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Evaluation of Origin Ensemble algorithm for image reconstruction for pixelated solid-state detectors with large number of channels

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Abstract

The Voxel Imaging PET (VIP) Pathfinder project intends to show the advantages of using pixelated solid-state technology for nuclear medicine applications. It proposes designs for Positron Emission Tomography (PET), Positron Emission Mammography (PEM) and Compton gamma camera detectors with a large number of signal channels (of the order of 10^6). For PET scanners, conventional algorithms like Filtered Back-Projection (FBP) and Ordered Subset Expectation Maximization (OSEM) are straightforward to use and give good results. However, FBP presents difficulties for detectors with limited angular coverage like PEM and Compton gamma cameras, whereas OSEM has an impractically large time and memory consumption for a Compton gamma camera with a large number of channels. In this article, the Origin Ensemble (OE) algorithm is evaluated as an alternative algorithm for image reconstruction. Monte Carlo simulations of the PET design are used to compare the performance of OE, FBP and OSEM in terms of the bias, variance and average mean squared error (MSE) image quality metrics. For the PEM and Compton camera designs, results obtained with OE are presented.

Keywords

Computerized Tomography (CT) and Computed Radiography (CR); Gamma camera, SPECT, PET PET/CT, coronary CT angiography (CTA); Medical-image reconstruction methods and algorithms, computer-aided so; Compton imaging

1 Introduction

This work presents three imaging algorithms using Monte Carlo simulations of designs for a PET detector, a PEM scanner and a Compton gamma camera as proposed by the Voxel Imaging PET (VIP) Pathfinder project [1]. For detectors without full angular coverage and a large number of channels, like the VIP PEM and the VIP Compton gamma camera design, traditional imaging algorithms like FBP and OSEM have disadvantages. The Origin Ensemble (OE) algorithm [2, 3] is presented as a possible alternative. Simulations on the VIP PET design, for which FBP and OSEM are the standard reconstruction algorithms, are used to make a comparison of OE, FBP and OSEM.

PET scanners are ring shaped detectors that detect hits from back-to-back gamma ray pairs originated from positron-electron annihilation. PEM scanners work on the same principle as PET, but concentrate their imaging abilities on a single body part which makes them especially useful for breast cancer detection. With PET and PEM the origin of the gamma pair and hence approximately of the emitted positron lies along the line-of-response (LOR) established by connecting the positions of the two hits for each event. By combining LORs of various events the locations of higher activity in the scanned object can be established. Compton gamma cameras use the kinematics of Compton scattering to localise the radioactive source. Gammas emitted by a radioactive source scatter inelastically in the scatter detector and, subsequently, are absorbed in the absorber detector (figure 1). The original gamma source is located on the surface of the Compton cone identified by the cone axis, constructed from the hit locations, and the scattering angle θ , obtained from:

$$\cos(\theta) = 1 - m_e c^2 \left(\frac{1}{E_\gamma - E_{scatter}} - \frac{1}{E_\gamma} \right) \quad (1.1)$$

where E_γ is the known energy of the original gamma and $E_{scatter}$ is the energy deposited in the scatter detector. However, due to the Doppler broadening effect [4] equation (1.1) is only approximately valid given that the initial momentum of the electrons in the scattering material cause an additional smearing of the scattering angle.

2 Experimental set-up

The aim of the VIP project is to show that using pixelated solid-state technology with a large number of signal channels for nuclear medicine detectors will improve both the energy and position resolution of the measurements with high efficiency compared to state-of-the-art crystal PET detectors [5]. For this purpose, a dedicated application specific integrated circuit (ASIC) is designed and produced by the project.

A single *VIP unit module* detector is made of 4 CdTe pixelated detectors, of 200 voxels each (figure 2) where each voxel has a size of $1 \times 1 \times 2 \text{ mm}^3$ and is connected to its own independent readout channel for the energy, position, and arrival time.

The *VIP PET scanner* [6] has an inner diameter of 42 cm, an outer diameter of 54 cm and an axial field of view of 25.4 cm. The ring-shaped detector consists of 66 sections, where each section contains 4 module blocks. A module block consists of 30 VIP unit modules and is made up out of 24000 voxels. Hence, the total number of voxels in the PET detector amounts to more than 6 million.

The *VIP PEM scanner* [7] consists of two scanning paddles, each with two heads (figure 3) of 80 VIP unit modules each. The heads are moved with 160 steps of 1.5 mm along the z-axis. Each paddle has a parallelepiped shape with size $160 \times 62 \times 240 \text{ mm}^3$ and the distance between the paddles is 40 mm. In total, the PEM detector consists of $2 \times 2 \times 80 \times 4 = 1280$ CdTe pixelated detectors and the total number of channels for the PEM scanner is 256000.

The scatter detector and the absorption detector of the *VIP Compton camera* [8] are made of pixelated Si and CdTe sensors respectively (figure 1). The scatter detector has a parallelepiped shape of size $380 \times 540 \times 26 \text{ mm}^3$ and a nominal depth of interaction of 2 cm. The absorption detector of the Compton camera, at a distance of 10 cm from the scatter detector, has a parallelepiped shape of size $380 \times 540 \times 62 \text{ mm}^3$ and a nominal depth of interaction of 4 cm. The total number of independent readout channels amounts to more than 3 million.

3 Image reconstruction algorithms

Filtered back-projection (FBP)

With FBP the measured data are represented in so-called sinograms in $r-\phi$ space. A Fourier transform, a frequency filter and, subsequently, an inverse Fourier transform are applied to the data before doing back-projection. The main drawback of FBP is that it requires a detector with an angular coverage of at least 180 degrees and hence cannot be used for PEM and Compton gamma cameras without producing artifacts in the final image. Also, in the case of a Compton camera, Compton cones are not easily represented in sinograms, which makes the application of filters not straightforward.

Ordered Subset Expectation Maximization (OSEM)

With OSEM [9] an image estimate is forward projected onto the detector and the projected data is compared with the real measured data. Next, a cost function is used to update the image estimate. These steps are repeated iteratively until a minimum error has been reached. The data can be organised in ordered subsets to optimise the algorithm. The main problem with OSEM for a Compton gamma camera with a large number of channels is that it requires an impractical large memory and CPU-time consumption. This is because OSEM uses a system matrix that maps probabilities P_{ij} for a signal coming from a voxel i in the field of view (FOV) to be detected in a certain detector bin j . Since the total number of detector bins in the system matrix depends on all possible combinations of all channels in the scatter detector with all channels in the absorber, the system matrix would be too large to be practical.

List-mode (LM) OSEM [10, 11], would significantly reduce this because, since the data is presented in list mode, the sum over all detector bins would be replaced by a sum over all detected events in the iterative update function. However, due to the large number of pixels in the FOV potentially contributing to a particular detected event (Compton cone), LM OSEM for a Compton gamma camera still would be very time consuming and only slowly converging.

The Origin Ensemble (OE) algorithm

OE [2] is straightforward to implement and relatively fast converging, independently of the number of channels.

At initialization, for each event a random position is assigned on the Compton cone or PET or PEM LOR and the event density matrix D_L stores the number of events for each FOV voxel location L . The only constraint made when assigning random positions on the cone or LOR, is to ensure that all the positions are inside the FOV. Additionally, the selected position might be varied according to the *resolution recovery* mechanism, explained below. Images are defined as a set of origins of events and at each step during iteration i , the current set of origins of events is described by the vector Y_i . During iteration, the following steps are done for each event:

- A new random location is selected on the corresponding cone or LOR.
- The new location L_{i+1} for an event at iteration $i+1$ is accepted with probability P , comparing the event density $D_{L_{i+1}}$ at the new location with the density D_{L_i} at the old location for this event:

$$P(Y_i \rightarrow Y_{i+1}) = \min \left[1, \frac{(D_{L_{i+1}} + 1)}{D_{L_i}} \right]$$

- When the new location for the event is accepted, the density matrix D is immediately updated accordingly before moving onwards to the next event.

Due to the stochastic nature of the algorithm, various trial runs have to be executed and the final result has to be an average of these trial runs. In our implementation for the Compton camera, during iteration, for each event a random vector on a cone pointing along the z-axis is rotated onto the real Compton cone and subsequently a random point on that vector is selected. To ensure that all events contribute to the convergence of the algorithm, the vector and the selected location along this vector must lie inside the FOV.

Because of the finite spatial resolution of the detectors the precise location of the radiation source will never be reached without the addition of a *resolution recovery* mechanism [3]. To counteract the *geometry resolution* due to the finite size of the detector voxels, the hit positions are varied uniformly in the range of the voxel-size (± 0.5 mm or ± 1 mm). In the case of the Compton camera, to recover from the *energy resolution*, an additional smearing of the measured energy deposition in the scatter detector is introduced with a factor of 1.7% at 511 keV [12]. Additionally, a smearing of 0.25% is applied to the Compton angle to counteract the *Doppler broadening* effect.

4 Results

All simulation data are obtained with the Geant4-based Architecture for Medicine-Oriented Simulations (GAMOS) package [13]. For FBP the SSRB-FBP2D utility from GAMOS and for OSEM the open source Software for Tomographic Image Reconstruction (STIR) toolkit [14] is used.

Image quality metrics

Using PET data it is possible to establish how well OE compares with conventional image reconstruction algorithms like FBP and OSEM. For this comparison, image quality metrics such as the bias, the variance and the average mean square error (MSE) are calculated, as defined with the following equations [15]. Bias is a measure of the accuracy of the image reconstruction algorithm:

$$\text{Bias} = \frac{1}{N} \sum_{i=1}^N (\bar{X}_i - X_i^{(T)})^2 \quad (4.1)$$

where N is the number of voxels in the selected FOV, $X^{(T)}$ is the *true* image and $\bar{X}$ is the average image over M images:

$$\bar{X}_i = \frac{1}{M} \sum_{j=1}^M X_i^{(j)} \quad (4.2)$$

where $X_{(j)}$ is the j th image in the set of M images. Variance is a measure of the consistency of the image reconstruction algorithm:

$$\text{Variance} = \frac{1}{MN} \sum_{j=1}^M \sum_{i=1}^N (X_i^{(j)} - \bar{X}_i)^2 \quad (4.3)$$

The MSE for image $X^{(j)}$ is defined as:

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^N \left(X_i^{(j)} - X_i^{(T)} \right)^2 \quad (4.4)$$

and the average MSE is the mean MSE over all M images and is the combination of bias and variance.

By varying the parameters of the different algorithms, the bias, variance and average MSE can be compared to find the optimum set-up for each of the algorithms by looking for the minimum average MSE.

PET

For a comparison of OSEM, FBP and OE, a Derenzo phantom [16] with PET was simulated and the image quality metrics explained above were calculated. The simulations were performed for data sets of $M = 20$ images, where each image was constructed from 1 million coincidences and different data sets were created with different parameters for each of the three image reconstruction algorithms. The *true* image of the Derenzo phantom is shown in figure 4. The FOV consisted of 160 bins in the two radial directions X and Y , with a voxel size of 0.25 mm and a range of -20 mm to 20 mm, while in the axial direction integrating over the entire length of the Derenzo rods.

The FBP algorithm was run with different values for the cutoff frequency either for the Hamming filter or for the Butterworth filter (with order 6) [17]. The image quality metrics versus cutoff frequencies are shown in figure 5. For both filters, the optimal value for the cutoff frequency is 0.4, where the average MSE is at the lowest value.

The OSEM algorithm was tested with a varying number of iterations. The resulting bias and average MSE image quality metrics versus number of iterations are shown in figure 5. The bias has a minimum value when running for 60 iterations, whereas the average MSE shows a minimum value for 40 iterations. Since the point with lowest average MSE also has lower variance, 40 iterations was taken as the optimal value for OSEM.

The OE algorithm was tested by varying the number of iterations while averaging over 20 trials for each run. The resulting bias and average MSE image quality metrics are shown in figure 5. The lowest values for the bias and the MSE are found after 20 iterations. Additionally, data sets with OE were constructed for 20 iterations but with a varying number of trials. Figure 6 shows that when OE is run with 170 trials, the image quality metrics reach a plateau and do not seem to improve significantly after that.

For the final image made with OSEM after running for 40 iterations, an additional smoothing filter was applied, as illustrated in figure 7. For the final image with OE, after running the algorithm for 20 iterations, and summing over 170 independent trial runs, an additional 3d-median smoothing filter was applied.

The images and the corresponding line profiles obtained with optimal parameters for each of the algorithms are shown in figures 8 and 9 and summarized in table 1. Here, both the line profiles as the average MSE values in table 1 show that FBP has the best performance for PET with a Derenzo phantom. However, both OSEM and OE show to be valid algorithms when FBP could not be used (as is the case for PEM and Compton camera). The line profiles show that all three algorithms can achieve a spatial resolution of better than 1 mm, corresponding to the diameter of the smallest rod. Due to the relative small number of bins in the FOV chosen for this study, the reconstruction times for all algorithms were of the order of a few minutes. With an increasing number of FOV bins, OSEM would get increasingly slower. An additional simulation was performed for OE, using a NEMA NU-4

standard point-like source [18]. This study gave values for the FWHM of the point-spread function (PSF) comparable to results obtained with FBP and OSEM: of the order of 0.5 mm in the x and y direction and of the order of 1 mm in the z direction (reflecting the PET voxel-size which is twice as large in the z direction).

PEM

The OE results on PEM data with a Derenzo phantom (figure 10) were obtained by taking the average of 5 independent trial runs. As with the PET data, OE is able to detect rods with a diameter of 1 mm.

The value for the FWHM of the PSF with a NEMA NU-4 point-like source is about 1.0 mm, which is similar to results obtained with standard reconstruction techniques [7].

Although more studies should be done on PEM data with the OSEM algorithm, these results show that OE is an algorithm to be considered for PEM image reconstruction, especially since FBP would be more problematic due to the lack of 180 degree angular coverage.

Compton camera

The Compton camera results (figure 11) are obtained by taking the average of 20 independent trial runs of 1000 iterations of OE on simulation data with 4 point-like 511 keV gamma sources on the corners of a cube with sides of 10 mm length. The OE resolution recovery mechanism now deals with all three main contributions to the spatial resolution. For the PSF spatial resolution for the Compton gamma camera, the FWHM from the x and y line profiles is 2 mm and from the z line profile is 2.4 mm.

5 Conclusions

An extensive image quality analysis has been performed to make a quantitative comparison of the performance of OE, FBP and OSEM on simulation data with a Derenzo phantom and a PET scanner. This analysis shows that although FBP clearly has a better performance, OE is a suitable candidate as an alternative image reconstruction algorithm. Additionally, it was found that the spatial resolution of the PSF with OE is in between 0.5 mm and 1 mm (FWHM) for PET and about 1 mm for PEM, which is better than products currently in the market can deliver. The results justify investigating OE as a potentially suitable image reconstruction algorithm for PEM and Compton camera data where FBP cannot be used and (in the case of Compton camera) (list mode) OSEM might be relatively slow.

OE needs an additional resolution recovery technique to compensate for the finite spatial resolution, especially in the case of the Compton camera where the Doppler broadening effect is a major contribution. After incorporating this mechanism, OE can produce 3D images of spatially distributed 511 keV gamma point-like sources in Compton camera data, with values for the FWHM of the PSF of the order of 2 mm.

Future work with OE will focus on the improvement of the resolution recovery mechanism (e.g. Doppler broadening) with Compton camera data and studying the effect of rotating the Compton camera. Also NEMA NU-4 image reconstruction quality tests and background studies will be done. Additionally, the list-mode OSEM algorithm will be implemented for the Compton camera to compare its performance with OE.

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Figure 1.
Schematic view of the VIP Compton camera.

Figure 2.
The VIP unit module.

Figure 3.
Schematic view of the VIP PEM.

Figure 4.
XY projection of the *true* image of the Derenzo phantom used for the image quality metrics.

Figure 5. Bias and average MSE versus cutoff, for FBP with a Hamming filter (top, left) and FBP with a Butterworth filter (top, right), and versus number of iterations for OSEM (bottom, left) and OE (bottom, right).

Figure 6. Image quality metrics bias and average MSE (left) and variance (right) for different number of OE trials after 20 iterations.

Figure 7. OSEM final images without (left) and with application of smoothing filter (right). The bottom row shows the effect of smoothing on the line profiles of the second largest rods.

Figure 8. Comparison of FBP with Hamming filter (cutoff 0.4) and FBP with Butterworth filter (cutoff 0.3), for a Derenzo phantom with PET, showing comparable results. The line profiles are taken along the bottom row of 6 rods with smallest diameter (1 mm).

Figure 9. Comparison of OSEM (40 iterations) and OE (20 iterations, 170 trials) for a Derenzo phantom with PET, showing comparable results. The line profiles are taken along the bottom row of 6 rods with smallest diameter (1 mm).

Figure 10. OE results with PEM with a Derenzo phantom. The line profile is taken along the bottom row of 6 rods with smallest diameter (1 mm).

Figure 11.
3D image and line profiles, using OE on Compton camera data with 4 point-like 511 keV sources on a cube with sides of 10 mm.

Table 1

Comparison of image quality metrics for FBP, OSEM and OE.

	bias:	variance:	average MSE:
FBP (Hamming cutoff 0.4)	957926	39760.5	997687
FBP (Butterworth cutoff 0.4)	956760	52353.6	1.00911e+06
OSEM (40 iterations)	920213	179473	1.09969e+06
OE (20 iterations, 20 trials)	1.3738e+06	250037	1.62384e+06
OE (20 iterations, 170 trials)	1.4039e+06	213736	1.61768e+06