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A Fast Iterated Conditional Modes Algorithm for Water-Fat Decomposition in MRI

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Abstract

Decomposition of water and fat in Magnetic Resonance Imaging (MRI) is important for biomedical research and clinical applications. In this paper, we propose a two-phased approach for the three-point water-fat decomposition problem. Our contribution consists of two components: (1) a background-masked Markov Random Field (MRF) energy model to formulate the local smoothness of field inhomogeneity; (2) a new Iterated Conditional Modes (ICM) algorithm accounting for high-performance optimization of the MRF energy model. The MRF energy model is integrated with background masking to prevent error propagation of background estimates as well as improve efficiency. The central component of our new ICM algorithm is the Stability Tracking (ST) mechanism intended to dynamically track iterative stability on pixels so that computation per iteration is performed only on instable pixels. The ST mechanism significantly improves the efficiency of ICM. We also develop a median-based initialization algorithm to provide good initial guesses for ICM iterations, and an adaptive gradient-based scheme for parametric configuration of the MRF model. We evaluate the robustness of our approach with high-resolution mouse datasets acquired from 7-Tesla MRI.

Keywords

Background masking; field inhomogeneity; MRF; iterated conditional modes(ICM); magnetic resonance imaging(MRI); Markov random field(MRF); median value initialization; stability tracking; water/fat

I. Introduction

Robust decomposition of water and fat in MR imaging has been under intensive study [1–8] for its significance in both biomedical research and clinical applications. In many settings, water in MR images acquired *in vivo* is of primary interest and fat signals have to be suppressed to reduce artifacts and increase underlying lesion conspicuity [9]. The decomposition of water and fat is also valuable for the diagnosis of obesity-induced diseases such as nonalcoholic fatty liver disease and bone marrow diseases.

The three-point method [1, 2] has been widely used to decompose water and fat components in MR. With the standard setup, the signal at an individual pixel p can be described as:

$$s_n(p) = \left(W(p) + F(p) e^{i2\pi\Delta f t_n} \right) e^{i2\pi\psi(p)t_n} + \varepsilon \quad (1)$$

In this model, the field map value $\psi(p)$ (in Hz) is the local magnetic resonance offset due to field inhomogeneity, $W(p)$ and $F(p)$ are intensities of water and fat respectively, Δf (in Hz) is the chemical shift between water and fat, t_n with $n = 1, 2, 3$ are uniformly spaced echo time (TE) shifts, and ε denotes the noise. In this simplified model, we can assume that the chemical shift Δf between water and fat is known *a priori* (for example, Δf is approximately 1051.2 Hz at 7 Tesla). Equivalently, (1) can be rewritten as

$$\underbrace{\begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix}}_S = \underbrace{\begin{pmatrix} e^{i2\pi\psi t_1} & e^{i2\pi(\Delta f + \psi)t_1} \\ e^{i2\pi\psi t_2} & e^{i2\pi(\Delta f + \psi)t_2} \\ e^{i2\pi\psi t_3} & e^{i2\pi(\Delta f + \psi)t_3} \end{pmatrix}}_{\Psi(\psi)} \cdot \underbrace{\begin{pmatrix} W \\ F \end{pmatrix}}_{\rho} + \varepsilon \quad (2)$$

Let $\Psi^+ = (\Psi^T \Psi)^{-1} \Psi^T$ denote the pseudo-inverse of the coefficient matrix Ψ in model (2). Then based on (2), water and fat intensities of a pixel can be readily calculated via

$$\rho = \Psi^+ \cdot S \quad (3)$$

whenever the field map value ψ is available. In this sense, the water-fat decomposition problem can be reduced to recovering field map values at each pixel.

Approaches recently proposed to address the field map recovery, and thus water and fat decomposition problem, include IDEAL [3], Region Growing [4], VARPRO [6], Algebraic Decomposition [7], and Graph Cut [8]. All these approaches share the same basic assumption of local spatial smoothness of field map, while differ essentially in how to harness such a smoothness property. For example, IDEAL [3] uses low-pass filtering to smooth the field map estimator, Region Growing [4] which was intended to enhance IDEAL uses extrapolation to rectify erroneous estimations, and Algebraic Decomposition [7] uses phase unwrapping techniques to generate estimation. VARPRO [6] formulates the local smoothness with a variable projection based Markov Random Field (MRF) energy function and employs the Iterated Conditional Modes (ICM) algorithm [10] to process the MRF energy minimization. VARPRO has demonstrated desirable noise performance and acceptable accuracy [6]. However, low efficiency on higher resolution acquisitions and accuracy in the presence of large field inhomogeneities [8] may be a significant barrier for VARPRO's wide adoption.

To overcome both the efficiency and accuracy problems associated with VARPRO, we propose a new approach to water and fat decomposition. Despite its root in VARPRO, our approach sports two distinctive features: (1) a more accurate Markov random field (MRF) energy model integrated with background masking; (2) an efficient ICM-based algorithm accounted for the MRF energy optimization.

The MRF energy model, which we call NLR-MRF, formulates the local smoothness property of field map as a nonlinear normed residual MRF energy function. Minimization of the NLR-MRF model gives rise to field map estimates at all pixels. The NLR-MRF model differs from the one used in VARPRO or elsewhere in its integration of background masking. Background masking is to mask pure noise regions out of the MR acquisitions so that estimate errors from noise regions could not propagate to object regions. Background

masking also enables improvement of efficiency by sparing computations on noise pixels. Two foreground extraction algorithms, one is morphology-based and the other GrabCut-based [12], are introduced for background masking.

Our new fast ICM algorithm, termed as ICM-ST (for ICM with Stability Tracking), provides efficient computation on the minimization of the NLR-MRF model. Compared to basic ICM algorithms, ICM-ST is unique in its agile stability tracking mechanism and automatic median-based initialization. The stability tracking mechanism is responsible for identifying pixel stability throughout all iterations. At each iteration, the energy function is computed only on pixels identified as instable instead of on all pixels like basic ICM algorithms do. With exponential decrease of the overall instability after the first few iterations, the time cost at each iteration is greatly reduced. Moreover, with appropriate parametric configuration, ICM-ST attains the very same accuracy as basic ICM algorithms. To address the sensitivity of ICM-ST to initialization, a median value based algorithm is developed. By assuming the statistical similarity of field inhomogeneity on the whole and unambiguous field maps as well as the field inhomogeneity's quasi-unimodality, the median initialization algorithm estimates the median of field inhomogeneity on unambiguous pixels and set it as the initial guess at each pixel.

The rest of the paper is organized as follows. First, the ICM-ST approach, including the NLR-MRF model with background masking, the stability tracking algorithm, the median initialization algorithm, and the gradient based regularization configuration scheme, is described in Section II. Experimental results are given in Section III. Discussions are presented in Section IV. The Appendix section gives a formal proof on the accuracy of the stability tracking mechanism.

II. Method

In this section, we describe the major components of the ICM-ST approach, including:

- i. The NLR-MRF model with background masking (Section II-A),
- ii. The stability tracking algorithm (Section II-B),
- iii. The median initialization algorithm (Section II-C),
- iv. The gradient based configuration scheme for regularization factors (Section II-D).

The whole ICM-ST algorithm is summarized in Section II-E.

A. NLR-MRF Model with Background Masking

1) The NLR-MRF Model—Let $\mathcal{P}$ denote the field map grid. In terms of the scanned object $\mathcal{P}$, consists of the foreground object region $\mathcal{F}$ and the background noise region $\mathcal{B}$. Assume $\psi_{\max}$ and $\psi_{\min}$ are an upper bound and a lower bound of the field inhomogeneity on $\mathcal{P}$, respectively. For a positive number m , let

$$I_m = \left\{ \psi_{\min} + \frac{k}{m} (\psi_{\max} - \psi_{\min}) : 0 \leq k \leq m \right\} \quad (4)$$

denote the evenly spaced set of the inhomogeneity interval $[\psi_{\min}, \psi_{\max}]$. For any configuration $\psi: \mathcal{P} \rightarrow I_m$ of $\mathcal{P}$ on I_m , we define the Non-Linear Residual MRF (NLR-MRF) energy function as:

$$E(\psi) = \underbrace{\sum_{p \in \mathcal{F}} D_p^l(\psi_p)}_{E_{data}} + \underbrace{\sum_{p \in \mathcal{F}} \mu_p \sum_{q \in \mathcal{N}_p \cap \mathcal{F}} \omega_{pq} |\psi_p - \psi_q|^l}_{E_{smooth}} \quad (5)$$

where, $l \geq 1$ is an integer, $\mathcal{N}_p$, μ_p , $W_p = (\omega_{pq})_{q \in \mathcal{N}_p}$ are the neighborhood, regularization factor and weight matrix of pixel p , respectively, and

$$D_p^l(\psi_p) = \|S - \Psi(\psi_p) \Psi(\psi_p)^+ S\|^l \quad (6)$$

is the l -normed non-linear residual at p . Note that $D_p^l(\psi_p)$ is a periodic function with the basic period of $1/(t_2 - t_1) (= 1/(t_3 - t_2))$ [4]. In (5), the data cost E_{data} measures the overall fitting of ψ with the acquired data, and the smoothness cost E_{smooth} enforces the local smoothness of the field map.

The most notable feature of our NLR-MRF formulation is its background masking: the energy formula (5) sums up only on foreground pixels $\mathcal{F}$ while all background pixels $\mathcal{B}$ are zeroed. Benefits of background masking for the energy formulation are twofold. The major benefit is the accuracy improvement. There is neither water nor fat component at any background pixel and so estimating based on the signal model (1) can be error prone. Without masking off background regions, estimation errors on these background pixels are likely to propagate to their foreground neighbors. The second benefit is the improvement of efficiency by sparing computation on background pixels.

The norm order l can be set as 1 or 2. The size of the neighborhood system $\{\mathcal{N}_p: p \in \mathcal{F}\}$ can impact the final recovery quality. As suggested in VARPRO [6], the 5×5 square neighborhood which we have adopted for this work is a good choice.

2) Background Masking Algorithm—Extraction of object regions from background for the NLR-MRF model (5) is relatively easy because of high contrast of object to noise regions in MRI images.

A background masking algorithm for MRI images is described in Algorithm 1. It is based on a series of morphological operations and thus straightforward to implement. Prior to these morphological operations, one sampling of the dataset needs to be selected as the template. Histogram-based methods (e.g., [11]) can be adopted to generate the dilation threshold in Step 2.

Morphological operations employed in Algorithm 1 may mask off object borders which are sometimes regions of interest (ROI). In this case, morphological dilation can be employed to minimize the masking error. Additionally, generic foreground extraction methods such as GrabCut [12] are also possible for MR background masking after some adaption. Fig. 1 shows the two masks generated separately by the Algorithm 1 and the adapted GrabCut method. No significant difference can be observed between them.

B. The Stability Tracking Algorithm

The Iterated Conditional Modes (ICM) algorithm [10] is one of the most widely used methods for MRF energy optimization. When directly applied to the NLR-MRF model (5), ICM updates the field map value at each iteration k and each pixel $p \in \mathcal{F}$ in the manner that

$$\psi_p^{new} = \arg \min_{\psi_p \in I_m} D_p^l(\psi_p) + \sum_{q \in \mathcal{N}_p \cap \mathcal{F}} \omega_{p,q} |\psi_p - \psi_q^{cur}|^l \quad (7)$$

where, ψ_q^{cur} denotes field map estimate of q at the $(k-1)$ -th iteration. This iterative process continues until sum of total difference between consecutive steps is below a preset threshold ε :

$$\sum_{p \in \mathcal{F}} |\psi_p^{new} - \psi_p^{cur}| < \varepsilon \quad (8)$$

Note that the data energy term $D_p^l(\psi_p)$ in formula (7) can be precomputed before all iterations start [6]. Therefore, the time cost of ICM on n iterations is

$$\mathcal{O}(|I_m| \times |\mathcal{N}_p| \times |\mathcal{F}| \times n) \quad (9)$$

where, $|I_m|$ is the number of discretization steps, $|\mathcal{N}_p|$ foreground size, and $|\mathcal{F}|$ the neighborhood size. This time cost is expensive even for images with moderate size [6].

The Stability Tracking algorithm, by tracing stability throughout iterations, manages to minimize the number of pixels (the $|\mathcal{F}|$ term in (9)) to be computed at each iteration. The stability tracking algorithm and related performance analysis are presented next.

1) Iterative Stability—First, we introduce some basic notions which are useful for description as well as analysis of the stability tracking algorithm. Let $\psi_p^{(k)}$ denote the field map value of a foreground pixel p generated by the k -th ICM iteration ($k \geq 1$) with formula (7). In particular, $\psi_p^{(0)}$ denotes the initial guess for p .

Definition A.1 A pixel p is called *stable* at iteration k ($k \geq 1$), if $\psi_p^{(k)} = \psi_p^{(k-1)}$.

From Definition A.1 and formula (7), it can be readily observed that

Property A.2 A pixel is stable at iteration k if all of its neighboring foreground pixels are stable at iteration k .

We next introduce the notions of cumulative stability number and recent stability.

Definition A.3 The *cumulative stable number* of a pixel p at iteration k is n , if p remains stable for the latest n iterations, i.e.,

$$\psi_p^{(k)} = \psi_p^{(k-1)} = \dots = \psi_p^{(k-n)} \quad \text{and} \quad \psi_p^{(k-n)} \neq \psi_p^{(k-n-1)} \quad (10)$$

We use $cus(p, k)$ to denote the cumulative stable number.

Definition A.4 A pixel p is *cumulatively stable* at iteration k with respect to a threshold integer cus_thr , if

$$cus(p, k) \geq cus_thr \quad (11)$$

2) The ST(k) Algorithm—From Property A.2, one can infer that the computation of formula (7) is at a pixel p is a waste if all p 's neighbors are known to be stable. Keeping track of stability information in each iteration therefore avoids on redundant computations.

In the ST(k) algorithm (see Algorithm 2), the two-dimensional lookup table CUS is set to keep track of all pixels' stability, where p denotes a pixel, k denotes the iteration index, and $CUS(p, k)$ represents the stability status of p by the k -th iteration. The close relationship between $CUS(p, k)$ and $cus(p, k)$ will be discussed later. A threshold value cus_thr is preset for identifying cumulative stability property in each iteration. To compute the estimate of a pixel p at the k -th iteration, the ST(k) algorithm first checking its stability by looking up the CUS table. If p is cumulatively stable (that is, $CUS(p, k-1) \geq cus_thr$), no further computation is needed for p and ST(k) jumps to the next pixel (line 2-3). Otherwise, formula (7) is calculated at p to obtain a new estimate (line 5). With the new estimate, stability of p within the current iteration is verified (line 6). If p is stable for the current iteration, the CUS number of p increments by β (line 7). Otherwise, the CUS number is decreased with the ratio of α (line 9); the CUS number of each neighbor of p also needs to be decreased in the same way (line 10-12). Finally, the value of ψ_p^{cur} is updated (line 13) so that it is plugged into subsequent computations on p 's posterior neighbors.

In Algorithm 2, all except the fifth line are intended to maintain the CUS structure. Compared to the time cost of computations on formula (7) of line 5, the maintenance overhead of the CUS structure, $\mathcal{O}(|\mathcal{N}_p| \times |\mathcal{F}| \times n)$, is ignorable. Initialization of CUS before starting the iteration ST(1) is:

$$CUS(p, 0) = 0, \quad \text{for each } p \in \mathcal{F} \quad (12)$$

meaning that none of the pixels in $\mathcal{F}$ is stable yet.

Note the distinction between the two numbers $CUS(p, k)$ and $cus(p, k)$: $CUS(p, k)$ is updated not only by computation at p , but also by computation at p 's neighboring pixels (line 10-12). It is not hard to verify that

$$CUS(p, k) \leq cus(p, k). \quad (13)$$

for any pixel p and iteration index k .

The update policy of the CUS structure (line 7-12) is well-known to be Additive-Increase Multiplicative-Decrease (AIMD) in networking congestion control research [13, 14]. The AIMD policy is critical to guarantee the accuracy of the ST algorithm. The AIMD parameters α should be constrained in the unit interval $[0, 1)$ and β no less than 1. A desirable configuration for them is: $\alpha = 0$ and $\beta = 1$.

In our ST algorithm, the asynchronous update strategy is employed to update field map values (line 13). Our evaluation shows that the ST algorithm equipped with the asynchronous updating runs about 16% faster than with the synchronous update (i.e., move $\psi_p^{cur} := \psi_p^{new}$ of line 13 to right above line 16) with almost identical results.

3) Accuracy with Stability Tracking—The cumulative stability threshold cus_thr is an important parameter for the ST(k) algorithm. Inappropriate setup of cus_thr leads to unpredictable accuracy. As shown in the Appendix, the ST algorithm configured with

$$cus_thr=2, \quad \alpha=0, \quad \text{and} \quad \beta=1 \quad (14)$$

yields the very same result as basic ICM algorithms at each iteration. Furthermore, according to our observation, there is no apparent change on field map estimates with $cus_thr = 1$.

4) Efficiency Improvement with Stability Tracking—The improvement on effectiveness of the ST algorithm on ICM iterations is determined by the size of the threshold cus_thr and the overall field map stability increasing speed.

According to Definition A.4, the cumulative instability ratio $r_{ins}(k)$ for iteration k is

$$r_{ins}(k) = \frac{\#\{p \in \mathcal{F} : CUS(p, k) < cus_thr\}}{\#\mathcal{F}} \quad (15)$$

where, $\#\{p \in \mathcal{F} : CUS(p, k) < cus_thr\}$ is the number of instable pixels, and $\#\mathcal{F}$ is the number of all foreground pixels. Apparently, the ratio $r_{ins}(k)$ is proportional to the time cost at iteration k and smaller $r_{ins}(k)$ implies more amounts of time that can be saved by the stability tracking. Regression analysis on a family of 60 mouse datasets demonstrates the approximate exponential decreasing of $r_{ins}(k)$ after the first cus_thr iterations, that is,

$$r_{ins}(k) = \xi^{k-cus_thr}, \quad \text{for } k \geq cus_thr + 1 \quad (16)$$

for some constant ξ . Fig. 2 presents linear fittings of $\log(r_{ins}(k))$ on 6 different datasets. The following analysis of efficiency improvement is based on the exponential decreasing assumption of (16).

Each iteration in basic ICM algorithms requires the same amount of time, say t_0 . Therefore, the time cost for n basic ICM iterations is $T_{ICM} = n \cdot t_0$. On the other hand, let $T(k)$ denote the time cost of ST(k). Then $T(k) = t_0$ for $k \leq cus_thr$ and $T(k) = \xi^{k-cus_thr} t_0$ for $k > cus_thr$. Thus, the total time cost for N ST iterations is

$$\begin{aligned} T_{ST} &= cus_thr \cdot t_0 + \sum_{k=cus_thr+1}^n t_0 \cdot \xi^{k-cus_thr} \\ &\approx cus_thr \cdot t_0 + \frac{\xi}{1-\xi} t_0 \end{aligned}$$

and the ratio ρ that stability tracking mechanism accelerates ICM iterations is

$$\rho = \frac{T_{ICM}}{T_{ST}} = n / \left(cus_thr + \frac{\xi}{1-\xi} \right) \quad (17)$$

In the special case of $cus_thr = 2$, the acceleration ratio is

$$\rho = \frac{1-\xi}{2-\xi} n \quad (18)$$

C. The Median Initialization Algorithm

ICM-based techniques are sensitive to initial guesses and perform well only with appropriate initialization [10, 31]. However, due to the high dimensionality of the smoothness cost terms in MRF formulation, it is difficult to find a practical initialization method. ICM-ST initialized with methods proposed in [6, 10, 31] are erroneous according to our tests.

The median initialization algorithm for ICM-ST is to estimate and apply the median of unambiguous pixels to initialize the ICM iterations at each pixel. Herein, unambiguous pixels refer to those pixels at which the signal model (1) has unique solution on the field map (thus unique solutions on the water and fat variables). Theoretically, pixels containing both fat and water are mostly unambiguous [4]. In addition, fat pixels are highly likely to be unambiguous with respect to the single-peak model (1), largely due to that the olefinic fat protons resonate at a frequency close to the water resonance [7].

The median initialization is based on the assumptions of statistical similarity and the quasi-unimodality on the whole and unambiguous fields, as have been constantly observed with our test. Finite Gaussian Mixture (FGM) models [15–17] have been widely used to formulate MR field inhomogeneity. As a sample from the whole field map, the unambiguous field inhomogeneity can also be modeled with FGM. Furthermore, these two types of FGM models are, to some extent, similar to each other, and the similarity extent is dependent on the density of unambiguous pixels. In the case of sufficiently large density of unambiguous pixels over the whole field, the likeliness of considerable distinction between these two FGM models is little, and consequently they have close probability density functions (PDF) of FGM and especially the median values. Therefore, the inhomogeneity median on the whole field map can be estimated from the unambiguous field map. An example illustrating this similarity is shown in Fig. 3(a), (b) and (c). Secondly, although general FGM models are usually multimodal, FGM probability density functions for moderate MR field inhomogeneity have great chance of containing a single dominant mode or a small family of dominant modes clustering within a small range (for example, in the two-sigma or three-sigma interval, as shown in Fig. 4). In other words, the field inhomogeneity is quasi-unimodal. Let ψ denote the correct reconstruction of field inhomogeneity and $\tilde{\psi}$ the new inhomogeneity reconstruction by ICM-ST initialized with ψ 's median. Then with our tests, $\tilde{\psi}$ would be almost identical to ψ as long as the latter is quasi-unimodality, or alternatively, the median values are good initial guesses for ICM iterations. Fig. 3(d) shows an instance for this identicalness. Quasi-unimodality assessment is generally a difficult problem. Although a robust method to automatically assess quasi-unimodality is not yet available, a few statistical techniques are applicable. For example, kernel density estimation [18–20] and DIP test [21–23] can be used to test unimodality of the field inhomogeneity. For multimodal field homogeneity, as a rule of thumb, the two-sigma and three-sigma rules can be applied to evaluate the closedness of dominant modes (see Fig. 4).

The harmonic retrieval analysis based approach covered in [7] can be applied to identify unambiguous pixels and further estimate their unique solutions. With this approach, the signal equations (1) at each pixel is first transformed to the quadratic equations of a as

$$\begin{cases} s_3(p) - (1+\lambda) s_2(p) \gamma + \lambda s_1(p) \gamma^2 = 0 \\ s_1(p)^* - (1+\lambda) s_2(p)^* \gamma + \lambda s_3(p)^* \gamma^2 = 0 \end{cases} \quad (19)$$

where $T = t_2 - t_1$, $\gamma = e^{2\pi i \psi T}$, $\lambda = e^{2\pi i \Delta f T}$, and the notation $*$ denotes the complex conjugate operation. For the sake of convenience, the solutions of (19) are called feasible solutions. Then pixels with unique feasible solution are identified as unambiguous pixels. It is noteworthy that due to noise contamination, straightforward calculation on (19) is error prone. Instead, more complicated techniques like the polynomial greatest common divisor (GCD) method [7, 24, 25] are more competent for this task.

The quasi-unimodality on the unambiguous field inhomogeneity may be corrupted when a considerable number of unambiguous pixels' solutions are phase wrapped at zero, as illustrated by Fig. 5(a). Therefore, prior to applying all unambiguous pixels' solutions to

estimate the median, it is important to test phase wrapping at zero and carry out phase unwrapping if necessary. This test can be performed with the statistic

$$X_0 = \frac{\#\{p \in \mathcal{U} : -w \leq \hat{\psi}(p) \leq w\}}{\#\mathcal{U}} \quad (20)$$

Herein, $\mathcal{U}$ denotes the set of all unambiguous pixels, $\hat{\psi}(p)$ the unambiguous solution at p , and $[-w, w]$ the window around 0, and thus X is the ratio of the number of unambiguous pixels with the solution neighboring 0 to the total number of unambiguous pixels. Given a significance level ε , $X_0 < \varepsilon$ implies occurrence of significant phase wrapping. Throughout our tests, ε has been fixed as 0.10 and w the width of 5 discretization steps, as shown in Fig. 5. Note that the two equations in (19) are periodic of $1/T$ with respect to ψ , and so phase unwrapping can be simply done by either adding $1/T$ to all negative inhomogeneity solutions or alternatively subtracting all positive inhomogeneity solutions with $1/T$.

By assuming validity of the quasi-unimodality on field inhomogeneity, the median initialization algorithm is summarized in Algorithm 3.

D. Gradient-Based Configuration of Regularization Factor

The MRF regularization factor μ in model (5) regulates the relative weight of data cost to smoothness cost. Undersized regularization values often result in under-smooth recoveries, while oversized values leads to results almost identical to initial guesses. Regularization configuration usually depends on both the MRF energy formulation and the optimization algorithm. Some configuration methods can be referred to in [6, 8, 26, 27].

The gradient-based regularization configuration scheme for the NLR-MRF model (5) and the ICM-ST algorithm is based on controlling the ratio of data cost gradient to smoothness cost gradient within a reasonable range. Given a field map estimate ψ_p of pixel p , we use $\nabla D_p^l(\psi_p)$ and $\nabla S_p(\psi_p)$ to separately denote the absolute value of data cost and smoothness cost gradients, where,

$$S_p(\psi_p) = \mu_p \sum_{q \in \mathcal{N}_p \cap \mathcal{F}} \omega_{pq} |\psi_p - \psi_q|^l \quad (21)$$

Despite the high dimensionality of $S_p(\psi_p)$, we can use the local smoothness feature of field map to estimate its gradients at each pixel. In fact, since field map values of p with its neighboring pixels should be nearly the same, with $\varrho(p)$ denoting the approximate value, we have

$$S_p(\psi_p) \approx \mu_p \sum_{q \in \mathcal{N}_p \cap \mathcal{F}} \omega_{pq} |\psi_p - \varrho_p|^l = \mu_p \omega |\psi_p - \varrho_p|^l$$

where $\omega = \sum_{q \in \mathcal{N}_p \cap \mathcal{F}} \omega_{pq}$ is the sum of neighboring weights and constant with respect to p and ψ . Thus, the gradient of $S_p(\psi_p)$ is

$$\nabla S_p(\psi_p) = l \mu_p \omega |\psi_p - \varrho_p|^{l-1} \quad (22)$$

Then the gradient ratio $\rho(\psi_p)$ of data cost to smoothness cost is

$$\rho(\psi_p) = \frac{\nabla D_p^l(\psi_p)}{\nabla S_p(\psi_p)} = \frac{\nabla D_p^l(\psi_p)}{l\mu_p\omega|\psi_p - \varrho_p|^{l-1}} \quad (23)$$

Alternatively, the regularization factor at p is

$$\mu_p = \frac{\nabla D_p^l(\psi_p)}{l\omega\rho(\psi_p)|\psi_p - \varrho_p|^{l-1}} \quad (24)$$

Equation (24) provides very useful heuristic for regularization factor configuration. Generally, the gradient ratio $\rho(\psi_p)$ can be set to be some number less than 10. The term $|\psi_p - \varrho_p|$ vanishes for the 1-normed NLR-MRF models, and can be replaced by $|\psi_{max}|$ in 2-normed models. $\nabla D_p^l(\psi_p)$ can be evaluated by the mean value of all absolute gradients of the data cost $D_p^l(\psi_p)$.

E. The ICM-ST Algorithm

The complete ICM-ST algorithm, integrated with background masking, median value initialization, stability tracking, as well as gradient based configuration of regularization factors, is summarized in Algorithm 4.

III. Results

A. MRI Acquisition and Methods

High resolution shifted spin echo scans were acquired on a Bruker BioSpec 7T/30cm System. A T1-weighted rapid acquisition with relaxation enhancement (RARE) sequence with varying echo asymmetry delays was used to achieve $\{\pi/6, 5\pi/6, 9\pi/6\}$ radian shifts between fat and water. Other typical parameters include field of view (FOV) of 102×40 to 102×50 mm, matrix size = 512×256 , 4 averages, pulse repetition time (TR) = 1087 ms/gated, echo time (TE) = 9.1 ms, echo asymmetry delays of $\{79.3, 396.4, 713.6\}$ μ s. Three groups of whole-body datasets, named HF1, HF2, and HF3, were acquired from 3 mice, each group corresponding to one mouse. Each group contains 20 datasets and three echoes were used in each dataset.

Algorithms were implemented on a desktop computer with 2.66GHz Intel Core2 CPU, 4GB RAM, MATLAB 2010a 64 bit, without using any parallel computing technique. We re-implemented the VARPRO [6] and IDEAL [3] in MATLAB since their original implementations were not available. Note that our IDEAL implementation was only based on the specification in [3] and lacked of region growing [4] or any other improving techniques later proposed.

Major parameters for ICM-ST and VARPRO were configured as follows. The norm order $l = 1$. The neighborhood system was 5-by-5 square weighted with the inverse of distance. Field map values were bounded in the range of $\pm 1/\Delta T$ where $\Delta T = 0.3172 \times 10^{-3}$ is the effective time shift. Moreover, after the field map median ζ was generated with the median value initialization algorithm, the bound interval was further centralized at ζ , that is, the effective lower/upper bounds of field map were: $\psi_{min} = -1/\Delta T - \zeta$, $\psi_{max} = 1/\Delta T + \zeta$. Following [6], the discretization numbers m was set to 300. The AIMD parameters for stability tracking were set as: $\alpha = 0$, $\beta = 1$, $cus_thr = 2$. Following the gradient based configuration formula (24), the regularization factor μ_p at each foreground pixel p was set to be $mean(\nabla D_p^l(\mathcal{S}_m))/\omega$.

B. Evaluation of Efficiency

We tested the proposed ICM-ST, VARPRO, and VARPRO with background masking for efficiency comparison. Fig. 6 shows the overall time costs of these three methods on the three dataset groups. The number of iterations for both VARPRO and VARPRO with background masking was set to 40, and the cumulative instability ratio for ICM-ST was constantly 0.001 (which in most of cases required more than 40 iterations to achieve). For ICM-ST, recoveries on datasets with moderate object region size required about 105 seconds which included the time cost for data precomputation (about 23 seconds), calculation of the regularization parameter μ (about 4 seconds) and initialization (about 14 seconds). Downsampling the images to 256×128 could decrease the initialization time cost to less than 6 seconds while not causing significant bias on the median estimates. For the VARPRO method, the average time cost was above 1000 seconds. For VARPRO with background masking, the average time cost on datasets with moderate object region size required about 390 seconds. With respect to IDEAL, noise pixels on the background generally required much more iterations than those foreground pixels. By setting the increment $\Delta\psi$ to be 5 Hz ([3]), we observed that IDEAL was more time consuming than ICM-ST. However, when applied with the background masking, IDEAL converged within 40 seconds.

C. Evaluation of Accuracy

All 60 datasets contained in the groups of HF1, HF2 and HF3 were tested on ICM-ST and IDEAL for accuracy evaluation. Based on the location and the number of pixels at which water-fat swapping occurs in reconstructions, three types of error scales were graded as follows:

- *good*: no visible water-fat swapping is observed
- *fair*: water-fat swapping occurs at object boundaries, and the number of error pixels is no more than 50
- *poor*: the number of pixels is more than 50, or water-fat swapping occurs inside object regions.

Note that recoveries at the fair scale can be easily corrected by either manual editing or simple smoothing processing.

The reconstruction accuracy of ICM-ST on the three datasets is shown in Fig. 7(a). Here, ICM-ST was uniformly initialized with the medians calculated from the median algorithm. The field inhomogeneity on datasets varied in different range. The average variation width of field inhomogeneity on the HF1, HF2 and HF3 groups are 1926 Hz, 1509 Hz and 2288 Hz, respectively. Totally there were 6 reconstructions at the fair error level and 3 ones at the poor error level (see Fig. 8(a), 8(b), 8(c)). The poor reconstruction shown in Fig. 8(a) was from the HF1 group. The most severe water-fat swapping occurred near to the mouse heart area, containing 118 error pixels. The correct field inhomogeneity varied from -460 Hz to 1957 Hz. The other two poor reconstructions, as shown in Fig. 8(b), 8(c), were both from the HF3 group.

The accuracy performance of ICM-ST can be improved after we substituted the uniform initialization with the block-wise initialization (see Fig. 7(b)). The block-wise median initialization method divides an image into multiple blocks and each block is independently initialized by the median algorithm. The block-wise initialization is effective when the field inhomogeneity varies in an extremely large range. For Fig. 8(b), the correct field map varies from -877.6 Hz to 1749.9 Hz, with the global median 488.7. However, the upper part and the lower part (divided by the green line) have distinctively different distributions: the minimum, median and maximum of field map values on the upper part are $\{-877.6, -120.9,$

383.6} while these quantities on the lower part are $\{-205.0, 530.7, 1749.9\}$, respectively. The difference between the global median value 488.7 and the median value -120.9 on the upper part is too large for ICM-ST iterations to converge every pixel to the correct estimate. By contrast, the block-wise initialization with the upper and lower part separately being -120.9 and 530.7 enables ICM-ST to converge every pixel correctly (Fig. 8(e)). Likewise, with the block-wise initialization applied for the field map in Fig. 8(c), ICM-ST yielded the correct reconstruction shown in Fig. 8(f). With the block-wise initialization improvement, the water-fat swapping contained in the poor reconstruction from the HF1 group was also greatly alleviated, as shown in Fig. 8(d).

IDEAL exhibited poor performance with the mouse datasets. For each dataset, IDEAL was initialized with 10 different values around that generated by the median algorithm and the best reconstruction was selected for evaluation. But water-fat swapping was still widely observed in most of reconstructions. A representative reconstruction with severe water and fat swapping is presented in Fig. 9. Water-fat swapping can be clearly observed from Fig. 9(a) and Fig. 9(b). For ICM-ST, although the field inhomogeneity ranges on the order of 2000, it still correctly reconstructed the water and fat images.

Initialization is crucial for ICM-ST and other gradient-based methods to converge to correct estimates. A few schemes, such as the all-zero (i.e., set all pixels to 0) [6] and the ‘winner-take-all’ (i.e., take data cost minimizers as the initial guess) [10, 31], have been proposed for ICM initialization. Besides, the ‘greatest-feasible-solution’ initialization [7], which initializes a pixel with its greatest feasible solution, appears tempting for ICM-ST. However, all these three initialization methods, when tested on all datasets, failed to provide initial guess for ICM-ST. Fig. 10 shows reconstructions of ICM-ST initialized with these three methods as well as the median initialization algorithm. The field homogeneity variation width is about 1600 Hz. ICM-ST successfully reconstructed the water and fat images (Fig. 10(a), 10(b)) when initialized with the value 1095.9 generated by the median initialization algorithm. However, initialized with any of the other three methods, a large amount of severe water-fat swapping within the object region, as labeled in the water images Fig. 10(d), 10(g), 10(j), could be easily observed.

IV. Discussion

We have mainly focused on the simplified signal model (1) in which the multiple peaks of the fat signal and T_2^* decay of water and fat components are ignored. Algorithmic extension of our method to handle these more accurate signal models is straightforward. Consider the multipeak fat model with the fat signal having M distinct peaks ([8, 28]):

$$s_n(p) = \left(W(p) + F(p) \sum_{m=1}^M r_m(p) e^{i2\pi\Delta f_m t_n} \right) e^{i2\pi\psi(p)t_n} \quad (26)$$

where $n = 1, 2, \dots, N$, $r_1(p) + r_2(p) + \dots + r_M(p) = 1$ for each individual pixel p , and all frequency shifts $\{\Delta f_m\}_{m=1}^M$ of fat peaks are known. In this model, three fat peaks are generally considered (i.e., $M = 3$). In addition, the relative fractions $\{r_m\}_{m=1}^M$ can be measured from separate calibration [28] so that we can assume that they are also known. Under these assumptions, the multipeak model (26) contains the same three types of unknown parameters as the simplified model (1) and thus $N = 3$ acquisitions are sufficient to perform the water-fat decomposition estimating. By replacing the matrix Ψ in Equation (6) at each foreground pixel with

$$\hat{\Psi} = \begin{pmatrix} e^{i2\pi\psi t_1} & e^{i2\pi\psi t_1} & \sum_{m=1}^3 r_m e^{i2\pi\Delta f_m t_1} \\ e^{i2\pi\psi t_2} & e^{i2\pi\psi t_2} & \sum_{m=1}^3 r_m e^{i2\pi\Delta f_m t_2} \\ e^{i2\pi\psi t_3} & e^{i2\pi\psi t_3} & \sum_{m=1}^3 r_m e^{i2\pi\Delta f_m t_3} \end{pmatrix} \quad (27)$$

we still have the NLR-MRF energy model whose minimization can be efficiently handled by our ICM-ST algorithm.

The T_2^* decay of water and fat components can cause severely erroneous estimates of the field map [8], but it is excluded from the simplified signal model (1). Following the common assumption that water and fat components in an individual pixel have the same decay rate $R_2^* = 1/T_2^*$ [29], the signal model becomes

$$s_n(p) = \left(W(p) + F(p) e^{i2\pi\Delta f t_n} \right) e^{i2\pi\psi(p)t_n} e^{-R_2^*(p)t_n} \quad (28)$$

Where $n = 1, 2, \dots, N$. The NLR-MRF energy model remains valid by substituting the matrix Ψ in the data cost equation (6) with

$$\tilde{\Psi} = \begin{pmatrix} e^{i2\pi(\psi + iR_2^*)t_1} & e^{i2\pi(\Delta f + \psi + iR_2^*)t_1} \\ e^{i2\pi(\psi + iR_2^*)t_2} & e^{i2\pi(\Delta f + \psi + iR_2^*)t_2} \\ \dots & \dots \\ e^{i2\pi(\psi + iR_2^*)t_N} & e^{i2\pi(\Delta f + \psi + iR_2^*)t_N} \end{pmatrix} \quad (29)$$

As indicated in [30], the estimation of ψ and R_2^* in the NLR-MRF model can be decoupled because of their uncorrelatedness in the Cramer-Rao bound analysis. Furthermore, a similar decoupling adaption of ICM-ST as the decoupled VARPRO method in [30] can be used to estimate ψ and R_2^* .

Initialization is of great importance to ICM-ST's reconstruction accuracy. We have demonstrated how the median value based initialization method is useful with field map reconstruction of a large group of high field MR datasets. However, further investigation is required to make this method prevalent. Note that this method is based on two critical assumptions: high density of unambiguous pixels over the whole field, and the quasi-unimodality of the field inhomogeneity. Low density of unambiguous pixels may result in significant distinction between the whole and the unambiguous field inhomogeneity and thus the median estimated from the unambiguous field is greatly biased. The quasi-unimodality may not be true for clinical MR acquisitions on human bodies. Moreover, we lack of the precise formulation of quasi-unimodality and some robust test technique for it. To relieve the median initialization from the quasi-unimodality constraint, one prospective boosting approach is to partition the whole field into multiple unimodal blocks such that the median initialization works at each block, as illustrated by the preliminary block-wise initialization method. We will keep on investigating how to adaptively configure the partition in our future research.

Another limitation of our approach is on the NLR-MRF energy model. With the integration of background masking, the NLR-MRF model is more accurate than the MRF-based model used by VARPRO [6] and some other methods. But this model only accounts for the two general estimating criteria of minimizing fitting error and penalizing local non-smoothness,

while other important features such as inhomogeneity variation speed are ignored. As noted in [8], this may degrade the accuracy of in clinical acquisitions with rapid variation.

Graph cut and belief propagation based methods, in addition to ICM, have proven to be robust for MRF energy minimization such fields as computer vision and image processing ([31, 32]). The approach proposed in [8] is based on graph cut techniques, and the belief propagation algorithm has been used in [33]. It will be intriguing to investigate and compare these methods in the setting of MR water and fat decomposition.

V. Conclusion

This paper has presented a novel method, named ICM-ST, for robust decomposition of water and fat in MRI. ICM-ST outperforms VARPRO with the use of background masking, stability tracking and median initialization. Background masking is integrated into the MRF energy function to gain improvement for both efficiency and accuracy. The stability tracking mechanism eliminates redundant computation in ICM iterations. The median initialization provides good initial guesses for the ICM iterative computation. Evaluation on 60 mouse datasets demonstrates the robustness of ICM-ST on high-field MRI.

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Appendix

In this part, we show that for both synchronous and asynchronous updating policies, the AIMD configuration of

$$\alpha=0, \quad \beta=1, \quad \text{cus_thr} \geq 2 \quad (30)$$

makes each ST(k) iteration yield the very same results as the k -th ordinary ICM iteration.

First of all, it is easy to observe that for both synchronous and asynchronous updating, it always holds that

Lemma 1. $CUS(p, k) = \text{cus}(p, k)$ for any pixel p and iteration index $k \geq 0$.

Let $\mathcal{N}_p^-$ denote the neighboring pixels of p which are computed prior to the pixel p at each iteration (Fig. 11(a)), and $\mathcal{N}_p^+$ denote the posterior neighboring pixels of p (Fig. 11(b)). Then for both asynchronous and synchronous updating,

Lemma 2. Given a pixel p with $CUS(p, n) = t$,

1. $\text{cus}(q, n) = t - 1$ for any $q \in \mathcal{N}_p^-$;
2. $\text{cus}(q, n) = t$ for any $q \in \mathcal{N}_p^+$.

Proof: For (1), assume there exists some $q \in \mathcal{N}_p^-$ with $cus(q, n) = t-2$. Then there is an index k such that $n-t+2 \leq k \leq n$ and q is not stable at iteration k . Thus, $CUS(p, k) = 1$. Then $CUS(p, n) = CUS(p, k) + (n-k) = 1 + t-2 = t-1$. Thus $CUS(p, n) = t-1$, contradicting the assumption that $CUS(p, n) = t$. 2 can be proved with similar argument.

Let us use $\psi(t, k)$ to denote the field map value yielded by the ST(k) iteration with the threshold value $cus_thr = t$. Let $\hat{\psi}(k)$ denote the value yielded by the k -th ordinary ICM iteration. Moreover, $\psi_p(t, k)$ and $\hat{\psi}_p(k)$ denote the values of $\psi(t, k)$ and $\hat{\psi}(k)$ at pixel p , respectively. Then for both updating policies, under the configuration of (30), we have

Theorem 3. Assume $\alpha = 0, \beta = 1$, and $cus_thr = t \geq 2$. Then for each iteration index $k \geq 0$,

$$\psi(t, k) = \hat{\psi}(k) \quad (31)$$

Proof: We will present an inductive proof for asynchronous updating, since the argument for synchronous updating is very similar.

Note that $\psi_p(t, 0)$ and $\hat{\psi}_p(0)$ are both equal to the initial guess at p , and so the equation (31) holds for $k = 0$.

Now assume the equation $\psi(t, k) = \hat{\psi}(k)$ holds for all indices $k \leq n$. We need to prove $\psi(t, n+1) = \hat{\psi}(n+1)$. Assume the iterating order on all voxels is $p_1 \rightarrow p_2 \rightarrow p_3 \rightarrow \dots \rightarrow p_{|\mathcal{F}|}$. Then we inductively show that

$$\psi_{p_j}(t, n+1) = \hat{\psi}_{p_j}(n+1) \quad (32)$$

always holds for $j=1, 2, \dots, |\mathcal{F}|$. For notational simplicity, we use $\psi_{q,n}^{cur}$ and $\hat{\psi}_{q,n}^{cur}$ to denote the iterated term ψ_q^{cur} in formula (7) for the $(n+1)$ -th iteration, respectively. According to formula (7), to prove (32), we only need to verify $\psi_{q,n}^{cur} = \hat{\psi}_{q,n}^{cur}$ for each neighbor q of p_j .

First, consider $j = 1$. If $CUS(p_1, n) < t$, the ST algorithm recomputes the label of p_1 at iteration $n+1$ using formula (7). Now that p_1 is the first pixel computed at iteration $n+1$, all its neighbors are posterior to it and their estimates have not been updated yet. Thus, for each $q \in \mathcal{N}_{p_1}$,

$$\psi_{q,n}^{cur} = \psi_q(t, n) = \hat{\psi}_q(n) = \hat{\psi}_{q,n}^{cur} \quad (33)$$

Therefore, $\psi_{p_1}(t, n+1) = \hat{\psi}_{p_1}(n+1)$. If $CUS(p_1, n) = t$, the ST algorithm skips computation at p_1 and so $\psi_{p_1}(t, n+1) = \psi_{p_1}(t, n) = \hat{\psi}_{p_1}(n)$ by induction. From Lemma 2, for any $q \in \mathcal{N}_{p_1}$,

$$cus(q, n) \geq CUS(p_1, n) = t \quad (34)$$

Since $t \geq 2$, it follows that $\psi_q(t, n) = \psi_q(t, n-1)$. With the inductive assumption,

$$\begin{aligned} \hat{\psi}_{q,n}^{cur} &= \hat{\psi}_q(n) = \psi_q(t, n) = \psi_q(t, n-1) \\ &= \hat{\psi}_q(n-1) = \hat{\psi}_{q,n-1}^{cur} \end{aligned} \quad (35)$$

Therefore, it follows from formula (7) that $\hat{\psi}_{p_1}(n+1) = \hat{\psi}_{p_1}(n) = \psi_{p_1}(t, n+1)$.

Assume $\psi_{p_j}(t, n+1) = \hat{\psi}_{p_j}(n+1)$ holds for all $j < m$. To finish proving equation (32), we only need to show that

$$\psi_{p_m}(t, n+1) = \hat{\psi}_{p_m}(n+1) \quad (36)$$

For this, first consider the case $CUS(p_m, n) < t$. In this case, for any $q \in \mathcal{N}_{p_m}^-$,

$$\psi_{q,n}^{cur} = \psi_q(t, n+1) = \hat{\psi}_q(n+1) = \hat{\psi}_{q,n}^{cur} \quad (37)$$

And for any $q \in \mathcal{N}_{p_m}^+$,

$$\psi_{q,n}^{cur} = \psi_q(t, n) = \hat{\psi}_q(n) = \hat{\psi}_{q,n}^{cur} \quad (38)$$

Since equation (37) and (38) say that $\psi_{q,n}^{cur} = \hat{\psi}_{q,n}^{cur}$ holds for any $q \in \mathcal{N}_{p_m}$, it follows from formula (7) that

$$\psi_{p_m}(t, n+1) = \hat{\psi}_{p_m}(n+1) \quad (39)$$

Now consider the other case that $CUS(p_m, n) = t$. In this case, the ST algorithm skips computation of formula (7) at p_m . So

$$\psi_{p_m}(t, n+1) = \psi_{p_m}(t, n) = \hat{\psi}_{p_m}(n) \quad (40)$$

Clearly, equation (35) still holds for all voxels $q \in \mathcal{N}_{p_m}^+$. As for any $q \in \mathcal{N}_{p_m}^-$, it follows from Lemma 2 that $ras(q, n) = t - 1$. Thus, similar to previous discussion on p_1 , we still have the equation (35). Hence, with formula (7), it still holds that

$$\hat{\psi}_{p_m}(n+1) = \hat{\psi}_{p_m}(n) = \psi_{p_m}(t, n+1).$$

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Fig. 1. Background masking

(a) a mouse whole-body image acquired with 7-Tesla MRI. It is selected as the masking template. (b) the mask produced by the morphological operation based Algorithm 1. (c) the mask yielded by the adapted GrabCut. In the GrabCut method, each slice of a three-point MRI dataset is set as a color channel, and the whole image is taken as the region of interest to spare human interaction.

Fig. 2.
Linear fittings of $\log(r_{ins}(k))$ on 6 datasets. The $\log(r_{ins}(k))$ lines are in green, and the linear fitting lines are in red.

Fig. 3. The whole/unambiguous field inhomogeneity similarity

(a) the “gold standard” whole field map. (b) the unambiguous field map with unambiguous pixels identified by the HR based method [7]. The unambiguous percentage is 76.8%, and the median and standard variance are separately 1909.4 and 309.9. (c) The quasi-unimodal probability density functions (PDF) of the whole field map and unambiguous map. The median and standard variance pair on the whole field map is (1930.5, 307.3), while it is (1909.4, 309.9) on the unambiguous field. (d) The difference of the “gold standard” field map in (a) with the ICM-ST reconstruction initialized with the median 1930.5.

Fig. 4. Quasi-unimodality of field inhomogeneity probability density

The quasi-unimodality is roughly assessed with the two-sigma and three-sigma rules. The two-sigma intervals for (a), (b), (c) and (d) are $806 \pm 251\text{Hz}$, $-411 \pm 643\text{Hz}$, $1096 \pm 328\text{Hz}$ and $-344 \pm 353\text{Hz}$, respectively; corresponding, the two-sigma are 94.3%, 93.7%, 95.1% and 95.5%. The three-sigma intervals for (a), (b), (c) and (d) are $806 \pm 377\text{Hz}$, $-411 \pm 964\text{Hz}$, $1096 \pm 492\text{Hz}$ and $-344 \pm 529\text{Hz}$, respectively; and the three-sigma percentages separately are 97.9%, 99.7%, 99.2% and 99.5%.

Fig. 5. Phase unwrapping for median estimat

(a) The histogram of the unambiguous solutions which are directly obtained with the HR-based method. Phase wrapping apparently occurred at zero. The window size $w = 105$ is the width of 5 discretization steps, $\varepsilon = 0.10$ and $X_0 = 0.028$. Since $X_0 < \varepsilon$, the test confirms the occurrence of phase wrapping. (b) The phase wrapped histogram of (a). Before phase unwrapping, the median is 864.2; after phase wrapped, the median is 1318.6. (c) A histogram of unambiguous solutions. There is no significant phase wrapping occurring. With $w = 105$, the statistic $X_0 = 0.141 > \varepsilon$ and so the test confirms no occurrence of phase wrapping.

Fig. 6. Efficiency comparison of ICM-ST to VARPRO

(a) time costs on the HF1 datasets. (b) time costs on the HF2 datasets. (c) time costs on the HF3 datasets. The number of iterations on both methods of VARPRO and VARPRO with background masking was set as 40. For VARPRO with background masking, the time cost varies in a large range because the object region size differs in the datasets.

Fig. 7. Accuracy evaluation of ICM-ST

(a) accuracy with the median initialization algorithm. (b) accuracy with the block-wise initialization algorithm which divides an image into multiple blocks and initialize each block independently with the median algorithm. The three types of error scales, good, fair, and poor are graded according to the number and the location of error pixels.

Fig. 8. Improvement of ICM-ST with the block-wise initialization

(a) the poor reconstruction from the HF1 datasets. The water-fat swapping at the mouse heart region contained 119 error pixels. Initialized with the block-wise algorithm (the image was divided into 3 blocks (labeled by the green lines), the number of error pixels declined to 39. (b) one of the two erroneous reconstructions at the poor level from the HF3 datasets. The water-fat swapping containing 59 pixels at the mouse head boundary was removed with the block-wise initialization. (c) the other erroneous reconstruction at the poor level from the HF3 datasets. Severe reconstruction errors occurred at two locations: one was near to the mouse area, and one at the bottom of the object boundary. By dividing the image into two blocks (labeled by the green line), water-fat swapping at these two locations vanished. (d) the less erroneous reconstruction of (a). (e) the corrected reconstruction of (b) with the block-wise initialization. (f) the corrected reconstruction of (c) with the block-wise initialization.

Fig. 9. Reconstruction failure of IDEAL

The erroneous reconstructions of water, fat and field map are shown (a), (b) and (c), respectively. Water-fat swappings are only labeled in the water image (a). (d), (e) and (f) are the correct water, fat and field map reconstructions by ICM-ST.

Fig. 10. Reconstructions of ICM-ST with different initialization methods

(a), (b), (c) the correct water, fat and field map reconstructions based on the median initialization method. (d), (e), (f) the reconstructions based on the 'greatest-feasible-solution' method proposed for Algebraic Decomposition [7]. (g), (h), (i) the reconstructions based on the 'winner-take-all' method. (j), (k) and (l) are reconstructions based on the all-zero method used by VARPRO [6]. Water-fat swapping is only labeled in the water images.

p	+	×	×	×
+	+	×	×	×
×	×	×	×	×
×	×	×	×	×
×	×	×	×	×

(a)

×	×	×	×	×
×	-	-	-	×
×	-	p	+	×
×	+	+	+	×
×	×	×	×	×

(b)

Fig. 11. Two 3×3 neighborhood configurations of p

In (a), p is the first pixel computed at each iteration. All of its neighbors are posterior to it.

In (b), p has both prior and posterior neighbors. -: prior neighbor of p ; +: posterior neighbor of p ; ×: not neighboring to p .

Algorithm 1**Background Masking**

-
- 1: morphologically dilate the template image.
 - 2: threshold the dilated image and a binary image is yielded.
 - 3: fill in holes in the binary image.
 - 4: morphologically erode the hole-filled binary image.
 - 5: perform morphological closing.
-

Algorithm 2Stability Tracking ST(k) at iteration $k - 1$

```

1: for all  $p$  in  $F$  do
2:   if  $CUS(p, k-1) < cus\_thr$  then
3:      $CUS(p, k) := cus\_thr$ ;
4:   else
5:     compute  $\psi_p^{new}$  with formula (7);
6:     if  $\psi_p^{new} = \psi_p^{cur}$  then
7:        $CUS(p, k) := \min(CUS(p, k) + \beta, cus\_thr)$ ;
8:     else
9:        $CUS(p, k) := \alpha \cdot CUS(p, k)$ ;
10:    for all  $q \in N_p \cap F$  do
11:       $CUS(q, k) := \alpha \cdot CUS(q, k)$ ;
12:    end for
13:     $\psi_p^{cur} := \psi_p^{new}$ ;
14:    end if
15:  end if
16: end for

```

Algorithm 3

Median Value Initialization

-
- 1: calculate feasible solutions γ on all foreground pixels, and identify unambiguous pixels.
 - 2: for each unambiguous pixel, calculate its fundamental phase angle $2\pi\psi T$ in the range $(-\pi, \pi]$ and draw out ψ .
 - 3: test phase wrapping on ψ 's and perform phase unwrapping if necessary.
 - 4: calculate the median and set it as the initial value at each foreground pixel.
-

Algorithm 4**ICM with Stability Tracking (ICM-ST)**

-
- 1: mask the background.
 - 2: pre-compute the data cost at each pixel.
 - 3: estimate the median of the unambiguous field.
 - 4: calculate the regularization factors.
 - 5: repeat the ST(k) algorithm until the instable ratio $r_{ins}(k)$ falls below the error threshold ε , i.e.

$$r_{ins}(k) < \varepsilon \quad (25)$$

-
- 6: recover water and fat components with formula (3).
-