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Two-Channel Transparency-Optimized Control Architectures in Bilateral Teleoperation With Time Delay

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Abstract

This paper introduces transparency-optimized control architectures (TOCAs) using two communication channels. Two classes of two-channel TOCAs are found, thereby showing that two channels are sufficient to achieve transparency. These TOCAs achieve a greater level of transparency but poorer stability than three-channel TOCAs and four-channel TOCAs. Stability of the two-channel TOCAs has been enhanced while minimizing transparency degradation by adding a filter; and a combined use of the two classes of two-channel TOCAs is proposed for both free space and constrained motion, which involve switching between two TOCAs for transition between free space and constrained motions. The stability condition of the switched teleoperation system is derived for practical applications. Through the one degree-of-freedom (DOF) experiment, the proposed two-channel TOCAs were shown to operate stably, while achieving better transparency under time delay than the other TOCAs.

Index Terms

Bilateral control; switched delay system; teleoperation; telerobotics; time delay; transparency-optimized control architecture (TOCA); two-channel architecture

I. Introduction

Is *transparency theoretically achievable* by *two-channel* control architecture in bilateral teleoperation? If so, what are its *structures*, *characteristics*, and *realizable forms*? This paper addresses these questions and issues by presenting a set of two-channel architectures that can theoretically achieve transparency by expounding on its worth and drawbacks, and by proposing application schemes.

Transparency refers to a match between the impedance perceived by the operator and the environment impedance [1]. Transparency is regarded as a major performance objective in bilateral teleoperation [2].

There have been many studies that investigate control architectures to improve transparency [1], [3]–[11], [13]–[15]. Among them, the transparency-optimized control architecture (TOCA) is noteworthy because it theoretically achieves perfect transparency [3], [4].

For this reason, several TOCAs have been reported, and they can be classified according to the *number* of communication channels employed between the master and the slave: four-channel TOCA [1], [4] and three-channel TOCA [3]. A four-channel TOCA was first proposed [1] and followed by an extended version that used local force feedback [4]. Based on these TOCAs, two classes of three-channel TOCAs were derived: an operator-force-compensated (OFC) TOCA and an environment-force-compensated (EFC) TOCA [3]. In addition, there has been a recent study introducing several architectures based on existing TOCAs [5].

Despite superior transparency achievability, the existing TOCAs suffer transparency degradation from time delay. Since time delay is unavoidable, the transparency degradation was regarded as a limitation [3]. Hence, these TOCAs still leave room for transparency improvement.

Having introduced four channels and then three channels consecutively, the literature points to *two-channel* TOCAs as a direction of extension. To our knowledge, however, a two-channel TOCA—the simplest possible form of TOCAs—has not been proposed yet. Of course, there have been many two-channel control architectures [6]–[12] such as the one using unilateral control approach [6], a compensator-based scheme to enhance transparency [7], [8], and stability-guaranteed schemes under time delay [9], [10]. However, these architectures are unable to achieve transparency [6]–[12], and cannot be classified as TOCAs.

This paper introduces a set of two-channel TOCAs, thereby showing the existence of such TOCAs. We found that the proposed two-channel TOCAs achieve transparency better than existing TOCAs, but they suffer stability problems in some cases, demanding a remedy. This led us to propose a modified version of the two-channel TOCA with further stability analysis. Moreover, a combined use of the two TOCAs is proposed for practical application followed by stability analysis using a theorem regarding the stability of a switched linear delay system [16].

II. Two-Channel Transparency-Optimized Control Architecture

After providing a background of the TOCA and introducing the existing TOCAs and related research, we will show the existence of two classes of two-channel TOCA.

A. Background of TOCA

The TOCA is a control architecture that is designed to achieve transparency. The existing TOCAs have been reported as condition sets of the general teleoperator architecture that meet the transparency condition [3], [4]. Provided below is a brief background regarding the transparency condition and the general teleoperator architecture.

The transparency condition was defined as follows [1]:

$$Z_t = Z_e \quad (1)$$

where Z_t denotes the transmitted impedance perceived by the operator and Z_e the environment impedance. The transmitted impedance above can be represented in terms of hybrid parameters ($h_{ij}(i, j = 1, 2)$) [3], [4]

$$Z_t = h_{11} - \frac{h_{12}h_{21}Z_e}{1+h_{22}Z_e}. \quad (2)$$

Substituting (2) into (1) leads to the following conditions [17]:

$$h_{11}=0; h_{22}=0; h_{12}=1; h_{21}=-1. \quad (3)$$

The general teleoperator architecture, a block diagram for describing overall teleoperation system, was reported as a unified framework for bilateral teleoperation [1]–[4] (see Fig. 1). Thanks to this framework, most control architectures can be expressed in terms of condition-sets made of its parameters.

From the definition of hybrid parameters [17] and Fig. 1, the hybrid parameters have been expressed as parameters of the general teleoperator architecture as follows [4]:

$$\begin{aligned} h_{11} &= \frac{Z_{cm}Z_{cs} + C_1C_4}{C_{61}Z_{cs} - C_3C_4} \\ h_{12} &= \frac{C_2Z_{cs} - C_4C_{51}}{C_{61}Z_{cs} - C_3C_4} \\ h_{21} &= -\frac{C_3Z_{cm} + C_1C_{61}}{C_{61}Z_{cs} - C_3C_4} \\ h_{22} &= \frac{C_{51}C_{61} - C_2C_3}{C_{61}Z_{cs} - C_3C_4} \end{aligned} \quad (4)$$

where Z_m denotes the master impedance; Z_s the slave impedance; C_k ($k = 1 \sim 6$), C_m , and C_s the parameters of general teleoperator architecture; with $Z_{cm} = Z_m + C_m$, $Z_{cs} = Z_s + C_s$, $C_{51} = 1 + C_5$, and $C_{61} = 1 + C_6$. Note that equations in are used to find TOCAs, the condition-sets that meet the transparency condition (3).

B. Existing TOCAs and Related Research

The **four-channel** TOCA was proposed by using four communication channels between the master and the slave [1]

$$\begin{cases} C_1 = Z_s + C_s \neq 0 \\ C_2 = 1 \\ C_3 = 1 \\ C_4 = -Z_m - C_m \neq 0. \end{cases} \quad (5)$$

Afterwards, the TOCA (5) was extended by supplementing local force feedback (C_5 and C_6) as follows [4]:

$$\begin{cases} C_1 = Z_s + C_s \neq 0 \\ C_2 = 1 + C_6 \neq 0 \\ C_3 = 1 + C_5 \neq 0 \\ C_4 = -Z_m - C_m \neq 0. \end{cases} \quad (6)$$

In this paper, the four-channel TOCA refers to (6) because (5) is a subset of (6). The TOCAs above, (5) and (6), led to the well-known statement: *all four communication channels are necessary to achieve transparency* [1], [2].

Three-channel TOCAs were derived from (6) by using local force feedback [3]. Depending on where the force feedback is made, there are two classes of three-channel TOCAs: operator-force-compensated (OFC) and environment-force-compensated (EFC) architectures.

In order to elaborate, the OFC is derived by using the local force feedback (C_6) in the *master*, which eliminates the slave force feedforward channel ($C_2 = 0$) and thus constructing a three-channel TOCA [3]

$$\begin{cases} C_1 = Z_s + C_s \neq 0 \\ C_2 = 0; C_6 = -1 \\ C_3 = 1 + C_5 \neq 0 \\ C_4 = -Z_m - C_m \neq 0. \end{cases} \quad (7)$$

The EFC, on the other hand, is obtained by using the local force feedback in the *slave* (C_5), which eliminates the master force feedforward channel ($C_3 = 0$) [3]

$$\begin{cases} C_1 = Z_s + C_s \neq 0 \\ C_2 = 1 + C_6 \neq 0 \\ C_3 = 0; C_5 = -1 \\ C_4 = -Z_m - C_m \neq 0. \end{cases} \quad (8)$$

Three-channel TOCAs, (7) and (8), show that transparency is still achievable with three channels, providing *counter-examples* to the former statement that *four channels* are necessary to achieve transparency [3]. This implies that two-channel TOCAs might be realizable, which we have attempted to implement and will present in Section II C.

It is noteworthy that several control architectures were reported based on the existing TOCAs above [5]. In order to improve stability under time delay, the communication channels were modified for these architectures [5]. On the other hand, we found that this modification makes achieving transparency difficult. More details are presented in Appendix A.

C. Two-Channel TOCAs

The realization of three-channel TOCAs has inspired us to try two-channel TOCAs, the simplest among TOCAs in bilateral teleoperation. In order to investigate the existence of two-channel TOCA, we have first categorized two-channel control architectures into four types: P-F, F-P, P-P, and F-F. The nomenclature is based on which information, among position (P) or force (F), is transferred from the slave to the master and from the master to the slave [4], [12], [18]. For each type, we attempt to derive a two-channel TOCA based on (3) and (4).

The definition of P-F type sets the channel constraints $C_1 = C_2 = 0$. Substituting these constraints into (4), one can find that there *exists* such a two-channel architecture that satisfies (3), that is transparency-optimized P-F control architecture (TOPF)

$$\begin{cases} C_1=0; C_s=-Z_s \\ C_2=0; C_6=-1 \\ C_3=1+C_5 \neq 0 \\ C_4=-Z_m-C_m \neq 0. \end{cases} \quad (9)$$

In addition to the elimination of the slave force feedforward channel ($C_2 = 0$), slave local position controller (C_s) eliminates master position feedforward channel ($C_1 = 0$).

Likewise, F-P type involves the channel constraints $C_3 = C_4 = 0$. Similarly to P-F type, transparency-optimized F-P control architecture (TOFP) can be obtained as follows:

$$\begin{cases} C_1=Z_s+C_s \neq 0 \\ C_2=1+C_6 \neq 0 \\ C_3=0; C_5=-1 \\ C_4=0; C_m=-Z_m. \end{cases} \quad (10)$$

In addition to the elimination of master force feedforward channel ($C_3 = 0$), master local position controller (C_m) eliminates slave position feedforward channel ($C_4 = 0$).

For P-P ($C_2 = C_3 = 0$) and F-F ($C_1 = C_4 = 0$) types, we found that there exists no solution that meets (3).

Since proposed TOPF (9) and TOFP (10) show the existence of two-channel TOCAs, the previous statements can now be updated to the following: *two communication channels are sufficient to achieve transparency*.

In order to implement two-channel TOCAs, it is vital either for the slave local position controller (C_s) to achieve $-Z_s$ for TOPF, or for the master local position controller (C_m) to achieve $-Z_m$ for TOFP. This requires accurate estimation of Z_s and Z_m . Note that the existing TOCAs also require the estimation. Equations (6)–(8) reveal that proper selection of C_1 and C_4 can only be embodied through accurate estimation of both Z_s and Z_m .

The proposed two-channel TOCAs are distinct in that they are the only architectures that can achieve transparency whereas the other two-channel architectures cannot [6]–[12].

III. Characteristics of Two-Channel TOCAs

Under no time delay in the communication channels, there is no difference in transparency achievability or stability among TOCAs with four-channels or three or two.

However, differences do arise when there is time delay, which is practically unavoidable in teleoperation. Therefore, the transparency and the stability of two-channel TOCAs under time delay are analyzed in comparison with three-channel and four-channel TOCAs.

A. Transparency of Two-Channel TOCAs Under Time Delay

Since (1) cannot be achieved under time delay, the *delayed impedance matching* [3] or *ideal transparency* [5] has been considered as the best transparency achievable under the existence of time delay (T), as follows:

$$h_{11}=0; h_{22}=0; h_{12}=e^{-iT_s}; h_{21}=-e^{-iT_s} \quad (11)$$

where $i = 1$ or -1 . Substituting (11) into (2), one can obtain that $Z_t = e^{-iT_s} Z_c$, defined as the delayed impedance matching [2].

The hybrid parameters with time delay are slightly altered from (4) [4]

$$\begin{aligned} h_{11} &= \frac{Z_{cm}Z_{cs} + e^{-2T_s}C_1C_4}{C_{61}Z_{cs} - e^{-2T_s}C_3C_4} \\ h_{12} &= \frac{e^{-T_s}(C_2Z_{cs} - C_4C_{51})}{C_{61}Z_{cs} - e^{-2T_s}C_3C_4} \\ h_{21} &= -\frac{e^{-T_s}(C_3Z_{cm} + C_1C_{61})}{C_{61}Z_{cs} - e^{-2T_s}C_3C_4} \\ h_{22} &= \frac{C_{51}C_{61} - e^{-2T_s}C_2C_3}{C_{61}Z_{cs} - e^{-2T_s}C_3C_4}. \end{aligned} \quad (12)$$

Substituting (9) and (10) into (12), one can obtain the hybrid parameters of the **two-channel TOCAs** under time delay

$$\begin{cases} h_{11}=0; h_{22}=0; h_{12}=e^{T_s}; h_{21}=-e^{T_s} & \text{for TOPF} \\ h_{11}=0; h_{22}=0; h_{12}=e^{-T_s}; h_{21}=-e^{-T_s} & \text{for TOFP.} \end{cases} \quad (13)$$

Similarly, substituting (7) and (8) into (12), we can determine the parameters for the **three-channel TOCAs** [3]

$$\begin{cases} h_{11}=\frac{(1-e^{-2T_s})Z_{cs}}{e^{-2T_s}C_{51}}; h_{22}=0; h_{12}=e^{T_s}; h_{21}=-e^{T_s} & \text{for OFC} \\ h_{11}=\frac{(1-e^{-2T_s})Z_{cm}}{C_{61}}; h_{22}=0; h_{12}=e^{-T_s}; h_{21}=-e^{-T_s} & \text{for EFC.} \end{cases} \quad (14)$$

In the same way, the hybrid parameters of the **four-channel TOCA** under time delay can be obtained by substituting (6) into (12) as follows:

$$\begin{aligned} h_{11} &= \frac{(1-e^{-2T_s})Z_{cm}Z_{cs}}{C_{61}Z_{cs} - e^{-2T_s}C_{51}Z_{cm}} \\ h_{22} &= \frac{(1-e^{-2T_s})C_{51}C_{61}}{C_{61}Z_{cs} - e^{-2T_s}C_{51}Z_{cm}} \\ h_{12} &= e^{-T_s} + \frac{e^{-T_s}(1-e^{-2T_s})C_{51}Z_{cm}}{C_{61}Z_{cs} - e^{-2T_s}C_{51}Z_{cm}} \\ h_{21} &= -e^{-T_s} - \frac{e^{-T_s}(1-e^{-2T_s})C_{61}Z_{cs}}{C_{61}Z_{cs} - e^{-2T_s}C_{51}Z_{cm}}. \end{aligned} \quad (15)$$

In order to compare transparency, each of (13)–(15) is compared with the delayed impedance matching (11), for it is regarded as the best transparency under time delay [3]. Clearly the two-channel TOCAs in (13) can exactly achieve (11) while the other two cannot. Although (14) is more similar to (11) than (15), h_{11} of (14) is still different from that of (11), leaving room for the improvement.

B. Stability of Two-Channel TOCAs Under Time Delay

Since Nyquist criterion was used for stability analysis of the three-channel TOCAs [3], this paper also employed it to investigate stability characteristics of two-channel TOCAs. Without any assumption on time delay, the stability condition is obtained by a characteristic equation due to coupled stability [19]. Here, the characteristic equation of teleoperation system was reported as follows [3]:

$$\Delta = 1 + \frac{Z_h}{Z_t} = 0 \quad (16)$$

where Z_h denotes the operator impedance.

Based on (16), the characteristic equations of the proposed two-channel TOCAs are derived from (2) and (13) as follows:

$$\Delta' = \begin{cases} 1 + \frac{Z_h}{Z_e} e^{-2Ts} = 0 & \text{for TOPF} \\ 1 + \frac{Z_e}{Z_h} e^{-2Ts} = 0 & \text{for TOFP.} \end{cases} \quad (17)$$

All the open-loop poles are in the left-half-plane because Z_h and Z_e are passive, and the number of clockwise encirclements of the Nyquist plot around point $(-1, 0)$ has to be nonpositive for a stable closed-loop system [3]. Since the time delay (e^{-Ts}) creates a monotonically growing negative phase of $-2T\omega$ [3], the stability conditions of the two-channel TOCAs are obtained from (17) as follows:

$$|Z_e(j\omega)| > |Z_h(j\omega)| \forall \omega \text{ for TOPF} \quad (18)$$

$$|Z_h(j\omega)| > |Z_e(j\omega)| \forall \omega \text{ for TOFP.} \quad (19)$$

Note that the stability conditions of the three-channel TOCAs were reported as follows [3]:

$$\begin{cases} \left| \frac{Z_e(j\omega) + \frac{Z_{cs}}{C_{51}}}{Z_h(j\omega) + \frac{Z_{cm}}{C_{61}}} \right| > \left| \frac{Z_h(j\omega) - \frac{Z_{cs}}{C_{51}}}{Z_e(j\omega) - \frac{Z_{cm}}{C_{61}}} \right| \forall \omega \text{ for OFC} \\ \left| \frac{Z_h(j\omega) + \frac{Z_{cm}}{C_{61}}}{Z_e(j\omega) - \frac{Z_{cm}}{C_{61}}} \right| > \left| \frac{Z_e(j\omega) - \frac{Z_{cm}}{C_{61}}}{Z_h(j\omega) - \frac{Z_{cm}}{C_{61}}} \right| \forall \omega \text{ for EFC.} \end{cases} \quad (20)$$

In contrast to (20), conditions (18) and (19) have *no tunable gain*; the stability of the two-channel TOCAs is solely determined by Z_h and Z_e . Hence, the stability of two-channel TOCAs can only be tested given specific models of Z_h and Z_e .

The model of the operator impedance (Z_h) has been represented as mass-spring-damper system in many papers [20]–[25] as summarized in Table I. In this paper, the operator impedance is represented as follows:

$$Z_h \in \left\{ \begin{array}{l} m_h s + b_h + \frac{k_h}{s} : \\ 0.027 \leq m_h \leq 4.54 \\ 0.67 \leq b_h \leq 29.5, 16.45 \leq k_h \leq 568 \end{array} \right\} \quad (21)$$

where m_h denotes the mass, b_h the damper, and k_h the stiffness of the operator's arm.

The environment impedance (Z_e) has been regarded as a spring-damper (or pure spring) model [1], [7], [15], [21], [26]

$$Z_e = b_e + \frac{k_e}{s} \quad (22)$$

where b_e denotes the damper, and k_e the stiffness of the environment.

Using (21) and (22), we plotted in Fig. 2 the magnitude of Z_h —both its maximum and minimum—and that of Z_e for both a soft environment ($0.2 + 1/s$) and a hard one ($20 + 1000/s$). In Fig. 2, the TOFP with the soft environment satisfies the stability condition (19), but is inappropriate in the hard environment because condition (19) cannot meet even if $k_e > 1.27$.

As to TOPF, Fig. 2 clearly displays that the stability condition (18) can never be met because the curve of $|Z_e|$ always crosses with the curve of $|Z_h|$. Of course, conditions (18) and (19) are conservative (delay-independent), and Fig. 2 stems from (18) and (19) with the worst case consideration of operator impedance. These results show that stability of the two-channel TOCAs is worse than that of the three-channel TOCAs and indicate the necessity of improving the stability as well as employing a less conservative method for stability analysis.

C. Discussion

So far we have shown that two channels are sufficient to achieve transparency. Further, the two-channel TOCAs yield better transparency under time delay than the TOCAs employing more channels. One may wonder why this happens or how the number of channels affects transparency.

Its effect can be observed by comparing each hybrid parameter in (13)–(15). Transparency becomes worse because of a nonzero term due to time delay ($1 - e^{-2Ts} \neq 0$) and the nonzero gains (Z_{cm} , Z_{cs} , C_{51} , and C_{61}). The number of nonzero gains in turn is the direct outcome of the number of channels as (6)–(10) clearly show.

Observe how the term without time delay ($1 - e^{-2Ts} = 0$) cleans up the gains in (14) and (15). It then becomes evident that increasing the number of channels introduces *more impeding elements* to transparency that does not appear when there is no time delay. On the other hand, the additional channels and gains are necessary to enhance stability. Take the existing three-channel TOCAs, OFC (7), and EFC (8), as an example. An inspection of their stability conditions (20) in comparison with (18) and (19) reveals that Z_{cs}/C_{51} and Z_{cm}/C_{61} are *crucial to stabilizing* otherwise unstable systems through the tuning of C_m , C_s , C_5 , and C_6 [3]. Note at the same time in (14) that Z_{cs}/C_{51} and Z_{cm}/C_{61} are also *crucial to degrading transparency*. Here we find a classic example of the *tradeoff* between transparency and stability. This tradeoff also explains the poor stability of the two-channel TOCAs and points to the direction of enhancing stability while minimizing the degradation of transparency.

Since the two-channel TOCAs require the estimation of Z_m and Z_s , one may think the modeling errors can cause instability. However, the modeling errors do not cause instability as the following. For TOPF, when modeling errors ($e_m = Z_m - \hat{Z}_m$; $e_s = Z_s - \hat{Z}_s$) exist, $C_s = -\hat{Z}_s$ instead of $C_s = -Z_s$. Then, the hybrid parameters of the TOPF are slightly altered from (13)

$$h_{11} = \frac{e^{2Ts} e_s}{C_{51}}; h_{22} = 0; h_{12} = e^{Ts}; h_{21} = -e^{Ts} \quad (23)$$

where e_s denotes the modeling error of Z_s . Contrast to (13), h_{11} becomes nonzero due to the error e_s . From (16) and (23), the stability condition of the TOPF is obtained as follows:

$$\left| Z_e(j\omega) + \frac{e_s}{C_{51}} \right| > |Z_h(j\omega)| \forall \omega. \quad (24)$$

In (24), one can see that the stability of the TOPF is improved due to e_s . Likewise, the existence of e_m enhances the stability of the TOPF.

IV. Application Schemes of Two-Channel TOCAs

In order to make the two-channel TOCAs more practical, the following issues are addressed: 1) enhancement of the stability of TOPF and 2) use of the stability-enhanced TOPF in combination with TOPF.

A. Modification of TOPF

Recall that the stability of TOPF cannot be guaranteed by using (18). To improve the stability of TOPF while minimizing degradation of transparency, we have proposed TOPF in (9)—termed the modified TOPF (mTOPF)—as the following:

$$\begin{cases} C_1=0; C_5=-Z_s \\ C_2=0; C_6=-1 \\ C_3=(1+C_5)Q \neq 0 \\ C_4=-Z_m-C_m \neq 0 \end{cases} \quad (25)$$

with

$$Q = \frac{1}{\left(\frac{s}{\omega_n}\right)^2 + \frac{2\zeta s}{\omega_n} + 1} \quad (26)$$

where Q denotes a filter for stabilization; ω_n the cut-off frequency; ζ the damping ratio. The only difference between mTOPF and TOPF lies in that C_3 of mTOPF is $(1 + C_5)Q$ whereas that of TOPF is $1 + C_5$.

1) Stability of mTOPF—From (2), (12), (16), and (25), the characteristic equation of mTOPF is obtained as follows:

$$\Delta' = 1 + \frac{QZ_h}{Z_e} e^{-2Ts} = 0. \quad (27)$$

Similarly to (18) of TOPF, the stability condition of mTOPF is obtained from (27) as follows:

$$|Z_e(j\omega)| > |Q(j\omega)Z_h(j\omega)| \forall \omega. \quad (28)$$

In comparison with (18), the stability condition (28) contains tunable gains (ω_n and ζ) due to Q (26).

In the same way as Fig. 2, we have constructed Fig. 3 including Q . Owing to the effect of Q , (28) holds up with the hard environment in Fig. 3. Recall that (19) did not hold up when TOPF was used with the same hard environment (see Fig. 2).

To be more detailed, we have obtained the stability diagrams of mTOPF displayed using (21), (22), (26), and (28) (see Fig. 4). Fig. 4(a) shows the minimum k_e and b_e that meet the stability condition (28) with respect to ω_n , whereas Fig. 4(b) shows the same with respect to ζ . From Fig. 4, one can see that: 1) the stability of TOPF has been greatly enhanced through mTOPF and 2) mTOPF is appropriate for operations with hard environments.

The stability of mTOPF still cannot be guaranteed in free motion or in contact with soft environments, e.g., $k_e < 717$, with the conservative stability condition.

2) Transparency of mTOPF—In order to check the transparency degradation due to the stability enhancement through mTOPF, we have obtained its hybrid parameters under time delay by substituting (25) into (12) as follows:

$$h_{11}=0; h_{22}=0; h_{12}=\frac{e^{Ts}}{Q}; h_{21}=-e^{Ts}. \quad (29)$$

Comparing (13) and (29), one finds that the transparency of mTOPF degrades because of the change in h_{12} due to Q . Our purpose is to minimize the amount of degradation so that mTOPF may preserve the advantage of transparency over the existing TOCAs. Hence, we have compared the transparency of mTOPF with that of the existing TOCAs—especially three-channel TOCAs. Recall that from (14) and (15) it was deduced that the transparency degradation of the three-channel TOCAs is less severe than that of the four-channel TOCA.

This comparison was numerically performed by using a well-known performance index for transparency, *impedance error measure* [27], [28], with an extensive set of environments ($0 < b_e < 1000$, $600 < k_e < 200\,000$ in (22)). Note that we set $k_e = 600$ considering the stability-guaranteed range of environment in mTOPF (see Fig. 4). Since this measure is positive and smallest-is-best [27], the following criteria are employed to evaluate transparency. If $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$, transparency of mTOPF is better than that of the three-channel TOCAs with

$$\varepsilon_1 = I_{A_OFC} - I_{A_mTOPF}; \varepsilon_2 = I_{A_EFC} - I_{A_mTOPF}$$

where I_{A_OFC} , I_{A_EFC} , and I_{A_mTOPF} denote the impedance error measures of OFC, EFC, and mTOPF, respectively. Here, the operator impedance was set to the average values of (21), and time delay was set to 500 ms. For Q in mTOPF, we chose $\omega_n = 2$ and $\zeta = 1$. The gains of three-channel TOCAs were tuned for best transparency according to [3].

The comparison results are displayed in Fig. 5. The results show that transparency of mTOPF is better than that of the three-channel TOCAs in the large range of environment. The exception is limited to a range of extremely hard environments (e.g., $k_e > 160\,000$) under OFC [see Fig. 5(a)]. This result is not exhaustive, but is sufficient to show that enhancing stability of mTOPF does not degrade transparency too much to preserve its advantage in most environments.

3) Simplicity of mTOPF—It is easy to observe that Z_{cs}/C_{51} (and Z_{cm}/C_{61}) in (20) enhances stability and degrades transparency for OFC (and EFC) just as Q in (28) does for mTOPF. It is easier to implement Q than Z_{cs}/C_{51} . In order for OFC to meet (20), four gains (C_5 , m_s , b_s , and k_s) should be selected that satisfy the following complex inequality conditions [3]:

$$\begin{aligned} m_h - \frac{2m_s}{C_{51}} &< 0 \\ k_h - \frac{2k_s}{C_{51}} &< k_e \\ \left(b_e + \frac{b_s}{C_{51}}\right)^2 &> \left(b_h - \frac{b_s}{C_{51}}\right)^2 + 2 \left[\left(\frac{m_s}{C_{51}}\right) \left(k_e + \frac{k_s}{C_{51}}\right) - \left(m_h - \frac{m_s}{C_{51}}\right) \left(k_h - \frac{k_s}{C_{51}}\right) \right]. \end{aligned}$$

Unless transparency evaluated with these gains is satisfactory, the gains should be selected again from the three inequalities. The iteration procedure could be tedious and difficult.

By comparison, (28) and the meaning of Q in (26) make the implementation far easier. Given target environments, ζ and ω_n can be selected for stability by using Fig. 4. Since transparency degradation decreases as Q approaches unity, transparency improves as ζ is reduced while ω_n is increased. Therefore, candidates of ζ and ω_n are selected as the smallest ζ and the largest that ω_n guarantee stability.

B. Combined Use of mTOPF and TOPF

The two-channel TOCAs should be able to carry out with both stability and transparency for tasks that require both free motion and constrained motion with the environment. Recall that TOPF can handle soft environments (including free space) whereas mTOPF can handle hard environments (see Fig. 4). Since these two architectures are complementary, it is reasonable to combine the two architectures, thereby *uniting* the respective stability range of TOPF and mTOPF into a much enlarged one.

1) Switching Between mTOPF and TOPF—To be more specific, we employ both mTOPF and TOPF, switching from one to the other according to the environment being handled: TOPF for free space or a soft environment; and mTOPF for a hard environment.

Each of mTOPF (27) and TOPF (17) can be expressed as a state equation, in the same way as [5]. Since the combined use involves switching between the two TOCAs, the whole system can be regarded as the following switched delay system of which the stability is well analyzed in [16]:

$$\dot{x}(t) = A_\sigma x(t) + E_\sigma x(t-2T) + B_\sigma \tau_{op} \quad (30)$$

with

$$\begin{aligned} A_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{\omega_n^2 k_e}{b_e} & a_{32} & a_{33} \end{pmatrix} \\ E_1 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{\omega_n^2 k_h}{b_e} & -\frac{\omega_n^2 b_h}{b_e} & -\frac{\omega_n^2 m_h}{b_e} \end{pmatrix} \\ B_1 &= \begin{pmatrix} 0 \\ 0 \\ \frac{\omega_n^2}{b_e} \end{pmatrix} \\ A_2 &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\frac{k_h}{m_h} & -\frac{b_h}{m_h} \end{pmatrix} \\ E_2 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{k_e}{m_h} & -\frac{b_e}{m_h} \end{pmatrix} \\ B_2 &= \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{m_h} \end{pmatrix} \end{aligned}$$

where $a_{32} = -w_n(2\zeta k_e + w_n b_e)/b_e$; $a_{33} = -(k_e + 2\zeta w_n b_e)/b_e$; $x = [x_m \dot{x}_m \ddot{x}_m]^T$; x_m denotes the master (or operator) position; τ_{op} the operator torque, and $\sigma = 1$ (mTOPF) or 2 (TOPF), which denotes switching signal [16].

Here, σ is used to set up a switching rule between mTOPF and TOPF as follows. If $F_s > F_{th}$, σ is 1 (mTOPF). Otherwise σ is 2 (TOPF), where F_s denotes the force sensed at the slave; F_{th} the force threshold value, designed by user. Although F_{th} is smallest-is-best, it should be chosen larger than the noise level or the resolution level of the force sensor in the slave to prevent unintended switching.

A different switching rule can be used that switches the controller based on the estimation of the environment stiffness (K_e). The switching rule, however, will need to estimate K_e from the environment model represented by (22).

2) Stability of Combined Use of mTOPF and TOPF—In spite of the enlarged stability range, the combined use still cannot meet the conservative (delay-independent) stability conditions (19) and (28) in constrained motion with soft environments. Moreover, it is well known that instability may be caused by the switching between two subsystems [29]. Based on a less conservative (delay-dependent) method for switched delay system [16], we now give asymptotical stability for the combined use of mTOPF and TOPF.

Assumption 1 [2], [4], [5], [9], [10]: In bilateral teleoperation, the input generated by the operator (i.e., the operator torque) does not cause instability itself.

Theorem A: Assume (30) satisfies Assumption 1. Suppose constant $T \leq t_m/2$ holds and

there exist matrices $P_i > 0$, $Q_i > 0$, $R_i > 0$ and $\begin{bmatrix} X_{11}^i & X_{12}^i \\ * & X_{22}^i \end{bmatrix} \geq 0$, and any matrices Y_i and U_i with appropriate dimensions such that

$$\begin{bmatrix} \phi_{11}^i & \phi_{12}^i & t_m A_i^T R_i \\ * & \phi_{22}^i & t_m E_i^T R_i \\ * & * & -t_m R_i \end{bmatrix} < 0$$

$$\begin{bmatrix} X_{11}^i & X_{12}^i & Y_i \\ * & X_{22}^i & U_i \\ * & * & -e^{-\alpha t_m} R_i \end{bmatrix} \geq 0, i \in \sigma.$$

Then, system (30) is *asymptotically stable* for any arbitrary switching signals, where

$$\begin{aligned} P_i &= P_j, Q_i = Q_j, R_i = R_j \text{ for all } i, j \in \sigma, \\ \phi_{11}^i &= A_i^T P_i + P_i A_i + Y_i^T + Q_i + t_m X_{11}^i \\ \phi_{12}^i &= P_i E_i - Y_i + U_i^T + t_m X_{12}^i \\ \phi_{22}^i &= -U_i - U_i^T - e^{-\alpha t_m} Q_i + t_m X_{22}^i. \end{aligned}$$

Proof: Assumption 1 enables to regard B_σ in (30) as 0 insofar as stability is concerned. Therefore (30) is asymptotically stable, if the following system is asymptotically stable [5]:

$$\dot{x}(t) = A_\sigma x(t) + E_\sigma x(t - T_d) \quad (31)$$

where $T_d = 2T$. Since constant $T \leq t_m/2$, the following conditions on T_d are satisfied:

$$0 \leq T_d \leq t_m; \dot{T}_d = 0$$

which confirm that (31) is a particular case of (A1) of Theorem 1 in [16] when $\tau = t_m$ and $d = 0$.

Further, Remark 2 in [16] proves that (31) is asymptotically stable subject to *arbitrary* switching signals, provided that $\mu = 1$ and $\alpha = 0$ from Theorem 1 in [16]. Therefore, Theorem A can immediately be proved by the particular case of Theorem 1 in [16] when $\tau = t_m$, $d = 0$, $\mu = 1$, and $\alpha = 0$.

Theorem A is less conservative than (19) and (28) because it is a delay-dependent method (see t_m in the theorem). However, recalling the stability diagram of the mTOPF based on Nyquist criterion (see Fig. 4), one may still wonder that solutions for satisfying theorem A exist, especially with soft environments. In order to answer the question, we numerically performed extensive stability analysis based on the theorem with numerous samples in broad range of environments, and found that the combined use is asymptotically stable if the maximum time delay ($t_m/2$) is limited. More details are presented in Appendix B.

V. Experiments

In order to verify two-channel TOCAs and their application schemes, we conducted 1-DOF experiments using the following experimental setup: as the master—a PHANTOM-type device with a ATI mini force sensor [see Fig. 6(a)]; as the slave—a SCARA-type manipulator with a roller and a ATI Gamma force sensor [see Fig. 6(b)]; as the environment—a silicon wall with a stiffness of about 15000 N/m [see Fig. 6(b)]. Note that each one joint of the master/slave, indicated in Fig. 6, was locked for 1-DOF experiments.

All TOCAs—four-channel, three-channel (OFC/EFC), and two-channel—were implemented in QNX, a real-time operating system based on PC, with a sampling rate of 1 kHz. The time delay between the master and the slave was set to be 500 ms, and acceleration was obtained by numerical differentiation with a 30 Hz low-pass filter.

The parameters of the general teleoperator architecture are listed in Table II for all the TOCAs implemented. For fair comparison, all TOCAs used the same parameter values if possible; the exceptions were that some parameters were fixed because of the constraints in (6)–(10).

The scenario for the experiment consists of two kinds of motion: free motion (0–50 s) and constrained motion with the silicon wall (50–80 s). In performing these motions, the operator tried his best to make the master track a predefined position profile, which is identical for all TOCAs.

As to the combined use of the two-channel TOCAs, TOFP was activated in the free motion, which is switched to mTOPF in the constrained motion. For the switching rule of this combined use, the force threshold (F_{th}) was set to 0.5 N based on the resolution of the force sensor attached at the slave.

The experimental results of all the TOCAs are displayed in Figs. 7–9: Fig. 7 shows the *overall* response, whereas Fig. 8 closes up the tracking errors in the *free motion* and Fig. 9 in the *constrained motion*. It is noteworthy in Fig. 7(c) that EFC goes unstable during constrained motion. EFC is unsuitable for this constrained motion because Z_e of the silicon wall ($k_e \approx 15\,000$) is much larger than Z_h represented as (21). This instability of EFC confirms (20) and the results in [3].

Fig. 8 displays that two-channel TOFP yields position tracking error between the master and the slave approximately the same as the three-channel TOCAs do and much smaller than the

four channel TOCAs. Regarding force tracking error between the two, the two-channel TOFP was much better than all the other TOCAs. Although a negligible level of force is expected in the free motion, significant level of force was perceived both in the three-channel and four-channel TOCAs, making the desired movement more difficult. Since the position and force tracking performance in free motion implies transparency, the two-channel TOFP demonstrates far superior transparency.

In the interpretation of Fig. 9 it is helpful to recall from (1) that the transmitted impedance is meaningful to check transparency in constrained motion. Since the velocity and acceleration are negligible in contact with a stiff wall, only the stiffness—the static version of impedance—is dominant in the transmitted impedance. Hence, the stiffness evaluated by the position/force of the master was used to compare transparency instead of force tracking error.

Fig. 9 displays that position tracking performance of the two-channel mTOPF is similar to that of the three-channel TOCAs and much better than the four-channel TOCA. As to the transmitted stiffness, the two-channel mTOPF is the closest to the environment stiffness, 15 000 N/m (see Fig. 9). This result confirms the prediction in Fig. 5 that mTOPF still preserves superiority over the other TOCAs.

The overall experimental results consistently show that the transparency of two-channel TOCAs is better than the existing TOCAs under time delay. Moreover, thanks to the combined use of mTOPF and TOFP, they can stably conduct teleoperation tasks that involve free motion, constrained motion, and their transition.

VI. Conclusion

This paper has proposed two classes of two-channel TOCAs. In comparison with the existing TOCAs, these two-channel TOCAs can achieve improved transparency under time delay. From the application viewpoint of the two-channel TOCAs, TOPF was modified to enhance its stability (mTOPF), and the combined use of mTOPF and TOFP was proposed for teleoperation tasks with free and constrained motion together. The proposed theorem gave asymptotical stability for the combined use. Through the 1-DOF experiment, the proposed two-channel TOCAs and their application schemes were verified and compared with the existing TOCAs. The two-channel TOCAs (mTOPF and TOFP) are only feasible by employing a switching controller between the two. A single two-channel TOCA does not have a satisfactory stability region as described in Section III.

This paper does not consider possible uncertainties, such as modeling error, force sensor noise, and local delay. For the sake of more practical research, the effect of the uncertainties in TOCAs will be important.

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References

1. Lawrence DA. Stability and transparency in bilateral teleoperation. *IEEE Trans Robot Autom.* Oct; 1993 9(5):624–637.
2. Hokayem PF, Spong MW. Bilateral teleoperation: An historical survey. *Automatica.* 2006; 42(12): 2035–2057.

3. Hashtrudi-Zaad K, Salcudean SE. Transparency in time-delayed systems and the effect of local force feedback for transparent teleoperation. *IEEE Trans Robot Autom.* Feb; 2002 18(1):108–114.
4. Hashtrudi-Zaad K, Salcudean SE. Analysis of control architectures for teleoperation systems with impedance/admittance master and slave manipulators. *Int J Robot Res.* 2001; 20(6):419–445.
5. Aziminejad A, Tavakoli M, Patel RV, Moallem M. Transparent time-delayed bilateral teleoperation using wave variables. *IEEE Trans Control Syst Technol.* May; 2008 16(3):548–555.
6. Reboulet, C.; Plihon, Y.; Briere, Y. Interest of the dual hybrid control scheme for teloperation with time delays. *Proc. Experimental Robot. IV, 4th Int. Symp. Experimental Robot*; 1995. p. 498–506.
7. Fite KB, Speich JE, Goldfarb M. Transparency and stability robustness in two-channel bilateral telemanipulation. *ASME J Dyn Syst, Meas, Control.* 2001; 123:400–407.
8. Fite KB, Shao L, Goldfarb M. Loop-Shaping for transparency and stability robustness in bilateral telemanipulation. *IEEE Trans Robot Autom.* Jun; 2004 20(3):620–624.
9. Anderson RJ, Spong MW. Bilateral control of teleoperators with time delay. *IEEE Trans Autom Control.* May; 1989 34(5):494–501.
10. Imaida T, Yokokohji Y, Doi T, Oda M, Yoshikawa T. Ground-space bilateral teleoperation of ETS-VII robot arm by direct bilateral coupling under 7-s time delay condition. *IEEE Trans Robot Autom.* Jun; 2004 20(3):499–511.
11. Salcudean SE, Hashtrudi-Zaad K, Tafazoli S, DiMaio SP, Reboulet C. Bilateral matched-impedance teleoperation with application to excavator control. *IEEE Control Syst Mag.* Dec; 1999 19(6):29–37.
12. Aliaga I, Rubio A, Sanchez E. Experimental quantitative comparison of different control architectures for master-slave teleoperation. *IEEE Trans Control Syst Technol.* Jan; 2004 12(1):2–11.
13. Baier H, Schmidt G. Transparency and stability of bilateral kinesthetic teleoperation with time-delayed communication. *J Intell Robot Syst.* 2004; 40(1):1–22.
14. Salcudean SE, Zhu M, Zhu WH, Hashtrudizaad K. Transparent bilateral teleoperation under position and rate control. *Int J Robot Res.* 2000; 19:1185–1202.
15. Hirche, S.; Bauer, A.; Buss, M. Transparency of haptic telepresence systems with constant time delay. *Proc. IEEE Conf. Control Appl*; 2005. p. 328–333.
16. Sun X-M, Zhao J, Hill DJ. Stability and L2-gain analysis for switched delay systems: A delay-dependent method. *Automatica.* 2006; 42:1769–1774.
17. Hannaford B. A design framework for teleoperators with kinesthetic feedback. *IEEE Trans Robot Autom.* Aug; 1989 5(4):426–434.
18. Brooks, TL. Telerobotic response requirements. *Proc. IEEE Int. Conf. Syst., Man, Cybern*; 1990. p. 113–120.
19. Colgate JE, Hogan N. Robust control of dynamically interacting systems. *Int J Control.* 1988; 48:65–88.
20. Hogan, N. Controlling impedance at the man/machine interface. *Proc. IEEE Int. Conf. Robot. Autom*; 1989. p. 1626–1631.
21. Kazerooni H, Tsay T-I, Hollerbach K. A controller design framework for telerobotic systems. *IEEE Trans Control Syst Technol.* Jan; 1993 1(1):50–62.
22. Tsuji T, Morasso PG, Goto K, Ito K. Human hand impedance characteristics during maintained posture. *Biologic Cybern.* 1995; 72:295–305.
23. Rahman, MM.; Ikeura, R.; Mizutani, K. Investigating the impedance characteristic of human arm for development of robots to co-operate with human operators. *Proc. IEEE Int. Conf. Syst., Man, Cybern*; 1999. p. 676–681.
24. Flemmer, H.; Eriksson, B.; Wikander, J. Control design for transparent teleoperators with model parameter variation. *Proc. IEEE Int. Conf. Robot. Autom*; 2002. p. 2956–2961.
25. Tsuji T, Takeda Y, Tanaka Y. Analysis of mechanical impedance in human arm movements using a virtual tennis system. *Biologic Cybern.* 2004; 91:295–305.
26. Çavusoglu MC, Sherman A, Tendick F. Design of bilateral teleoperation controllers for haptic exploration and telemanipulation of soft environments. *IEEE Trans Robot Autom.* Aug; 2002 18(4):641–647.

27. Kron A, Schmidt G. Stability and performance analysis of kinesthetic control architectures for bimanual telepresence systems. *J Intell Robot Syst.* 2006; 46(1):1–26.
28. Chang PH, Kim J. Telepresence index for bilateral teleoperations. *IEEE Trans Syst, Man, Cybern B, Cybern.* 10.1109/TSMCB.2011.2160849
29. Mahapatra, S.; Zefran, M. Stable haptic interaction with switched virtual environments. *Proc. IEEE Int. Conf. Robot. Autom;* 2003. p. 1241-1246.

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Appendix

A. Discussion on the Existing 4CH/3CH Architectures Developed for Enhancing Transparency

Recently, there has been a study to enhance transparency under time delay [5]. This study developed a new framework, passivity-based delay-compensated 4CH teleoperation system—a combination of the general teleoperator architecture (see Fig. 1) and time-delay compensated channels [5]. In order to enhance transparency, the existing TOCAs, (6)–(8), were directly employed in this framework, thus a *4CH architecture* and two classes of *3CH architecture* were proposed [5]. Since the time-delay compensated channels cause these architectures to lose the transparency condition (1) [5], they cannot be regarded as TOCA. Note that none of the two-channel architecture were proposed in [5].

Under time delay it is more obvious that the 4CH/3CH architectures leave room for improvement on transparency. The hybrid parameters under time delay for the 4CH architecture [5] are represented as follows:

$$\begin{aligned} h_{11} &= \frac{W_t(Z_{cs}^2 - b^2 Z_{cm}^2)}{D} \\ h_{22} &= \frac{W_t(b^2 C_2^2 - C_3^2)}{D} \\ h_{12} &= \frac{2bW_t e^{-Ts}(Z_{cm}C_3 + Z_{cs}C_2)}{D} \\ h_{21} &= -h_{12} \end{aligned} \quad (32)$$

where $D = b(W_t + 2)(Z_{cs}C_2 + Z_{cm}C_3) - W_t(bC_2Z_{cm} + C_3Z_{cs})$; $W_t = W^2 e^{-2Ts} - 1$; with W denoting a first-order low-pass filter [5], and with b the characteristic wave impedance [5]. Similar to (15), (32) shows that the 4CH architecture cannot achieve the delayed impedance matching (11), which is the best transparency achievable under time delay. In order to achieve the delayed impedance matching, [5] proposed additional conditions of b . These conditions, however, are only valid under the assumption that both the master and the slave

are represented as *masses* [5]. This assumption is far from realistic and sets the transparency limitation of 4CH architecture under time delay.

As to 3CH architectures [5], the hybrid parameters under time delay are represented as follows:

$$\begin{cases} h_{11}=0; h_{22}=\frac{W_t}{2Z_{cm}}; h_{12}=We^{-Ts}; h_{21}=-We^{-Ts} \\ h_{11}=0; h_{22}=-\frac{W_t}{2Z_{cs}}; h_{12}=We^{-Ts}; h_{21}=-We^{-Ts}. \end{cases} \quad (33)$$

Even though the low-pass filter is not considered ($W=1$), h_{22} of (33) is still different from that of (11), which shows that the 3CH architectures cannot achieve the delayed impedance matching, either.

B. Stability Analysis of Combined Use of mTOPF and TOPF

In order to analyze stability of the combined use of mTOPF and TOPF based on theorem A, two thousands samples in the following broad range of environments were picked up:

$$0 < b_e < 100; 1.27 < k_e < 200\,000.$$

Recall that the stability of the TOPF can be guaranteed by using the conservative condition (19) if $k_e < 1.27$. It is noteworthy that 52% of the total samples were in the range of soft environments.

Based on the samples above, the following extensive stability analysis was performed by using the theorem. For each sample, we first tried to search suitable matrices ($P, Q, R, X_1, X_2, Y_1, Y_2, U_1$, and U_2) to meet the stability conditions in theorem A under 10 s of maximum time delay. If there was no solution in a certain number of trials, the maximum delay decreased to next level ($5 \rightarrow 1 \rightarrow 0.5 \rightarrow 0.1 \rightarrow 0.05 \rightarrow 0.01$ s), and we tried to search again. This loop was terminated when the suitable matrices were obtained (asymptotically stable), or after we could not find any solution in spite of 0.01 s maximum delay (no stability guarantee). Note that the operator impedance was set to the maximum values in (21) for worst case consideration, and we set $\omega_n = 2$ and $\zeta = 1$ for mTOPF.

In contrast to the stability analysis based on Nyquist criterion, the result based on the proposed theorem was very positive; all two thousands samples were asymptotically stable with up to 0.5 s time delay. In detail, the maximum allowable time delay for guaranteeing stability at all environment samples is displayed in Fig. 10. Although the allowable delay is large enough (10 or 5 s) in large range of environments [see Fig. 10(a)], it is remarkably limited (0.5 s) in some samples, such as soft environments [see Fig. 10(b)]. Thanks to the delay-dependent theorem, it can be verified that the proposed combined use is asymptotically stable under limited time delay, even with soft environments as well as switching between mTOPF and TOPF.

Fig. 1.
General teleoperator architecture [3], [4].

Fig. 2. Maximum-minimum magnitude of operator impedance with magnitude of some environment impedances.

Fig. 3. Maximum-minimum magnitude of filtered operator impedance with the magnitude of environment impedance.

Fig. 4. Stability diagram of mTOPF based on Nyquist criterion (a) with respect to ω_n , $\zeta = 1$ and (b) with respect to ζ , $\omega_n = 2$ (Hz).

Fig. 5. Transparency comparison between mTOPF and three-channel TOCAs. (a) Comparison between mTOPF and OFC. (b) Comparison between mTOPF and EFC.

Fig. 6.
Experimental setups: master, slave and environment. (a) Master; (b) Slave, Environment.

Fig. 7.

Experimental results of TOCAs: Transmitted and environment stiffness was calculated during the constrained motion. (a) Four-channel TOCA. (b) Three-channel TOCA: OFC. (c) Three-channel TOCA: EFC. (d) Two-channel TOCA: mTOPF and TOFP.

Fig. 8.

Experimental comparison in free motion: Position/force tracking errors are defined as $x_m(t) - x_s(t)$, and $f_s(t) - f_m(t)$, where x_m , x_s , f_s , and f_m denote the master position, the slave position, the slave force, and the master force, respectively.

Fig. 9.
Experimental comparison in constrained motion.

Fig. 10. Maximum allowable time delay for asymptotic stability based on Theorem A. (a) Broad range of environments. (b) Range of soft environments.

TABLE I

Prior Knowledge of Operator Impedance From Literatures

	Mass (Kg)	Damper (Ns/m)	Stiffness (N/m)
Hogan, 1989 [20]	0.8	5.5	568
Kazarooni, 1993 [21]	3.97~4.54	4.57~6.83	10~12.5
Tsuji, 1995 [22]	0.388~2.509	1.83~24.52	16.45~486.82
Flemmer, 2002 [23]	0.027	0.67~0.81	12~32
Tendick, 2002 [24]	(0)*~0.1	4.2~(infinite)*	200~(2000)*
Tsuji, 2004 [25]	1.2~2.6	13.4~29.5	42.2~275.1

Note that () * values are excluded to decide (21) owing to their extremeness.

TABLE II

Parameters of General Teleoperator Architecture Under All TOCAs Implemented in the Experiments

	4ch	3ch OFC	3ch EFC	2ch mTOPF	2ch TOPF
C_1		$2s+2000+10000/s$		0^*	same to 4ch
C_2	1	0^*	1	0^*	1
C_3	10	10	0^*	$10Q^*$	0^*
C_4		$-0.01s-40-200/s$			0^*
C_5	9	9	-1^*	9	-1^*
C_6	0	-1^*	0	-1^*	0
C_m		$39+200/s$			$-0.01s-1^*$
C_s		$1460+10000/s$		$-2s-40^*$	same to 4ch

* These parameter values are fixed because of the constraint in (6)~(10).

Note that $\omega_\eta = 2$ Hz, $\zeta=1$ in the filter Q in 2ch mTOPF.