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Preserving the excitation profile of small flip angle RF pulses in the presence of rapid transverse relaxation

Jun Shen*

Molecular Imaging Branch, National Institute of Mental Health, Bethesda, MD 20892, USA

Abstract

Degradation of excitation profile of selective RF pulses by rapid transverse relaxation has been a long-standing concern. In this report we demonstrate that transverse relaxation can be incorporated into small flip angle RF pulse design based on the linear response theory. Small flip angle pulses that were designed without considering transverse relaxation effects can be transformed for a predefined pulse duration/ T_2 ratio. The transformed pulses, within the realm of the linear response theory, produce the same transverse frequency response as if there were no relaxation.

Keywords

RF pulse design; Linear response theory; Relaxation effects

1. Introduction

Frequency selective radiofrequency (RF) pulses play a pivotal role in many modern nuclear magnetic resonance techniques such as multi-slice two-dimensional magnetic resonance imaging and selective high-resolution NMR spectroscopy in liquids. Because of the complexity of the Bloch equations that govern the behavior of RF pulses, most frequency selective RF pulses are designed without considering the effect of relaxation. However, in many studies involving in vivo magnetic resonance imaging and spectroscopy as well as solution NMR spectroscopy of large molecules T_1 is much longer than T_2 . For example, the T_1 of adenosine triphosphate (ATP) α , β and γ ^{31}P signals is approximately 2–3 s in skeletal muscle while their T_2 is 0.007–0.023 s [1]. At 7 T, the brain white matter T_1 and T_2 are ~ 1.0 [2] and 0.027 s [3], respectively. Often one finds that the longitudinal relaxation time is much longer than the duration of the required RF pulses (T_p). As a result, the effect of longitudinal relaxation on the frequency selectivity of RF pulses can be safely ignored [1]. In contrast, the time scale of transverse relaxation in a variety of situations is comparable to that of the selective RF pulses. Consequently, the integrity of selective RF pulses is often significantly compromised by rapid transverse relaxation [1,4]. Degradation of excitation profile of selective RF pulses by rapid transverse relaxation has been a long-standing concern, especially for long pulses such as multidimensional spatial pulses [5] and spatial-spectral pulses [6]. Although parallel transmission techniques can shorten multidimensional, spatially selective pulses [7] it would be highly desirable to preserve the frequency selectivity of RF pulses in the presence of rapid transverse relaxation by pulse design.

A few attempts have been made in the literature [8–12] to address the effect of rapid transverse relaxation on the frequency selectivity of long soft pulses including directly

*Address: Molecular Imaging Branch, National Institute of Mental Health, Bldg. 10, Rm. 2D51A, 9000 Rockville Pike, Bethesda, MD 20892-1527, USA. Fax: +1 301 480 2397. shenj@intr.nimh.nih.gov.

incorporating relaxation terms into RF pulse design [10–12]. Obviously, one may shorten or truncate the RF pulses as much as possible [9,13] in order to reduce signal loss caused by relaxation at the expense of frequency selectivity. Another approach is to restrict the deviation of magnetization from equilibrium until the final portion of the RF shape [11]. Both approaches are to limit the exposure of the magnetization to relaxation effects during the execution of the pulse. Alternatively, relaxation terms can be retained in the Bloch equations for pulse design using numerical methods such as simulated annealing [10] and optimal control [12] to iteratively minimize the deviation of the actual magnetization profile from a predefined magnetization profile.

A very useful class of frequency selective pulses are the small flip angle pulses [5,6,14,15] where the linear response theory applies. Under the linear response approximation, the frequency profile of the excited magnetization is proportional to the resonant Fourier component of the applied RF pulse. A typical example of such pulses is a truncated sinc pulse which gives an approximate top-hat frequency profile with a linear phase [14]. The design of multidimensional spatial pulses and spatial-spectral pulses are also based on the principle of the linear response theory [5,6]. The key insight of the current work came from the observation that the response of a harmonic oscillator remains linear in the presence of damping. The analogous situation in magnetic resonance is the excitation of a spin system with T_2 , $T_p \ll T_1$ by small flip angle pulses. Because the response of a spin system with rapid transverse relaxation to excitation by small flip angle pulses remains linear, Fourier transform can be utilized to design small flip angle RF pulses in the presence rapid transverse relaxation. This line of reasoning also led to a fast transform that converts existing small flip angle RF pulses into ones that preserve excitation frequency selectivity in the presence of rapid transverse relaxation [16].

2. Theory

Although the general aspects of the linear response theory are well known we will derive here the response of magnetization with T_2 , $T_p \ll T_1$ to excitation by small flip angle pulses to explicitly reveal the necessary assumptions. For simplicity, we will start without considering relaxation effects.

Assume that a small flip angle RF pulse ($B_1(t)$) lasts from $t = 0 - T_p$ and the initial state of the magnetization is $M_{xy}(0) = 0$. In addition, we assume that $M_z \ll M_{xy}$ during the execution of the RF pulse ($t = 0 - T_p$). As a result of these assumptions we can set $M_z = M_z(0) = M_0$ and integrate from 0 to T_p :

$$M_{xy}(\Delta\omega; T_p) = iM_0 \int_0^{T_p} \gamma B_1(t) \exp(i\Delta\omega t) dt \quad (1)$$

where $\Delta\omega$ and γ denote frequency offset and gyromagnetic ratio, respectively. The above result is the well known linear response limit [14,15]. Within the limit of linear response theory, designing a small flip angle RF pulse without considering relaxation effects is achieved simply by inverting Eq. (1).

To investigate the linear response in the presence of rapid transverse relaxation we first reformulate the Bloch equations in the case of T_2 , $T_p \ll T_1$ into

$$\frac{dM_+(\Delta\omega, t)}{dt} = -\left\{i\Delta\omega + \frac{1}{T_2}\right\} M_+(\Delta\omega, t) + i\gamma B_{1+}(t) M_z \quad (2)$$

where the transverse magnetization $M_+(\Delta\omega, t) \equiv M_x + iM_y$ with explicit dependence on frequency offset $\Delta\omega$, the complex RF pulse $B_{1+} \equiv B_{1x} + iB_{1y}$. Within the limit of the linear

response theory, M_z in Eq. (2) will be replaced by M_0 and Eq. (2) is converted into an integrable form:

$$\frac{d}{dt} \left\{ M_+(\Delta\omega, t) \exp \left[\left(i\Delta\omega + \frac{1}{T_2} \right) t \right] \right\} = iM_0\gamma B_{1+} \exp \left[\left(i\Delta\omega + \frac{1}{T_2} \right) t \right] \quad (3)$$

Integrating Eq. (3) from 0 to T_p :

$$M_+(\Delta\omega, T_p) = iM_0 \exp \left(-i\Delta\omega T_p \right) \int_0^{T_p} \gamma B_{1+}(t) \times \exp \left(\frac{t-T_p}{T_2} + t\Delta\omega \right) dt \quad (4)$$

In essence, Eq. (4) gives the linear response approximation to the solution of the Bloch

equations containing the T_2 terms, which states that $M_+(\Delta\omega, T_p)$ and $\exp \left(\frac{t-T_p}{T_2} \right) \gamma B_{1+}(t)$ form a Fourier transform pair for time-independent $\Delta\omega$. For $T_p \ll T_2$, Eq. (4) is reduced to the well known Eq. (1).

In the most general case, $\Delta\omega(t) = \gamma \mathbf{G}(t) \cdot \mathbf{r} + \Delta\omega^c$, where $\mathbf{G}(t)$ is the field gradient vector, $\mathbf{r}$ the position vector, and $\Delta\omega^c$ the static frequency offset due to, e.g., chemical shift and magnetic susceptibility. For the general case, Bloch equations incorporating T_2 relaxation is given by

$$\frac{dM_+(\mathbf{r}, \Delta\omega^c, t)}{dt} = - \left\{ i(\gamma \mathbf{G}(t) \cdot \mathbf{r} + \Delta\omega^c) + \frac{1}{T_2} \right\} M_+(\mathbf{r}, \Delta\omega^c, t) + i\gamma B_{1+}(t) M_0 \quad (5)$$

Again, M_0 , the longitudinal magnetization, is assumed to be unaffected by B_{1+} [5,14,15]. Eq. (5) can be solved by integration over 0– T_p with the initial condition $M_+(\mathbf{r}, \Delta\omega, 0) = 0$:

$$M_+(\mathbf{r}, \Delta\omega^c, T_p) = i\gamma M_0 \int_0^{T_p} e^{(t-T_p)/T_2} B_{1+}(t) e^{1 \int_{T_p}^t (\gamma \mathbf{G}(s) \cdot \mathbf{r} + \Delta\omega^c) ds} dt \quad (6)$$

Both Eqs. (4) and (6) indicate that the same $M_+(\Delta\omega, t)$ is obtained when

$$B_{1+}(t, T_2) = e^{(T_p-t)/T_2} B_{1+}(t, T_2 = \infty) \quad (7)$$

where $B_{1+}(t, T_2 = \infty)$ is the small flip angle RF pulse designed without considering transverse relaxation. Eq. (7) is the desired fast transform that converts a small flip angle pulse designed without considering transverse relaxation ($B_{1+}(t, T_2 = \infty)$) into one that produces the same $M_+(\mathbf{r}, \Delta\omega^c, t)$ in the presence of T_2 relaxation (under the small flip angle approximation). Because of the linearity of Eqs. (4) and (6), the RF waveform of $B_{1+}(t, T_2 = \infty)$ is valid for any arbitrary small flip angles. By the scaling property of Bloch equations, the excitation profile is a function of the T_p/T_2 ratio for the same RF waveform.

3. Methods

Numerical simulations of Bloch equations containing both T_1 and T_2 relaxation terms were performed using either a C routine (Numerical Recipe in C. second ed., Cambridge University Press, Cambridge, 1988) containing a Runge Kutta driver with adaptive stepsize control written by Dr. David Horita or an in-house Matlab routine (R2009b, MathWorks, Natick, Massachusetts, USA). The Matlab routine uses the hard pulse approximation method [17] modified by incorporation of a multiplicative relaxation matrix $[\exp(-\delta t/T_2) \ 0 \ 0; \ 0 \ 0 \ 0]$

$\exp(-\delta t/T_2) \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$ for each pulse segment. The validity of the C routine was previously verified experimentally by comparing the measured and simulated frequency responses of several shaped RF pulses under the condition $T_p > T_1, T_2$ [4]. The in-house relaxation matrix method was verified numerically by comparing with the frequency responses calculated by the Runge Kutta integration method.

The pulses analyzed in this work were the following: rectangular, Gaussian, sinc of various side lobes, and the polychromatic selective pulse series [18,19]. For brevity, only results from the sinc pulse with four lobes on each side and a 17-frequency component polychromatic pulse were shown here.

4. Results and discussion

To illustrate Eq. (7), we first use a symmetric sinc pulse [11] with four lobes on each side for the extreme case of $T_p/T_2 = 4$. Fig. 1 top row shows the result of Bloch simulation for $T_2 = \infty$. Flip angle = $\pi/4$. As expected from the linear response theory (see Eq. (1)), an approximately “top hat” frequency profile for M_{xy} is obtained. Note that because of the

finite magnitude of the flip angle, $M_z (= \sqrt{1 - M_{xy}^2})$ is significantly perturbed. In addition, the frequency selectivity in M_z is coupled to that of M_{xy} when $T_2 = \infty$. The middle row shows the degraded M_{xy} and M_z profiles for $T_p/T_2 = 4$ using the original sinc pulse and Bloch equations containing the T_2 terms. For M_{xy} , there are significant loss in the magnitude of magnetization in the passband and excitation of transverse magnetization in the stopband. This behavior of M_{xy} in the stopband can be understood by rewriting Eq. (4) into an approximate convolution operation:

$$M_+(\Delta\omega, T_p) \approx M_+(\Delta\omega, T_p, T_2 = \infty) \exp\left(-\frac{T_p}{T_2}\right) \otimes \frac{\exp\left[\left(\frac{1}{T_2} + i\delta\omega\right)T_p\right] - 1}{\frac{1}{T_2} + i\Delta\omega} \quad (8)$$

where $\otimes$ denotes convolution. The convolution operation corrupts the original top hat frequency response profile of M_{xy} . The loss of frequency selectivity of M_z in the stopband can be easily understood: outside the transition band, the magnetization deviates from its equilibrium position during the execution of a pulse, leading to irreversible loss of total

magnetization, and therefore a loss of M_z as well. Note that $M_z = \sqrt{1 - M_{xy}^2}$ no longer holds for a finite T_2 . The major difference between the loss of frequency selectivity in M_{xy} and M_z is that the former, resulted from the convolution operation, is reversible, as shown by Eq.

(7). Essentially the convolution effect of the time-domain filter $\left(\frac{t - T_p}{T_2}\right)$ is eliminated by multiplying the RF waveform by the inverse of the identical time-domain filter.

When the transformed RF waveform (Fig. 2A, red, whose area is the same as the original sinc pulse) is used, the top hat profile of M_{xy} is preserved despite the extremely fast transverse relaxation condition ($T_p/T_2 = 4$). As expected, the longitudinal magnetization outside and close to the transition band cannot be fully preserved regardless of RF pulse design (see Fig. 1, bottom-right panel). Fig. 2A compares the waveform of the original sinc pulse (blue) and its transform by Eq. (7) (red). Note that, to fully recover the loss of transverse signals due to T_2 relaxation in Fig. 1 for the extreme case of $T_p/T_2 = 4$, Eq. (7) requires large deviations of magnetization from thermal equilibrium which are outside the realm of the linear response theory. The frequency response of the transformed sinc pulse (Fig. 2A, red) as a function of the area of its waveform is given in Fig. 2B. Since Eq. (7) is originated from the linear response theory, the transformed sinc pulse generates a transverse

magnetization profile with an approximately linear phase (frequency offset -----* $T_p/2$) that can be refocused by field gradient reversal during slice selection.

The assumptions of the linear response theory imply very weak RF field and that $M_z \approx M_0$ and $M_{xy} \approx 0$. In reality, the nonlinear Bloch equations are essentially bilinear [20]. A bilinear system borders a linear and a nonlinear system. It has been observed, both experimentally and by numerical simulation of Bloch equations, that Eq. (1) gives fairly accurate prediction of M_{xy} for flip angles up to 30° or larger [15]. In the case of a sinc 90° pulse the linear response theory reasonably predicts a top hat-shaped transverse frequency profile although the magnitude of M_{xy} (and M_z) on- and close to resonance is way off. Therefore, one can use the small flip angle/linear response assumptions to design RF pulses to cancel the effect of T_2 relaxation on transverse magnetization profile with the understanding that the longitudinal magnetization cannot be accurately predicted by the linear response theory. As seen in Fig. 1, there is a non-uniform response in M_z . Unfortunately, this is unavoidable regardless of pulse shapes: because of rapid T_2 loss (especially so for the extreme case of $T_p/T_2 = 4$), magnetization cannot be fully restored to M_0 once it deviates from it.

The success of the linear response assumptions in the design of small flip angle RF pulses suggest that the RF pulse waveforms transformed by Eq. (7) are also expected to generate consistent frequency response profiles under a certain threshold of RF field strengths for the same T_p/T_2 ratio. This is confirmed by Fig. 2B where the waveform area of the transformed sinc pulse (Fig. 2A, red) was systematically varied from 50% to 150% of the waveform area of the original sinc pulse (Fig. 2A, blue). It should be noted, however, the transform given by Eq. (7) may cause large deviation of magnetization from equilibrium during the execution of the pulse, therefore, leading to effects similar to large flip angles. When this is the case, the threshold of RF field strengths where the linear response assumptions apply is reduced. As seen in Fig. 2B, the passband frequency response becomes noticeably non-uniform when the waveform area of the transformed sinc pulse reaches 150% of that of the original sinc pulse. In a different context, Norris observed that rapid transverse relaxation reduces the insensitivity of the hyperbolic secant pulses to variations in B_1 .

In Introduction, we have given justifications for designing a class of RF pulses that are suitable for $T_2 \sim T_p \ll T_1$. There are other situations where both T_1 and T_2 relaxation processes are rapid. For example, most contrast agents in magnetic resonance imaging are designed for shortening T_1 (and therefore T_2 since $T_2 \ll T_1$). Inclusion of T_1 relaxation in RF pulse design requires the use of the full, nonlinear Bloch equations. The proposed method relies on the linearization of Bloch equations (Eqs. (1), (4), and (6)) and does not apply to significantly nonlinear situations. Using Bloch equation containing both T_1 and T_2 terms, we have investigated the effects of T_1 relaxation ($T_1 \gg T_2$) on the frequency response of the RF pulses generated by Eq. (7) (data not shown). As expected, when $T_1 \gg T_p$, RF pulses generated by Eq. (7) behave as if $T_1 = \infty$. The omission of the T_1 terms in RF pulse design is an obvious drawback. From a technical point, Eqs. (2) and (5) are not integrable unless M_z is replaced by M_0 . Partially because of the omission of the T_1 terms the Bloch equations are approximately linear for small flip angle excitations, leading to rational design of RF pulses. To the best of our knowledge the method proposed here is the only one that gives a closed-form solution to RF pulse shapes that are immune to transverse relaxation.

As a second example, we use a polychromatic selective pulse [18,19] which was designed in the frequency domain based on the linear response theory. The pulse waveform is given in Fig. 4A (blue). The pulse waveform was synthesized using 17-frequency components with a magnitude profile given by $1/(2\gamma)^{1/2}$. The relative frequency displacement between neighboring components is $0.5/T_p$. As shown by Kupce and Freeman [18], such a frequency

displacement suppresses the side lobes of the sinc response from each individual frequency component, resulting in an approximately top-hat absorption mode excitation response for M_y (see Fig. 3, top row; $T_2 = \infty$ flip angle = $\pi/8$) while the dispersive response (M_x) can be purged using a hard $\pi/2$ quadrature pulse [18,19].

The middle panel of Fig. 3 shows the degraded M_y and M_z profiles for $T_p/T_2 = 2$ using the original polychromatic pulse. The transformed polychromatic waveform with the same area was depicted in Fig. 4A (red). As shown by the bottom panel of Fig. 3, the transformed pulse (Fig. 4A, red) essentially restored the original frequency response of M_y although $T_p/T_2 = 2$. Interestingly, for the polychromatic pulse, rapid T_2 relaxation smoothes out the ripples in the M_y frequency response as a result of the convolution operation of Eq. (8). As predicted by Eqs. (4) and (6), the ripples reappeared in the M_y frequency response of the transformed pulse. The frequency response of the transformed polychromatic pulse as a function of the area of its waveform is given in Fig. 4B. Similar to the transformed sinc pulse, the passband frequency response of the transformed polychromatic pulse holds reasonably well when its waveform area was varied from 0% to 150% of the waveform area of the original polychromatic pulse (Fig. 4A, blue).

5. Conclusions

We have demonstrated that transverse relaxation can be easily incorporated into RF design based on the linear response theory. Any small flip angle pulse that was crafted without considering transverse relaxation effects can be transformed into one that produces the same frequency response for a predefined T_p/T_2 ratio as long as the transformed pulse remains a small flip angle pulse. Since small flip angle pulses are widely used in magnetic resonance imaging where rapid transverse relaxation is prevalent especially at high magnetic field it is expected that the principle outlined in this report will be helpful in design and use of a variety of small flip angle pulses.

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Fig. 1.

Top row: Bloch simulation results for $T_2 = \infty$ using a truncated sinc pulse with four side lobes on each side. Flip angle = $\pi/4$. Middle row: M_{xy} and M_z profiles for $T_p/T_2 = 4$ using the original sinc pulse (see Fig. 2A, blue). Bottom row: M_{xy} and M_z profiles for $T_p/T_2 = 4$ using the transformed sinc pulse (see Fig. 2A, red). The actual frequency scale is for $T_p = 40$ ms. For the same RF waveform the excitation profile is a function of the T_p/T_2 ratio. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 2.

(A) Blue: waveform of a truncated sinc pulse (blue) with four side lobes on each side. Red: Transformed sinc pulse according to Eq. (7) for $T_p/T_2 = 4$. The pulse area of the transformed pulse is normalized to that of the original sinc pulse. (B) M_{xy} excited by the transformed sinc pulse (A, red) as a function of the area of its waveform (normalized to the waveform area of the original sinc pulse). Blue: pulse area = 0.5; Red: pulse area = 1.0; Black: pulse area = 1.5. The actual frequency scale is for $T = 40$ ms.

Fig. 3.

Top: Bloch simulation results for $T_2 = \infty$ using the polychromatic selective pulse depicted in Fig. 4A (blue). Flip angle = $\pi/8$. Middle: M_y profile for $T_p/T_2 = 2$ using the original polychromatic pulse. Bottom: M_y profile for $T_p/T_2 = 2$ using the transformed polychromatic pulse (Fig. 4A, red). The actual frequency scale is for $T_p = 40$ ms. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 4.

(A) Blue: waveform of a polychromatic selective pulse with 17-frequency components and a magnitude profile given by $1(2)_71$. The relative frequency displacement between neighboring components is $0.5/T_p$. Red: transformed polychromatic selective pulse according to Eq. (7) for $T_p/T_2 = 2$. The pulse area of the transformed pulse is normalized to that of the original polychromatic pulse. (B) M_y excited by the transformed polychromatic pulse (red) as a function of the area of its waveform (normalized to the waveform area of the original polychromatic pulse). Blue: pulse area = 0.5; red: pulse area = 1.0; black: pulse area = 1.5. The actual frequency scale is for $T_p = 40$ ms.