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Conductivity and permittivity imaging at 3.0T

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Abstract

Tissue conductivity and permittivity are critical to understanding local radio frequency (RF) power deposition during magnetic resonance imaging (MRI). These electrical properties are also important in treatment planning of RF thermotherapy methods (e.g. RF hyperthermia). The electrical properties may also have diagnostic value as malignant tissues have been reported to have higher conductivity and higher relative permittivity than surrounding healthy tissue.

In this study, we consider imaging conductivity and permittivity using MRI transmit field maps (B_1^+ maps) at 3.0 Tesla. We formulate efficient methods to calculate conductivity and relative permittivity from 2-dimensional B_1^+ data and validate the methods with simulated B_1^+ maps, generated at 128 MHz. Next we use the recently introduced Bloch-Siegert shift B_1^+ mapping method to acquire B_1^+ maps at 3.0 Tesla and demonstrate conductivity and relative permittivity images that successfully identify contrast in electrical properties.

Keywords

Magnetic resonance imaging; Tissue conductivity; Tissue permittivity; Hyperthermia; Oncology

INTRODUCTION

The conductivity and permittivity of tissue are critical to understanding local radio frequency (RF) power deposition, also known as local specific absorption rate (local SAR) during magnetic resonance imaging. This local distribution of RF power has come under increasing study with higher field imaging and with the use of multiple transmitters, two trends that have resulted in a number of numerical studies to evaluate the potential for concentrated RF power deposition (i.e. local hot spots) in the body (1, 2).

Tissue electrical properties are also important in the therapeutic application of heat using radio frequency, for example, RF hyperthermia. During the application of RF power, the temperature rise at a given location in the body is proportional to the tissue conductivity and the square of the electric field strength at that location. The electric field strength and distribution in turn depends on the conductivity and permittivity values of the tissue. Therefore, tissue electrical properties are important parameters in hyperthermia treatment planning and optimization (3).

Tissue electrical parameters may also have diagnostic value as the conductivity and permittivity of malignant tissue have been reported to be higher than surrounding healthy tissue (4–6).

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Therefore, a method that non-invasively estimates the conductivity and permittivity in a clinically acceptable time has multiple benefits, namely in evaluating RF safety, in RF therapeutic methods and potentially in diagnosing tissue abnormality.

In prior work, electrical impedance tomography has been applied to estimate tissue electrical properties (7). In this method, electrodes are placed on the surface of the body, currents applied by one set of electrodes, resulting voltages recorded by another set of electrodes and an inverse problem is solved to estimate the conductivity and permittivity of tissue within the body. However, in addition to the inconvenience of using electrodes, electrical impedance tomography leads to computational issues as the inverse problem is ill-posed.

In more recent work, estimation of tissue electrical properties using the spatial variation of transmit magnetic field in MRI (B_1^+ maps) was studied in (8). However, the proposed method required, in addition to the transmit field (B_1^+), the receiver sensitivity (B_1^-) and the z -component of the RF magnetic field (H_z), which are not typically available. In a following study, this method was extended to use only B_1^+ data and an integral form for calculating electrical properties from B_1^+ maps was presented (9). The gradient of the B_1^+ is integrated over a closed volume to estimate the electrical properties with this approach.

An alternative approach is to use the second derivative of B_1^+ . In early work, equations for the conductivity and relative permittivity of a layered object were presented in terms of the second derivative of B_1^+ in a layered model (10). This has been extended to arbitrary volume of constant electrical properties using second partial derivatives of B_1^+ in x , y and z directions (Laplacian) in (11–14).

In this work, we formulate the Laplacian based calculation of conductivity and permittivity using central difference equations. This calculation can be carried out using B_1^+ maps acquired in 2-dimensional slices. These equations are validated by estimating the conductivity and permittivity of a human head model using B_1^+ maps generated from electromagnetic simulation at 128MHz, corresponding to 3.0T imaging. Next, the recently introduced Bloch-Siegert B_1^+ mapping (15) technique is used to generate B_1^+ maps at 3.0T and the algorithm is evaluated for a head phantom.

THEORY

In this section, a theoretical relationship between the spatial variation of B_1^+ , and conductivity and permittivity is given. Using the phasor notation for time harmonic fields, the circularly polarized, transmit magnetic field $B_1^+(\mathbf{x})$ at location $\mathbf{x} = (x, y, z)$ is given by,

$$B_1^+(\mathbf{x}) = \frac{\mu}{2} (H_x(\mathbf{x}) + jH_y(\mathbf{x})) \quad [1]$$

where μ is the permeability, H_x and H_y are the complex x and y components of the magnetic field (16). The spatial variation of B_1^+ can be related to electrical properties starting with Ampere's Law and Faraday's Law and using the divergence equation (11). However, a concise derivation, using the Helmholtz equation is given here (13). In a source free region of constant electrical properties, the x and y components of the magnetic field satisfies (17)

$$\nabla^2 H_x(\mathbf{x}) + \omega^2 \mu \epsilon(\mathbf{x}) \left(1 + \frac{\sigma(\mathbf{x})}{j\omega \epsilon(\mathbf{x})} \right) H_x(\mathbf{x}) = 0 \quad [2]$$

$$\nabla^2 H_y(\mathbf{x}) + \omega^2 \mu \varepsilon(\mathbf{x}) \left(1 + \frac{\sigma(\mathbf{x})}{j\omega \varepsilon(\mathbf{x})} \right) H_y(\mathbf{x}) = 0 \quad [3]$$

Where $\nabla^2(\cdot)$ is the Laplace operator, ω is the frequency in radians/s, μ is the permeability, ε is the permittivity, ($\varepsilon = \varepsilon_r \varepsilon_0$, where ε_r is the relative permittivity, $\varepsilon_0 = 8.8542 \cdot 10^{-12}$ F/m is the free space permittivity), and σ is the conductivity. The permeability μ is assigned $4\pi \cdot 10^{-7}$ H/m to all tissue types. Then performing [2] + [3], using the definition of B_1^+ and recognizing the linearity of Laplace operator, we obtain

$$\nabla^2 B_1^+(\mathbf{x}) + \omega^2 \mu \varepsilon(\mathbf{x}) \left(1 + \frac{\sigma(\mathbf{x})}{j\omega \varepsilon(\mathbf{x})} \right) B_1^+(\mathbf{x}) = 0 \quad [4]$$

Then by separating the real and imaginary components of the equation, we arrive at

$$\varepsilon_r(\mathbf{x}) = \frac{-1}{\omega^2 \mu \varepsilon_0} \operatorname{Re} \left\{ \frac{\nabla^2 B_1^+(\mathbf{x})}{B_1^+(\mathbf{x})} \right\} \quad [5]$$

$$\sigma(\mathbf{x}) = \frac{1}{\omega \mu} \operatorname{Im} \left\{ \frac{\nabla^2 B_1^+(\mathbf{x})}{B_1^+(\mathbf{x})} \right\} \quad [6]$$

Where the material properties of relative permittivity (ε_r) and conductivity (σ) are given in terms of the transmit field B_1^+ .

METHODS

Central difference equations

To formulate equations for discrete data, expand equation [5]:

$$\varepsilon_r(\mathbf{x}) = \frac{-1}{\omega^2 \mu \varepsilon_0} \operatorname{Re} \left\{ \frac{1}{B_1^+(\mathbf{x})} \left(\frac{\partial^2}{\partial x^2} B_1^+(\mathbf{x}) + \frac{\partial^2}{\partial y^2} B_1^+(\mathbf{x}) + \frac{\partial^2}{\partial z^2} B_1^+(\mathbf{x}) \right) \right\}$$

Assuming that the volume of interest is discretized in steps of $(\Delta x, \Delta y, \Delta z)$ such that the location $\mathbf{x} = (i\Delta x, j\Delta y, k\Delta z)$ can be represented as (i, j, k) , partial second derivatives can be calculated as (18)

$$\begin{aligned} \frac{\partial^2}{\partial x^2} B_1^+(\mathbf{x}) &: \frac{B_1^+(i+2, j, k) - 2B_1^+(i, j, k) + B_1^+(i-2, j, k)}{4\Delta x^2} \triangleq \delta_x(i, j, k) \\ \frac{\partial^2}{\partial y^2} B_1^+(\mathbf{x}) &: \frac{B_1^+(i, j+2, k) - 2B_1^+(i, j, k) + B_1^+(i, j-2, k)}{4\Delta y^2} \triangleq \delta_y(i, j, k) \\ \frac{\partial^2}{\partial z^2} B_1^+(\mathbf{x}) &: \frac{B_1^+(i, j, k+2) - 2B_1^+(i, j, k) + B_1^+(i, j, k-2)}{4\Delta z^2} \triangleq \delta_z(i, j, k) \end{aligned}$$

Then the permittivity is:

$$\widehat{\varepsilon}_r(i, j, k) = \frac{-1}{\omega^2 \mu \varepsilon_0} \operatorname{Re} \left\{ \frac{\delta_x(i, j, k) + \delta_y(i, j, k) + \delta_z(i, j, k)}{B_1^+(i, j, k)} \right\} \quad [7]$$

Similarly, the conductivity is

$$\widehat{\sigma}(i, j, k) = \frac{1}{\omega\mu} \text{Im} \left\{ \frac{\delta_x(i, j, k) + \delta_y(i, j, k) + \delta_z(i, j, k)}{B_1^+(i, j, k)} \right\} \quad [8]$$

The equations show that to estimate relative permittivity and conductivity at location (i, j, k) , we require B_1^+ data at $(i, j, k), (i \pm 2, j, k), (i, j \pm 2, k), (i, j, k \pm 2)$ points, for a total of 7 data points (19). Note the equations use next to nearest neighbor points in the calculation and not the nearest neighbor points.

Estimation of relative permittivity from B_1^+ magnitude and conductivity from B_1^+ phase

Although the relative permittivity and conductivity are estimated using complex B_1^+ , relative permittivity is mostly dependent on the magnitude of B_1^+ and therefore can be calculated using only B_1^+ magnitude maps and the conductivity is mostly dependent on the phase of B_1^+ and therefore can be calculated by using only B_1^+ phase maps (9, 13).

For permittivity imaging with only B_1^+ magnitude maps,

$$\varepsilon_r(\mathbf{x}) \approx \frac{-1}{\omega^2 \mu \varepsilon_0} \left\{ \frac{\nabla^2 |B_1^+(\mathbf{x})|}{|B_1^+(\mathbf{x})|} \right\} \quad [9]$$

and therefore, the estimate can be carried out as

$$\widehat{\varepsilon}_r(i, j, k) \approx \frac{-1}{\omega^2 \mu \varepsilon_0} \left\{ \frac{\delta_x(i, j, k) + \delta_y(i, j, k) + \delta_z(i, j, k)}{|B_1^+(i, j, k)|} \right\} \quad [10]$$

where the partial derivative terms $\delta_x(i, j, k), \delta_y(i, j, k)$ and $\delta_z(i, j, k)$ are also calculated using the magnitude of B_1^+ .

For conductivity imaging with only B_1^+ phase maps, with the phase of B_1^+ defined as

$$\sigma(\mathbf{x}) \approx \frac{1}{\omega\mu} \text{Im} \left\{ \frac{1}{\cos(\varphi(\mathbf{x})) + j \sin(\varphi(\mathbf{x}))} \nabla^2 (\cos(\varphi(\mathbf{x})) + j \sin(\varphi(\mathbf{x}))) \right\} \quad [11]$$

and therefore the estimate can be carried out as

$$\widehat{\sigma}(i, j, k) \approx \frac{1}{\omega\mu} \text{Im} \left\{ \frac{1}{\cos(\varphi(i, j, k)) + j \sin(\varphi(i, j, k))} (\delta_x(i, j, k) + \delta_y(i, j, k) + \delta_z(i, j, k)) \right\} \quad [12]$$

Here the partial derivatives $\delta_x(i, j, k), \delta_y(i, j, k)$ and $\delta_z(i, j, k)$ are also calculated using $\cos(\varphi(i, j, k)) + j \sin(\varphi(i, j, k))$

Use of a *skip factor* in the calculation of Laplacian

Additive noise in B_1^+ maps can corrupt the estimation of the Laplacian and therefore, the accuracy of electrical properties estimate. This issue can be addressed by evaluating the Laplacian over a larger volume. Define an integer *skip factor* ($sf=1, 2, 3, \dots$). Then the partial derivatives can be estimated as

$$\begin{aligned}\delta_x(i, j, k) &= \frac{B_1^+(i+2*sf, j, k) - 2B_1^+(i, j, k) + B_1^+(i-2*sf, j, k)}{4(sf*\Delta x)^2} \\ \delta_y(i, j, k) &= \frac{B_1^+(i, j+2*sf, k) - 2B_1^+(i, j, k) + B_1^+(i, j-2*sf, k)}{4(sf*\Delta y)^2} \\ \delta_z(i, j, k) &= \frac{B_1^+(i, j, k+2*sf) - 2B_1^+(i, j, k) + B_1^+(i, j, k-2*sf)}{4(sf*\Delta z)^2}\end{aligned}$$

As the *skip factor* increases, the numeric values of the difference terms increase relative to the noise, leading to better estimates.

Electromagnetic simulations

In order to verify the equations, B_1^+ maps were generated from an electromagnetic simulation (HFSS, Ansoft Corp, Pittsburgh, PA, USA). A human head model was used in a birdcage type transmit coil(20) to obtain H_x and H_y in the axial planes. These were processed to find B_1^+ using equation [1]. Then using equations [5] and [6], the relative permittivity and conductivity were estimated and compared with known electrical properties from the model.

The transmit coil model was a 16-rung, shielded birdcage coil, diameter 28cm, rung length 28cm and rung to shield spacing of 3.5cm. Each rung had a unit amplitude current source at the center of rung. The phases of the current sources were assigned a step increment from rung to rung, equal to $2\pi/16$, to generate a circular polarized magnetic field. The frequency of each current source was 128MHz and the coil model simulated a low pass birdcage coil operating in a 3.0T imaging system.

The human head model had 5 components with each component representative of sections of the human head anatomy and assigned a tissue type with corresponding electrical properties. The components and the electrical properties were *brain_frontal* ($\epsilon_r=67.93$, $\sigma=0.54$ S/m), *cerebellum* ($\epsilon_r=83.42$, $\sigma=0.82$ S/m), *eyes_left* ($\epsilon_r=69.07$, $\sigma=1.51$ S/m), *eyes_right* ($\epsilon_r=69.07$, $\sigma=1.51$ S/m), and *body_exterior* ($\epsilon_r=64.7$, $\sigma=0.74$ S/m). The model electrical properties resolution was 4mm in x , y , and z directions. The model and a *cut-plane* view of the central axial plane are shown in Fig. 1. This plane intersected the components *brain_frontal* and *body_exterior*.

Following the convergence of the finite element simulation in HFSS, the magnetic field quantities H_x and H_y were exported in a 24cm $\times$ 24cm axial plane region using a 128 $\times$ 128 sampling grid. This region and grid corresponded to a typical 128 $\times$ 128, 24cm field of view (FOV) MR image. In addition to the central axial slice, magnetic field data were exported for two slices at 5mm spacing in the superior direction (+ z -direction) and two slices at 5mm spacing in the inferior direction ($-z$ -direction), for a total of 5 slices.

The H_x and H_y data were imported to Matlab (Mathworks, Natick, MA, USA) and B_1^+ maps were calculated for each slice using equation [1]. The relative permittivity and conductivity of the central axial slice were estimated using B_1^+ data from the central axial slice, axial slice 10mm in the superior direction and axial slice 10mm in the inferior direction. The discretization steps in this case were $\Delta x = 1.875$ mm, $\Delta y = 1.875$ mm, and $\Delta z = 5$ mm. The equations [5] and [6] were then evaluated in Matlab. At each pixel, the calculation results were accepted based on prior knowledge, discarded otherwise. For example, the relative permittivity of tissue in the human body is typically in the range of 1 – 100, so results outside this range were discarded. Similarly, conductivity values outside the range of 0 – 4 S/m were discarded.

Phantom experiments

Next, a head phantom (Phantom Labs, Salem, NY, USA) was used in experimental validation of the method. The head phantom had two compartments; one inner, smaller compartment (20mm diameter, 35mm long) and an outer compartment. A photo of the phantom is shown in Fig. 2.

In the first experiment, the phantom was filled with non-conducting fluids, B_1^+ magnitude data acquired using the recently introduced Bloch-Siegert shift B_1^+ mapping method (15) and relative permittivity was estimated.

In the second experiment, the inner compartment was filled with a salt solution, B_1^+ magnitude and phase data acquired and both relative permittivity and conductivity were estimated.

Head phantom – non-conducting fluids

The outer compartment of the head phantom was filled with 50% (volume) iso-propanol and 50% distilled water solution. The inner compartment was filled with distilled water. In order to reduce T1 relaxation time, copper sulphate was added to both solutions until concentration was 1g/liter of copper sulphate in the final prepared solution.

The relative permittivity based on literature is 18 for iso-propanol and 81 for distilled water (21). Using the dielectric mixture formula ((22), power law, $\beta=1/2$), we expected the 50% mixture in outer compartment to have a relative permittivity of 43.8.

The head phantom was placed in a 16 rung, quadrature transmit and receive head coil (Model 5182872 HDx, GE Healthcare, Waukesha, WI, USA). The phantom was aligned such that the central axial plane bisected the inner compartment of the phantom.

Then data for Bloch-Siegert shift based B_1^+ mapping were acquired in a 3.0T scanner (Discovery MR 750, GE Healthcare, Waukesha, WI, USA) for three axial plane slices with the following parameters: FOV 24cm, slice thickness 5mm, spacing 5mm, resolution 128×128, TE 30ms, TR 800ms. At each slice, the B_1^+ mapping pulse sequence generated two off resonance images (± 4 kHz) using 6ms long Fermi pulses to induce B_1^+ dependent phase shifts in the images. From the two off resonance images, a phase difference image was calculated in Matlab, square root calculated and scaled to obtain the B_1^+ map for each slice (15).

The B_1^+ data were then used in the calculation of relative permittivity. At each pixel, the calculation results were accepted if within 1 – 100 range, discarded otherwise. The *skip factors* were used to improve the estimates. However, to estimate electrical properties using only the three axial plane B_1^+ maps, the Laplacian was evaluated on a larger *in plane* area, rather than in a larger *volume*, by using *skip factors* in the calculation of *x* and *y* partial derivatives only.

The relative permittivity images were calculated for *skip factors* 1, 2 and 3 and the images were combined to obtain the final image. At any given pixel, an estimate from higher *skip factor* replaced an estimate from lower *skip factor*.

Head phantom – conducting fluids

In the next experiment, the inner compartment was filled with a salt solution (9 g/liter) and 1g/liter of copper sulphate. Data for Bloch-Siegert shift based B_1^+ mapping were acquired for three axial plane slices with the following parameters: FOV 24cm, slice thickness 6mm, spacing 6mm, resolution 128×128, TE 30ms, TR 800ms.

In order to estimate conductivity, B_1^+ phase, in addition to B_1^+ magnitude is required. The phase of a spin echo image in a switched mode, quadrature birdcage coil has been shown to be approximately half of B_1^+ phase (8, 13, 14). Under axial rotational and mirror symmetry, this relationship was shown to be exact (23).

Spin echo images were acquired for all three slices with the same alignment, but with TR 400ms. The phase encoding direction was A/P (anterior/posterior) in these images.

The spin echo images have a linear phase term and an additional constant phase, unrelated to phase variation due to the imaging sample properties. An efficient algorithm to remove the linear phase term using the autocorrelation of image pixels and the constant phase term using histogram analysis was presented in (24). This method was used to correct spin echo image phase.

The B_1^+ magnitude and phase were combined to obtain complex B_1^+ , and conductivity and relative permittivity images were constructed. As in the previous section, *skip factors* 1, 2 and 3 were used in the calculation of x and y partial derivatives, and the three images from each *skip factor* were combined to obtain final conductivity and relative permittivity images.

RESULTS

Electromagnetic simulations

The magnitude and phase of simulated B_1^+ for the central axial slice are shown in Fig. 2. The calculated electrical properties images for the central axial slice are shown in Fig. 3. The left hand figure shows the estimated conductivity and the right hand figure shows the estimated relative permittivity. A comparison of the model electrical properties and estimated electrical properties is given in Tables 1 and 2. The estimated values are in good agreement with model parameters.

Phantom experiments

Head phantom – non-conducting fluids—The B_1^+ maps for the head phantom filled with 50% mixture of iso-propanol and distilled water (outer compartment) and distilled water (inner compartment) are shown in Fig. 5. The relative permittivity images calculated from these B_1^+ maps are shown in Fig. 6. The images for each of the *skip factor* 1, 2, and 3 are shown and the images demonstrate the improvement as the *skip factor* is increased. The final relative permittivity image, after combining the images for each *skip factor*, is shown in Fig. 7. The image shows the lower relative permittivity of the outer compartment and the higher relative permittivity of the inner compartment of the head phantom.

Head phantom – conducting fluids—The B_1^+ maps for the head phantom filled with 50% mixture of iso-propanol and distilled water (outer compartment) and salt water (inner compartment) are shown in Fig. 8.

The original spin echo image phase and the phase after removing the linear and constant phase terms are shown in Fig. 9.

The calculated conductivity images for *skip factor* 1, 2, and 3 are shown in Fig. 10. The conductivity images incrementally improve as the *skip factor* increases. There is similar improvement in the permittivity images, but are not shown as they are similar to Fig. 6.

The final conductivity and relative permittivity images, after combining the images for *skip factors* 1, 2, and 3 are shown in Fig. 11. The images show the higher conductivity and the higher relative permittivity of the inner compartment.

DISCUSSION AND CONCLUSION

The MRI B_1^+ based electrical properties imaging offers a non-invasive and computationally efficient method as the B_1^+ maps are obtained in non-invasive imaging routines and conductivity and relative permittivity images are calculated using difference equations.

Conductivity and relative permittivity estimated from simulated B_1^+ maps showed good agreement with model electrical properties. The variance of the estimated values was highly correlated to the size of the hexahedral shaped mesh used in the finite element simulation. The variance decreased as the finite element mesh size was reduced. The results presented here were obtained by restricting the largest mesh element in the region of interest to be no larger than 5mm. This turned out to be an acceptable engineering compromise between the accuracy of electrical properties images and the required computer memory and the convergence time for the simulation.

The use of a *skip factor* in the calculation of Laplacian was an effective method to improve the estimates in the presence of noise in B_1^+ maps, yet retained the computational efficiency of the calculation. In general, the electrical properties estimates improved as the *skip factor* was increased. However, when larger *skip factors* were used, poor estimates at material boundaries occurred. As invalid estimates were discarded, the image boundaries became smaller. The calculation of electrical properties images for *skip factors* 1, 2, 3 and combining the images turned out to be a good compromise between accuracy and image resolution as the phantom experiments successfully identified higher conductivity and higher relative permittivity in a localized region of $2\text{cm} \times 3.5\text{cm}$ in an average head size volume.

The results point to the feasibility of imaging conductivity and relative permittivity in a non-invasive and computationally efficient manner using Bloch-Siegert shift based B_1^+ mapping and central difference based equations.

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Figure 1.
Human head model in birdcage coil (left), *cut-plane* view of the central axial plane (right)

Figure 2.
Photo of the phantom. The outer compartment and inner compartment can be filled with different fluids

Figure 3.
Simulation results: B1⁺ magnitude (left, μ Tesla) and phase (right, degrees) at central axial plane

Figure 4. Conductivity image (left, S/m), and relative permittivity image (right) obtained from simulated $B1^+$ data

Figure 5. $|B_1^+|$ in central axial plane (center) and in axial planes ± 10 mm in S/I directions (right, left, units μ Tesla). The phantom inner compartment has distilled water and outer compartment has 50% mix of iso-propanol and water

Figure 6. Permittivity image with Laplacian calculated over larger and larger areas ($sf=1$ (left), $sf=2$ (center), and $sf=3$ (right))

Figure 7. Final relative permittivity image (left). The inner compartment has distilled water and the outer compartment has 50% mix of iso-propanol and water. The estimated values on a line that runs across the image, through the inner compartment, is shown on right

Figure 8.

$|B_1^+|$ in central axial plane (center) and in axial planes $\pm 12\text{mm}$ in S/I directions (right, left, units μT). The phantom inner compartment has a salt solution and outer compartment has 50% mix of iso-propanol and water

Figure 9.

Spin echo phase for the three slices (top, degrees), after removing linear and constant phase terms (bottom, degrees). Note: The color scales are different in top and bottom images.

Figure 10. Conductivity images with Laplacian calculated over larger and larger areas ($sf=1$ (left), $sf=2$ (center) and $sf=3$ (right))

Figure 11. Conductivity image (left, S/m) and the permittivity image (right) for the central axial plane; inner compartment has a salt solution (distilled water with 9g/liter of salt) and outer compartment has 50% mix of iso-propanol and water

TABLE 1

Estimated values of Relative Permittivity

Component	Model	Estimated	
		Mean	Std. dev.
<i>brain_frontal</i>	67.93	66	10
<i>body_exterior</i>	64.7	59.9	15

TABLE 2

Estimated values of conductivity

Component	Model (S/m)	Estimated	
		Mean (S/m)	Std. dev.
<i>brain_frontal</i>	0.54	0.57	0.16
<i>body_exterior</i>	0.74	0.75	0.245