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The geometric response function for convergent slit-slat collimators

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Abstract

We have derived an analytic geometric transfer function (GTF) for the convergent slit-slat collimator that treats the parallel slit-slat collimator as a special case. The effective point spread function (EPSF) is then derived from the GTF through the Fourier transform. The results of these derivations give an accurate description of the complete geometric response for a slit-slat collimator that includes the effects of the shape and orientation of the slit and slats. We have also derived exact and approximate sensitivity formulas and spatial resolution formulas using the effective point spread function.

1. Introduction

Slit-Slat (SS) collimation is often used in SPECT systems. It is comprised of two physically separate components: a slit plate and a stack of slats. The slit plate is parallel to the axis of rotation of the SPECT system. The slats are parallel to a transverse plane or converge to a focal line that lies in the transverse plane. This method of collimation was first described as part of a dedicated brain SPECT system (Rogers et al. 1982). The SS collimator has been used subsequently in other SPECT systems (Rogers et al. 1988, Juni 2003, Walrand et al. 2005, Chang et al. 2006, Chang et al. 2008). Recent publications have shown the theoretical and experimental performance of SS collimation (Wieczorek and Goedicke 2006, Metzler et al. 2006).

With axially-parallel slats, SS collimation is operated in a slice-collimated (2D) mode, where axial collimation is provided by the parallel slats. Collimation in the transverse plane is accomplished by the slit, which acts as a 1D pinhole that can magnify or minify its projection by adjusting the geometry parameters. With axially-convergent slats, SS collimation is operated in a three-dimensional (3D) mode, which provides higher sensitivity than parallel slats between the focal line of the slats and the slit plate. This collimation geometry is a variation of cone-beam collimation with inherent tradeoffs among the properties of sensitivity, resolution, field of view and other considerations (Hawman and Hsieh 1987, Gullberg et al. 1992).

The characteristics of collimator are usually described in terms of sensitivity and spatial resolution with a radioactive point source placed in the field of view. Collimator septa complicate the expressions by causing small but significant (usually) periodic fluctuations in the collimator response function due to the septa shadowing effect and penetration. The sensitivity and spatial resolution of a collimator are better described by the effective point spread function (EPSF), which is the inverse Fourier transform of the geometric transfer function (GTF). The EPSF was originally proposed for parallel hole collimators (Metz et al. 1980) and extended for fan-beam, cone-beam and slat collimators (Tsui and Gullberg 1990, Frey et al. 1998, Staelens et al. 2005). The EPSF averages over all translated positions of the

collimator holes with respect to the point source. In this paper, we use similar methods to derive the EPSF for a slit-slat collimator.

The EPSF can be useful in collimator-response compensation in both analytic and iterative image reconstruction algorithms (Tsui et al. 1988, Xia et al. 1995, Formiconi et al. 1989, Frey and Tsui 2006). Based on the EPSF, we derived sensitivity and spatial resolution formulas for the SS collimation. These functions are crucial in comparing SS collimation performance to other collimation types and in identifying applications that would benefit from using a SS collimation; for example, the recently introduced dedicated cardiac C-SPECT systems (Chang et al. 2006, Chang et al. 2008) and the human brain SPECT system (Mahmood et al. 2008). In addition, this information will aid in designing of collimators and developing of simulations and image reconstruction algorithms. Accorsi *et al.* have recently reported a sensitivity formula for the SS collimation that is similar to what we have derived using a different method (Accorsi et al. 2008). However, it is only one component in the larger scope of work presented here.

2. Geometric Response Function

The schematic diagram of a slit-slat collimator is illustrated in figure 1. Following the method in (Accorsi et al. 2008), we let the x and y axes lie in the plane of the slit, with the y axis running along the centerline of the slit plate and the x axis perpendicular. The z axis is perpendicular to the slit plane. Assume the slit has width w in x direction and is infinite in the y direction. We locate a point source of gamma radiation at $P = (x_p = h \cot \theta, y_p, z_p = h)$ in the quadrant of space $x, z \geq 0$. The planes of the converging slats intersect at the focal line given by $(y = 0, z = F - s)$, where F is the focal length of the slat assembly, and s is the distance between the slit plane and entrance plane of the slats. On the entrance plane, the slats are separated by a distance $d + t$, d is the free space between two septa, and t is their thickness. The slat collimator has a thickness (or height) of a , and the distance between the back of the slats and the imaging plane is B . h and $b = h + s$ are the distances from the point source to the slit plane and slat entrance plane respectively.

Our geometric response function (GRF) derivation for SS collimator uses the same approach as in (Metz et al. 1980, Tsui and Gullberg 1990, Frey et al. 1998). For each collimator hole, we define front and back aperture functions and find the projection of these aperture functions onto the imaging plane. The overlap between the aperture functions projected onto the imaging plane is the area where a gamma ray that passes through that collimator hole can be detected. This process is repeated for the other holes. By averaging over all collimator holes, we can find an average geometric response function (GRF) for the collimator system.

A slit-slat collimator uses a slit to achieve transverse collimation and slats to achieve axial collimation, which makes a separable point spread function in x, y direction,

$$\psi(x, y; x_p, y_p) = K(x, y) \psi_x(x, x_p) \psi_y(y, y_p) \quad (1)$$

where,

$$K(x, y) = \frac{H+B}{4\pi \left[(H+B)^2 + (x-x_p)^2 + (y-y_p)^2 \right]^{3/2}} \quad (2)$$

Here $H = h + s + a = b + a$, the multiplicative factor $K(x, y) = \cos^3 \phi (4\pi(H+B)^2)$ models the detection efficiency of the detector, and $\cos \phi = (H+B)/R$ with $R = [(H+B)^2 + (x-x_p)^2 + (y-y_p)^2]^{1/2}$.

$(y - y_p)^2]^{1/2}$. This $\cos^3\phi$ dependence can be interpreted as the product of an inverse square decrease with distance, providing a $\cos^2\phi$ dependence, and multiplied by a $\cos\phi$ dependence due to the obliquity between the rays and the detector plane.

The point spread function (PSF) in x direction can be obtained from the projection of the slit aperture function $a_{\text{slit}}(x)$. The projection is done from the source location, $P = (x_p, y_p, h)$, onto the imaging plane which is located at $z = -(a + s + B)$. The projection is scaled by

$\frac{H+B}{h}$, and its center is translated to $x_0 = -\frac{a+s+B}{h}x_p$. So the PSF in x direction can be written as,

$$\psi_x(x; x_p) = a_{\text{slit}}\left(\frac{h}{H+B}x + \frac{a+s+B}{H+B}x_p\right) \quad (3)$$

Due to the shadowing effect of the slit collimator septa, the amount of radiation that passes through a single slit hole depends on the exact source position relative to the position of the hole in y direction. So the PSF in y direction cannot be obtained directly from the projection of the slit aperture functions. To avoid the difficulties of space variance, we calculated the effective PSF (EPSF) using a similar approach to that of (Metz et al. 1980, Tsui and Gullberg 1990). To compute the EPSF, the slit collimator hole positions are averaged over the collimator face. Figure 2 shows the schematic illustration of projections from a given slit hole. The slit hole is defined by the front (or entrance) and the back (or exit) aperture functions. The overlap region of these aperture functions on the imaging plane denotes an *open window* for the detection of gamma-rays emitted from a source. The cross-hatched region shown in figure 2 is an example of this overlap from a convergent slit hole.

Assuming the slit hole is centered at y' in the plane of the front face of the slit collimator, the projections of the front and back slit aperture functions onto the imaging plane at the position $\mathbf{r} = (x, y)$ are given by,

$$a_{\text{slat}}^f(y, y'; y_p) = a_{\text{slat}}\left(\frac{b}{H+B}y + \frac{a+B}{H+B}y_p - y'\right) \quad (4)$$

$$a_{\text{slat}}^b(y, y'; y_p) = a_{\text{slat}}\left(\frac{F}{F+a} \frac{b+a}{H+B}y + \frac{F}{F+a} \frac{B}{H+B}y_p - y'\right). \quad (5)$$

Note that the above are derived by assuming the walls of the slit holes are tapered to focus at the same focal line ($y = 0, z = F - s$). If the slit hole is moved to all possible locations y' during image formation, the resulting total fluence for the slit at $\mathbf{r} = (x, y)$ can be obtained by integrating the product of the two projected aperture functions,

$$\psi_y(y; y_p) = \frac{1}{N} \int_{-\infty}^{+\infty} a_{\text{slat}}^f(y, y'; y_p) a_{\text{slat}}^b(y, y'; y_p) dy'. \quad (6)$$

Here $N = \int_{-\infty}^{+\infty} a_{\text{slat}}(y') dy'$ is a normalization factor. By substituting equations (4) and (5) in equation (6) and letting

$$\sigma_y = y' - \frac{b}{H+B}y - \frac{a+B}{H+B}y_p \quad (7)$$

we get,

$$\psi_y(y; y_p) = \frac{1}{N} \int_{-\infty}^{+\infty} a_{\text{slat}}(-\sigma_y) a_{\text{slat}}(y_T - \sigma_y) d\sigma_y \quad (8)$$

where,

$$y_T = \frac{a}{H+B} \frac{F-b}{F+a} \left(y - \frac{F+a+B}{F-b} y_p \right) \equiv m_y (y - y_0) \quad (9)$$

Here, $m_y = \frac{a}{H+B} \frac{F-b}{F+a}$ and $y_0 = \frac{F+a+B}{F-b} y_p$. The Fourier transform of $\psi_y(y; y_p)$ is given by,

$$\Psi_y(v_y) = \int_{-\infty}^{+\infty} \psi_y(y; y_p) e^{-j2\pi v_y y} dy = \frac{1}{N} e^{-j2\pi y_0 v_y} \frac{1}{|m_y|} \left| A_{\text{slat}} \left(\frac{v_y}{m_y} \right) \right|^2 \quad (10)$$

The Fourier transform of $\psi_x(x; x_p)$ is given by,

$$\Psi_x(v_x) = \int_{-\infty}^{+\infty} \psi_x(x; x_p) e^{-j2\pi v_x x} dx = e^{-j2\pi x_0 v_x} \frac{1}{|m_x|} A_{\text{slit}} \left(\frac{v_x}{m_x} \right) \quad (11)$$

Here, $m_x = \frac{h}{H+B}$ and $x_0 = -\frac{a+s+B}{h} x_p$. $A_{\text{slat}}(v_y)$ and $A_{\text{slit}}(v_x)$ are the Fourier transforms of aperture functions $a_{\text{slat}}(y)$ and $a_{\text{slit}}(x)$, respectively. Putting equation (10) and (11) into equation (1) and taking two-dimensional Fourier transform, the total geometric transfer function can be written as,

$$\Psi(v_x, v_y) \approx \frac{K_0}{N} \frac{1}{|m_x m_y|} e^{-j2\pi x_0 v_x} e^{-j2\pi y_0 v_y} A_{\text{slit}} \left(\frac{v_x}{m_x} \right) \left| A_{\text{slat}} \left(\frac{v_y}{m_y} \right) \right|^2. \quad (12)$$

Here $K_0 = K(x_0, y_0)$, we used that the value of $K(x, y)$ given by equation (2) is essentially a constant when $\Psi(x, y; x_p, y_p)$ is non-zero near (x_0, y_0) (Metz et al. 1980, Tsui and Gullberg 1990). Without penetration, the aperture functions of slit and slat can be modeled as rectangle functions, *i.e.* they are $a_{\text{slit}}(x) = \Pi(x/w)$ and $a_{\text{slat}}(y) = \Pi(y/d)$ respectively. Here $\Pi(x)$ is,

$$\Pi(x) = \begin{cases} 1, & \text{for } |x| < 1/2 \\ \frac{1}{2}, & \text{for } |x| = 1/2 \\ 0, & \text{for } |x| > 1/2 \end{cases} \quad (13)$$

The Fourier transform of $\Pi(y/d)$ is $d \text{sinc}(dv_y)$. The sinc function is defined by $\text{sinc}(v) = \sin(\pi v)/(\pi v)$. The normalization factor N in equation (6) then becomes $N = d$. When taking penetration into account, an effective aperture function $a_{\text{slit}}^{\text{eff}}(x)$ will be used instead of $a_{\text{slit}}(x)$, and the effective aperture function of the slit collimator can be written as,

$$a_{\text{slit}}^{\text{eff}}(x) = \begin{cases} 1, & \text{for } |x| \leq \frac{w}{2} \\ e^{-\mu \kappa(|x| - \frac{w}{2})}, & \text{for } |x| > \frac{w}{2} \end{cases} \quad (14)$$

Here, μ is the linear attenuation coefficient of the slit material, and $\kappa(|x| - \frac{w}{2}) = \Delta L$ gives the length of the intersection path of gamma rays with the slit material. As shown in the appendix, the Fourier transform of effective aperture function of the slit can be written as,

$$A_{\text{slit}}^{\text{eff}}(v_x) = \frac{2\pi\mu\kappa v_x \cos(\pi w v_x) + \mu^2 \kappa^2 \sin(\pi w v_x)}{\pi v_x (\mu^2 \kappa^2 + 4\pi^2 v_x^2)} \quad (15)$$

Putting equation (15) and other parameters (m_x , m_y , x_0 and y_0) into equation (12), the final geometric transfer function can be given by,

$$\begin{aligned} \Psi(v_x, v_y) &\approx K_0 d \frac{(H+B)^2}{ah} \frac{F+a}{F-b} \exp\left(j2\pi \frac{a+s+B}{h} x_p v_x\right) \exp\left(-j2\pi \frac{F+a+B}{F-b} y_p v_y\right) \\ &\times \frac{2\pi \frac{H+B}{h} \mu\kappa v_x \cos\left(\pi \frac{H+B}{h} w v_x\right) + \mu^2 \kappa^2 \sin\left(\pi \frac{H+B}{h} w v_x\right)}{\pi \frac{H+B}{h} v_x (\mu^2 \kappa^2 + 4\pi^2 (\frac{H+B}{h})^2 v_x^2)} \\ &\times \text{sinc}^2\left(\frac{H+B}{a} \frac{F+a}{F-b} d v_y\right). \end{aligned} \quad (16)$$

Using $A_{\text{slat}}(v_y) = d \text{sinc}(d v_y)$ and $N = d$, and taking the inverse Fourier transform of equation (10), the line spread function (LSF) in the y direction can be written as,

$$\psi_y(y; y_p) = \Lambda\left(\frac{m_y(y - y_0)}{d}\right). \quad (17)$$

Here, we used the equation that the inverse Fourier transform of $\text{sinc}^2(v_y)$ is $\Lambda(y)$, and $\Lambda(y)$ is,

$$\Lambda(y) = \Pi * \Pi(y) = (1 - |y|) \Pi\left(\frac{y}{2}\right). \quad (18)$$

Then, the total point spread function in equation (1) becomes,

$$\psi(x, y; x_p, y_p) = K(x, y) a_{\text{slit}}^{\text{eff}}\left(\frac{m_x(x - x_0)}{w}\right) \Lambda\left(\frac{m_y(y - y_0)}{d}\right). \quad (19)$$

Without penetration, the point spread function in equation (19) can be given simply by,

$$\psi(x, y; x_p, y_p) = \frac{\Pi\left(\frac{h}{(H+B)w} \left(x + \frac{a+s+B}{h} x_p\right)\right) \Lambda\left(\frac{h}{(H+B)w} \left| \frac{F-b}{F+a} \right| \left(y - \frac{F+a+B}{F-b} y_p\right)\right)}{4\pi(H+B)^2 \left[1 + \left(\frac{x-x_p}{H+B}\right)^2 + \left(\frac{y-y_p}{H+B}\right)^2\right]^{3/2}} \quad (20)$$

In most scenarios where penetration is not a concern, we can simplify the geometric transfer function by just letting the attenuation coefficient $\mu \rightarrow \infty$, and then the effective aperture function in equation (14) becomes $\Pi(x/w)$, and its Fourier transform in equation (15) becomes $w \text{sinc}(w v_x)$. We can also consider penetration by defining the *effective width* of the slit w_{eff} instead of w as

$$w_{\text{eff}} = w + \frac{2}{\mu\kappa}. \quad (21)$$

Typical profiles of PSF and GTF are given by figures 3 and 4. For any two-dimensional object located on a plane parallel to the imaging plane, the detected image by the slit-slat collimator can be obtained from the PSF. Suppose that a two-dimensional object distribution

$O(x_p, y_p)$ is placed at a distance h from the slit plane. Then the detected image can be written as

$$I(x, y) = \int_{-\infty}^{+\infty} O(x_p, y_p) \psi(x, y; x_p, y_p) dx_p dy_p. \quad (22)$$

For a three-dimensional object, the object can be imagined to have many slices parallel to the imaging plane each with different h . Thus the detected image can be obtained by taking an additional integral over the variable h .

3. Sensitivity

Sensitivity is typically defined as the fraction of gamma rays emitted by a point source of radiation that passes through the collimator and reaches the detector (Cherry et al. 2003). This quantity excludes effects such as detection efficiency, dead time, etc., and it has thus been referred to as the geometrical efficiency of the collimator system (Accorsi et al. 2008). The total sensitivity, g , can be obtained by integrating the point spread function of a point source over the surface of the imaging plane $D = [-L_x/2, L_x/2] \times [-L_y/2, L_y/2]$.

$$g = \frac{d}{d+t} \iint_D \psi(x, y; x_p) dx dy \quad (23)$$

Putting equation (20) into equation (23), we get

$$g = \frac{d}{4\pi(d+t)(H+B)} \int_{-L_x/2}^{L_x/2} \Pi\left(\frac{m_x}{w}(x-x_0)\right) \varphi_x dx \quad (24)$$

with

$$\varphi_x = \frac{1}{H+B} \int_{-L_y/2}^{L_y/2} \Lambda\left(\frac{m_y}{d}(y-y_0)\right) \left[1 + \left(\frac{x-x_p}{H+B}\right)^2 + \left(\frac{y-y_p}{H+B}\right)^2\right]^{-3/2} dy \quad (25)$$

Change variables with $\xi = \frac{x-x_0}{H+B}$ and $\zeta = \frac{y-y_0}{H+B}$, then equations (24) and (25) become,

$$g = \frac{d}{4\pi(d+t)} \int_{\eta_1}^{\eta_2} \varphi_\xi d\xi \quad (26)$$

and

$$\varphi_\xi = \int_{\lambda_1}^{\lambda_2} \left(1 - \frac{|\zeta|}{\lambda}\right) \left[1 + \left(\xi + \frac{x_p}{h}\right)^2 + \left(\zeta + \frac{y_p}{F-b}\right)^2\right]^{-3/2} d\zeta \quad (27)$$

where

$$\eta_1 \equiv \max\left[-\eta, \frac{-L_x - 2x_0}{2(H+B)}\right], \quad \eta_2 \equiv \min\left[\eta, \frac{L_x - 2x_0}{2(H+B)}\right] \quad (28)$$

$$\lambda_1 \equiv \max\left[-\lambda, \frac{-L_y - 2y_0}{2(H+B)}\right], \quad \lambda_2 \equiv \min\left[\lambda, \frac{L_y - 2y_0}{2(H+B)}\right] \quad (29)$$

and

$$\eta = \frac{w}{2h}, \quad \lambda = \frac{d}{a} \left| \frac{F+a}{F-b} \right|. \quad (30)$$

For an infinitely large detector in y direction, *i.e.* $L_y \rightarrow \infty$, then $\lambda_2 = -\lambda_1 = \lambda$. Actually, this condition is not always necessary. When the point source is not near the focal point, the slit assembly satisfies $d \ll a$, *i.e.* $\lambda \ll 1$; this also gives $\lambda_2 = -\lambda_1 = \lambda$. Integration of equation (27) over ζ is possible via the identity (Gradshteyn and Ryzhik 2007)

$$\int \frac{P+Q\zeta}{(R+(\zeta-\zeta_0)^2)^{\frac{3}{2}}} d\zeta = \frac{(P+Q\zeta_0)(\zeta-\zeta_0)-QR}{R\sqrt{R+(\zeta-\zeta_0)^2}} \quad (31)$$

which results in

$$\varphi_\xi = \frac{1}{\lambda} [T(\xi, \lambda) + T(\xi, -\lambda) - 2T(\xi, 0)] \quad (32)$$

where

$$T(\xi, \zeta) = \frac{\sqrt{1+(\xi-\xi_0)^2+(\zeta-\zeta_0)^2}}{1+(\xi-\xi_0)^2} \quad (33)$$

Here ξ_0 and ζ_0 are given in equation (39). The integration in equation (26) is the essential integral of $T(\xi, \zeta)$, and we define $m(\xi, \zeta)$ as the indefinite integral of $T(\xi, \zeta)$ with respect to variable ξ , and hence $m(\xi, \zeta)$ can be given by

$$m(\xi, \zeta) = (\zeta - \zeta_0) \tan^{-1} \frac{(\xi - \xi_0)(\zeta - \zeta_0)}{\sqrt{1+(\xi-\xi_0)^2+(\zeta-\zeta_0)^2}} + \log \left[\sqrt{1+(\xi-\xi_0)^2+(\zeta-\zeta_0)^2} + (\xi - \xi_0) \right]. \quad (34)$$

Finally, the general sensitivity formula of the slit-slat collimator can be given by

$$g = \frac{d}{4\pi(d+t)\lambda} [m(\eta_2, \lambda) + m(\eta_2, -\lambda) - 2m(\eta_2, 0) - m(\eta_1, \lambda) - m(\eta_1, -\lambda) + 2m(\eta_1, 0)] \quad (35)$$

Equation (35) gives an exact general sensitivity formula for a slit-slat collimator, which is valid for the whole region. For point sources in special regions, the formula can be simplified. For example, consider a point source on a line perpendicular to both the focal line of the slats and the centerline of the slit plate (the lines intersect both the focal line of slat and the centerline of slit plate). We refer to this as the *principal line*, which in our geometry is the z axis. Thus,

$$g_{\text{pri}} = \frac{d}{\pi(d+t)} \left[\tan^{-1} \frac{\eta\lambda}{\sqrt{1+\eta^2+\lambda^2}} + \frac{1}{2\lambda} \log \frac{\sqrt{1+\eta^2+\lambda^2}+\eta}{\sqrt{1+\eta^2+\lambda^2}-\eta} - \frac{1}{2\lambda} \log \frac{\sqrt{1+\eta^2}+\eta}{\sqrt{1+\eta^2}-\eta} \right] \quad (36)$$

The first term in (36) is the exact formula for the solid angle subtended by the normalized rectangular support of the point spread function $[-v, v] \times [-\lambda, \lambda]$ (Fischbeck and Fischbeck 1987). The second and third terms are caused by the triangular shape of the point spread function in the y direction.

If the point source is put in different regions, we can use some simple formulas to give an accurate approximation for the sensitivity. Following Accorsi *et al*, we define the region in

which λ is sufficiently small that $\frac{d}{a} \left| \frac{F+a}{F-b} \right| \ll 1$ as the off-focus region (Accorsi et al. 2008). We also define the region in which v is sufficiently small that $h \ll w$ as the regular region. This condition implies $v_2 = -v_1 = v \rightarrow 0$.

The standard region is defined as the intersection of the off-focus region and the regular region. This condition implies that $v \rightarrow 0$ and $\lambda \rightarrow 0$. Thus, in the standard region, the m functions in (35) can be expanded to third orders in a Taylor series with respect to λ and η .

$$g_{\text{std}} = \frac{d}{4\pi(d+t)} \frac{2\lambda\eta}{(1+\xi_0^2+\zeta_0^2)^{3/2}} (1-\epsilon) \quad (37)$$

where

$$\epsilon = \frac{(1+\xi_0^2-4\zeta_0^2)2\eta^2 + (1+\xi_0^2-4\zeta_0^2)\lambda^2}{4(1+\xi_0^2+\zeta_0^2)^2} \quad (38)$$

To simplify the equation (37), we introduce two parameters: azimuth angle γ_0 and elevation angle ϕ_0 , as shown in figure 5. Here $P = (x_p, y_p, h)$ is the position of point source, $Q = (x_0, y_0, -a-s-B)$ is the center of the EPSF, and P' is the projection of P onto the imaging plane, *i.e.* the xy plane. The angles θ and β are same as in figure 1. The direction of vector $(x_p - x_0, y_p - y_0, H+B)$ is $(\cos\phi_0 \cos\gamma_0, \cos\phi_0 \sin\gamma_0, \sin\phi_0)$, so we can easily get,

$$\begin{aligned} \xi_0 &= \frac{x_p}{h} = \cot\theta = \cot\phi_0 \cos\gamma_0 \\ \zeta_0 &= -\frac{y_p}{F-b} = \cot\beta = \cot\phi_0 \sin\gamma_0. \end{aligned} \quad (39)$$

Then equation (37) becomes

$$g_{\text{std}} = \frac{wd^2}{4\pi ah(d+t)} \left| \frac{F+a}{F-b} \right| \sin^3\phi_0 (1-\epsilon) \quad (40)$$

and

$$\epsilon = \frac{\sin^2\phi_0}{4} \left[\left(1 - 5\cot^2\theta \sin^2\phi_0\right) \frac{w^2}{2h^2} + \left(1 - 5\cot^2\beta \sin^2\phi_0\right) \left(\frac{d}{a} \frac{F+a}{F-b}\right)^2 \right]. \quad (41)$$

In most situations, the first term in equation (40) can give an accurate sensitivity estimation for the standard region, and the second term can be used to calculate the relative error. The first term of equation (40) is identical to equation (19) in (Wieczorek and Goedicke 2006) and to equation (24) in (Accorsi et al. 2008). When the point source is in the principal line, the sensitivity formula is the geometric mean of the sensitivity for pinhole and cone-beam collimators (Metzler et al. 2006, Cherry et al. 2003).

In the off focus region far from the slit plate, only the condition $\lambda \ll 1$ is satisfied. If we expand equation (35) in a Taylor series with respect to λ , we can get the sensitivity formula for the off-focus region,

$$\begin{aligned} g_{\text{off}} &= \frac{d^2}{4\pi a(d+t)} \left| \frac{F+a}{F-b} \right| \frac{\sin^3\phi_0}{1-\cot^2\theta \sin^2\phi_0} [C_2 - C_1] \\ &= \frac{d^2}{4\pi a(d+t)} \left| \frac{F+a}{F-b} \right| \sin^2\beta \sin\phi_0 [C_2 - C_1] \end{aligned} \quad (42)$$

where

$$C_{2,1} = \frac{(\eta_{2,1} - \cot\theta)}{\sqrt{1 - 2\eta_{2,1}\cot\theta\sin^2\phi_0 + \eta_{2,1}^2\sin^2\phi_0}} \quad (43)$$

Equation (42) is essentially the same as equation (22) in (Accorsi et al. 2008), and one easily can verify that $\sin^2\phi_0/(1 - \cot^2\theta \sin^2\phi_0) = \sin^2\beta$. At points closer to the slit plate, $h \ll w$, which implies that $\eta \rightarrow +\infty$, and we call this region near field region as in (Accorsi et al. 2008). A simpler formula can be derived from (42) for the near field region using $\eta \rightarrow +\infty$.

$$g_{\text{nf}} = \frac{d^2}{2\pi a(d+t)} \left| \frac{F+a}{F-b} \right| \sin^2\beta (1 - \varepsilon_1) \quad (44)$$

Where

$$\varepsilon_1 = \frac{1}{2\eta^2 \sin^2\beta} \quad (45)$$

From equation (44), we see that the sensitivity near the slit plate is independent of the incidence angle θ and only depends on angle β or y_p . In the near field region, the slit has no effect on the collimation; only the slat works.

In the regular focus region far from the focal line of slat assembly, only the condition $\eta \ll 1$ is satisfied. If we expand equation (35) in a Taylor series with respect to η , we can get the sensitivity formula for the regular region,

$$g_{\text{reg}} = \frac{wd}{4\pi h(d+t)} \frac{a}{d} \left| \frac{F-b}{F+a} \right| \frac{\sin^2\theta}{\sin\phi_0} [T_1 + T_2 - 2] \quad (46)$$

Where

$$T_{2,1} = \sqrt{1 \pm 2\lambda \cot\beta \sin^2\phi_0 + \lambda^2 \sin^2\phi_0} \quad (47)$$

When the source approaches the focal line of the slat, even for the infinitely large detector, $L_y \rightarrow +\infty$, equation (46) does not diverge for $b \rightarrow F$ (which implies $\lambda \rightarrow +\infty$). At the focus, the sensitivity formula becomes,

$$g_{\text{foc}} = \lim_{\lambda \rightarrow \infty} g_{\text{reg}} = \frac{wd}{2\pi h(d+t)} \sin^2\theta \quad (48)$$

On the focal line, the slats have no effect, and the sensitivity has a $\sin^2\theta$ dependence.

The formulas with different conditions are derived for different regions. A summary of the relevant formulas and their regions applicability is provided in figure 6.

4. Spatial Resolution

Spatial resolution is defined as the FWHM of the system point spread function, and it always includes collimator resolution and intrinsic resolution. Scatter and penetration are not considered initially. For the collimator resolution, the transverse resolution is provided by the slit, and the axial resolution is provided by the slat. The resolution for the collimator in the detector plane can be given by the geometric point spread function in (20),

$$R_{tr}^d = \frac{(h+f)w}{h}, \quad R_{ax}^d = \frac{(h+f)d}{a} \left| \frac{F+a}{F-b} \right| \quad (49)$$

where $f = s + a + B$ is the distance from the slit plate to the detector plane, and $h + f = H + B$. The magnification factor from the patient plane to the detector can be easily given by,

$$m_{tr} = -\frac{f}{h}, \quad m_{ax} = \frac{F+a+B}{F-b}. \quad (50)$$

Here the negative value of the magnification factor in the transverse direction is caused by the inverse fan-beam (1D pinhole) collimation. So the system resolution in the transverse direction is given by

$$R_{tr} = \sqrt{\left(\frac{R_{tr}^d}{|m_{tr}|}\right)^2 + \left(\frac{R_i}{|m_{tr}|}\right)^2} = \sqrt{\frac{(h+f)^2 w^2}{f^2} + \left(\frac{h}{f} R_i\right)^2} \quad (51)$$

Here, R_i is the intrinsic spatial resolution. The system resolution in the axial direction can be given by

$$R_{ax} = \sqrt{\left(\frac{R_{ax}^d}{|m_{ax}|}\right)^2 + \left(\frac{R_i}{|m_{ax}|}\right)^2} = \frac{F+a}{F+a+B} \sqrt{\frac{(h+f)^2 d^2}{a^2} + \left(\frac{F-b}{F+a} R_i\right)^2}. \quad (52)$$

When the focal length $F \rightarrow \infty$, equation (51) and (52) are the same as equations (1) and (2) in (Metzler et al. 2006), and the same as equations (5) and (6) in (Novak et al. 2008). When penetration is considered, we need to put the effective slit width w_{eff} from equation (21) and effective slat hole width $a_{eff} = a - 2/\mu$ instead of w and a into equations (51) and (52) to calculate the resolution (Accorsi et al. 2008, Novak et al. 2008).

5. Simulations

Based on the point spread function, we gave general and accurate sensitivity formulas for a slit-slat collimator. Experimental measurements and simulation of the sensitivity for a parallel or convergent slit-slat assembly recently have been published (Metzler et al. 2006, Accorsi et al. 2008). Here we follow the collimator parameters in (Metzler et al. 2006, Accorsi et al. 2008). The collimator parameters are $w = 2.03$ mm, $d = 1.27$ mm, $t = 0.11$ mm, $a = 34$ mm, $s = 80$ mm, $F = 400$ mm and $B = 0$ mm. We assume a point source on the principal line, so $x_p = 0$, $y_p = 0$, *i.e.* $\theta = \pi/2$. Figure 7 gives the results. For the simulations, we considered the cases of a finite detector ($L_x = 398.72$ mm, $L_y = 242.08$ mm) and an infinite detector ($L_x = 9999$ mm, $L_y = 9999$ mm). These are labeled Simulations 1 and 2, respectively, in figure 7.

From figure 7, we can easily see that in the near field and near focus regions, the approximate formulas give a slight overestimation of the sensitivity. This is because in these regions, the truncated nature of a finite detector decreases the sensitivity. In the standard region, the finite support of the point spread function is always included in the relatively large detector area, so there is no difference between finite and infinite detectors. In the focal region, the formulas for the regular region(46), the whole region (35)/(36) and Simulation 2 agree very closely as shown in the inset of figure 7. More importantly, they do not diverge at the focus even with infinitely large detector, which is not the case with the method in (Accorsi et al. 2008).

With the particular collimator setup and distance $h = 50$ mm, the profiles of the point spread function are given in figure 3. The FWHM of PSF in the transverse direction is $R_{tr}^d = 6.66$ mm, and the FWHM of PSF in the axial direction is $R_{ax}^d = 9.85$ mm. The intrinsic detector resolution is taken to be $R_f = 3.5$ mm, the system resolution in the transverse direction is $R_{tr} = 3.30$ mm, and the system resolution in axial direction is $R_{ax} = 6.50$ mm.

6. Conclusion

We have derived the closed EPSF and geometric response function for SS collimator in this paper. Based on this EPSF, the general formulas for the sensitivity and spatial resolution are likewise derived. Simplified sensitivity formulas are also given in different regions based on Taylor series expansions. The formulas show that the sensitivity has dependence on both the incidence angle and the distance between the point source and the slit plane, while the spatial resolution is constant in a plane at a fixed distance from the collimator. The formulas derived here are beneficial to the design and development of high performance SPECT systems, such as our recently proposed dedicated C-SPECT cardiac imaging systems (Chang et al. 2006, Chang et al. 2008).

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Appendix A. Proof of Equation (15)

The effective aperture function in equation (14) can be written as $a_{\text{slit}}^{\text{eff}}(x) = a_0(x) + a_1(x)$, with $a_0(x) = \Pi(x/w)$, and

$$a_1(x) = \begin{cases} 0, & \text{for } |x| \leq \frac{w}{2} \\ e^{-\mu K(|x| - \frac{w}{2})}, & \text{for } |x| > \frac{w}{2} \end{cases} \quad (\text{A.1})$$

The Fourier transform of $a_0(x)$ is $A_0(v_x) = w \text{sinc}(wv_x)$, where again, $\text{sinc}(v) = \sin(\pi v)/(\pi v)$. The Fourier transform of $a_1(x)$ is

$$\begin{aligned} A_1(v_x) &= \left(\int_{-\infty}^{-w/2} + \int_{w/2}^{+\infty} \right) e^{-\mu K(|x| - \frac{w}{2})} e^{-j2\pi x v_x} dx \\ &= e^{j\pi w v_x} \int_{-\infty}^0 e^{\mu K x - j2\pi x v_x} dx + e^{-j\pi w v_x} \int_0^{+\infty} e^{-\mu K x - j2\pi x v_x} dx \\ &= \frac{e^{j\pi w v_x}}{\mu K - j2\pi v_x} + \frac{e^{-j\pi w v_x}}{\mu K + j2\pi v_x} \\ &= \frac{2\mu K \cos(\pi w v_x) - 4\pi v_x \sin(\pi w v_x)}{\mu^2 K^2 + 4\pi^2 v_x^2} \end{aligned} \quad (\text{A.2})$$

Using the Linearity of the Fourier transform, Putting $A_0(v_x)$ and $A_1(v_x)$ together, we can get equation (15).

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Figure 2.
Schematic diagram of imaging geometry of slit collimator

Figure 3.
Profiles of point spread function

Figure 4.
Profiles of geometric transfer function

Figure 5.
The coordinate system and the angles for vector $\vec{QP}$

Figure 6.
Summary of equations and conditions of applicability

Figure 7.

Dependence of sensitivity on the distance h . The off-focus, regular region, standard region, whole region and near field formulas are respectively, (42), (46), (40), (35)/(36) and (44). Simulations 1 and 2 correspond to the direct solid angle calculation with finite and infinite detectors.