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Adiabatic Quantum Computation: Coherent Control Back Action

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Abstract

Though attractive from scalability aspects, optical approaches to quantum computing are highly prone to decoherence and rapid population loss due to nonradiative processes such as vibrational redistribution. We show that such effects can be reduced by adiabatic coherent control, in which quantum interference between multiple excitation pathways is used to cancel coupling to the unwanted, non-radiative channels. We focus on experimentally demonstrated adiabatic controlled population transfer experiments wherein the details on the coherence aspects are yet to be explored theoretically but are important for quantum computation. Such quantum computing schemes also form a back-action connection to coherent control developments.

Keywords

Adiabatic Quantum Computing; Ultrafast pulse shaping; coherent control

INTRODUCTION

One of the main developments in the implementation of quantum computing has been in addressing an atomic or molecular ensemble, such as the one which addresses nuclear spin in bulk molecular systems [1]. However, typical problems in experimental implementation of quantum computing exist in the complexity of the experimental setup and in scaling the number of qubits [2]. Other implementation approaches are being pursued to find ways to circumvent the existing problems. Optical approaches could be attractive but for the rapid decoherence timescales. Minimizing decoherence or lengthening of decoherence timescales is also an important challenge towards the realization of the decades old dream of using photons as “reagents” in chemical reaction and has doomed the attempts to do “laser selective chemistry”. The intramolecular relaxation processes, such as Intramolecular Vibrational Relaxation (IVR), is the most important contributor to decoherence even in isolated molecules. Restriction of IVR by optical schemes [3] can be an attractive route towards selective excitation in large molecular systems. Although attractive, most of the photon-mediated theoretical approaches towards restricting IVR (also called “photon-locking”) use complicated pulse shapes that are yet to be demonstrated in the laboratory due to stringent requirements of intensity and precision.

Use of adiabatic evolution for optical quantum computation has recently become an attractive approach due to its inherent robustness [4,5,6,7,8]. In this framework, logical implementation of quantum gates uses the language of ground states, spectral gaps and Hamiltonians wherein a quantum gate represents a device which performs a unitary transformation on selected qubits within a fixed period of time. Thus, a computational

procedure in the adiabatic quantum computation model is a continuous time evolution of a time-dependent Hamiltonian with limited energetic resources.

Our previous paper [8] showed that an important aspect of the adiabatic quantum computation model lies in addressing an atomic or molecular ensemble and hence in robust implementation. These were based on our earlier experimental demonstration on selective population transfer in a two-level and a three-level system [9,10]. However, the issue of coherence aspects of such control processes has not been addressed before and that is main topic of our present paper since coherence issues form a critical aspect in the quantum computing paradigm. With the rise in interest in quantum-control spectroscopy [11], it is an important interdisciplinary development to be pursued.

FORMALISM

An ultrafast laser pulse can be represented as a coherent superposition of many monochromatic light waves within a range of frequencies that is inversely proportional to the duration of the pulse. Thus, for instance, a 40fs pulse at 800nm that is commercially available has a spectrum as broad as 30nm. Possibilities of manipulating such an ultrafast coherent bandwidth are nontrivial as it lasts for such an ultrashort duration wherein no modulators work. A creative solution to the problem of slow modulators is the indirect pulse shaping in the frequency domain. In the time domain the filter is characterized by a time response function $g(t)$. The output of the filter $E_{out}(t)$ in response to an input pulse $E_{in}(t)$ is given by the convolution of $E_{in}(t)$ and $g(t)$, such that, $E_{out}(t) = E_{in}(t) \otimes g(t)$. In the frequency domain, the filter is characterized by its frequency response $G(\omega)$, i.e., $E_{out}(\omega) = E_{in}(\omega) \times G(\omega)$, where $E_{in}(t)$, $E_{out}(t)$ and $g(t)$ and $E_{in}(\omega)$, $E_{out}(\omega)$ and $G(\omega)$, respectively are Fourier transform pairs [12]. With a delta function input pulse, the input spectrum $E_{in}(\omega)$ is unity and the output spectrum is equal to the frequency response of the filter, and thus, due to Fourier transform relations generation of a desired output waveform can be accomplished by implementing a filter with the required frequency response. Another attractive approach is the use of simple chirped pulses, which, by contrast, have been produced routinely at very high intensities and at various different wavelengths for many applications, including selective excitation of molecules in coherent control and we discuss further on that now for various physical models.

Two-Level Model

The simplest model describing a molecular system is an isolated two-level system or ensemble without relaxation or inhomogeneities. This simple model often turns out to be a very practical model for most systems interacting with ultrashort laser pulses as the magnitude of the relaxation processes are immensely large as compared to the light-matter interaction time. Let us apply a linearly polarized laser pulse of the form $E(t) = \epsilon(t)e^{i[\omega t + \phi(t)]}$ to a simple two-level system with $|0\rangle \rightarrow |1\rangle$ transition, where $|0\rangle$ and $|1\rangle$ represent the ground and excited eigenlevels, respectively, of the field-free Hamiltonian (Fig. 1).

The laser carrier frequency or the center frequency for pulsed lasers is ω . We have $\epsilon(t)$ and $\phi(t)$ as the instantaneous amplitude and phase. We can define the rate of change of instantaneous phase, $\dot{\phi}(t)$ as the frequency-sweep. If we expand the instantaneous phase function of $E(t)$ as a Taylor series with constants b_n , we have

$$\begin{aligned}\phi(t) &= b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 + \dots \\ \dot{\phi}(t) &= b_1 + 2b_2 t + 3b_3 t^2 + 4b_4 t^3 + 5b_5 t^4 + \dots \quad (1) \\ \dot{\phi}(t) &= \sum_{n=1} n b_n t^{(n-1)}\end{aligned}$$

In an earlier paper [13], we have proposed the use of simple chirped pulses, which, by contrast, have been produced routinely at very high intensities and at various different wavelengths for many applications, including selective excitation of molecules in coherent control. Establishing this generalization enables us to treat all possible chirped pulse cases by exploring the effects of each of the terms in Eqn. (1) initially for a simple two-level system and then extend it to the multilevel situation for a model five-level system of anthracene molecule, which has been previously investigated with complicated shaped-pulses [3]. We use a density matrix approach by numerically integrating the Liouville

equation $\frac{d\rho(t)}{dt} = \frac{i}{\hbar} [\rho(t), H^{FM}(t)]$ for a Hamiltonian in the rotating Frequency Modulated (FM) frame of reference. $\rho(t)$ is a 2×2 density matrix whose diagonal elements represent populations in the ground and excited states and off-diagonal elements represent coherent superposition of states. The Hamiltonian for the simple case of a two-level system under the effect of an applied laser field can be written in the FM frame for N-photon transition [14]

as, $H^{FM} = \hbar \begin{pmatrix} \Delta + N\dot{\phi}(t) & \frac{\Omega_1}{2} \\ \frac{\Omega_1^*}{2} & 0 \end{pmatrix}$. The time derivative of the phase function, $\dot{\phi}(t)$, appears as an additional resonance offset over and above the time-independent detuning $\Delta = \omega_R - N\omega$, while the direction of the field in the orthogonal plane remains fixed. We define the multiphoton Rabi Frequencies as complex conjugate pairs: $\Omega_1(t) = k(\mu_{eff} \cdot \mathbf{E}(t))^N / \hbar$ and $\Omega_1^*(t) = k(\mu_{eff}^* \cdot \mathbf{E}^*(t))^N / \hbar$, where k is a proportionality constant having dimensions of $(\text{energy})^{(1-N)}$, which in SI units would be $\text{Joule}^{(1-N)}$. For the $|1\rangle \rightarrow |2\rangle$ transition, $\omega_R = \omega_2 - \omega_1$ is the single-photon resonance frequency. We have assumed that the transient dipole moment of the individual intermediate virtual states in the multiphoton ladder result in an effective transition dipole moment, μ_{eff}^N , which is a product of the individual N virtual state

dipole moments, $\mu_N, \left(i.e., \mu_{eff}^N = \prod_n \mu_n \right)$. This approximation is particularly valid when intermediate virtual level dynamics for multiphoton interaction can be neglected [15]. Such approaches have helped us to look at the population dynamics. Here, we also use the off-diagonal elements of the density matrix evolution and the underlying dressed states to discuss the coherence aspects of the associated dynamics.

Multi-Level IVR Model

We now extend the two-level formalism into a multilevel situation involving intramolecular vibrational relaxation (IVR). In the conventional zeroth order description of intramolecular dynamics, the system can be factored into an excited state that is radiatively coupled to the ground state, and nonradiatively to other bath states that are optically inactive (Fig. 2).

These “dark” states have no radiative transition moment from the ground state as determined by optical selection rules [16]. They can belong to very different vibrational modes in the same electronic state as the “bright” state, or can belong to different electronic manifolds. These dark states can be coupled to the bright state through anharmonic or vibronic couplings. Energy flows through these couplings and the apparent bright state population disappears. Equivalently, the oscillator strength is distributed among many eigenstates. The general multilevel Hamiltonian in the FM frame for an N-photon transition ($N > 1$), expressed in the zero-order basis set, is:

$$H = \hbar \begin{pmatrix} |0\rangle & |1\rangle & |2\rangle & |3\rangle & |4\rangle & \dots \\ 0 & \Omega_1(t) & 0 & 0 & 0 & \dots \\ \Omega_1^*(t) & \delta_1(t) & V_{12} & V_{13} & V_{14} & \dots \\ 0 & V_{12} & \delta_2(t) & V_{23} & V_{24} & \dots \\ 0 & V_{13} & V_{23} & \delta_3(t) & V_{34} & \dots \\ 0 & V_{14} & V_{24} & V_{34} & \delta_4(t) & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad (2)$$

where, $\Omega_1(t)$ (and its complex conjugate pair, $\Omega_1^*(t)$) is the transition matrix element expressed in Rabi frequency units, between the ground state $|0\rangle$ and the excited state $|1\rangle$. The background levels $|2\rangle, |3\rangle, \dots$ are coupled to $|1\rangle$ through the matrix elements $V_{12}, V_{23}, \dots$ etc. Both Rabi frequency $\Omega_1(t)$ and detuning frequency [$\delta_{1,2,\dots} = \Delta_{1,2,\dots} + N \dot{\phi}(t)$] are time dependent (the time dependence is completely controlled by the experimenter). In general, the applied field would couple some of the dark states together, or would couple $|1\rangle$ to dark states, and thus, the V_{ij} terms would have both an intramolecular, time independent component and a field-dependent component. As an alternative to Eqn. (2), the excited states' submatrix containing the bright state $|1\rangle$ and the bath states $|2\rangle, |3\rangle, \dots$ can be diagonalized to give the eigenstate representation containing a set of Δ_i' as diagonal elements and corresponding Ω_i' as off-diagonal elements. The eigenvalues of such a time-dependent Hamiltonian representation is often referred to as the dressed states of the system. Such a representation corresponds closely to what is observed in conventional absorption spectroscopy. As long as the intensity of the field is very low ($|\Omega_i'| \ll \Delta_i'$) the oscillator strength from the ground state (and hence the intensity of the transition, which is proportional to $|\Omega_i'|^2$) is distributed over the eigenstates, and the spectrum mirrors the distribution of the dipole moment. On the other hand, a pulsed excitation creates a coherent superposition of the eigenstates within the pulse bandwidth. Physically, in fact, the presence of the dark states has been key to the loss of selectivity of excitation to a specified bright state. Interestingly such a process essentially is a Hadamard operation in Quantum Computing Language as this enables us to produce equal superposition between the ground and excited states which form the qubits.

Another common situation with short pulses is a ladder excitation situation where the individual excited states undergo dephasing through a coupled energy structure with states $|0\rangle, |1\rangle, |2\rangle, \dots$ in the zero-order basis as shown in Fig. 2b. Such a model of IVR is often referred to as the tier model and is common in polyatomic molecules and in most rovibrational states [17] and can be represented by the following Hamiltonian:

$$H^{FM} = \hbar \begin{pmatrix} |0\rangle & |1\rangle & |2\rangle & |3\rangle & |4\rangle & |5\rangle & |6\rangle & |7\rangle & |8\rangle & |9\rangle \\ \hline 0 & \Omega_1(t) & \Omega_2(t) & \Omega_3(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ \Omega_1^*(t) & \delta_1(t) & V_{12} & V_{13} & V_{14} & V_{15} & 0 & 0 & 0 & 0 \\ \Omega_2^*(t) & V_{12} & \delta_2(t) & V_{23} & V_{24} & V_{25} & V_{26} & V_{27} & 0 & 0 \\ \Omega_3^*(t) & V_{13} & V_{23} & \delta_3(t) & 0 & 0 & V_{36} & V_{37} & V_{38} & V_{39} \\ 0 & V_{14} & V_{24} & 0 & \delta_4(t) & 0 & 0 & 0 & 0 & 0 \\ 0 & V_{15} & V_{25} & 0 & 0 & \delta_5(t) & 0 & 0 & 0 & 0 \\ 0 & 0 & V_{26} & V_{36} & 0 & 0 & \delta_6(t) & 0 & 0 & 0 \\ 0 & 0 & V_{27} & V_{37} & 0 & 0 & 0 & \delta_7(t) & 0 & 0 \\ 0 & 0 & 0 & V_{38} & 0 & 0 & 0 & 0 & \delta_8(t) & 0 \\ 0 & 0 & 0 & V_{39} & 0 & 0 & 0 & 0 & 0 & \delta_9(t) \end{pmatrix} \quad (3)$$

A short pulse laser can optically couple the states $|1\rangle$, $|2\rangle$, $|3\rangle$ etc. to the ground state $|0\rangle$ with respective transition matrix elements expressed in Rabi units as $\Omega_1(t)$, $\Omega_2(t)$, $\Omega_3(t)$, etc. and their corresponding complex conjugates. The background levels $|4\rangle$, $|5\rangle$, $|6\rangle$, etc. are coupled to the optically excited states $|1\rangle$, $|2\rangle$ and $|3\rangle$ through the matrix elements V_{14} , V_{15} , V_{24} , etc. Such a molecular system can become useful for realizing qubits effectively if these large number of optically coupled states can be accessed simultaneously as has been in case of the atomic system of Rydberg state of cesium [18]. However, the difficulty in extending this scheme to the molecular system starts at the very first step of initializing the qubits due to high decoherence of the possible qubit states as in the gedanken system Hamiltonian presented in Eqn. (3). Thus an adiabatic scheme is necessary.

COHERENCE CORRELATIONS

Effects of the pulse profile and the frequency sweep on the population are usually interpreted in the framework of adiabatic rapid passage (ARP) [9,12]. In ARP, the barriers to total population inversion in real systems are removed by either modulating the laser pulse so that its spectrum ‘sweeps’ through the distribution of resonances of the atoms or molecules in the ensemble or by perturbing the atomic or molecular energy levels via an electric field (i.e., a Stark shift) so that they shift through resonance with the irradiating laser field. If this frequency sweep is sufficiently slow, the transition is said to be ‘adiabatic’. If the rate at which the frequency of the laser pulse changes with time is slow compared with the precession rate of μ (and fast compared with the relaxation rates of the material system), μ is said to “adiabatically follow” the pseudofield vector, thereby giving rise to complete population transfer. Herein, with the help of the models discussed in the earlier sections, we probe the real part, the imaginary part, the phase angle and the absolute values of the off-diagonal elements of the density matrix model propagations and compare it with the expected population dynamics that our model has been efficiently successful in corresponding to the experimental results [4,8,12,13].

In order to understand the behavior of the off-diagonal elements of the density matrix, we first explore the simplest two-level model as depicted in Fig. 1. In Fig. 3, we show that in the temporal evolution of the density matrix for an unchirped pulse excitation, only the imaginary part of the off-diagonal elements has non-zero values which gets switched to the real part for a perfectly adiabatic chirped pulse excitation! Thus, the highlighting issue is the fact that the off-diagonal elements of density matrix are either a complete imaginary number for the unchirped case or an almost real number for the adiabatic cases with a maximum value of $1/2$. In case of a non-adiabatic chirped pulse excitation, the real and imaginary components of the off-diagonal elements are highly oscillating (Fig. 3(d)). However, there is no rapid oscillatory behavior under either of the adiabatic cases of complete inversion (ARP)

or coherent superposition (half-ARP), and the evolution is smoothly characterized by the real part of the off-diagonal elements (Fig. 3(b) and (c)). The maximum possible absolute value of the coherence terms (off-diagonal elements of the density matrix) is always 0.5 or less and while it is possible to have similar absolute values for both the chirped versus unchirped cases, their complex character are in quadrature (Fig. 4). The corresponding dressed state character is always an equal mix of the ground and excited states of the two-level system with a raw value of 0.5 throughout its time evolution while in case of ARP, it follows one or the other state and its value remains as either 1 or zero throughout the time evolution (Fig. 4(d)). Thus, we find that for the classic case of the “half- π ” pulse versus the adiabatic half-passage scenario, the absolute value of the final coherence is the same for the two cases, namely $\frac{1}{2}$, i.e., that there is complete coherence between the two concerned states. This difference can be correlated to the precession angle cone under the adiabatic condition remains very small as it undergoes the half-passage while the precession cone for the half- π pulse essentially is along the equator in the Bloch or Feynman vector picture representing the essential differences in the two coherent phases! We are, therefore, able to correlate the phase of the complex coherence term in the density matrix to the precession angle cone.

With this correlation let us now look into the off-diagonal elements of density matrix of the multilevel system. In all the different IVR models discussed in the previous section, essentially the population of the initial bright state gets redistributed to the “dark” states as shown in Fig. 5(a). Under adiabatic conditions it is possible to restrict this IVR process into either generating an equally populated bright state distribution which can be represented as a Hadamard transform (Fig. 5(b)) or “photon-lock” the ground and the bright states with no participation of the dark states (Fig. 5(c)). Let us focus on the comparison of effects of the transform-limited pulse to that of the adiabatically chirped pulse case (Fig. 5(a) and (c)) and try to determine if the same correlations as developed in the two-level situations prevail in such a multi-level case.

The population evolution of the multi-level system is dictated by the dressed state character and the corresponding eigen-energy evolution and their crossings as shown in Fig. 6. At low powers where almost no population transfer occur, there is minimal interaction of the eigen-energies corresponding to the “dark” states although dressed state character switching (DS1, DS2, DS3, DS4, DS5 and DS6 corresponding to the six level model being used) is already evident (Figs. 6(a) and (b)). As soon as the power is high enough to induce population transfer, the interaction increases and the dressed state characters undergo several switching and multiple eigen-energies interact (Fig. 6(c) and (d)) which is evidenced in terms of population redistribution and IVR (Fig. 5(a)). However, under the adiabatic condition, the dressed states due to the ground and the bright states never interact with the dressed states corresponding to the “dark states” (Fig. 6(e)) and as such there is no population leakage to the dark states (Fig. 5(c)). The eigen-energy distribution as shown in Fig. 6(f) also reaffirms these results.

With these backgrounds, we probe into the off-diagonal elements of the density matrix for the 6-level IVR model that we have discussed in detail in Fig. 6. We first focus on the adiabatically decoupled model with quadratic chirped Gaussian pulse excitation since it had the simplest dressed state distribution and non-crossing eigen-energies (Fig. 6(e) and (f) respectively) and find that our two-level generalization that the real part has most of the contribution to the off-diagonal elements of density matrix under the adiabatic condition holds (Fig. 7).

While the two-level characteristic of the multi-level system is generated due to half-ARP chirped pulses and the contribution of the dark-states are minimized, the major contribution

of the real part in the values of the off-diagonal elements of density matrix has been shown in Fig. 7. This further reinforces the argument that the approach to control IVR through half-ARP essentially succeeds in creating a coherent superposition of a two-level system from a multilevel system. Using the same arguments, the Hadamard gate generating adiabatic pulse (Fig. 5(b)) manages to generate identical set of coherently coupled two-level systems.

However, the situation is not as simple in case of the unchirped pulses as shown in Fig. 8. At low enough power, wherein the population transfer is minimal, some evidence of the more imaginary part characteristic of the off-diagonal elements of density matrix can still be argued (Figs. 8(a) and (b)) since it is reminiscent of the two-level system interacting with unchirped pulses. As the power increases, the two-level character is further reduced as the coupling to the dark states overwhelms and there is no preference of the real versus the imaginary part of the off-diagonal elements of density matrix as shown in Figs. 8(d) and (f), indicating efficient IVR process.

Thus the imaginary component of the off-diagonal elements of density matrix can be treated as a measure of the two-level character in a multilevel system for unchirped pulse excitation and the real component of the off-diagonal elements of density matrix can be treated as a measure of the two-level character in a multilevel system for chirped pulse excitation.

CONCLUSIONS

Evolution of the off-diagonal elements of density matrix can be an important parameter for determining the coherence prevailing in the system. The change in the nature of the off-diagonal elements of density matrix between unchirped and adiabatic chirped pulse case is an important indication of the change in the mechanism of the coherence process underlying the evolution of the system. For a multilevel system, it is only the adiabatic pulses that can generate and maintain coherence between states as is demonstrated here through the evolution of the off-diagonal elements of density matrix and as such forms an attractive approach for both robust quantum computing processing as well as coherent control for multilevel systems. Further research on this novel idea is currently in progress in the author's laboratory.

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Figure 1. Schematic of an ultrafast laser excitation for a simple two-level system, $|0\rangle \rightarrow |1\rangle$ through (a) single-photon, (b) two-photon and (c) multi-photon process.

(a)

(b)

Figure 2. (a) Schematic of IVR process in anthracene molecule based on the data extracted from experimental measurements in Ref. [16]. (b) Tier-level model of coupled IVR system commonly seen in many polyatomic systems as well as several rovibrational states.

(a)

(b)

(c)

Figure 3.

(a) Time evolution of the off-diagonal elements of density matrix and their corresponding eigen-energies under the effect of a Transform-limited Gaussian pulse with pulse area of π that results in complete population inversion on resonance conditions. (b) Time evolution of the off-diagonal elements of density matrix and their corresponding eigen-energies under the effect of a linearly chirped Gaussian pulse which results in ARP. (c) Time evolution of the off-diagonal elements of density matrix and their corresponding eigen-energies under the effect of a quadratic chirped Gaussian pulse which results in half-ARP. (d) Highly oscillatory nature of time evolution of the real and the imaginary parts of the off-diagonal elements of density matrix for a non-adiabatic chirped Gaussian pulse. The corresponding oscillation in the absolute value is minimal.

(a)

(b)

(c)

(d)

Figure 4.

(a) Time evolution of the absolute value of the off-diagonal elements of density matrix under the effect of a Transform-limited Gaussian pulse with different pulse areas. (b) Time evolution of the absolute value of the off-diagonal elements of density matrix under the effect of linearly chirped Gaussian pulse at different Rabi Frequencies to illustrate adiabatic versus non-adiabatic situations. Inset shows the onset of adiabaticity as a function Rabi Frequency. (c) Same as in (b) for quadratic chirped Gaussian pulse. (d) Time evolution of the dressed-state character (DS1, DS2) for the unchirped versus chirped pulse cases contrasting the adiabatic and non-adiabatic situations. This shows the importance of coherence and reaffirms the correlation of adiabatic following. Note that half-ARP is effective during the pulse unlike the ideal $\frac{1}{2}$ π pulse which leaves the system in coherent superposition.

(a)

(b)

(c)

Figure 5.

(a) Population time evolution for a six-level IVR model with a single bright state as discussed in Fig. 1(a) under the effect of a transform-limited Gaussian pulse at low-enough Rabi Frequency that only results in non-adiabatic situations. Notice the complete redistribution of population to the non-optically coupled (dark) states indicating IVR. (b) Population time evolution for a seven-level IVR model with multiple bright states as discussed in Fig. 1(b) under the effect of an adiabatically chirped Gaussian pulse that generates equal population distribution of the bright states with no population leaking to the dark states. (c) Population time evolution for the same model as in (a) under the effect of a quadratic chirped Gaussian pulse under adiabatic condition that results in photon-locking between the ground the bright state with no population leaking to the dark states indicating restriction of IVR.

(a)

(b)

(c)

(d)

(e)

(f)

Figure 6.

(a) Evolution of the dressed state character for a six-level IVR model with a single bright state under the effect of a transform-limited Gaussian pulse at very low Rabi Frequency (Rabi frequency = 0.1) that results in almost no population transfer. Notice the switching of dressed state characters (DS1, DS2, DS3, DS4, DS5, DS6) in spite of minimal population transfer. (b) Evolution of eigen-energy under the same conditions as in (a). (c) Evolution of the dressed state character for the same model as in (a) with a transform-limited Gaussian pulse at low Rabi Frequency (Rabi frequency = 1) that results in some population transfer. (d) Evolution of eigen-energy under the same conditions as in (c) showing that the interactions are strong enough to result in distribution of population to dark states (Fig. 5(a)). (e) Same as in (c) under adiabatic condition with quadratic chirp pulse. No switching of the dressed state character occurs and IVR is restricted (Fig. 5(c)). (f) Corresponding eigen-energy evolution for (e) with time also does not show any crossing between the different eigen-energies.

(a)

(b)

(c)

Figure 7.

Time evolution of (a) real, (b) imaginary and (c) absolute value of the off-diagonal elements of density matrix under the effect of an adiabatic quadratic chirped Gaussian pulse for a six-level IVR model with a single bright state. As in the two-level case, we find that the real part has most of the contribution to the off-diagonal elements of density matrix under such an adiabatic condition.

(a)

(b)

(c)

(d)

(e)

(f)

Figure 8.

Time evolution of real, imaginary and absolute values (a, b, c and d, e, f, respectively) of the off-diagonal elements of density matrix under the effect of very low power (Rabi frequency = 0.1) and medium power (Rabi frequency = 1) transform-limited Gaussian for the same IVR model for which IVR restriction was demonstrated in Fig. 7. It becomes increasingly difficult to make correlations as in the two-level case that the imaginary part mostly contributes to the off-diagonal elements of the density matrix under transform-limited conditions.