crack parameters $\mathbf{X}$ and their corresponding RMS. The crack identification problem based on the constructed Kriging model can be stated as follows:

$$\begin{cases} \text{find } \mathbf{x}^* \\ \min \mathbf{w}^T (\mathbf{rms}^*(\mathbf{x}^*) - \mathbf{rms}^{(\text{Target})}) = \sum_{l=1}^q w_l (\text{rms}_l^*(\mathbf{x}^*) - \text{rms}_l^{(\text{Target})}) \\ \text{subject to } \mathbf{lb} < \mathbf{x}^* < \mathbf{ub} \end{cases} \quad (23)$$

where $\mathbf{rms}^*$ is a vector of the predicted root mean square values at a new set of crack parameters $\mathbf{x}^*$ related to the locations of crack tips, and $\mathbf{rms}^{(\text{Target})}$ represents the given ones which may be measured on the real cracked plate. $\mathbf{lb}$ and $\mathbf{ub}$ denote the lower and upper bounds. $\mathbf{w}$ is the weighting imposed on the components of $\mathbf{rms}^*$. In this study, all its element are set to 1.

To search for the global optimal solution effectively, the SPSO algorithm [37] is employed for crack identification based on the initial Kriging model. When the weighting is set to 0, the simple updating procedure of the traditional PSO can be expressed as

$$S_i(t+1) = S_i(t) + c_1 r_1 [P_i(t) - S_i(t)] + c_2 r_2 [P_g(t) - S_i(t)] \quad (24)$$

where c_1 and c_2 are two positive constants called acceleration coefficients, P_i and P_g are local and global best locations, respectively, and r_1 and r_2 are random numbers in the interval (0,1). If

Fig. 4 Flow chart of crack identification process

$P_g = P_i$, $S_i(t+1) = S_i(t)$. As such, the particle at the global best position will stop evolution. To improve the global searching ability, this algorithm randomly generates an extra particle labeled j with position S_j to continue evolution in the search domain. It means that at least one particle is generated in the searching domain randomly to improve the global searching ability and final searching quality of PSO algorithm.

When the optimal solution from the initial Kriging model is obtained, the FE analysis should be carried out by using these parameters to assess the accuracy of the initial Kriging model. Owing to the small number of samples, the initial surrogate model should be updated. The optimal solution and its corresponding RMS values by FE analysis will be then inserted into the sample database. The Kriging model will be rebuilt by this point-adding process until it is sufficiently accurate according to *SMC* and *EISE* criterion. The proposed method can be organized as a surrogate model updating and optimizing estimation procedure as illustrated

Table 2 Parameters setting of the SPSO algorithm

Parameters	Values
Swarm size	15
Maximum swarm	300
c_1 and c_2	1.0

in Fig. 4. The identification of crack tip locations consists of the following steps:

- Step 1: Generate initial sampling parameters $\mathbf{X}$ (crack tip locations) and run the FE analysis program to obtain the corresponding output $\mathbf{RMS}$.
- Step 2: Construct the initial Kriging surrogate model with samples and output RMS values obtained in Step 1.
- Step 3: Find the optimal parameters $\mathbf{x}_k^*$ by minimizing discrepancies between the targeted values $\mathbf{rms}^{(\text{Target})}$ and the calculated $\mathbf{rms}^*$ by means of SPSO algorithm based on the initial Kriging surrogate model, and set the iterative index $k = 1$.
- Step 4: Check criteria: If the current RMS values predicted by ANSYS and the one based on Kriging surrogate model satisfy the given criteria, then stop updating surrogate model and predict crack parameters with this surrogate model; else, go to Step 5.
- Step 5: Add $\mathbf{x}_k^*$ and $\mathbf{rms}(\mathbf{x}_k^*)$ into the initial sampling database $\mathbf{X}$ and $\mathbf{RMS}$ generated in Step 1, respectively, then update the current Kriging model and update the iterative index: $k = k + 1$.
- Step 6: Loop to Step 3 and repeat the process till the criteria are satisfied, and output the optimal crack parameters.

Simulations

To demonstrate the effectiveness and efficiency of the presented method, a numerical study of a cantilever plate with an internal crack is carried out. The plate under consideration has a square area of $1.4 \text{ m} \times 1.4 \text{ m}$ and thickness of 0.01 m . The material properties of the steel plate are: Young's modulus $E = 210 \text{ GPa}$, Poisson's ratio $\nu = 0.29$, and density $\rho = 7850 \text{ Kg/m}^3$. All the DOFs on the cantilever side are fixed in the FE model. Fifty samples are firstly generated via Latin Hypercube design [38] and prepared for the construction of the initial Kriging model by varying

Fig. 5 Schematic of six cases considered in simulations

Table 3 Comparison of actual and identified locations of crack tips

Case	Tip 1 (x_1, y_1)		Tip 2 (x_2, y_2)	
	Actual	Identified	Actual	Identified
1	(0.6, 0.6)	(0.6042, 0.6029)	(0.8, 0.8)	(0.8067, 0.8025)
2	(0.6, 0.5)	(0.6043, 0.4998)	(1.0, 0.7)	(1.0032, 0.7042)
3	(0.5, 0.9)	(0.4993, 0.8987)	(0.7, 0.6)	(0.7026, 0.5963)
4	(0.5, 0.5)	(0.4941, 0.4983)	(0.8, 0.5)	(0.8000, 0.4989)
5	(0.7, 0.5)	(0.7097, 0.5047)	(0.7, 0.8)	(0.7036, 0.7989)
6	(0.8, 1.1)	(0.7998, 1.1042)	(0.9, 0.9)	(0.9002, 0.8956)

the locations of crack tips. Forced vibration tests are simulated assuming a random white-noise force spectrum with constant amplitude $0.004 \text{ kgf}^2/\text{Hz}$ acting at four specified points, respectively. Consequently, sixteen RMS values can be extracted and expressed as a 1×16 response vector for each case, so that $q = 16$. The concerned frequency range selected to compute the RMS is chosen between 1 Hz and 180 Hz. The Kriging model is constructed with $\mathbf{f}(\mathbf{x}^i)$ in Eq. (7) taken to be a constant one for any sample.

Once the initial Kriging surrogate model has been constructed, the SPSO algorithm is used to estimate the optimal crack parameters based on the surrogate model by minimizing the objective function with optimization parameters given in Table 2. Meanwhile, the accuracy of the current surrogate model is checked according to the given criteria ($SMC > 0.996$ and $EISE < 0.005$) to decide whether the model should be updated.

It is expected that there would be some deviation due to noise originating from environment as well as electronic devices. To simulate the actual harmonic responses $\mathbf{y}_\omega$ measured by experiment, it is assumed that the responses are contaminated by noise in the form of [39]

$$\mathbf{y}_{\omega M} = (1 + d n_M) \mathbf{y}_{\omega S} \quad (25)$$

where d denotes the normally distributed random values between -1 and 1 ; the noise level $n_M = 0.03$ is used. $\mathbf{y}_{\omega M}$ and $\mathbf{y}_{\omega S}$ represent measured and simulated harmonic responses, respectively.

Results and Discussion

Six cases are considered to assess the performance of the proposed method, as shown in Fig. 5. These cases cover typical crack situations. The actual and identified crack parameters are reported in Table 3. Very good agreement between the actual and the identified locations of crack tips based on the Kriging surrogate model is obtained. It is worth noting that the proposed method provides a correct identification of crack tip locations even for noisy cases, and it can be seen that the Kriging surrogate model seems to be insensitive to random noise.

Fig. 6 Evolution of objective function with the Kriging model updating during crack identification

(a)

(b)

Fig. 7 (a) The SMC values of the final Kriging model; (b) the EISE values of the final Kriging model

Figure 6 illustrates the convergence of the objective function with the Kriging model updating during crack identification. The horizontal axis is the number of updating steps and the vertical axis is the value of the objective function. The relative error of the identified nondimensional crack parameters are within $\pm 0.62\%$. The cost values seem to be reduced in general with the iteration of surrogate model updating, which illustrates the process of surrogate model updating is essentially a continuous improvement in describing the changing trends of the surrogate model around the current optimal solution. It is worth noting that after certain steps, some cost values are increasing (e.g., $k = 5$ in case 1, $k = 3, 4, 7$ in case 3, and so on), which indicates another minimum may exist around this optimal solution. That is to say, the current solution, which will be selected as an additional sampling point, may not be the global minimum. Consequently, it is stated that the accuracy of the surrogate model and the global searching ability of the algorithm are equally important in the whole crack identification process.

The values of SMC and $EISE$ at the last iterative step for each case are shown in Fig. 7 and it can be found that the final Kriging model, after several updating steps, is accurate enough for the estimation of crack parameters. Figure 8 compares, for each of the considered cases, the RMS values predicted by the final Kriging model and corresponding FE model.

Fig. 8 Comparison of RMS values predicted by the final Kriging model and FE model for each case

To investigate the effect of sampling size on the precision of the identified results, all the cases are recalculated by using (75, 100, 125, 150, 175, 200) samples given in the Appendix, respectively. The identified crack parameters by using 100 and 200 samples are listed in Table 4. In addition, the required surrogate model updating steps are compared in Fig. 9. It seems that the initial sampling sizes do not directly affect the precision of the

identified crack parameters, but do change the required updating steps. From Fig. 9, it can be seen that the optimal solution can be found with $k = 1$ when 200 samples are initially used for the construction of the surrogate model so that there is no need to carry out the updating process in Cases 1, 3, and 6, while the initial Kriging model cannot provide the “best” solution for other cases. In this sense, the surrogate model is not perfect for describing the

Table 4 The identified crack parameters by using 100 and 200 samples

Case	100 samples		200 samples	
	Tip 1 (x_1, y_1)	Tip 2 (x_2, y_2)	Tip 1 (x_1, y_1)	Tip 2 (x_2, y_2)
1	(0.6052, 0.6029)	(0.7987, 0.8025)	(0.6041, 0.6033)	(0.7983, 0.8042)
2	(0.6041, 0.5042)	(1.0029, 0.6963)	(0.6087, 0.4987)	(0.9983, 0.6938)
3	(0.4998, 0.9079)	(0.7083, 0.5998)	(0.5032, 0.9041)	(0.7053, 0.6078)
4	(0.4967, 0.4936)	(0.8033, 0.5032)	(0.4998, 0.4983)	(0.8026, 0.5042)
5	(0.7067, 0.5052)	(0.7043, 0.7963)	(0.7067, 0.5026)	(0.6997, 0.8000)
6	(0.8053, 1.0997)	(0.9000, 0.8967)	(0.8000, 1.1072)	(0.9063, 0.9003)

Fig. 9 The required steps of surrogate model updating with different initial sampling sizes

relation between crack parameters and RMS values around the optimal solution. As such, more samples should be used and a process of surrogate model updating should be then considered.

All the results indicate that the Kriging model is a powerful tool to give a quantitative estimation of how the locations of crack tips affect RMS values. The actual crack parameters can be determined by this method.

Conclusions

An effective method based on a Kriging surrogate model for crack identification in a plate is presented in the paper. The results clearly show its usefulness and accuracy. Some appealing features of the presented method include: (1) the Kriging model is applied to provide a simple relation between crack parameters and corresponding RMS values to avoid the remeshing problem at every iterative step of optimization, (2) the values of RMS at specified points of the plate are used as output responses to construct the surrogate model, which is impervious to random noise and has seldom been used in crack identification, and (3), PEM, instead of traditional spectral analysis, is used to produce accurate results and reduce computational cost.

It should be mentioned that only simulated noise is considered in this work. There may be some undesired discrepancies (also known as “model error”) between FE model predictions and experimental results due to inevitable uncertainties in material properties and boundary conditions. FE model updating may be a better way to solve this problem. Therefore, the model updating strategy should be considered before crack identification for use in real applications. In addition, only one crack was considered in this paper. More cracks with more complex shapes also may appear in practical problems. As cracks can be complex, more crack parameters should be considered to describe the characterization of intercrossing between two or more arbitrary cracks. This issue will be discussed in the future.

Appendix

Table 5 200 Samples of crack parameters for use in simulations

No.	ξ_1	η_2	ξ_1	η_2	No.	ξ_1	η_2	ξ_1	η_2
1	0.776675	0.228961	0.232999	0.573867	101	0.662108	0.182655	0.15674	0.760524
2	0.173691	0.65281	0.400621	0.752202	102	0.845756	0.226297	0.194824	0.633815
3	0.235376	0.3939	0.214739	0.512827	103	0.315063	0.562044	0.204671	0.78427
4	0.459799	0.239722	0.763473	0.617469	104	0.476799	0.669308	0.530532	0.347806
5	0.612833	0.80243	0.646607	0.530977	105	0.390775	0.720292	0.810401	0.393351
6	0.805125	0.421395	0.706449	0.60946	106	0.809375	0.776189	0.676266	0.77579
7	0.501944	0.199455	0.581676	0.771605	107	0.827703	0.666568	0.65224	0.536167
8	0.413228	0.306574	0.502514	0.299502	108	0.242487	0.648841	0.731183	0.588631
9	0.230774	0.443439	0.257584	0.55646	109	0.38292	0.178319	0.451602	0.185201
10	0.787916	0.825945	0.55002	0.462808	110	0.657346	0.571413	0.157289	0.294304
11	0.299883	0.513167	0.269756	0.768656	111	0.405405	0.705349	0.471604	0.658419
12	0.839726	0.643616	0.627843	0.798758	112	0.666906	0.844826	0.227658	0.195968
13	0.594791	0.809173	0.27284	0.793196	113	0.464955	0.77791	0.234321	0.384532
14	0.761064	0.157454	0.151921	0.700615	114	0.505656	0.289435	0.509953	0.673236
15	0.652765	0.847134	0.789918	0.405421	115	0.247675	0.745154	0.164831	0.367726
16	0.742154	0.368994	0.446121	0.303082	116	0.268876	0.385916	0.185974	0.68743
17	0.555965	0.531884	0.813177	0.730592	117	0.798857	0.223399	0.546719	0.277665
18	0.793514	0.707428	0.602487	0.807757	118	0.71837	0.591658	0.826361	0.7584
19	0.21229	0.728594	0.610008	0.451515	119	0.709945	0.595865	0.592179	0.321043
20	0.513304	0.232789	0.309387	0.474869	120	0.226943	0.466045	0.381408	0.779234
21	0.203887	0.789919	0.620941	0.849887	121	0.565835	0.300177	0.600547	0.450631
22	0.611047	0.520455	0.748193	0.567431	122	0.812946	0.351513	0.824032	0.251005
23	0.324947	0.505521	0.841718	0.194534	123	0.695798	0.516185	0.821255	0.585901
24	0.526442	0.372284	0.843519	0.310955	124	0.581488	0.262565	0.672023	0.726713
25	0.257035	0.799399	0.743047	0.33811	125	0.469576	0.347919	0.722809	0.454819
26	0.631162	0.754057	0.805913	0.824943	126	0.569706	0.766152	0.372368	0.307474
27	0.562291	0.629844	0.266143	0.73592	127	0.37941	0.215985	0.846802	0.679939

Table 5 Continued

No.	ξ_1	η_2	ξ_1	η_2	No.	ξ_1	η_2	ξ_1	η_2
28	0.770648	0.26898	0.55608	0.162841	128	0.834454	0.353455	0.5762	0.550603
29	0.487116	0.417074	0.543673	0.323013	129	0.624764	0.210307	0.727983	0.476713
30	0.332589	0.44756	0.249003	0.171729	130	0.336635	0.459328	0.832881	0.415191
31	0.45274	0.256566	0.642702	0.205466	131	0.606053	0.769669	0.795792	0.628044
32	0.602517	0.3271	0.351426	0.490084	132	0.157954	0.29126	0.689026	0.274954
33	0.152823	0.687684	0.832328	0.434298	133	0.543826	0.523634	0.778693	0.690146
34	0.28894	0.383031	0.636868	0.540454	134	0.737576	0.250842	0.327865	0.603799
35	0.292972	0.840221	0.786121	0.564372	135	0.220375	0.557878	0.280689	0.625408
36	0.350557	0.83628	0.706735	0.285884	136	0.264544	0.637862	0.221142	0.639024
37	0.52849	0.253713	0.402207	0.315234	137	0.31847	0.4114	0.369828	0.2708
38	0.427012	0.277327	0.564044	0.365202	138	0.821187	0.721871	0.464611	0.748667
39	0.301923	0.245441	0.315033	0.667254	139	0.395683	0.187887	0.386022	0.502688
40	0.638262	0.744321	0.390768	0.408638	140	0.846821	0.735826	0.738108	0.417825
41	0.668351	0.321833	0.379407	0.527449	141	0.432688	0.684639	0.422917	0.613657
42	0.539124	0.606508	0.174216	0.390074	142	0.765162	0.816444	0.283454	0.497225
43	0.252743	0.175789	0.276521	0.481073	143	0.482326	0.659229	0.485906	0.345324
44	0.227637	0.74068	0.753533	0.335316	144	0.660726	0.564873	0.291682	0.150316
45	0.278486	0.491827	0.766463	0.533655	145	0.422332	0.608702	0.535995	0.765954
46	0.348306	0.714706	0.761197	0.229411	146	0.682546	0.320304	0.775623	0.493814
47	0.785131	0.589628	0.783242	0.15991	147	0.177391	0.782094	0.678731	0.223684
48	0.591955	0.312304	0.201801	0.339362	148	0.428654	0.342701	0.814742	0.561132
49	0.418417	0.574553	0.426327	0.65637	149	0.189884	0.312898	0.261289	0.642028
50	0.206565	0.44611	0.176017	0.693431	150	0.539114	0.373271	0.18959	0.786728
51	0.197012	0.309552	0.170573	0.841562	151	0.663486	0.398328	0.832032	0.324188
52	0.726204	0.553707	0.816673	0.208101	152	0.328318	0.203646	0.725712	0.150841
53	0.31204	0.623068	0.65009	0.739731	153	0.808451	0.270246	0.204367	0.269542
54	0.308569	0.599419	0.262618	0.427756	154	0.827141	0.417981	0.241632	0.363033
55	0.490825	0.38952	0.366837	0.562943	155	0.318513	0.564798	0.384803	0.839301
56	0.633718	0.168492	0.396407	0.843571	156	0.74826	0.536351	0.280369	0.296139
57	0.218171	0.696783	0.498989	0.259558	157	0.456841	0.19827	0.521399	0.462991
58	0.829071	0.543048	0.625771	0.577627	158	0.52236	0.435947	0.734319	0.176715
59	0.83855	0.374061	0.19118	0.397824	159	0.36009	0.704185	0.6402	0.698044
60	0.474617	0.154177	0.342734	0.606941	160	0.598656	0.684157	0.376655	0.830167
61	0.496935	0.833538	0.615279	0.811455	161	0.434711	0.229296	0.395256	0.631879
62	0.731919	0.550803	0.341575	0.257419	162	0.401131	0.277646	0.846417	0.411253
63	0.618522	0.261599	0.685502	0.164436	163	0.505577	0.460801	0.676532	0.332329
64	0.387959	0.469673	0.726758	0.504077	164	0.692367	0.294949	0.488353	0.542595
65	0.545871	0.439221	0.410756	0.642334	165	0.383552	0.589859	0.596909	0.670533
66	0.446803	0.16351	0.572483	0.356918	166	0.220958	0.475308	0.341654	0.233326
67	0.533954	0.274016	0.665645	0.176321	167	0.611768	0.842566	0.701942	0.814721
68	0.644685	0.577619	0.244647	0.697537	168	0.171299	0.6233	0.748501	0.738267
69	0.795858	0.293502	0.252669	0.483683	169	0.797445	0.169575	0.564935	0.434247
70	0.188273	0.818634	0.468042	0.179342	170	0.276053	0.732775	0.155336	0.524935
71	0.584041	0.765794	0.289666	0.718102	171	0.156268	0.494655	0.460463	0.422859
72	0.71496	0.194239	0.636448	0.239214	172	0.654377	0.502987	0.575018	0.356575
73	0.740018	0.21808	0.803491	0.214001	173	0.350735	0.238014	0.434894	0.771037
74	0.750485	0.615328	0.457939	0.818617	174	0.268672	0.582248	0.662168	0.610016
75	0.282209	0.732501	0.714132	0.378123	175	0.735409	0.797534	0.174109	0.187321
76	0.690471	0.315547	0.259826	0.621388	176	0.726481	0.334787	0.622679	0.679518
77	0.239146	0.700373	0.838646	0.289544	177	0.699717	0.394891	0.479833	0.23759
78	0.628768	0.235988	0.597079	0.245189	178	0.212718	0.792392	0.548942	0.589832
79	0.166171	0.34378	0.224362	0.326163	179	0.846737	0.645392	0.293162	0.281498
80	0.441258	0.269689	0.578191	0.401952	180	0.253882	0.771375	0.236477	0.746692
81	0.193117	0.692206	0.534251	0.813034	181	0.778691	0.673539	0.770576	0.394645
82	0.169013	0.640294	0.434883	0.731333	182	0.582074	0.759074	0.619172	0.506356
83	0.517293	0.481613	0.320858	0.268772	183	0.483479	0.150392	0.349509	0.483479
84	0.775046	0.497436	0.218111	0.409272	184	0.290337	0.528592	0.792385	0.219286
85	0.722933	0.484522	0.735818	0.218036	185	0.472785	0.72506	0.428402	0.477501
86	0.64744	0.426635	0.161599	0.553717	186	0.168373	0.835333	0.346091	0.580231
87	0.261947	0.401748	0.333129	0.220796	187	0.198455	0.499656	0.168255	0.494663
88	0.294312	0.379063	0.493431	0.748128	188	0.71688	0.278078	0.508355	0.729699
89	0.822102	0.586478	0.713455	0.373996	189	0.329163	0.648701	0.150354	0.22051
90	0.727535	0.360497	0.211914	0.816604	190	0.509512	0.365084	0.63525	0.797674
91	0.701465	0.396119	0.460424	0.71192	191	0.412176	0.632494	0.460584	0.538695
92	0.704756	0.681813	0.311257	0.170315	192	0.818125	0.669669	0.782175	0.509823
93	0.154748	0.424756	0.40874	0.522335	193	0.807218	0.409181	0.329597	0.36878
94	0.43588	0.358479	0.517253	0.377446	194	0.766141	0.190561	0.727294	0.313739
95	0.18164	0.365031	0.377458	0.825882	195	0.676368	0.179702	0.256857	0.214399
96	0.619957	0.455842	0.348705	0.581334	196	0.706915	0.21736	0.372272	0.837284
97	0.367894	0.692856	0.522896	0.707345	197	0.571129	0.808378	0.600023	0.39947
98	0.579562	0.434359	0.669039	0.439167	198	0.378954	0.67745	0.246789	0.600311
99	0.35496	0.462231	0.354461	0.684341	199	0.26532	0.537561	0.848866	0.562301
100	0.678733	0.812148	0.526218	0.281236	200	0.361861	0.157113	0.535204	0.781166

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