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Incentives for Optimal Multi-level Allocation of HIV Prevention Resources

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Abstract

HIV/AIDS prevention funds are often allocated at multiple levels of decision-making. Optimal allocation of HIV prevention funds maximizes the number of HIV infections averted. However, decision makers often allocate using simple heuristics such as proportional allocation. We evaluate the impact of using incentives to encourage optimal allocation in a two-level decision-making process. We model an incentive based decision-making process consisting of an upper-level decision maker allocating funds to a single lower-level decision maker who then distributes funds to local programs. We assume that the lower-level utility function is linear in the amount of the budget received from the upper-level, the fraction of funds reserved for proportional allocation, and the number of infections averted. We assume that the upper level objective is to maximize the number of infections averted. We illustrate with an example using data from California, U.S.

Keywords

HIV; AIDS; resource allocation; public policy

1. INTRODUCTION

HIV prevention funds are often allocated at multiple levels. For example, a national level decision maker may allocate funds to regional decision makers who then distribute funds to local organizations, risk groups, or programs. The Global Fund to Fight AIDS, Tuberculosis and Malaria distributes funds to different countries. Funds received by countries may then be allocated to regional decision makers or specific programs. In the U.S., the Centers for Disease Control and Prevention (CDC) disperse funds to various community planning groups. These groups then distribute the funds to local programs or risk groups.

Many decision makers use equity-based heuristics like proportional allocation to allocate HIV prevention funds (Valdiserri et al., 1995; Holtgrave et al., 2000; Johnson-Masotti et al., 2000). Some examples of proportional allocation includes dividing the budget equally among competing programs, prioritizing resources among programs without considering the effectiveness of targeted programs, allocating in proportion to HIV prevalence, incidence, or population size.

Numerous researchers have developed mathematical programming models of resource allocation at a single level of decision-making. These include linear programming models (Ruiz, 2001; Earnshaw et al., 2007), integer programming models (Stinnett and Paltiel,

1996; Cromwell et al., 1998; Sendi and Al, 2003) and dynamic programming models (van Zon and Kommer, 1999; Zaric and Brandeau, 2002; Bala and Mauskopf, 2006). Many researchers have combined epidemic modeling with optimization techniques (Zaric and Brandeau, 2001a, b; Brandeau, 2001; Brandeau et al., 2003).

Since equity heuristics are often used, several authors have looked at equity versus efficiency tradeoffs at multiple-levels of decision-making process (Lasry et al., 2007; Zaric and Brandeau, 2007). Lasry et al. (2007) modeled a two-level decision making process combining epidemic modeling with optimization technique in the context of Sub-Saharan Africa in which an upper level allocates funds to the lower level, who further distribute the funds to two sub-populations, each funding two different prevention programs. Zaric and Brandeau (2007) examined a two-level decision-making process with multiple lower-level decision makers and multiple sub-populations. The model was based on 40 U.S. states with three risk groups per state. The authors concluded that optimal allocation at the lower level is likely to yield greater gains than optimal allocation at the higher level. Thus, upper-level decision makers, such as donor organizations, should develop incentives to promote optimal allocation at the lower level. We expand on the work of Zaric and Brandeau (2007) by investigating an incentive program to encourage optimal allocation at the lower level.

Incentives may encourage optimal behaviour. Fuloria and Zenios (2001) considered incentives in a health care delivery system in which the provider of health-care services is given incentives by the purchaser to motivate the provider. Su and Zenios (2006) looked at incentives in kidney allocation problems and showed a considerable increase in the efficiency of the allocation process due to incentives. However, we are not aware of any research developing incentives to promote optimal allocation of HIV prevention resources.

In this paper, we model a two-level resource allocation problem in which the upper-level decision maker uses incentives to promote optimal allocation by the lower-level decision maker. Our study attempts to answer the following questions. Under what situations does giving incentives to the lower-level decision maker help encourage optimal allocation at the lower level? What is the optimal level of incentives? The rest of the paper is organized as follows: Section 2 provides the mathematical description of the model. Section 3 introduces mathematical analysis of the model and illustrates with a numerical example. Concluding comments are provided in Section 4.

2. THE MODEL

We developed a single period model of a two-level decision-making process. There is a single decision maker at each level and a fixed time horizon of length $T > 0$. The lower-level decision maker has two sets of decisions: 1. What proportion of the funds received to allocate based on equity?, and 2. How should funds be allocated to programs? The upper-level decision maker has one decision, which is the amount to allocate to the lower level. We assume that the upper level decision maker uses an incentive scheme to encourage the lower-level decision maker reduce the fraction of funds reserved for equity by making the amount received by the lower level dependent on this fraction.

In particular, the lower level decision maker chooses r ($0 \leq r \leq 1$), the fraction of the funds to be reserved for proportional allocation, and then distributes amount y_j , $j = 1, 2, \dots, m$ to program j . We assume that one program is available for each risk group, that the programs do not interact, and that the costs and benefits scale linearly. The upper-level decision maker chooses a fraction f , $0 \leq f \leq 1$, which we refer to as the “strength of the incentive.” The upper-level decision maker has a total budget B and allocates an amount Z to the lower level using the following equation:

$$Z(r)=B(1-rf). \quad (1)$$

Thus, the lower level receives the total budget B when $f=0$ and receives $(1-r)$ times the budget B when $f=1$. When $r=0$, all funds are reserved for optimal allocation and the lower level receives B . When $r=1$, all the funds are reserved for proportional allocation and the lower level receives $B(1-f)$.

We formulate and solve this problem as a dynamic program in which the time sequence is as follows: the upper level chooses f ; then the lower level chooses r for the given value of f ; then the lower level determines y_j for the given values of f using backward induction and present the details in reverse time sequence.

2.1 The Lower-level Model

Step 3—Let h_j be the number of HIV infections prevented per dollar invested in program j over time T . Let n_j be the size of the risk group j and $N=\sum_j n_j$. We assume that the programs have been indexed so that $h_1 > h_2 > \dots > h_m$. This allows the optimization problem to be formulated as a knapsack problem. The total number of HIV infections averted, IA , is given by the following equation:

$$IA=h_1y_1+h_2y_2+\dots+h_my_m. \quad (2)$$

In the last step of the dynamic program, given r and f , the total number of infections averted is found by solving the following linear programming problem:

$$\begin{aligned} \max_{y_1, y_2, \dots, y_m} \quad & IA=h_1y_1+h_2y_2+\dots+h_my_m \\ \text{S.t.} \quad & \sum_{j=1}^m y_j \leq Z=B(1-rf) \\ & y_j \geq rZ \frac{n_j}{N} \end{aligned}$$

This is similar to the “Knapsack LP” formulation at the lower level in Zaric and Brandeau (2007), and the resulting optimal solution is of the following form:

$$\begin{aligned} y_1 &= Z - rZ \frac{n_2}{N} - \dots - rZ \frac{n_m}{N}, \\ y_j &= rZ \frac{n_j}{N}, \quad j=2, \dots, m. \end{aligned} \quad (3)$$

$$IA(r)=Z(h_1-rk)=B(1-rf)(h_1-rk),$$

$$\text{where } k=(h_1-h_2)\frac{n_2}{N}+\dots+(h_1-h_m)\frac{n_m}{N}.$$

Note that the result of step 3 is the function $IA(r)$ which is quadratic in r (3). Thus, the solution at the lower level will depend on the coefficient of r^2 in $IA(r)$.

Step 2—In the second step, the lower-level decision maker chooses r to maximize his utility function. We consider a utility function that can capture the preference of the lower-level decision maker for proportional allocation. We consider a linear form for the utility function, $U_L(r)$

$$U_L(r) = aZ(r) + br + cIA(r). \quad (4)$$

The parameters $a, b, c > 0$ represent the weights, for budget, equity, and infections averted. We assume that values of a, b , and c are known by both the lower and upper-level decision makers. The lower-level resource allocation problem is written as:

$$L_L: \max_r U_L(r) = aZ(r) + br + cIA(r) \quad (5)$$

$$\text{S.t. } 0 \leq r \leq 1 \quad (6)$$

The lower level solves L_L to obtain $r^*(f)$ and $U_L^*(r^*(f))$.

2.2 The Upper-level Model

Step 1—We assume that the objective at the upper level is to maximize the number of infections averted, as has been assumed in many other studies, (e.g., Ruiz, 2001; Zaric and Brandeau, 2001a; Earnshaw et al., 2007; Zaric and Brandeau, 2007). The upper-level resource allocation problem in step 1 is written as follows:

$$IA_L: \max_f IA(r^*(f)) \quad (7)$$

$$\text{S.t. } 0 \leq f \leq 1 \quad (8)$$

$$r^*(f) = \arg \max (L_L(r)) \quad (9)$$

3. ANALYSIS

Before stating some analytical results we define three threshold values,

$$f_t^L = \frac{b - cBk}{B(a + c(h_1 - k))}, b_{ut}^L = B(a + ch_1), \text{ and } b_{lt}^L = cBk.$$

Proposition 1—For problem L_L with three populations:

- i. U_L is a convex function of r and therefore the optimal solution is either $r^* = 0$ or $r^* = 1$.
- ii. If $f \leq f_t^L$ then $r^* = 1$.
- iii. If $f > f_t^L$ then $r^* = 0$.

Proof—See Appendix.

Corollary 1—When $b > b_{ut}^L$ then $r^* = 1$ and when $b < b_{lt}^L$, then $r^* = 0$.

Proof—Note that $h_1 > k$. Thus,

$$B(a+c(h_1-k)) > 0. \quad (10)$$

First, consider the case where $b > b_{ut}^L = B(a+ch_1)$.

Subtracting cBk from both sides yields $b - cBk > B(a+ch_1 - ck) = B(a+c(h_1 - ck))$.

Dividing both sides by $B(a+c(h_1 - ck))$ yields

$$\frac{b-cBk}{B(a+c(h_1-k))} = f_t^L > 1.$$

From proposition 1, part (ii) we know that $f \leq f_t^L \Rightarrow r^* = 1$. Since $0 < f < 1$ and $f_t^L > 1$, $b > b_{ut}^L \Rightarrow r^* = 1$.

Next, consider the case where $b < b_{it}^L = cBk$.

Thus, $b - cBk < 0$.

Dividing both sides by $B(a+c(h_1 - ck))$ yields $\frac{b-cBk}{B(a+c(h_1-k))} = f_t^L < 0$.

From Proposition 1, part (iii) we know that $f > f_t^L \Rightarrow r^* = 0$.

Part (i) of Proposition 1 says that over the region of r the lower-level utility function is a convex function of r and hence an extreme point solution is optimal. Part (ii) says that all the funds received from the upper level are reserved for proportional allocation if the strength of the incentive is less than threshold value f_t^L . In this case, the lower level would choose to allocate entire budget proportionally if the strength of the incentive is below f_t^L . Conversely, part (iii) says that all funds are reserved for optimal allocation if the strength of the given incentive is greater than f_t^L . In this case, the lower-level decision maker will always choose to allocate the entire budget optimally if the strength of the incentive is above f_t^L .

We next present results that characterize the optimal solution to the upper-level resource allocation problem.

Proposition 2—For problem IA_L with three populations:

- i. If $b \leq b_{it}^L$ then $r^* = 0$ at the lower-level, IA_L is independent of f and any f is optimal. Thus, $IA^*(r^*(f)) = Bh_1$.
- ii. If $b > b_{ut}^L$ then $r^* = 1$ at the lower-level, $IA_L = B(1-f)(h_1 - k)$ and $r^* = 0$ is optimal. Thus, $IA^*(r^*(f)) \equiv B(h_1 - k)$.
- iii. If $b_{it}^L < b \leq b_{ut}^L$ and $f > f_t^L$ then $r^* = 0$ at the lower-level and any $f^* > f_t^L$ is optimal. $IA^*(r^*(f)) \equiv B(h_1 - k)$.

The proof of Proposition 2 is straightforward and omitted. Part (i) of Proposition 2 says that any level of incentive is optimal if the coefficient of r in the lower-level linear utility function is less than b_{it}^L . In this case, the lower-level decision maker will always choose to allocate the entire budget optimally even without incentives from the upper level. Part (ii)

says that if the coefficient of r is greater than b_{ur}^L , then the lower-level decision maker will always allocate all funds proportionally, regardless of any incentives provided by the upper level. Part (iii) says that the level of incentive provided should be greater than f_t^L if the coefficient of r is in between b_{ur}^L and b_{lr}^L . For b between these levels, a choice of incentive above f_t^L would ensure optimal allocation.

4. EXAMPLE

We created a numerical example to illustrate. We used data from California for three risk groups ($m=3$): injection drug users (IDUs); and heterosexuals (HET); men who have sex with men (MSM). Risk group 1 consists of 17,759 IDUs, risk group 2 consists 12,167 HET, and risk group 3 consists 121,128 MSM (California Department of Public Health, 2008). We obtained estimates of the potential cost and effectiveness of interventions in each population from elsewhere (Zaric and Brandeau, 2007). We calculated the number of infections averted per dollar invested in each intervention using a formula published elsewhere (Zaric and Brandeau, 2007) as 0.00012, 0.000046, and 0.0000088, in risk groups 1, 2, and 3, respectively. We estimate that approximately \$36,000,000 of the CDC's \$455,000,000 budget is allocated to California.

To solve for a , b , and c , in the lower-level utility function we set $b=1$ and assumed $U_L(r=0)=\gamma$ and $U_L(r=1)=1$, $0<\gamma<1$. In the base case we assumed that $\gamma=0.5$. Estimates of a , b , and c are shown in Table 1.

Figures 1, 2, and 3 contain three graphs showing the value of f_t^L for a budget ranging from \$10 million to \$50 million. Each graph has three lines for three different values of c relative to the base case estimates shown in Table 1. The three graphs show the impact of three different values of parameter a .

In all graphs, f_t^L is decreasing in B . This implies that as the budget increases, the lower-level decision maker will choose optimal allocation for a smaller incentive. When the budget is large enough, $f_t^L=0$, suggesting that no incentive is needed in these cases. The threshold f_t^L is decreasing in a and c , suggesting that the lower level will choose optimal allocation if it places high value on the number of infections averted and on the budget received from the upper level. f_t^L is less than one for B less than \$25 million in the first graph, \$17 million in the second graph, and \$10 million in the third graph. Thus for a smaller budget it may be impossible to induce $r^*=0$ at the lower level.

Figures 4 and 5 contain two graphs showing the impact on b_{ur}^L and b_{lr}^L for different values of parameter c and a . Each graph has two lines for values of parameter c varying from 0.0001 to 0.001 and a varying from 0.00000001 to 0.0000001. The two lines divide both graphs into three regions where the area below b_{lr}^L , in between b_{lr}^L and b_{ur}^L , and above b_{ur}^L corresponds, respectively, to optimal allocation, either optimal or proportional allocation depending on provided incentives, and proportional allocation of HIV prevention funds, respectively. Figure 4 suggests that with an increase in the preference for preventing infections, the lower level will choose optimal allocation without incentive or with smaller incentive. Figure 5 suggests that with increase in the budget, the lower level will choose optimal allocation for the given incentive.

5. DISCUSSION

We considered a two-level resource allocation problem where the objective at the upper level is to maximize the total number of HIV infections averted and the objective at the lower level is to maximize a utility function that contains terms for infections averted, budget, and equity. We showed that the objective function at the lower level is convex in the fraction of the funds reserved for proportional allocation and therefore the optimal allocation has either all or none of the funds reserved for equity. The choice of all or none depends on several factors including the level of the incentive provided by the upper level and the coefficient of the proportional allocation term in the lower-level utility function.

Our goal was to demonstrate the significance of incentives to encourage optimal allocation of HIV prevention funds. Thus, an incentive scheme incorporating the lower level decision maker's concerns about equity and the upper levels choice about the strength of incentive was proposed to help illustrate the above objective. However, different incentive schemes could be developed. Our analysis can be generalized for different incentive schemes by adjusting for the upper and lower-level utility functions.

Our analysis has limitations. We considered a single region at the lower level. However, it could be extended to multiple regions. A short time horizon model was considered which does not incorporate epidemic dynamics. Models for longer time horizons could be developed. These would need to consider epidemic dynamics and may lead to different allocation decisions. We assumed that the upper and lower-level decision makers have complete knowledge of each other's decisions and the parameter values, which could be valid if detail proposals are received from the lower level and the incentive scheme is made known by the upper level. However, if information was incomplete then a probability distribution about the lower level decisions could be incorporated. We assumed that benefits of intervention scale linearly with respect to amounts invested. This assumption is common in cost effectiveness analysis but may not always be valid.

Little information is available about the utility functions of regional decision maker in practice. Our analysis demonstrates the value of such information to help make optimal decisions at the upper level. The results of this evaluation may be useful for aid and other organizations to obtain the most impact from limited budgets.

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APPENDIX

We define the following terms before presenting the proofs:

$$\text{Let } f_t^L = \frac{b - cBk}{B(a + c(h_1 - k))}, b_{ut}^L = B(a + ch_1), \text{ and } b_{it}^L = cBk.$$

Proposition 1

For problem L_L with three populations:

- (i) U_L is a convex function of r and therefore, the optimal solution r^* is either 0 or 1.
- (ii) If $f \leq f_t^L$ then $r^* = 1$.
- (iv) If $f > f_t^L$ then $r^* = 0$.

Proof

- i. The lower-level utility function is $aZ + br + cIA$. When $Z = B(1 - rf)$ and $IA(r) = B(1 - rf)(h_1 - rk)$ are substituted into this we obtain:

$$L_L: \max_r U_L = aZ + br + cIA = aB(1-rf) + br + cB(1-rf)(h_1 - rk) \\ \text{s.t. } 0 \leq r \leq 1$$

The second derivative of U_L with respect to r is given by,

$$U_L'' = \frac{d}{dr} U_L' = \frac{d}{dr} [aB(-f) + b + cB(-f(h_1 - rk) - k(1 - rf))] = 2cBfk \geq 0.$$

Since $U_L'' \geq 0$, U_L is a convex function of r and the optimal solution is either $r^* = 1$ or 0.

$$\text{ii. } U_L(0) = aB + cBh_1 \text{ and } U_L(1) = aB(1-f) + b + cB(1-f)(h_1 - k).$$

$$\text{If } U_L(1) \geq U_L(0) \text{ then } r^* = 1 \text{ and if } U_L(1) < U_L(0) \text{ then } r^* = 0. \quad (11)$$

Note that $B(a + c(h_1 - k)) > 0$ because $h_1 > k$.

$$\text{If } f \leq f_t^L = \frac{b - cBk}{B(a + c(h_1 - k))} \text{ then}$$

$$\begin{aligned} f(B(a + c(h_1 - k))) &\leq b - cBk \\ aBf + cBfh_1 - cBfk &\leq b - cBk \\ \text{Thus, } 0 &\leq -aBf + b - cBfh_1 + cBfk - cBk. \quad (12) \\ \Rightarrow aB + cBh_1 &\leq aB(1-f) + b + cB(1-f)(h_1 - k) \\ &\Rightarrow U(0) \leq U(1) \end{aligned}$$

Thus, $U(0) \leq U(1)$ if $f \leq f_t^L$.

iii. This follows by reversing the original inequality in part (ii).

Figure 1. Threshold f_t^L versus budget for different values of c with a at half the base case value.

Figure 2.

Threshold f_t^L versus budget for different values of c and a at the base case value.

Figure 3. Threshold f_t^L versus budget for different values of c and a at 2 times the base case value.

Figure 4.

Thresholds b_{ut}^L and b_{lt}^L for different values of parameter c

Figure 5.

Thresholds b_{ut}^L and b_{lt}^L for different values of parameter a .

TABLE 1

Base case parameter values

Parameter	Base Case Values
<i>a</i>	0.00000004
<i>b</i>	1
<i>c</i>	0.0004