Voicing produced by a constant velocity lung source

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An investigation is made of the influence of subglottal boundary conditions on the prediction of voiced sounds. It is generally assumed in mathematical models of voicing that vibrations of the vocal folds are maintained by a constant subglottal mean pressure p_l , whereas voicing is actually initiated by contraction of the chest cavity until the subglottal pressure becomes large enough to separate the vocal folds. The problem is reformulated to determine voicing characteristics in terms of a prescribed volumetric flow rate Q_o of air from the lungs—the evolution of the resulting time-dependent subglottal mean pressure $\bar{p}_-(t)$ is then governed by glottal mechanics, the aeroacoustics of the vocal tract, and the influence of continued contraction of the lungs. The new problem is analyzed in detail for an idealized mechanical vocal system that permits precise specification of all boundary conditions. Predictions of the glottal volume velocity pulse *shape* are found to be in good general agreement with the traditional constant- p_l theory when p_l is set equal to the time averaged value of $\bar{p}_-(t)$. But, in all cases examined the constant- p_l approximation yields values of the mean flow rates Q_o and sound pressure levels that are smaller by as much as 10%.

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I. INTRODUCTION

Numerical simulations of voiced speech usually determine the glottal flow in terms of a prescribed constant or slowly varying subglottal mean pressure (for recent examples see: Zhang *et al.*, 2002; Zhao *et al.*, 2002; Rosa *et al.*, 2003; de Vries *et al.*, 2003; Tao *et al.*, 2007; Luo *et al.*, 2008; Link *et al.*, 2009; Zheng *et al.*, 2011). The magnitude of the corresponding mean volume flow rate Q_o of air from the lungs is then deduced from the results of the simulation. In reality, however, the lung cavity contracts at a more or less fixed rate Q_o , and the subglottal driving pressure is time dependent and determined by flow continuity and the excess $Q(t) - Q_o$ of the glottal volume velocity $Q(t)$ over Q_o (t denotes time).

The constant pressure hypothesis was used in previous work by McGowan and Howe (2012) on Level I source-tract interactions [for which the motion of the vocal folds is prescribed; Titze (2008)], the assumption being that the pressure of air flowing from the lungs was approximately constant at the glottis, at least prior to any back-reaction from the subglottal system. In this paper we show that this assumption is unnecessary, and that an arguably more natural and rigorous boundary condition is the specification of the flow rate Q_o at the lungs without regard to the nature of the resulting pressure behind the glottis. This approach is more suited to modeling the practical situation where voicing is initiated by contraction of the chest cavity until the subglottal pressure becomes large enough to force apart the vocal folds—the subsequent evolution of the subglottal pressure and its mean value are then determined by the equations of motion,

subject to the influence of continued contraction of the lung cavity.

We describe in this paper how this procedure can be incorporated into the reduced complexity equation for the glottal volume velocity Q . This is the “Fant equation” introduced in Fant’s (1960) pioneering analysis of the problem. It determines Q in terms of forcing by the subglottal and supraglottal pressures and by the hydrodynamic pressures produced by jet formation at the glottis and jet interactions with vocal tract structures. The latter includes the false folds, whose contribution has not hitherto appeared explicitly in the Fant equation. McGowan and Howe (2010) argued from a simplified model that the false folds have a negligible impact on the glottal pulse amplitude, which is in agreement with the numerical predictions of Zhang *et al.* (2002) and Zheng *et al.* (2011).

On the other hand, several studies have revealed that structures within the supraglottal tract can substantially modify the voice source, either by changing the inertia of the glottal flow, or the spectrum of the supraglottal pressure. Titze (1994) and Titze and Story (1997) have pointed out that proper geometrical adjustment of the lower supraglottal tract (the epilarynx tube, the piriform sinuses, the pharynx) plays a crucial role in voice quality control. A relatively narrow epilarynx tube, for example, appears to promote interactions between higher formants and the glottal flow and produces rippling of the volume velocity pulse profile (Titze and Story, 1997). At the very least one might expect the false folds to produce an acoustic mass that could skew the glottal pulse beyond that calculated in their absence. Indeed, analytical modeling of source-tract interaction (McGowan and Howe, 2012) has confirmed the existence, established previously by Rothenberg (1981), Ananthapadmanabha and Fant (1982),

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The modified Fant equation is derived in Sec. II for the vocal tract model of Fig. 1. The level I numerical procedure for solving the equation is discussed in Sec. III. Illustrative numerical results are analyzed (Sec. IV), and a comparison is made with predictions of the constant-pressure-driven Fant equation. The influence of the false folds on the skewing of the glottal pulse is also briefly discussed.

The voice source is usually of sufficiently low frequency to permit the supraglottal tract to be modeled as a plane-wave guide. We shall therefore consider the idealized mechanical vocal system of Fig. 1, consisting of a nominally hard-walled upper tract of length L and cross-sectional area

Voicing is initiated by contraction of the lung cavity causing a rise in subglottal pressure. In the model of Fig. 1, steady contraction of the cavity is achieved by a piston-like motion of the lower end [at $x_1 = -(L_s + H)$ in Fig. 1] with constant volume velocity Q_o . This results in a very low Mach number flow into the upper tract that is interrupted by the periodic opening and closing of the glottis. The air emerges into the upper tract as a succession of “puffs” of volume velocity $Q(t)$, the latter being equivalent to the effective monopole source strength of the sound radiated into the supraglottal tract.

Fant's (1960) treatment of voicing was based on an equation for the glottal flow, which may be regarded as

locally incompressible, and where the inertia of the fluid column in motion through the glottis is balanced against an aggregate of forces consisting of a constant subglottal overpressure p_l , back-pressure associated with turbulence losses and interactions with the upper tract, and viscous forces at the walls. Subsequent analyses involving increasingly sophisticated applications of this basic equation have been discussed extensively in the literature and reviewed by McGowan and Howe (2012). Howe and McGowan (2011) derived the general form of Fant's equation from the equations of aerodynamic sound. For the model of Fig. 1 it takes the form

$$\rho_o \bar{\ell} \frac{dQ}{dt} + \rho_o \int_V (\nabla Y \cdot \omega \wedge \mathbf{v})(\mathbf{y}, t) d^3 \mathbf{y} = A(p_- - p_+), \quad (1)$$

where ρ_o is the mean air density, $\mathbf{v} \equiv \mathbf{v}(\mathbf{y}, t)$ is the velocity at $\mathbf{y}$ and time t in the fluid volume V , $\omega = \text{curl } \mathbf{v}$ is the vorticity, and $p_{\pm} \equiv p_{\pm}(t)$ are, respectively, the pressures just downstream of the false folds and upstream of the glottis.

The auxiliary function $Y(\mathbf{y}, t)$ is a solution of Laplace's equation $\nabla^2 Y = 0$ that defines the unique velocity potential of a hypothetical incompressible flow from $y_1 < 0$ to $y_1 > 0$ through the glottis and false folds. It is normalized to have unit speed in the positive y_1 direction within the uniform upper and lower tracts (where $\partial Y / \partial y_1 = 1$), with normal velocity $\partial Y / \partial y_n = 0$ on the instantaneous configuration of the glottis and vocal tract walls (see Howe, 2002, for a detailed discussion). The length $\bar{\ell} \equiv \bar{\ell}(t)$ is the effective column length of fluid involved in unsteady motion through the glottis, given by

$$\bar{\ell}(t) = \int \left(\frac{\partial Y}{\partial y_1}(\mathbf{y}, t) - 1 \right) dy_1, \quad (2)$$

where the integration path passes in the positive direction through the glottis between sufficiently distant points within the lower and upper tracts at which $\partial Y / \partial y_1 = 1$.

Contributions to Eq. (1) from additional surface integrals involving $Y(\mathbf{y}, t)$ have been neglected. These represent surface viscous forces within the glottal region and the influence of normal motions of the glottis wall and neighboring tissue, which were shown by Howe and McGowan (2012) to be responsible for small modifications of the glottal pulse profile; they have been ignored for the purpose of the present discussion.

C. The vortex force

The integral

$$F(t) = \rho_o \int_V (\nabla Y \cdot \omega \wedge \mathbf{v})(\mathbf{y}, t) d^3 \mathbf{y} \quad (3)$$

represents the drag force $F(t)$ exerted on the glottis and false folds by vortex structures ("turbulence") in the flow (Howe, 2002, Sec. 4.4.2). This force acts to modulate the airstream through the glottis. It is equal and opposite to the vortex-surface interaction force on the fluid in the glottal region,

and its evaluation requires a detailed knowledge of the complex flow near the glottis. This is not normally available, but the dominant contribution will obviously be supplied by the mean characteristics of the glottis jet, and for the configuration of Fig. 1 we can put $F = F_g + F_f$, where F_g, F_f , respectively, denote the force components produced respectively by jet interactions with the glottis and the false folds.

The glottis component F_g can be evaluated approximately by use of a simple quasi-static, "free-streamline" model of the jet (Howe and McGowan, 2011; McGowan and Howe, 2012). The jet in the vicinity of the idealized glottis is modeled as in Fig. 2, where the vorticity is confined to thin vortex sheet shear layers at the outer edge S_J of the jet. Then $\omega \wedge \mathbf{v} = (1/2)U_\sigma^2 \delta(s_\perp) \mathbf{n}$, where $U_\sigma(t)$ is the jet velocity just inside the shear layer (constant along the free streamline), $s_\perp$ is the distance measured in the direction of the outward unit normal $\mathbf{n}$ from the jet, and the vorticity is convected at half the free streamline velocity. Figure 2 illustrates schematically the family of "streamlines" near the glottis of the velocity potential $Y(\mathbf{y}, t)$. The main contribution to the integral is from the section of the jet close to the glottis, where these streamlines cut across the edge of the jet and spatial variations of U_σ can be neglected, so that

$$\begin{aligned} \frac{F_g}{\rho_o} &\equiv \int_{V(t)} \frac{\partial Y}{\partial \mathbf{y}} \cdot \omega \wedge \mathbf{v} d^3 \mathbf{y} \approx \frac{U_\sigma^2(t)}{2} \oint_{S_J} \frac{\partial Y}{\partial \mathbf{y}} \cdot \mathbf{n} dS \\ &= \frac{(A - \sigma A_g) U_\sigma^2}{2}, \end{aligned} \quad (4)$$

where $\sigma = \sigma(t)$ is the jet contraction ratio, and the surface integral is just equal to the net flux $(A - \sigma A_g)$ through the jet boundary of the hypothetical flow defined by the velocity potential $Y(\mathbf{y}, t)$.

This result for F_g does not depend on the precise functional form of $Y(\mathbf{y}, t)$. However, some knowledge of the behavior of $Y(\mathbf{y}, t)$ in the vicinity of the false folds is necessary to calculate the component F_f in terms of the vorticity

FIG. 2. Local streamline pattern of the hypothetical flow through the glottis defined by the velocity potential $Y(\mathbf{y}, t)$ intersecting the vortex sheet boundary of the idealized jet.

convecting between the folds. In principle $Y(\mathbf{y}, t)$ can be found by numerical integration of Laplace's equation. But more insight is obtained by use of the following approximation (Rayleigh, 1945, Sec. 308; Howe, 2002, p. 80)

$$Y(\mathbf{y}, t) \simeq \int_0^{y_1} \frac{A}{S(\xi, t)} d\xi, \quad (5)$$

where $S(\xi, t)$ is the cross-sectional area at any point $y_1 = \xi$ within the entire vocal tract at time t . In our case $S(y_1, t)$ is time dependent only within the glottis ($-\ell_g < y_1 < 0$), where $S(y_1, t) = A_g(t)$.

Equation (5) is strictly valid when the cross-sectional area varies "slowly" with position in the duct. But it nonetheless provides a good and physically correct approximation when used to evaluate the force integral [Eq. (3)]. Formula (5) determines only the component $\partial Y / \partial y_1$ of ∇Y . The equation of continuity can be used to obtain an improved approximation to ∇Y , and for the present two-dimensional duct geometry one finds (Rayleigh, 1945, Sec. 308)

$$\frac{\partial Y}{\partial y_1} = \frac{A}{S(y_1, t)}, \quad \frac{\partial Y}{\partial y_2} = -y_2 \frac{\partial}{\partial y_1} \left(\frac{A}{S(y_1, t)} \right). \quad (6)$$

The jet emerging from the glottis expands laterally and can be expected to "wet" the surfaces of the false folds. The lateral velocities are small, however, compared with the axial mean speed, and to evaluate F_f it will be assumed that the vorticity convects parallel to the duct axis through the section of cross-sectional area A_f between the false folds at the mean speed Q_o/A_f . Thus

$$F_f(t) \simeq -\rho_o \int y_2 (\boldsymbol{\omega} \wedge \mathbf{v})_2(\mathbf{y}, t) \left(\frac{A}{A_f} - 1 \right) \times (\delta(y_1 - \ell_f) - \delta(y_1 - \ell_t)) d^3 \mathbf{y}, \quad (7)$$

where the main contributions to F_f arise from vortex-surface interactions at the leading and trailing edges, respectively, $x_1 = \ell_f, \ell_t$, of the idealized false folds, where ∇Y is singular.

However, vorticity interacting with a "trailing edge" $x_1 = \ell_t$ (labeled B in Fig. 1) would in practice induce the shedding of *new* vorticity into a wake. When this shedding is ignored the force calculated from Eq. (7) has two components, from vortex elements passing the leading and trailing edges. The overall force, however, also involves a contribution from the shed vorticity. A similar problem arises in the calculation of the force produced by a turbulent "gust" in mean flow past an airfoil. In that case it is known that in a first approximation the force component attributed to the wake is *equal and opposite* to that generated by the gust at the trailing edge (Howe, 1976, 2002, Sec. 6.3). This happens because shedding acts to *smooth* the trailing edge flow, removing potential flow singularities that would otherwise occur. Therefore the leading order effect of the wakes of the false folds can be determined, without knowledge of details of the shed vorticity, merely by deleting the singularity $\delta(y_1 - \ell_t)$ from the integrand of Eq. (7) and ignoring the contribution to the remaining integral from the wake

vorticity. Of course, the trailing edge of a false fold is actually tapered and much smoother than the sharp corner B of Fig. 1 (see Fig. 1 of Zheng *et al.*, 2011). But separation of the mean flow must still occur in this region, which will again reduce its contribution to F_f . The wake was neglected in the analogous false folds problem discussed by McGowan and Howe (2010), where surface wetting by the mean jet was ignored.

If lateral expansion of the jet is ignored over the relatively short distance $\sim \ell_f$ between the glottis and the false folds, the integration in Eq. (7) can be performed by means of the vortex sheet approximation used above for Eq. (4), to obtain

$$\frac{F_f(t)}{\rho_o} \simeq -\sigma \left(\frac{A}{A_f} - 1 \right) \left[\frac{A_g(t) U_\sigma^2(t)}{2} \right], \quad (8)$$

where the square brackets [] indicate that the enclosed quantity is to be evaluated at the "retarded" time at which the vorticity passing the leading edge $x_1 = \ell_f$ at time t emerged from the glottis.

Combining this result with F_g of Eq. (4), we find

$$\begin{aligned} \frac{F(t)}{\rho_o} &\equiv \int_V (\nabla Y \cdot \boldsymbol{\omega} \wedge \mathbf{v})(\mathbf{y}, t) d^3 \mathbf{y} \\ &\simeq \frac{(A - \sigma A_g(t)) U_\sigma^2(t)}{2} - \sigma \left(\frac{A}{A_f} - 1 \right) \left[\frac{A_g(t) U_\sigma^2(t)}{2} \right]. \end{aligned} \quad (9)$$

The contribution from the false folds is smaller than that from the glottis by a factor of order A_g/A_f . It is also of opposite sign, and represents the effect of a "suction" force at the leading edge of the false folds acting on the mean flow in the $+x_1$ direction, whereas the glottis component F_g opposes the motion through the glottis. This reduces the overall flow resistance through the glottis produced by vortex-surface interactions, in agreement with numerical simulations reported by Zhao *et al.* (2002), and should therefore reduce the subglottal pressure required to maintain a given mean volume velocity Q_o .

D. The pressures p_+ and p_-

The pressure fluctuations p_+ in Eq. (3) are determined by the volume source of strength $Q(t)$ at the glottis of the flow into the upper tract. Standard acoustic analysis for the supraglottal duct of Fig. 1 (cf. Murray and Howe, 2012) provides the Fourier representation

$$p_+ = -\frac{i\rho_o c_o}{2\pi A} \int \int_{-\infty}^{\infty} Q(\tau) \frac{\sin(k_o \bar{L}) e^{-i\omega(t-\tau)}}{\cos(k_o \bar{L})} d\omega d\tau, \quad (10)$$

where $k_o = \omega/c_o$, c_o is the speed of sound, and the length $\bar{L}$ is equal to the interior duct length L suitably augmented to account for the end-correction of the open mouth.

The pressure p_- near the glottis in the lower tract is governed by the net volumetric rate of inflow, comprising the steady inflow Q_o due to lung contraction and the outflow $Q(t)$ through the glottis. By making the usual assumptions of continuity of pressure and volume velocity at the junction $x_1 = -L_s$, we find

$$p_- = \frac{i\rho_o c_o}{2\pi A} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (Q_o - Q(\tau)) \frac{[A \cos(k_o L_s) \cos(k_o H) - A_L \sin(k_o L_s) \sin(k_o H)] e^{-i\omega(t-\tau)}}{[A \sin(k_o L_s) \cos(k_o H) + A_L \cos(k_o L_s) \sin(k_o H)]} d\omega d\tau. \quad (11)$$

In both of these formulas causality requires the path of integration with respect to ω to pass above all singularities (simple poles) of the integrands.

These expressions are strictly applicable in the absence of damping. Thus, the undamped poles for the upper tract pressure p_+ [Eq. (10)] occur at the resonance frequencies $\omega = \omega_n = (n - 1/2)\pi c_o / \bar{L}$ ($-\infty < n < \infty$). Damping in the upper tract is produced by thermo-viscous wall-losses, open-end radiation (consistent with the idealized model of Fig. 1), and flexural motions of the tract walls. These effects cause the poles to be shifted into the lower complex plane, perturbing in general both their real and imaginary parts (Stevens, 1998). In a first approximation we can write

$$\omega = \pm \omega_n - i\epsilon_n, \quad n \geq 1, \quad \omega_n = \left(n - \frac{1}{2}\right) \frac{\pi c_o}{\bar{L}}, \quad \epsilon_n > 0. \quad (12)$$

Evaluation by residues of the integral [Eq. (10)] then gives

$$p_+ = \rho_o c_o^2 \sum_{n=1}^{\infty} Z_n(t), \quad (13)$$

where

$$Z_n = \frac{2}{AL} \int_{-\infty}^t Q(\tau) \cos \omega_n(t - \tau) e^{-\epsilon_n(t-\tau)} d\tau, \quad n \geq 1. \quad (14)$$

The integrand of Eq. (11) for the subglottal pressure p_- contains a simple pole at $\omega = 0$, which yields the contribution $\bar{p}_-$, say, given by

$$\bar{p}_- = \rho_o c_o^2 W_0(t), \quad W_0(t) = \frac{1}{V} \int_{-\infty}^t (Q_o - Q(\tau)) d\tau, \quad (15)$$

where $V = AL_s + A_L H$ is the mean volume of the lung cavity and the subglottal tract.

When $A_L \gg A$ and $\omega \neq 0$

$$\begin{aligned} & \frac{A \cos(k_o L_s) \cos(k_o H) - A_L \sin(k_o L_s) \sin(k_o H)}{A \sin(k_o L_s) \cos(k_o H) + A_L \cos(k_o L_s) \sin(k_o H)} \\ & \simeq -\frac{\sin(k_o L_s)}{\cos(k_o L_s)}. \end{aligned}$$

The undamped subglottal resonance frequencies therefore correspond approximately to the quarter-wave modes of a duct of length L_s open at one end. When the influence of damping is included the perturbed resonance frequencies are taken in the form

$$\omega = \pm \omega_n^s - i\epsilon_n^s, \quad n \geq 1, \quad \omega_n^s = \left(n - \frac{1}{2}\right) \frac{\pi c_o}{L_s}, \quad \epsilon_n^s > 0, \quad (16)$$

so that the net subglottal pressure p_- , including the zero frequency pole contribution $\bar{p}_-$, becomes

$$p_- = \rho_o c_o^2 \left(W_0(t) + \sum_{n=1}^{\infty} W_n(t) \right), \quad (17)$$

where

$$W_n(t) = \frac{2}{AL_s} \int_{-\infty}^t (Q_o - Q(\tau)) \times \cos \omega_n^s(t - \tau) e^{-\epsilon_n^s(t-\tau)} d\tau, \quad n \geq 1. \quad (18)$$

E. The Fant equation

The results of Eqs. (9) and (13) and (18) and the relation $Q = \sigma A_g U_\sigma$ now permit the Fant Eq. (1) for the idealized vocal system of Fig. 1 to be cast in the form

$$\begin{aligned} & \frac{\bar{\ell}}{A} \frac{dQ}{dt} + \left(1 - \sigma \frac{A_g}{A}\right) \frac{Q^2}{2\sigma^2 A_g^2(t)} - \left(\frac{A}{A_f} - 1\right) \left[\frac{Q^2(t)}{2\sigma A A_g(t)} \right] \\ & = c_o^2 \left(W_0(t) + \sum_{n=1}^{\infty} \{W_n(t) - Z_n(t)\} \right), \end{aligned} \quad (19)$$

where the glottal column length $\bar{\ell}$ is given by

$$\bar{\ell}(t) = \ell_g \left(\frac{A}{A_g(t)} - 1 \right) + (\ell_t - \ell_f) \left(\frac{A}{A_f} - 1 \right). \quad (20)$$

In general the time dependent variation of the glottis cross-section $A_g(t)$ is known, or determined by an equation that is to be solved simultaneously with Eq. (19). The solution $Q(t)$ of Eq. (19) vanishes identically in the absence of lung contraction, i.e., unless $Q_o \neq 0$ in the definition [Eq. (15)] of W_0 .

F. Interpretation of $\bar{p}_- = \rho_o c_o^2 W_0(t)$

Integration of the continuity equation $\partial \rho / \partial t + \rho_o \operatorname{div} v = 0$ over the entire volume V of the subglottal region, and use of the linear, adiabatic formula $\delta p = \delta \rho / c_o^2$, relating changes in pressure δp and the density $\delta \rho$, reveals that

$$\frac{V}{\rho_o c_o^2} \frac{d\bar{p}}{dt} = Q_o - Q(t), \quad (21)$$

where $\bar{p}(t)$ is the space-averaged subglottal pressure. Therefore

$$\bar{p} \equiv \bar{p}_- = \rho_o c_o^2 W_0(t). \quad (22)$$

It follows fairly obviously from Eq. (21) that during periodic voicing characterized by a limit cycle solution of the Fant Eq. (19), the mean volume velocity $\langle Q(t) \rangle \equiv Q_o$, where

$$\langle Q(t) \rangle = f_o \oint Q(t) dt,$$

the integration being over a complete period $1/f_o$ of the glottal motion.

Equations (21) and (22) also permit the Fant Eq. (19) to be re-cast in the form

$$\begin{aligned} \frac{d^2 \bar{p}_-}{dt^2} + \omega_H^2 \bar{p}_- = \omega_H^2 \rho_o \left\{ \left(1 - \sigma \frac{A_g(t)}{A} \right) \frac{Q^2(t)}{2\sigma^2 A_g^2(t)} \right. \\ \left. - \left(\frac{A}{A_f} - 1 \right) \left[\frac{Q^2(t)}{2\sigma A A_g(t)} \right] \right. \\ \left. + c_o^2 \sum_{n=1}^{\infty} \{W_n(t) - Z_n(t)\} \right\}, \quad (23) \end{aligned}$$

where $\omega_H = \sqrt{c_o^2 A / \mathcal{V} \ell}$ is the *instantaneous* value of the resonance frequency of the ‘‘Helmholtz resonator’’ formed by the entire subglottal region of volume $\mathcal{V}$ with a small opening at the glottis (Rayleigh, 1945). Equation (23) determines the space-averaged subglottal pressure $\bar{p}_-(t)$ produced by lung contraction, nonlinear jet forces, and acoustic modes within and outside the cavity.

G. The constant-pressure-driven Fant equation

Most discussions of the Fant equation assume that the motion is driven by a constant subglottal pressure p_l , rather than the contraction rate Q_o of the lungs. To derive this approximation the space-averaged subglottal pressure term $W_0(t)$ in Eq. (19) is replaced by its time-averaged value $\langle W_0(t) \rangle = \langle \bar{p}_-(t) \rangle / \rho_o \equiv p_l / \rho_o$. Similarly, we must set $Q_o = 0$ in the definition [Eq. (18)] of W_n . This yields the constant-pressure-driven Fant equation

$$\begin{aligned} \frac{\ell}{A} \frac{dQ}{dt} + \left(1 - \sigma \frac{A_g(t)}{A} \right) \frac{Q^2}{2\sigma^2 A_g^2(t)} - \left(\frac{A}{A_f} - 1 \right) \left[\frac{Q^2(t)}{2\sigma A A_g(t)} \right] \\ = \frac{p_l}{\rho_o} + c_o^2 \sum_{n=1}^{\infty} \{W_n(t) - Z_n(t)\}. \quad (24) \end{aligned}$$

$$\frac{A_g(t)}{A} = \begin{cases} a_0, & \text{for } 0 \leq f_o t - [f_o t] < f_o t_c \\ a_0 + \frac{a_1}{2} \left(1 - \cos \left(2\pi \frac{\{f_o(t - t_c) - [f_o t]\}}{1 - f_o t_c} \right) \right), & \text{for } f_o t_c < f_o t - [f_o t] < 1, \end{cases} \quad (25)$$

where $[f_o t]$ denotes the largest integer not exceeding $f_o t$. The open duty factor of the glottis is then equal to $1 - f_o t_c$.

B. The jet contraction ratio

Calculation for an orifice of uniform rectangular cross section indicates that the glottis-jet contraction ratio σ varies abruptly between 0.61 and 1.1 during opening and closing of the glottis (Howe and McGowan, 2010). This is consistent with the experimental findings of Park and Mongeau (2007), although the variation must actually be a smooth function of the time. Discontinuities associated with rapid jumps in σ will therefore be avoided by using in the Fant equation the following smoothed representation of the variation:

The value of the volumetric inflow velocity Q_o no longer provides a boundary condition for this equation and the ultimate voice source; its value must be determined from the solution of Eq. (24) and the formula $Q_o = f_o \oint Q(t) dt$.

H. Other special cases

The Fant Eq. (19) can be modified further to treat idealized representations of two alternative geometrical configurations that are frequently discussed in the literature.

- (1) *Exposed glottis*: The upper tract downstream of the false folds is removed. Then $p_+ = 0$ to a very good approximation, and the required Fant equation is obtained from Eq. (19) by discarding $\sum_{n=1}^{\infty} Z_n$.
- (2) *Non-reflective upper tract*: $c_o^2 \sum_{n=1}^{\infty} Z_n$ in Eq. (19) is replaced by $c_o Q(t)/A$. This is equivalent to modeling the upper tract as a semi-infinite duct.

III. THE LEVEL I APPROXIMATION

A. Prescribed glottal motion

The numerical analysis of the Fant Eqs. (19) and (24) will be based on Titze’s (2008) level I approach, whereby the character of the solution generated by contraction of the lungs is examined for prescribed cyclic variations of the glottis cross-section $A_g(t)$. This is done using the formula $A_g(t) = A \{a_0 + (1/2)a_1(1 - \cos(2\pi f_o t))\}$, where f_o Hz is the fundamental voicing frequency and a_0, a_1 are suitable coefficients. The coefficient a_0 must be assigned a small positive value ($\ll a_1$) to avoid instability in the numerical solution. The maximum open area is then $\sim a_1 A$.

In practice, however, the glottis closes for a finite time t_c , say, during each cycle. To model this, the following more general formula will be used

$$\sigma = 0.61 + \frac{0.49}{(a_0 + a_1)} \frac{A_g(t)}{A}. \quad (26)$$

A similar smoothing formula was used by Zanartu *et al.* (2007) to model vocal fold mechanics.

C. Numerical procedure

The Fant equation is solved by fourth order Runge-Kutta integration applied to Eq. (19) and to the corresponding system of equations satisfied by $W_0(t)$, $W_n(t)$, and $Z_n(t)$, $n \geq 1$, viz.,

$$\frac{dW_0}{dt} = \frac{Q_o - Q(t)}{\mathcal{V}}, \quad (27)$$

$$\left. \begin{aligned} \frac{dW_n}{dt} &= \frac{2(Q_o - Q(t))}{AL_s} - \epsilon_n^s W_n - \omega_n^s W'_n \\ \frac{dW'_n}{dt} &= \omega_n^s W_n - \epsilon_n^s W'_n \end{aligned} \right\} n = 1, 2, \dots, \quad (28)$$

where

$$\left. \begin{aligned} W'_n &= \frac{2}{AL_s} \int_{-\infty}^t \{Q_o - Q(\tau)\} \sin \omega_n^s(t - \tau) e^{-\epsilon_n^s(t - \tau)} d\tau \\ Z'_n &= \frac{2}{AL} \int_{-\infty}^t Q(\tau) \sin \omega_n(t - \tau) e^{-\epsilon_n(t - \tau)} d\tau, \end{aligned} \right\} n = 1, 2, \dots \quad (30)$$

(cf. [Howe and McGowan, 2011](#)).

The modal coefficients $W_n(t)$, $Z_n(t)$ govern the subglottal and supraglottal cavity mode contributions to the pressures $p_{\pm}$, and are small when $2\pi f_o \ll \omega_n^s$, ω_n . This means that it is usually permissible to truncate the infinite systems [Eqs. (28) and (29)] at $n \sim 3$, i.e., by taking account of the first three formants of the lower and upper tracts. The integration is started at $t=0$ subject to the initial conditions $Q=0$, $W_0 = W_n = W'_n = Z_n = Z'_n = 0$ ($n \geq 1$).

Periodic solutions of Eq. (19) must satisfy the condition

$$f_o \oint Q(t) dt = Q_o,$$

which provides a convenient check on convergence to a limit cycle solution.

IV. NUMERICAL RESULTS

A. Model vocal tract parameters

Sample solutions of the Fant Eq. (19) and of the constant-pressure-driven approximation [Eq. (24)] are now discussed. Typical parameter values for the simplified model of Fig. 1 are given in Table I, based on those used in the recent investigation of [Zheng et al. \(2011\)](#).

TABLE I. Parameter values for the ideal vocal system of Fig. 1.

Parameter	Value
Glottal length ℓ_g	3 mm
Glottis and duct span ℓ_s	15 mm
Glottis coefficient a_0	0.00005
Glottis coefficient a_1	0.1
Duct width w	20 mm
False fold ℓ_f	5 mm
False fold ℓ_r	20 mm
False fold A_f/A	0.34
Volume of subglottal cavity $\mathcal{V}$	4000 ml
Glottis frequency (nominal) f_o	120 Hz
Density of air ρ_o	1.23 kg/m ³
Speed of sound c_o	350 m/s

The first subglottal formant $\omega_1^s/2\pi \equiv F_{s1} \sim 620$ Hz ([Ishizaka et al., 1976](#)). Therefore for the purpose of illustration the three subglottal formants F_{s1} , F_{s2} , F_{s3} and their corresponding half-power bandwidths Δf are taken to be defined as in Table II ([Ishizaka et al., 1976](#); [Zanartu et al., 2007](#); [Howe and McGowan, 2012](#)).

These values are consistent with the simple rectilinear model of the subglottal tract of Fig. 1 and the frequency formula [Eq. (16)]. The half-power bandwidths supply the dissipation rates ϵ_n^s by means of the formula

$$\epsilon_n^s = \pi(\Delta f)_n. \quad (31)$$

Numerical results will be discussed for a notional neutral vowel phoneme corresponding to supraglottal formants $F1$, $F2$, $F3$ consistent with the rectilinear duct model of Fig. 1. They are defined along with their bandwidths (estimated from [Olive et al., 1993](#)) in Table III.

B. Influence of glottal frequency

The glottis volume velocity $Q(t)$ is determined by the Fant Eq. (19) entirely in terms of the prescribed value of the lung cavity contraction rate Q_o . The details of the glottal pulse depend on the acoustic properties of both the subglottal region (Table II) and the articulatory state of the upper vocal tract. Figure 3 depicts a survey of possible neutral vowel waveforms, each for fixed values of Q_o but for a range of increasing values of the glottis frequency f_o . In all cases $f_o t_c = 0.3$, so that the glottis duty factor is 0.7.

The contraction rate is fixed at $Q_o = 200$ cm³/s and f_o is increased in stages from 120 to 700 Hz. This interval encompasses the subglottal resonance at $F_{s1} = 620$ Hz and the first supraglottal formant $F1 = 500$ Hz. Each of the frames [Figs. 3(a)–3(j)] depicts three limit cycles of the glottal pulse $Q(t)/Q_o$.

TABLE II. Subglottal frequencies.

	F_{s1}	F_{s2}	F_{s3}
Frequency, Hz	620	1860	3100
Bandwidth Δf , Hz	200	150	100

TABLE III. Supraglottal formants.

	$F1$	$F2$	$F3$
Frequency, Hz	500	1500	2500
Bandwidth Δf , Hz	50	75	100

plotted against $f_o t$. In case (a) $f_o = 120$ Hz is well below the two resonance frequencies, and the smooth volume velocity pulse is slightly skewed to the latter half of the glottis open phase. This skewing is due almost entirely to the back-reaction on the glottis flow of the acoustic modes in the upper and lower tracts. The back-reactions are weak because $f_o \ll F_s 1, F1$, but the skewing is absent only when the contributions to Eq. (19) from modes in *both* tracts are ignored (cf. the discussion below of Fig. 4). The contribution to skewing from the augmentation of the slug length $\bar{\ell}$ produced by the narrowed passage between the false folds is much smaller (Rothenberg, 1981; Ananthapadmanabha and Fant, 1982; Fant, 1986; Titze, 1994; Titze and Story, 1997), and does not become noticeable until A_g/A is smaller than about 0.1 (Fig. 6).

FIG. 3. Illustrating the variation with f_o of limit cycle glottal pulse profiles $Q(t)/Q_o$ predicted by the Fant Eq. (19) for the conditions of Table I, when $Q_o = 200$ cm³/s, $f_o t_c = 0.3$. The subglottal resonances are defined as in Table II, and the calculations are performed for the neutral vowel defined by the upper tract formants of Table III.

FIG. 4. Illustrating the limit cycle variations (—) of $A_g(t)/A$, $Q(t)/Q_o$, $\bar{p}_-(t)$ Pa for $f_o = 120$ Hz, $f_o t_c = 0.3$, $Q_o = 200$ cm³/s. The mean subglottal pressure $p_l = 478.41$ Pa. The broken-line curve (---) is the non-skewed profile of $Q(t)/Q_o$ predicted when back reactions from acoustic modes in the upper and lower tracts are ignored. The dotted curve (•••) is the volume velocity predicted by the constant-pressure-driven Fant Eq. (24) for $p_l = 478.41$ Pa, for which $Q_o = 180.46$ cm³/s.

The appearance of a “knuckle” at the front of the glottal pulse when $f_o = 170$ Hz [labeled A in Fig. 3(b)] indicates the increasing influence of the first formant $F1$. The knuckle advances toward the peak of the volume velocity profile with increasing frequency [Figs. 3(c) and 3(d)]; the distinct double peak in Fig. 3(c) occurs at the subharmonic $f_o = 250$ Hz of $F1$. In Fig. 3(d) the secondary peak is maintained by subglottal interactions at the subharmonic 310 Hz of $F_s 1$. At higher frequencies the double peak disappears,

FIG. 5. Limit cycle variations (—) of $A_g(t)/A$, $Q(t)/Q_o$, $\bar{p}_-(t)$ Pa for $f_o = 250$ Hz, $f_o t_c = 0.3$, $Q_o = 273.78$ cm³/s. The mean subglottal pressure $p_l = 1000$ Pa. The dotted curve (•••) is the volume velocity predicted by the constant-pressure-driven Fant Eq. (24) for $p_l = 1000$ Pa, for which $Q_o = 258.09$ cm³/s.

FIG. 6. Volume velocity profile skewing produced by narrowing of the gap between the false folds. The figure compares limit cycle volume velocity profiles for $Q_o = 320 \text{ cm}^3/\text{s}$, $f_o = 120 \text{ Hz}$, $a_0 = 0.06$, $t_c = 0$ in the two cases $A_f/A = 0.1$ (—) and $A_f/A = 1$ (---).

and Fig. 3(e) reveals a new disturbance B associated with the first formant $F1$ advancing toward the velocity peak. At $f_o = 480 \text{ Hz}$ the formation of a secondary ripple C is evident. The combination leads to the triple peak profile of Fig. 3(g) at the resonance condition $f_o = F1$. The central peak in this profile is produced by interaction with $F2$; when the contribution from the second formant is omitted from Eq. (19) the resonant profile assumes a characteristic double peak form, with a deep central minimum that is typical of resonance forcing (Lighthill, 1978; Howe and McGowan, 2011). At higher frequencies the volume velocity profile becomes skewed to the first half of the glottal cycle. Resonance forcing at the subglottal frequency $f_o = F_s1$ [Fig. 3(i)] does not exhibit a double peak behavior, because of the heavy damping of the subglottal modes.

Figures 4 and 5 display the combined limit cycle variations of $A_g(t)/A$, $Q(t)/Q_o$, and the subglottal space-averaged pressure $\bar{p}_-(t)$ Pa for special cases of the neutral vowel. Figure 4 is for case (a) of Fig. 3, i.e., for $f_o = 120 \text{ Hz}$, $Q_o = 200 \text{ cm}^3/\text{s}$ and $f_o t_c = 0.3$, typical of quiet speech. The back-reaction of the upper and lower tract resonant modes produce skewing of the volume velocity profile to the latter half of the open phase of the glottis. The back-reaction is relatively mild because of the large disparity between the glottal frequency f_o and F_s1 and the first formant $F1$. In the absence of these modes there is no skewing of the corresponding $Q(t)/Q_o$ -profile (---), obtained by discarding $W_n, Z_n, n \geq 1$ in Fant Eq. (19) and the systems [Eqs. (28) and (29)]. The mean subglottal pressure $\langle \bar{p}_-(t) \rangle \equiv p_I = 478.41 \text{ Pa}$, as indicated by the broken line in the upper part of Fig. 4, and $\bar{p}_-(t)$ varies over a narrow range of about $\pm 15 \text{ Pa}$ about this mean.

The profiles in Fig. 5 are for $f_o = 250 \text{ Hz}$, $f_o t_c = 0.3$ and $Q_o = 273.78 \text{ cm}^3/\text{s}$, the latter having been adjusted to yield a mean subglottal pressure $p_I = 1000 \text{ Pa}$ —a value frequently used in voicing studies. The glottis frequency f_o is a subharmonic of the formant $F1 = 500 \text{ Hz}$, and the variation of the volume velocity $Q(t)/Q_o$ is strongly influenced by interaction with this resonant mode. The subglottal mean pressure $\bar{p}_-(t)$ exhibits a near saw-tooth cyclic waveform over $\pm 10 \text{ Pa}$ about the mean.

TABLE IV. Compared predictions of Eqs. (19) and (24).

f_o (Hz)	Q_o (cm^3/s)	Q'_o (cm^3/s)	p_I (Pa)	p'_I (Pa)
120	200.00	180.46	478.41	549.10
250	273.78	258.09	1000.00	1089.47

C. Solution of the constant-pressure-driven Fant equation

Inspection of Figs. 4 and 5 reveals that the departures of the space-averaged subglottal pressure $\bar{p}_-(t)$ from its mean value p_I do not exceed about $\pm 3\%$. It might therefore be surmised that corresponding predictions of the constant-pressure-driven Fant Eq. (24) should, in such cases, be similar or very close to those of the “exact” Eq. (19).

To examine this hypothesis $Q(t)$ has been calculated from Eq. (24) for the conditions of Figs. 4 and 5 by setting the pressure p_I in the equation respectively equal to the mean values $\langle \bar{p}_-(t) \rangle = 478.41$ and 1000 Pa calculated from the Fant Eq. (19). The numerical solution of Eq. (24) is then used to evaluate the corresponding mean volume velocity $Q'_o = f_o \oint Q(t) dt$, say, and thence the ratio $Q(t)/Q'_o$, which is plotted as the dotted curves ($\bullet \bullet \bullet$) in Figs. 4 and 5. This procedure is seen to yield an excellent approximation to the fractional volume velocity waveform $Q(t)/Q_o$ determined by Eq. (19).

However, in all cases the predicted mean volume velocity $Q'_o < Q_o$, as indicated in Table IV. To obtain equality of the mean volumetric flow rates (and therefore of the predicted speech sound pressure levels) it is necessary to increase the magnitude of the constant driving pressure p_I in Eq. (24) to the respective values labeled p'_I in the table.

D. Influence of the false folds on profile skewing

According to Rothenberg (1981), Ananthapadmanabha and Fant (1982), Fant (1986), Titze (1994), and Titze and Story (1997), constriction of the glottis flow by a narrowing of the lower end of the supraglottal tract is one factor that causes skewing of the volume velocity wave pulse. Narrowing can occur in the model of Fig. 1 by reducing the area A_f between the false folds, which has the effect of increasing the glottal column length $\bar{\ell}$ [see Eq. (20)], and therefore of slowing the initial rate of rise of the velocity pulse. This is illustrated in Fig. 6, which displays the waveforms for the conditions: $f_o = 120 \text{ Hz}$, $Q_o = 320 \text{ cm}^3/\text{s}$, and $t_c = 0$, when $a_0 = 0.06$ [corresponding to a maximum glottis area $A_g(t) = 18 \text{ mm}^2$] in the two cases $A_f/A = 0.1$ (—) and $A_f/A = 1$ (---).

V. CONCLUSION

Voiced speech arises from vibrations of the vocal folds produced by air forced to flow through the glottis by contraction of the lung cavity. However, mathematical representations of this mechanism have largely been formulated in terms of a prescribed subglottal pressure p_I applied to the folds, the amplitude of the pressure being fixed to accord with experiment. But experiment has also determined the

characteristic rate Q_o of volumetric airflow produced by lung contraction which flows through the glottis during voicing. The actual value of the subglottal pressure $\bar{p}_-(t)$ determined by Q_o is not constant, even when the lungs contract at a constant rate. The relation between these parameters has been investigated in terms of the Fant equation for an idealized mechanical vocal system that is simple enough to permit precise specification of all boundary conditions.

The approximate, constant-pressure-driven Fant equation in which p_l is set equal to $\langle \bar{p}_-(t) \rangle$ yields predictions of $Q(t)/Q_o$ that are generally in excellent agreement with those obtained from the exact equation. However, in all cases examined it is found that the absolute level of the mean flow rate Q_o calculated from the approximate equation can be up to 10% smaller than for the exact equation. This implies also that there would be corresponding discrepancies in the predicted sound pressure levels. The differences are admittedly small, and can be removed by suitably increasing by a few percent the driving pressure p_l in the approximate equation, but the conclusion suggests that it would be worthwhile to extend the present investigation to a more realistic model of the vocal system. Such calculations should be done at Titze's (2008) "level II," by including a separate equation of motion for the vocal fold vibrations, and should also incorporate a geometrically precise representation of vocal tract area variations, perhaps by use of concatenated cylindrical elements (Lighthill, 1978).

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