Probabilistic framework for assessing the ice sheet contribution to sea level change

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Previous sea level rise (SLR) assessments have excluded the potential for dynamic ice loss over much of Greenland and Antarctica, and recently proposed “upper bounds” on Antarctica’s 21st-century SLR contribution are derived principally from regions where present-day mass loss is concentrated (basin 15, or B15, drained largely by Pine Island, Thwaites, and Smith glaciers). Here, we present a probabilistic framework for assessing the ice sheet contribution to sea level change that explicitly accounts for mass balance uncertainty over an entire ice sheet. Applying this framework to Antarctica, we find that ongoing mass imbalances in non-B15 basins give an SLR contribution by 2100 that: (i) is comparable to projected changes in B15 discharge and Antarctica’s surface mass balance, and (ii) varies widely depending on the subset of basins and observational dataset used in projections. Increases in discharge uncertainty, or decreases in the exceedance probability used to define an upper bound, increase the fractional contribution of non-B15 basins; even weak spatial correlations in future discharge growth rates markedly enhance this sensitivity. Although these projections rely on poorly constrained statistical parameters, they may be updated with observations and/or models at many spatial scales, facilitating a more comprehensive account of uncertainty that, if implemented, will improve future assessments.

climate change | glaciology | decision-making | risk management

Quantifying the future rate of sea level rise (SLR) is critical for climate change mitigation and coastal planning efforts (1–4). However, the requirements for precision and robustness in SLR projections depend on the specific objectives, location, risk tolerance, and time horizon of decision makers (1, 4–7). A comprehensive risk management approach to sea level change would thus be greatly aided by a continuous probabilistic representation that allows the likelihood of upper bounds (i.e., SLR with a minimal chance of exceedance) to be generated and assessed self-consistently with best estimates (2, 4, 7).

Although probabilistic projections have been made for other sources of SLR (e.g., thermosteric expansion; refs. 8, 9) and other climate indicators (e.g., climate sensitivity; ref. 10), the contribution of continental ice sheets to SLR has generally been presented without an associated likelihood (2, 3, 8, 11). The development of probabilistic projections with process-based numerical ice sheet models is hindered by persistent physical uncertainties and computational constraints (8, 12, 13).

In the absence of comprehensive, process-based, models, ice sheet mass loss has been projected using expert-informed regional mass balance scenarios (3, 11). In these scenarios, the future Antarctic SLR contribution is almost entirely derived from Pine Island and Thwaites glaciers [where present-day mass loss is concentrated (14–17)] despite evidence that future discharge in other drainage basins remains uncertain. This evidence includes: (i) observations of ongoing dynamic change (17), (ii) the possibility of substantial changes in climatic boundary conditions (18, 19), and (iii) evidence that other basins may have contributed to previous episodes of SLR (20, 21). The ice loss scenarios of refs. 11 and 3 are presented without a probability of exceedance, making the relevance of excluded regions, and the interpretation of their upper bounds, problematic.

In this paper, we present a probabilistic framework in which Antarctic mass loss is calculated as the sum of regional contributions that are linked via statistical relationships (schematically illustrated, as applied in this paper, in Fig. 1). Including uncertainty in present-day and future discharge clarifies the quantitative impact of different subsets of drainage basins. With an upper bound defined by explicit exceedance probability, we highlight the uncertainties that dominate upper-bound SLR projections, so that these uncertainties may be systematically reduced by targeted observations and process-based modeling. We conclude with suggestions for future extensions of this framework.

1. Setting Up the Problem

We first segregate Antarctica into 18 major drainage basins (after ref. 15; Fig. 2). In each basin (b), we use a mass balance model in which accumulation (A^0 , in GT y^{-1}) remains constant and discharge (Q , in GT y^{-1}) grows continuously with a constant rate (α_b). This discharge growth rate is applied to a baseline discharge (Q^0) in the year t^0 :

$$Q_b(t) = Q_b^0(1 + \alpha_b(t - t^0)^n) \quad [1]$$

To account for uncertainty in Antarctica’s present-day mass balance (15, 16), we incorporate observational baselines (i.e., A^0 and Q^0) from two different analyses, here referred to as R08 (14) and Z11 (15) (Fig. 2 and Table 1). Uncertainty in future discharge is propagated to SLR projections by Monte Carlo (MC) sampling of α_b from a multivariate normal distribution where $\alpha \sim \mathcal{N}(\mu, \Sigma)$ (selected other distributions are presented in Fig. S1). In the remainder of this section, we describe the various α examined in this analysis; more details are available in *Materials and Methods*.

In basin 15 (B15), distributions for discharge growth rates [$\mathcal{N}(\mu_{15}, \sigma_{15}^2)$] are generated by fitting trends in observed discharge (22) to Eq. 1 with $n = 0.5$, 1.0, and 2.0 (square root, linear, and quadratic laws, respectively). Our choice to extrapolate historic trends is driven principally by weak constraints on B15’s future mass balance. However, ice sheets adjust slowly to climate forcing; their future SLR contribution will, to some degree, reflect an ongoing dynamic response to historic perturbations (20, 23). Furthermore, although changes in B15 discharge are not driven by a specific climate forcing or emissions scenario, they are largely consistent with process-based regional and continental-scale ice sheet model simulations (24–26).

Discharge growth rate distributions may be uniquely specified for non-B15 basins; however, data are limited for many basins, and specifying statistical parameters at this level of detail obscures this analysis. Instead, we assume discharge growth is temporally constant, linear ($n = 1.0$), has a zero mean ($\mu_b = 0$), and is equally

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Fig. 4. Sensitivity to the strength and extent of spatially correlated discharge growth. The upper-bound SLR contribution from all non-B15 basins are shown for three values of discharge growth rate uncertainty (σ : circles; $0.5\mu_{B15}$: asterisks; μ_{B15} : squares) at different correlation coefficients (gray: regional, ρ_r ; black: continental, ρ_a). In A, the upper bound is defined as SLR with a 10% probability of exceedance; B uses a 1% exceedance probability. The R08 baseline is used.

higher when discharge growth is correlated over the entire ice sheet (black symbols), and is highest at low values of ρ_a . This nonlinear response is more pronounced as the specified exceedance probability is lowered (Fig. 4B).

3. Discussion

Our results demonstrate that the total SLR contribution from Antarctica, as well as the relative importance of non-B15 basins, is sensitive to three key uncertainties: discharge growth trends in B15, observations of mass fluxes at subcontinental scales, and spatial covariance in discharge growth. These physical uncertainties interact with decision-making criteria; the dominant source of uncertainty influencing SLR projections changes depending on the likelihood of the outcome being considered. Median projections are strongly influenced by the observational baseline, unless discharge growth is expected to be high in B15. If non-B15 discharge is uncertain, or outcomes with a lower exceedance probability are required for decision-making, the importance of spatial correlation grows.

3.1. B15 Growth Laws. Although the quadratic growth law is a better fit for recent B15 discharge (Fig. 3A), and Antarctic mass loss (16) offers evidence that superlinear trends may apply over shorter periods, the physical basis underlying its extrapolation must be considered carefully. Observed decadal changes in B15 convolve long-term retreat (29) and recent stress perturbations due to melting and/or removal from bedrock pinning points (30, 31). Ignoring stabilizing feedbacks that manifest over longer timescales may result in dramatic overestimates of century-timescale SLR.

Recent numerical simulations do not support the maintenance of quadratic increases in ice flux over a century, giving constant or decreasing rates of ice loss following a short (decadal) adjustment period to forcing (24–26, 32, 33.). If the fraction of B15 ice flux discharged through Pine Island Glacier (PIG) remains constant, our linear growth law gives a median PIG SLR contribution comparable to the upper bound of a set of recent numerical simulations (2.7 cm) (24). In contrast, a quadratic law gives a median SLR contribution far greater than numerical simulations (>14 cm).

Although this evidence does not eliminate the possibility of accelerated trends, we judge that a linear growth law (with a large range of uncertainty) better represents current understanding. Reconciling whether the current superlinear growth in Antarctic discharge will be damped over long timescales, and how widespread the underlying processes are (so that these growth trends may be applied, or discounted, in other basins), remains an important area of focus for future observational and modeling efforts.

3.2. Present-Day Mass Balance. Median projections of SLR may be significantly underestimated if present-day mass imbalances persist. Although the simple continuation in discharge used here lacks a strong physical basis, because ice sheets are far from steady state (8, 17, 20), we suggest that it is more physically justified than an assumption of zero net SLR contribution. However, the optimal choice of observational baseline remains unclear, and projections vary if different subsets of the ice sheet are included in our analysis.

The importance of observational baselines is not limited to extrapolation-based projections; process-based models require validation. The large interregion differences shown here indicate that these datasets are unlikely to be sufficient for model validation at a smaller scale. Furthermore, if 100-y changes in discharge are relatively small or unlikely, the ongoing adjustment to long-term changes in climate is likely to play a large role in 21st-century SLR. Continental-scale models that use only anomalies from a “spin-up” ignore this signal (26). Continued efforts to rationalize (15, 16) and refine (34) observational techniques are necessary.

3.3. Spatially Correlated Discharge Growth. As shown in Fig. 4, in the presence of uncertainty a widespread, correlated response of many individual glacier systems may generate high SLR upper bounds. Their magnitude depends on the prescribed growth law, the number of uncertain basins, and the form of growth rate distributions (Fig. S1). The nonlinear response to the propagation of discharge growth uncertainty, and its dependence on correlation strength, may be enhanced if heavier-tailed distributions are judged to be more physically appropriate (26, 35).

Fig. 4 indicates that small differences in either correlation strength or spatial extent can influence upper bounds on SLR. Efforts to characterize past or present-day correlations in discharge may thus provide valuable constraints on SLR, particularly if moderate growth scenarios with spatial correlation are deemed more plausible than extreme rates of acceleration in single basins. Present-day observations (16, 34, 36), regional paleo-reconstructions (20, 37), and continental-scale models (12, 38,) may inform these efforts even if they do not provide a comprehensive basis for century-timescale SLR projections.

4. Conclusions

Using a unique framework for assessing the ice sheet contribution to sea level change, we have demonstrated that excluding potential dynamic changes outside basin 15 (B15, drained largely by Pine Island, Thwaites, and Smith glaciers) results in an underestimate of SLR by 2100 over the full range of outcomes needed for decision-making. With linear discharge growth rates and the R08 observational baseline, greater than 20% of Antarctica’s future SLR contribution is derived from non-B15 basins (Fig. 5). Depending on the specified exceedance probability, the non-B15 contribution may be larger than projected 21st-century changes in Antarctic surface mass balance (8). Even with relatively low discharge uncertainty and spatial correlation ($\sigma_b = 0.25\mu_{B15}$ and $\rho_r = 0.25$), roughly half of the 90th-percentile Antarctic SLR (~ 18 cm) originates in non-B15 basins. Although the Z11 observational baseline (Figs. S2–S4) gives a smaller non-B15 contribution because of compensating mass gains in East Antarctica, the fractional contribution from non-B15 basins also increases sharply with increasing uncertainty and/or a lower risk tolerance.

Previous estimates of upper bounds on SLR specify dramatic increases in B15 discharge, with 25- (ref. 3) and 40-cm (ref. 11) SLR equivalent arising from discharge in this basin. Our analysis indicates that a comparable contribution from non-B15 basins requires extremely widespread and uncertain ice discharge growth. However, we suggest that previously published upper bounds may have an implicit exceedance probability that is lower than 1%. At these low probabilities, the contribution of non-B15 basins is large even if uncertainty is relatively small (Fig. 5A); ignoring their possible contribution is inconsistent with the goal

Fig. 5. Sensitivity to the exceedance probability. In *A*, the upper-bound SLR contribution from all non-B15 basins is plotted as a function of the exceedance probability and discharge growth rate uncertainty. *B* shows the corresponding fraction of the Antarctic SLR contribution that is derived from non-B15 basins (assuming a linear growth law in B15). The R08 baseline is used; $\rho_r = 0.25$.

of bounding SLR. Furthermore, their exclusion may “anchor” SLR projections (6) and focus uncertainty reductions where their quantitative impact on SLR would be lower.

Because the sensitivity of SLR upper bounds to discharge uncertainty is heightened when discharge growth is spatially correlated or if lower-probability outcomes are explored, our results underscore the importance of a comprehensive assessment of the entire ice sheet that assigns likelihoods to scenarios of ice loss. Without a formal definition, upper bounds are of limited utility, given the strong sensitivity of SLR projections to this assumption and widely varying interpretations of acceptable risks.

5. Outlook

These results are limited by the convolution of external forcing and ice sheet dynamics into discharge growth laws, the aggregation of many physically distinct basins, and the highly abstracted statistical parameters used. However, in this framework, a hierarchy of regional statistical or dynamical models at many spatial scales (13, 39) may be incorporated and updated over time, traceably linking aggregated kinematic approaches with basin-specific, process-based model projections. Where observational data are insufficient or model-based results are less robust, uncertainty estimates may continue to be supplemented by physically based expert judgments (40, 41) (using, for example, bedrock elevation; ref. 42, and/or ocean thermal regimes; refs. 17, 18).

This framework is easily extended to incorporate different future climate and/or emissions scenarios by implementing scenario-dependent discharge (or accumulation) assumptions. Although this may be accomplished using a common relationship, perhaps between ice discharge and global or Southern Ocean temperature projections (19, 26), we suggest that this relationship be carefully examined, given evidence that current (17) and future (18) regional oceanography may be largely uncorrelated with global climate.

A complete accounting of sea level requires the inclusion of Greenland ice sheet dynamics and integration with other terms in the sea level budget (2, 8). Although the Greenland ice sheet is subject to differences in dominant physical processes (16) and may require different mass balance models, this basin-by-basin

framework remains applicable. With the inclusion of both present-day ice sheets, probability distributions of local sea level change may be derived by passing mass balance trends to appropriate solid-earth models (43). The statistical linkages between ice sheet drainage basins used in this paper could be extended to all sea level sources driven by common climate forcing (e.g., correlated discharge trends in Greenland and Antarctica, or regional thermoclastic and ocean dynamic changes driven by ice discharge; ref. 44).

Local coastal adaptation efforts require projections of sea level that involve physical processes operating over a wide range of timescales (2, 8). Until (and if) these sea level dynamics are captured in a robust, comprehensive, earth system model, this framework provides a viable means to include the contribution of the ice sheets in a more self-consistent, probabilistic manner.

Materials and Methods

Here, we project only the component of global mean sea level change (ΔSL , where positive values indicate SLR) resulting from changes in ice flux (discharge) across Antarctica’s grounding lines. Discharge is assumed to grow at a constant uncertain rate, and these growth rates are sampled from a multivariate normal distribution where $\alpha \sim \mathcal{N}_{18}(\mu, \Sigma)$, using an MC method.

B15 Discharge Growth Rates. We train square root, linear, and quadratic growth laws on the aggregate historical discharge of B15 (derived from five sets of satellite-derived velocity measurements of Pine Island, Thwaites, Smith, and “interstream” regions between 1974 and 2007) (22) using Eq. 1 with $n = 0.5, 1.0$, and 2.0 . The best fit for each growth law is shown with dashed lines in Fig. 3 (r^2 values are 0.75, 0.87, and 0.98 for $n = 0.5, 1.0$, and 2.0). Uncertainty in the fit is then used to inform the future growth rate distribution; for example, the linear trend of $1.46 \times 10^{-2} \text{ y}^{-1}$ has a $0.79 \times 10^{-2} \text{ y}^{-1} 1\sigma$ confidence interval. The growth rate distribution of B15 is then given by a normal distribution with $\mu_{15} = 1.46 \times 10^{-2} \text{ y}^{-1}$ and $\sigma_{15} \equiv \sqrt{\Sigma_{15,15}} = 0.79 \times 10^{-2} \text{ y}^{-1}$.

Basin Discharge Growth Rate Covariance Matrices. If regionally correlated growth is prescribed, basins are aggregated into regions B_r , where $r = 1, 2, 3, 4, 5$; each region contains the set of basins $B = \{b_1, b_2, \dots, b_n\}$ (Table 1). Σ is

given by a block-diagonal matrix where the blocks (regions) have a within-region correlation and no cross-region correlation:

$$\Sigma_r = \rho_r \sigma_{b_i} \sigma_{b_j} \quad [2]$$

The regional correlation coefficient (ρ_r) is equal to 1 if $i = j$. If growth rates are correlated across the continent,

$$\Sigma = \rho_a \sigma_{b_i} \sigma_{b_j} \quad [3]$$

and $i = j = 1, 2, \dots, 18$. The continental correlation coefficient (ρ_a) is equal to 1 if $i = j$.

Each MC simulation involves a specification of the SD of the growth rate in non-B15 basins ($0 \leq \sigma_b \leq 1.46 \times 10^{-2} \text{ y}^{-1}$), and either ρ_a or ρ_r . Here, σ_b and ρ_r are assumed to be the same for all five regions; future analyses may treat them independently. Growth rates used in representative basins for three different simulations (each with a 90th-percentile SLR outcome of $\sim 20 \text{ cm}$) are shown in Fig. S5.

Mass Balance Model, Sampling, and Aggregation. Each MC simulation contains 40,000 realizations (k) that use a unique growth rate (α_b^k) for each basin. The net mass balance of a basin is given by

$$\Delta SL_b^k(t) = -\frac{1 \text{ mm}}{361 \text{ GT}} \left[A_b^0 - Q_b^k(\alpha_b^k, t) \right] \quad [4]$$

where accumulation, A^0 (in GT y^{-1}), is constant, and discharge increases at a constant linear rate. In B15, discharge is given by

$$\begin{aligned} Q_{15}^k(t) &= 178 \left(1 + \alpha_{15}^k(t - 1974)^{0.5} \right) && \text{square root} \\ Q_{15}^k(t) &= 176 \left(1 + \alpha_{15}^k(t - 1974) \right) && \text{linear} \\ Q_{15}^k(t) &= 180 \left(1 + \alpha_{15}^k(t - 1974)^2 \right) && \text{quadratic} \end{aligned} \quad [5]$$

Only a linear extrapolation is used for the Z11 analyses, and $Q^{1974} = 163 \text{ GT y}^{-1}$. This assumes that the adjustments to R08 discharge applied in Z11 are constant in time.

For all other basins, the discharge is given by

$$Q_{b \neq 15}^k(t) = Q_{b \neq 15}^{2000} \left(1 + \alpha_{b \neq 15}^k(t - 2000) \right) \quad [6]$$

Discharge in the R08 and Z11 analyses is assumed to represent year 2000 discharge, Q^{2000} (Table 1). Q^{2000} and A^{2000} are always taken from the same analysis.

For each realization, the cumulative sea level contribution of each basin is aggregated by region,

$$\Delta SL_{B_r}^k = \sum_{n=1}^{b_n} \sum_{t=2000}^{2100} \Delta SL_b^k(t) \quad [7]$$

The aggregate sea level change over all regions across all realizations provides a cumulative distribution function for SLR.

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