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The Approximate Number System and its Relation to Early Math Achievement: Evidence from the Preschool Years

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Abstract

Humans rely on two main systems of quantification - one is non-symbolic and involves approximate number representations (known as the approximate number system or ANS), the other is symbolic and allows for exact calculations of number. Despite the pervasiveness of the ANS across development, recent studies with adolescents and school-aged children point to individual differences in the precision of these representations, which, importantly, have been shown to relate to symbolic math competence, even after controlling for general aspects of intelligence. Such findings suggest that the ANS, which humans share with nonhuman animals, interfaces specifically with a uniquely human system of formal mathematics. Other findings, however, point to a less straightforward picture, leaving open questions about the nature and ontogenetic origins of the relation between these two systems. Testing children across the preschool period, we found that ANS precision correlated with early math achievement, but, critically, that this relation was non-linear. More specifically, the correlation between ANS precision and math competence was stronger for children with lower math scores than for children with higher math scores. Taken together, our findings suggest that early-developing connections between the ANS and mathematics may be fundamentally discontinuous. Possible mechanisms underlying such non-linearity are discussed.

Keywords

approximate number system (ANS); mathematical cognition; analog magnitude; ratio effect; preschoolers; development

Introduction

Humans rely on basic intuitions of quantity to guide a variety of everyday decisions. When at the grocery store, for example, we identify (and avoid) the longest line to the cashier without having to explicitly count the number of people waiting in line or formally calculating the proportion of grocery items to people using learned principles of arithmetic. Also known as a “number sense,” such intuitions are rooted in a system of nonverbal number representation, whose origins and early development are not dependent on explicit instruction (e.g., Ansari, 2008; Feigenson, Dehaene, & Spelke, 2004). This system, henceforth referred to as the approximate number system (ANS), is shared with nonhuman

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animals and widespread across cultures (for review, see Dehaene, 2009). Unlike nonhuman animals, though, humans in many societies around the world supplement their number sense with a formal system of mathematics. Such a system consists of symbolic representations (e.g., number words and Arabic digits) as well as various quantitative concepts and formalized computational operations, all of which are learned over development, often via explicit instruction. Despite differences in scope and developmental trajectory, accumulating behavioral and neural findings suggest that the ANS and symbolic math processes do not operate independently, but rather are functionally intertwined (e.g., Halberda, Mazocco, & Feigenson, 2008; Lemer, Dehaene, Spelke, & Cohen, 2003; Piazza et al., 2010; for an alternative perspective, see Noël & Rousselle, 2011). Building on this research, the present paper addresses questions concerned with the nature and ontogenetic origins of the link between these two systems of quantification.

Approximate Number System (ANS)

The key signature of the ANS is its imprecision. Nonverbal representations of number are inherently approximate, with variance (i.e., “noise”) increasing linearly as a function of absolute numerical value (e.g., Buckley & Gilman, 1974; Gallistel & Gelman, 1992). This imprecision can be modeled as overlapping Gaussian distributions on an internal continuum (Halberda et al. 2008; Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004; Pica, Lemer, Izard, & Dehaene, 2004) and is captured by Weber’s law, which states that subjective differences in intensity between unequal stimuli are proportional to their objective intensities. Adults respond faster (and more accurately) the greater the ratio of numbers being compared (e.g., 8 vs. 4 compared to 8 vs. 7; Dehaene, 1992; Moyer & Landauer, 1967). Numerical comparisons of nonhuman animals similarly abide by Weber’s law (for review, see Nieder, 2005), providing support for the pervasiveness of the ANS.

The ANS has been shown to emerge early in human development and to increase in precision over the lifespan. Among infants, there is evidence that whereas newborns require a 3:1 ratio for discrimination of non-symbolic visual arrays (Izard, Sann, Spelke, & Streri, 2009), six-month-olds discriminate visual arrays (Xu & Spelke, 2000; see also Brannon, Abbott, & Lutz, 2004) and auditory sequences (Lipton & Spelke, 2003; van Marle & Wynn, 2009) that differ by a ratio of 2:1. Other experiments show that ANS precision is further refined during childhood (Droit-Volet, Clément, & Fayol, 2008; Halberda & Feigenson, 2008) and into adulthood, with adults across cultures detecting ratios as small as 10:9 (Halberda & Feigenson, 2008; Piazza et al., 2004; Pica et al., 2004).

Studies using neuroimaging techniques suggest that the ANS activates similar brain regions across development that, like behavioral discrimination, show modulation by ratio. At three months of age, there is greater neural activity in parietal and prefrontal cortices for smaller than larger ratios (Izard, Dehaene-Lambertz, & Dehaene, 2008). In children, the ANS activates the intraparietal sulcus (IPS) specifically, as well as the prefrontal cortex (PFC; Ansari & Dhital, 2006; Cantlon, Brannon, Carter, & Pelphrey, 2006; Holloway & Ansari, 2010). In comparison to children, adults display increased activity in the IPS (and decreased activity in the PFC) when processing visual displays of non-symbolic number (Ansari, 2008). Taken together, these studies suggest that increases in ANS precision over development are accompanied by specialization of the IPS during non-symbolic numerical processing.

Symbolic Math and its Relation to the ANS

In contrast to the ANS, a more formal, symbolic system of mathematics allows for reference to, and computations of, exact quantity. Although there is disagreement about whether the ANS is foundational for the emergence and development of symbolic math (Butterworth,

2011; Dehaene, 2009; Le Corre & Carey, 2007; Noël & Rousselle, 2011; Rips, Bloomfield, & Asmuth, 2008), accumulating evidence hints at interactions between these two systems at different points during development. When adults perform arithmetic calculations, there is parietal lobe activation, similar to areas recruited for comparisons of non-symbolic numbers (Dehaene et al., 1999). Furthermore, adults' performance on exact arithmetic problems suffers when non-symbolic, approximate numerical abilities are impaired by injury or temporary deactivation of the IPS (Cappelletti, Barth, Fregni, Spelke, & Pascual-Leone, 2007; Lemer et al., 2003). Developmental neuroimaging studies suggest that the ANS and symbolic math activate similar brain regions even in early childhood. Though weaker than that in adults, children show parietal activation when solving arithmetic problems (Davis et al., 2009), with the extent of activation shown to correlate with calculation accuracy (Meintjes et al., 2010).

Individual differences in ANS precision have been documented across the lifespan (Halberda & Feigenson, 2008; Mazzocco, Feigenson, & Halberda, 2011a; Piazza et al., 2010), even as early as infancy (Libertus & Brannon, 2010). Similarly, there are well-known individual differences in mathematical competence (Jordan, Kaplan, Oláh, & Locuniak, 2006). In extreme cases, low levels of math competence can lead to a diagnosis of dyscalculia (a learning disability specific to math), the prevalence of which is estimated at 5-7% in industrialized nations such as the United States and United Kingdom (e.g., Butterworth, 1999; Shalev, 2007). Given the importance of adequate mathematical functioning for technological societies, it is perhaps not surprising that a growing body of research has begun to examine the relation between individual differences in school-relevant math achievement and individual differences in ANS precision.

Using a longitudinal design, Halberda and colleagues (2008) showed that ANS precision in adolescents (14 years of age) predicted performance on standardized tests of symbolic calculation administered between kindergarten and 6th grade. Adolescents were tested on a number comparison task in which they judged whether there were more blue dots or more yellow dots in a spatially intermixed array, with ratios that varied from 2:1 to 8:7. Dot arrays were presented below the threshold possible for explicit counting (200 ms) to ensure processing by the ANS. Using psychophysical modeling to compute subject-specific ANS precision, based on the width of associated Gaussian distributions (also known as the Weber fraction or w estimate), adolescents with smaller w s (i.e., greater ANS precision) were found to have higher math scores as children. Additional research has yielded similar results when assessing ANS precision in childhood (Gilmore, McCarthy, & Spelke, 2010; Libertus, Feigenson, & Halberda, 2011; Mazzocco, Feigenson, & Halberda, 2011a; Wagner & Johnson, 2011), suggesting that the ANS may already connect to symbolic math early in development.

Other research, however, has produced mixed results. Holloway and Ansari (2009) found that although comparisons of Arabic numerals predicted math achievement in children aged 6 and 7 years, comparisons of dot arrays did not (see also De Smedt, Verschaffel, & Ghesquière, 2009), at least when using the magnitude of the distance effect as the measure of ANS precision. They suggested that the link between the ANS and school-relevant math might be indirect, mediated by experience with symbolic number representations (e.g., Arabic digits). Consistent with this possibility is evidence that children who perform poorly on math achievement tests (including children with dyscalculia) have difficulty recognizing the magnitudes underlying symbolic numbers (De Smedt & Gilmore, 2011; Iuculano, Tang, Hall, & Butterworth, 2008; Landerl & Kölle, 2009). In another study, Soltész, Szűcs, and Szűcs (2010) found no evidence of an association between the ANS and symbolic math concepts such as knowledge of the count sequence (e.g., number words between 1 and 10) in 4- to 6-year-olds. Such contradictory evidence raises questions about the reliability and

origins of the link between these two systems, and clearly necessitates the need for further experimentation.

Present Study

The present study addresses questions about the nature and early development of the link between nonverbal number representations, specifically ANS precision, and math achievement, including knowledge of numerical symbols and elementary arithmetic. Of particular interest is whether, and how, ANS precision relates to school-relevant math skills during the preschool years, when math instruction is only just beginning. From 3 to 5 years of age, children transition from early exposure to symbolic number representations to the acquisition of quantitative concepts and being able to perform exact (albeit simple) arithmetic computations (e.g., Gelman & Gallistel, 1978; Wynn, 1992). The preschool period is theoretically and practically important because it is when the ANS has been shown to increase in precision (e.g., Halberda & Feigenson, 2008) and when individual differences in symbolic math abilities are already detectable (Jordan, Kaplan, Locuniak, & Ramineni, 2007; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010; Siegler & Booth, 2004). Although recent studies examining the relation between ANS precision and symbolic math abilities have begun focusing on the preschool period, none have systematically examined development within this age range, instead assuming homogeneity for children between 3 and 5 years of age. For example, Libertus and colleagues (2011) reported that ANS precision predicted math competence in 3- to 5-year-olds, but they did not examine the strength of this relation separately for each of the age groups. Statistical analyses that collapse across age make it unclear whether important differences exist. Given the mixed findings in this literature, the present study adopts a different approach, focusing on the potentially important differences between groups of preschoolers. More specifically, we examine the relation between ANS precision and math competence separately by age group, and, as described next, separately for higher versus lower math achievers.

Existing studies have assumed a linear relation between ANS precision and math competence, but there is reason to believe that it may actually be non-linear, and, indeed, may only hold for children who are less proficient at math. For example, recent research on adolescents with dyscalculia (Mazzocco et al., 2011b; see also Piazza et al., 2010), identified as at or below the 10th percentile on standardized tests of math achievement, suggests that ANS precision in dyscalculics, compared to their non-dyscalculic peers, is lower than would be predicted by a linear function. Similar to the question of whether the link between ANS precision and math competence may vary over early development, we ask to what extent it varies in strength by the level of math competence. In other words, is the connection to the ANS stronger for lower compared to higher math-achieving children? In the present study, we examined this possibility directly.

As in previous studies (Halberda et al., 2008; Holloway & Ansari, 2009; Libertus et al., 2011), children in this study completed a computerized number task, where they judged which of two non-symbolic numbers (dot arrays) was larger. Statistical estimates of ANS precision for each child were based on predicted accuracy at an untested ratio. In addition, children completed a standardized test of math achievement appropriate for preschoolers (Test of Early Mathematics Ability-3rd edition, or TEMA-3; Ginsberg & Baroody, 2003), allowing us to examine the relation between ANS precision and mathematical reasoning early in development. As a contrastive control, we also included a measure of verbal competence. The majority of children completed a standardized test of receptive vocabulary (Peabody Picture Vocabulary Test-4th edition, or PPVT-4; Dunn & Dunn, 2007), unlike other research that used parental report of verbal competence (Libertus et al., 2011). Analyses included segmented (i.e., piecemeal) regression to examine whether the relation between ANS precision and math achievement varies non-linearly across high and low

achievers (i.e., higher vs. lower scoring groups on the TEMA-3). We predicted that ANS precision would improve over the preschool period, as documented in recent studies (e.g., Halberda & Feigenson, 2008), and that the relation to math achievement might be potentiated at the lower end, as suggested by recent findings with dyscalculic individuals (Mazzocco et al., 2011b).

Methods

Participants

Seventy-four children from 3 to 5 years of age (3-year-olds: $N = 24$, 9 girls, $M = 42.5$ months, $range = 37.7$ to 47.8 months; 4-year-olds: $N = 25$, 18 girls, $M = 52.4$ months, $range = 48.2$ to 59.9 months; 5-year-olds: $N = 25$, 13 girls, $M = 65.6$ months, $range = 60.0$ to 71.2 months) participated in this study. All children were given our computerized Number Discrimination Task (NDT) and the TEMA-3. Of the total sample, 55 children participated in a second testing session where they completed the PPVT-4; not all children were given the PPVT-4 because not all families were able to return for a second visit.¹ All children were recruited from the surrounding metropolitan community and testing for both sessions took place in a laboratory. Two additional children failed to complete the NDT and so were not included in the statistical analyses.

Tasks

Number Discrimination Task (NDT)—The NDT was created using a custom Visual Basic script (Microsoft Corp.) and administered on a Dell Vostro laptop equipped with an external touchscreen (MagicTouch, Keytec). Children sat approximately 40 cm from the computer screen (34×21 cm). On each trial, children judged which of two dot arrays contained the larger number, using a touchscreen stylus. Arrays were presented simultaneously and arranged vertically on the screen (see Figure 1); each array was surrounded by a black border of constant size (13.8×9.5 cm). Following previous work (Holloway & Ansari, 2009; Temple & Posner, 1998), ordinal judgments were made with respect to a fixed, reference number (8 dots). Comparison arrays were either larger (9–12 dots) or smaller (4–7 dots) than the reference number (half of each type), yielding eight different ratios (larger array divided by smaller array: 2.00, 1.60, 1.50, 1.38, 1.33, 1.25, 1.14, 1.13). Based on Weber's law, ratios closer to 1.00 should be more difficult to discriminate. We included only numbers greater than three to ensure processing by the ANS and sufficient variability of responses. Smaller numbers (< 4) invoke processes of subitization, which result in few, if any, differences in estimation (e.g., Revkin, Piazza, Izard, Cohen, & Dehaene, 2008).

Across trials, dots varied in size and position. Differences in size allowed for cumulative area to be varied orthogonally to number, such that the numerically larger array was either larger or smaller in cumulative area than the numerically smaller array, creating spatially congruent (e.g., larger number, larger cumulative area) and incongruent (e.g., larger number, smaller cumulative area) trials. Half the trials were congruent and half were incongruent (randomized order). Ratios of cumulative area for pairs of arrays mirrored the ratios used for number (e.g., number pairs varying on 1.33 ratio also varied in area by ratio of 1.33). Collapsing across ratio, position of the correct array (top or bottom) and spatial congruity (congruent or incongruent) were counterbalanced across trials. There were a total of 40 test trials (5 trials for each number ratio; randomized order). Performance on the NDT was found

¹Comparisons of children who did and did not complete the PPVT-4 revealed no significant differences in TEMA-3 scores, age, or overall accuracy on the NDT ($p > .25$), suggesting no systematic differences between the two groups.

to be reliable, Cronbach's $\alpha = .847$, using percent correct for each ratio as our within-task measure for each child.

Standardized Tests—Both the TEMA-3 (Ginsberg & Baroody, 2003) and PPVT-4 (Dunn & Dunn, 2007) were administered following a standardized protocol. The TEMA-3 is designed to measure general mathematical competence, including counting proficiency (e.g., knowing the order of number words from 1-10), cardinality (e.g., knowing that the word “four” refers to a set of four items), and elementary arithmetic (e.g., single digit addition or subtraction). This test has been normed on 1,219 children from 3 to 8 years of age, and it takes approximately 40 minutes to administer. The PPVT-4 measures receptive vocabulary of standard American English. This test has been normed on 3,540 children and adults from 2 to 90 years of age. It takes approximately 10 to 15 minutes to administer. Both tests have high internal consistency (both $r_s > .90$) and test-retest reliability (both $r_s > .80$; Dunn & Dunn, 2007; Ginsberg & Baroody, 2003). TEMA-3 and PPVT-4 scores reported in the Results section below are all age-standardized.

Procedure

Children were tested individually by an experienced experimenter in a quiet room. All children were given the NDT followed by the TEMA-3 in a single testing session. To familiarize children with the touchscreen stylus (used for responding on the NDT), children engaged in a warm-up game, which involved “catching a frog”; pictures of frogs appeared in random positions on the screen and children were required to quickly localize them by touching the pictures with the stylus. Instructions on how to respond on the NDT were conveyed via a short video, in which children observed two boxes containing a different number of “bubbles.” They were told that they needed to select the box with the most bubbles before they popped. Arrays remained onscreen until children made a response, but they were discouraged from verbally counting to ensure that they relied on approximate, nonverbal number representations (i.e., ANS processing). If children attempted to count, they were immediately interrupted by the experimenter and told that it was “not a counting game.” Children were given four practice trials, in which corrective feedback was provided. As noted above, there were 40 test trials, identical to the practice trials except that there was no feedback concerning accuracy. Across test trials, children were presented with animations and reminded of task instructions. Animations served to keep children motivated, and reminders ensured that they knew to select the array with the larger, not smaller, number of dots.

After completing the NDT, children took a short break and were given the TEMA-3. For each item of the TEMA-3, the experimenter read children a question. Children provided either a verbal or written response, depending on the question. For those children who participated in a second experimental session, they were given the PPVT-4. On each item, children were read a word and asked to select one of four pictures that matched.

Results

Performance on NDT

Preliminary analyses on overall accuracy revealed no gender differences for any age group (all $p_s > .5$); gender was thus excluded from subsequent analyses. A mixed-factor ANOVA with age (between-subjects) and ratio (within-subjects) as independent variables revealed significant main effects of age, $F(2, 71) = 24.128$, $p < .001$, $p^2 = .405$, and ratio (assumption of sphericity violated, $p = .038$, Greenhouse-Geisser corrected), $F(6.032, 428.305) = 6.805$, $p < .001$, $p^2 = .087$, on accuracy. The interaction between age and ratio was not statistically significant ($p > .2$). Follow-up analyses revealed that older children performed better than

younger children (5-year-olds > 4-year-olds > 3-year-olds, p s < .005, Bonferroni corrected; see Figure 2), consistent with previous findings of increasing ANS precision over development (Halberda & Feigenson, 2008). A linear trend analysis (using linear contrast for ratio) also revealed that, as predicted by Weber's law, accuracy was modulated by ratio, $F(1, 71) = 24.732$, $p < .001$, with better performance on ratios farther versus closer to 1.00, replicating previous findings in children (Halberda & Feigenson, 2008) and infants (Xu & Spelke, 2000).

Relation between ANS Precision and Math Ability

To capture ANS precision on the NDT, we fit a psychophysical function to percentage correct across each of the ratios tested. Following previous research (Halberda et al., 2008; Piazza et al., 2004; Pica et al., 2004), this function was based on a Gaussian model with one free parameter (w) reflecting the amount of variability within the ANS using the following equation: $.5 * [1 + \text{erf}(\log(x) / ((2) * w))]$. We then computed precision as the predicted accuracy at an untested ratio: 1.2. (The 1.2 ratio was selected because its placement on the steepest slope of the Gaussian curve guaranteed sufficient variability of estimates). Because responses based on number (rather than cumulative area) should be above the chance level of 50%, predicted accuracy was set to within 50% (chance performance) and 100% (perfect performance). Indeed, comparisons to chance confirmed that each age group performed significantly better than chance [3-year-olds: $M = 61.9\%$, $SEM = 3.0\%$, $t(23) = 4.01$; 4-year-olds: $M = 73.9\%$, $SEM = 2.6\%$, $t(24) = 9.30$; 5-year-olds: $M = 86.7\%$, $SEM = 2.0\%$, $t(23) = 18.71$; all p s < .001], and that performance was significantly above chance at each ratio (see Figure 2).

Although recent experiments (e.g., Halberda et al., 2008) have calculated ANS precision as the ratio at which participants exceed 75% accuracy, this approach was problematic within our sample. Performance was below 75% for several children, even at the easiest ratio (see also Inglis et al., 2011; Mazzocco et al., 2011a), requiring that estimates of the corresponding ratio be based on extrapolation beyond the range of tested ratios. Because statistical estimates are more valid for interpolated data (e.g., Cohen, Cohen, West, & Aiken, 2003), we instead estimated ANS precision by interpolating accuracy (percentage correct) for an untested ratio (1.2), thereby establishing predicted accuracy within the range of observed performance. Predicted accuracy from individually fitted psychophysical functions thus served as our metric of ANS precision and is used in the analyses below.

Following statistical approaches in previous research (e.g., Libertus et al., 2011), we first compared ANS precision to performance on the TEMA-3 by collapsing across age. TEMA-3 scores ranged from 72 to 141 with a mean of 112. Correlation analyses revealed a positive relation between ANS precision and TEMA-3 scores, $r(72) = .387$, $p = .001$, which suggests that children with more precise number representations performed better at math, confirming recent findings of a link between the ANS and a formal math system during the preschool years (Libertus et al., 2011; Mazzocco et al., 2011a). Although neither ANS precision, $r(53) = -.034$, $p > .8$, nor performance on the TEMA-3, $r(53) = .237$, $p > .08$, correlated significantly with PPVT-4 scores, for completeness and to assess the extent of specificity between ANS precision and math competence, we controlled for verbal competence in a hierarchical regression analysis. This analysis revealed that ANS precision was a significant predictor of TEMA-3 scores when controlling for performance on the PPVT-4, $B = .379$, $t(52) = 3.052$, $p = .004$, suggesting at least some specificity between the ANS and math competence, consistent with recent findings (e.g., Halberda et al., 2008). The same pattern was observed when raw overall accuracy was used instead of predicted accuracy; raw overall accuracy was a significant predictor of TEMA-3 scores, even when controlling for PPVT-4 scores, $B = .347$, $t(52) = 2.751$, $p = .008$.

In subsequent analyses, we assessed the relation between ANS precision and TEMA-3 scores at each age group (3-, 4-, and 5-year-olds; see Table 1). As mentioned in the Introduction, recent studies with preschoolers using similar procedures have *not* reported analyses broken down by age (Libertus et al., 2011; Mazzocco et al., 2011a), leaving open questions about how this relation develops during a period with significant changes in ANS precision and symbolic math skills. Although ANS precision and TEMA-3 scores remained significantly correlated when controlling for age, $r_p(71) = .313$, $p = .007$, [as well as both age and PPVT-4 scores, $r_p(51) = .312$, $p = .023$ (missing cases excluded, pairwise)], separate correlation analyses for each age group revealed clear differences. Data for 3- and 4-year-olds were consistent with the overall analysis; that is, there was a marginally significant correlation between ANS precision and TEMA-3 scores for 3-year-olds, $r(22) = .379$, $p = .068$, and a significant correlation for 4-year-olds, $r(23) = .399$, $p = .048$. The pattern for 5-year-olds, however, was nowhere near significant, $r(23) = .130$, $p > .5$.²

What might account for the null effect in the older age group? One possibility is that the link between the ANS and school-relevant math weakens over development (cf. Holloway & Ansari, 2009; Inglis et al., 2011). Positive findings in adolescents (Halberda et al., 2008) and college students (Lourenco, Bonny, Fernandez, & Rao, 2012; Paulsen, Woldorff, & Brannon, 2010), however, suggest otherwise. Another possibility, raised in the Introduction, is that the link between ANS precision and symbolic math depends on one's level of math achievement. More specifically, the lack of a correlation in our 5-year-old sample may be due to their relatively higher TEMA-3 scores (see Table 1 for breakdown by age). A one-way ANOVA with TEMA-3 scores as the dependent variable confirmed a significant effect of age group, $F(2, 71) = 3.087$, $p = .052$. Compared to 3- and 4-year-olds, 5-year-olds had higher standardized scores: 5- vs. 3-year-olds, $p = .038$, 5- vs. 4-year-olds, $p = .033$ (Fischer's Least Significant Difference test; Levene's test, $p > .05$). There were no such age differences for PPVT-4 scores, $F(2, 52) = 1.268$, $p > .2$. Correlation analyses revealed similar effects; age was positively correlated with TEMA-3 scores, $r(72) = .241$, $p = .039$, but not with PPVT-4 scores, $r(53) = -.109$, $p > .4$. Given that TEMA-3 scores are age-standardized and that 5-year-olds were sampled from the same pool of participants as 3- and 4-year-olds, the finding that 5-year-olds scored significantly higher on the TEMA-3 was unexpected. As noted above, research in dyscalculics hints at a non-linear relation between ANS precision and math competence. The confound between age and TEMA-3 scores in our own data made it especially critical that we examine whether the link to ANS precision varied in strength for higher versus lower scorers on the TEMA-3.

To test for this possibility, we conducted a segmented regression analysis. In this analysis, the relation between two variables is assumed to change discontinuously at a "breakpoint," resulting in two distinct lines of best fit before and after the breakpoint (e.g., Abedini & Shaghaghian, 2009). A non-linear function is thus approximated by separate linear segments. Evidence for a breakpoint is reflected in greater explained variance compared to a single linear model (Smith, 1979; see also Knell, 2009). To test for a discontinuous change between ANS precision and math competence, we fit two linear functions to our data, using a program (SegReg, Oosterbaan, 2011) to estimate the breakpoint, with TEMA-3 scores as the x-variable and ANS precision as the y-variable across the three age groups. The breakpoint estimate of 113.4 (TEMA-3 score) provided a better fit to the data than the single linear model, $F(1, 71) = 7.829$, $p = .007$ (see Figure 3), suggesting discontinuity at approximately the statistical median of TEMA-3 scores (114).

²When controlling for PPVT-scores at each age group, correlations between ANS precision and TEMA-3 scores were in the same direction as when not accounting for PPVT scores: 3-year-olds, $r_p(15) = .423$, $p = .091$; 4-year-olds, $r_p(15) = .420$, $p = .093$; 5-year-olds, $r_p(16) = .156$, $p = .537$.

In follow-up analyses, we examined the nature of this discontinuity by directly comparing children across the breakpoint. A comparison of ANS precision revealed that children scoring above the breakpoint (TEMA-3 > 113.4) had significantly higher levels of ANS precision than those scoring below the breakpoint (TEMA-3 < 113.4), Mann-Whitney $U = 338$, $z = 3.747$, $p < .001$ (see Table 2), as expected given the overall relation to math competence. Correlation analyses revealed a marginally positive relation between ANS precision and TEMA-3 scores for children below the breakpoint, $r(35) = .306$, $p = .066$, but absolutely no relation for those above the breakpoint, $r(35) = -.177$, $p > .2$ (see Figure 3). Furthermore, the strength of the correlation was significantly higher for children below versus above the breakpoint, Fisher r to z test, $z' = 2.04$, $p = .041$.³ Importantly, there was no significant difference in age across the two groups (see Table 2), Mann-Whitney $U = 545.5$, $z = 1.503$, $p > .1$, suggesting that the discontinuity holds for children of different ages and that the effect of age described above (i.e., 3- and 4-year-olds, but not 5-year-olds, showing correlations between ANS precision and TEMA-3 scores) was due to differences in the level of math achievement. That is, the higher a child's level of math competence, the weaker the link to ANS precision. There was no significant difference in PPVT-4 scores across the breakpoint, Mann-Whitney $U = 290.5$, $z = 1.384$, $p > .1$ (see Table 2). Although ANS precision was higher overall for higher math-achieving children, the connection between ANS precision and math achievement was stronger for those with lower math scores, suggesting a non-linear link between the two systems, not due to age or verbal intelligence.

But could differences across the breakpoint be due to statistical artifacts? One possibility is that the weaker correlation between ANS precision and TEMA-3 scores for higher math achievers could be due to less variability in performance, or deviations from normality, on either the NDT or TEMA-3, or both. Analyses, however, revealed no significant differences between children above versus below the breakpoint in either variability (Levene's test of homogeneity, both $ps > .05$) or normality (Kolmogorov-Smirnov test, both $ps > .1$; see Table 2) for either ANS precision or TEMA-3 scores.

General Discussion

The results of the present study provide support for the claim that the ANS, which humans share with nonhuman animals and which may form part of our innate cognitive endowment (e.g., Izard et al., 2009; Nieder, 2005), interfaces with a more formal, learned system of symbolic mathematics, unique to humans. Using non-symbolic number arrays and psychophysical modeling to capture individual differences in ANS precision, we found that preschoolers with more precise number representations were generally more mathematically competent, as assessed by a standardized test of early math achievement. Our findings converge with other evidence for a link between ANS processing and symbolic math that is specific to these two systems, shown by controls for verbal competence (this study) and other domain-general abilities such as speed of processing (e.g., Halberda et al., 2008), and that is functional from as early as the preschool years when exposure to symbolic math is just beginning (Gillmore et al., 2010; Libertus et al., 2011; Wagner & Johnson, 2011).

³One potential concern with this comparison is that the difference in correlations might be driven by the slightly negative correlation for the higher math achievers. Although we cannot rule out this possibility directly, we would suggest two reasons why this may not be the case. One is that the negative correlation for the higher math achievers was nowhere near significantly different from zero. Another reason is that our data may have actually underestimated the strength of the correlation in the lower math achievers. Given that these children were still at about the mean level of performance on the TEMA-3, and that research suggests that ANS precision is particularly low for dyscalculic individuals (Mazzocco et al., 2011b), the link between symbolic math competence and the ANS may be even stronger in children who score lower on standardized math tests than those who participated in the present study. Following up on these issues is clearly an important area for future research.

The results of the present study also add to the growing body of research in this area by providing preliminary evidence for a non-linear relation between the ANS and symbolic math. Using segmented regression, we found that ANS precision was less predictive of mathematical achievement in children with higher, compared to lower, TEMA-3 scores across the preschool period. Mazzocco and colleagues (2011b) recently demonstrated that adolescents with dyscalculia (achievement scores 10th percentile) had significantly noisier non-symbolic number representations than their non-dyscalculic counterparts, even compared to those considered low achieving (achievement scores between 11th and 25th percentile). There were no significant differences, however, in ANS precision among individuals without dyscalculia, despite greater variation in their achievement scores (> 11th percentile). Although correlations between ANS precision and math scores were not compared separately by achievement group (as in the present study), comparable levels of ANS precision among the higher math-achieving groups suggest non-linearity. Admittedly, children in the present study were far from dyscalculic. Our findings nevertheless suggest that the connection between the ANS and symbolic math may be fundamentally discontinuous. Below we speculate about how these systems might come to interact non-linearly.

One possibility is that children with higher scores on the TEMA-3 relied on a broader network of knowledge and skills than children with lower scores, including, but not limited to, the ANS. Studies using neuroimaging and neuropsychological techniques point to a distributed neural network of math reasoning, which encompasses both domain-specific and domain-general processes, and which re-organizes over development and with increasing expertise (for review, see Butterworth, Varma, & Laurillard, 2011). When solving calculation problems, adults who are prodigious calculators show neural activation that is both overlapping and distinct from that of non-experts (Pesenti et al., 2001). The additional areas of activation in prodigious calculators correspond to mechanisms of long-term episodic memory, proposed to expand working memory capacity during complex computations (Butterworth, 2006). Although the precise strategies used by children and adults are known to vary (Ashcraft, 1992; Siegler, 1987), higher math-achieving children may similarly benefit from performance enhancing strategies, above and beyond that afforded by ANS precision.

Mathematics is a complex, associative system that is organized hierarchically - more advanced concepts and operations building on more basic concepts and operations. For example, an understanding of the cardinal meaning of numerals builds on an appreciation of both magnitude and symbolic notation. As another example, the ability to perform arithmetic computations of Arabic numerals, even single digit calculation, assumes knowledge of individual number symbols and combinatorial relations among such symbols. It has been suggested that the connections between ANS processing and advanced math problems, such as the rank ordering of fractions and multi-digit arithmetic, are mediated by symbolic number knowledge and basic mathematical processes, such as the understanding of number words and Arabic numerals (Mazzocco et al., 2011a; see also Holloway & Ansari, 2009; Sasanguie, De Smedt, Defever, & Reynvoet, 2011) as well as the ordinal relations of numerical symbols (Lyons & Beilock, 2011). Children who score higher on standardized math tests such as the TEMA-3 typically solve more complex problems, with greater symbolic content and computational demands.⁴ Such problems may yield weak

⁴It is unlikely that the non-linear relation between ANS precision and math competence in our study was due to lower math-achieving children receiving more non-symbolic than symbolic content on the TEMA-3. The vast majority of items on this test are symbolic in nature. Only one item involves a completely non-symbolic comparison such that even the lowest math-achieving children in our sample would have received questions with symbolic content.

connections to the ANS, either because of factors that mediate the relation between the ANS and mathematical competence or that directly support specific math abilities.

The idea behind the mediating role of symbolic number knowledge is that math competence is supported by the mapping of symbolic number (e.g., number words and Arabic numerals) to numerical magnitude more than by ANS precision (for review, see Noël & Rousselle, 2011). Following from this, the non-linear pattern between ANS precision and math competence in our data could reflect higher math-achieving children relying more heavily on representations of symbolic number than on the ANS. Yet accumulating evidence that ANS precision predicts math competence in adult samples (e.g., Lourenco et al., 2012; Paulsen et al., 2010) suggests that access to symbolic number representations may not fully mediate the connections between the ANS and a formal system of mathematics. Of course, other basic aspects of mathematical processing could still play a role in mediating connections to the ANS. Future research would do well to examine the relative contributions of ANS precision and symbolic abilities to math competence over development.

Models of adult mathematical reasoning suggest various conditions under which the ANS may not be directly implicated in solving math problems (Dehaene, 1992). Familiar multiplication problems, for example, can be solved by access to long-term memory (i.e., rehearsed multiplication tables) rather than explicit calculation, with neural evidence suggesting activation associated with such factual recall that is distinct from the ANS (e.g., Dehaene & Cohen, 1997). As children become more proficient at using arithmetic facts to solve calculation problems, they may come to rely increasingly on retrieval strategies (e.g., Ashcraft, 1992; Siegler, 1987). Although preschoolers in the present study may not have relied on arithmetic facts per se, knowledge of symbolic number (Holloway & Ansari, 2008; Noël & Rousselle, 2011) and/or access to relevant computational strategies (e.g., Siegler, 1987; Torbeyns, Verschaffel, & Ghesquiere, 2005) may have contributed to performance on the TEMA-3. Such factors may play a greater role than ANS precision in accounting for individual differences in mathematical competence among the higher math-achieving preschoolers who received more complex problems to solve.

Other possible accounts of a non-linear relation between the ANS and more formal math abilities may reflect specific learning trajectories and environmental experiences. A common view of how humans come to learn formal mathematics is that symbolic number notation and knowledge of quantitative concepts and operations build on non-symbolic representations of numerosity in the ANS (e.g., Gallistel & Gelman, 1992; Nieder & Dehaene, 2009). This view has led to the suggestion that better ANS precision enhances math learning. Given the lack of direct evidence concerning causality, we are cautious about making proposals that rest on a specific causal direction. Nevertheless, our results suggest that any benefits to learning (or doing) math may not extend beyond a certain threshold of ANS precision, with increasingly precise number representations yielding little functional significance, perhaps because, as noted above, particular types of math reasoning may be less directly dependent on the ANS.

Even if better ANS precision enhances the learning of mathematics, there are other cognitive and social factors that are known to impact math achievement in the school years, including motivation (Cleary & Chen, 2009), math-related anxiety (Ashcraft, 2002), socioeconomic status (Jordan, Huttenlocher, & Levine, 1992), and the spatial organization of the “mental number line” (Siegler & Booth, 2004). Recent findings by Levine and colleagues suggest that, in young children, “number talk” by parents plays a critical role in the acquisition of early math concepts (Gunderson & Levine, 2011; Levine et al., 2010). In an analysis of longitudinal data, they found that toddlers (between 14 and 30 months) whose parents used number words in quantitatively meaningful ways became preschoolers (46 months of age)

with a better understanding of cardinality. Above we noted that better ANS precision could lead to more advanced math abilities. Another possibility is based on the reverse causal direction, with the quality and/or quantity of math-related input, even of the informal type experienced by young children, determining ANS precision. Such experiences could lead to higher overall ANS precision. Such experiences could also lead to higher mathematical competence, even in children with low ANS precision, by bypassing the ANS altogether and essentially serving as protection against noisy non-symbolic number representations.

To summarize, we have offered multiple suggestions to account for discontinuity between the ANS and school-relevant mathematics in preschool-aged children. These accounts include the mediating role of symbolic number knowledge, threshold effects on ANS precision, and the protective properties of math-related input. Although the focus here was on potential mechanisms in isolation, it is possible, indeed likely, that some combination of these factors could lead to non-linearity and might account for conflicting findings in the existing literature. A limitation of the present study is the relatively small sample sizes, especially when analyzed by higher- vs. lower-math achievers. Nevertheless, when taken together with recent research (e.g., Lyons & Beilock, 2011; Mazzocco et al., 2011a), the present findings point to complex interconnections between the ANS and a system of formal math. Much of this research, however, concerns only a limited set of mathematical processes, such as symbolic number identification and elementary arithmetic, typical of the preschool period. Beyond this period, humans gain access to complex mathematical concepts and operations, such as place value, long division, and Pythagorean Theorem. Future research would do well to examine how far the ANS reaches, and what role, if any, it has in increasingly advanced mathematical processing.

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Highlights

Research suggests a link between the approximate number system (ANS) & math skills.

The present study examined this link across the preschool period (3 to 5 years).

Children with greater ANS precision had higher standardized math scores.

Segmented regression analyses, however, revealed non-linearity.

Children with lower math scores showed stronger traces of ANS processing.

Figure 1.
Sample trial from the NDT (8 vs. 4 dots).

Figure 2.

Accuracy on the NDT as measured by the mean percentage of trials in which children correctly chose the numerically larger array. Error bars reflect ± 1 SEM. Collapsing across age, mean accuracy was above chance at each ratio (all p s < .001, Bonferroni corrected).

Figure 3. Scatterplot relating ANS precision on the NDT and mathematical ability (TEMA-3 scores) for children scoring above (right side of figure) versus below (left side of figure) the breakpoint. TEMA scores are centered to the breakpoint (i.e., individual score minus 113.4).

Table 1

Descriptive statistics of ANS precision (computed as predicted accuracy at 1.2 ratio) and test scores (standardized) by age group.

	3-year-olds			4-year-olds			5-year-olds		
	N	M	SEM	N	M	SEM	N	M	SEM
ANS	24	58.2%	1.8%	25	67.3%	2.5%	25	82.2%	2.7%
Precision									
TEMA-3	24	109.1	3.1	25	109	3.4	25	117.9	2.1
PPVT-4	18	123.8	3	18	116.5	4	19	120.5	2.6

Table 2
Comparisons of children scoring above and below the breakpoint (TEMA-3 score of 113.4)

	TEMA-3 Scores < 113.4				TEMA-3 Scores > 113.4			
	N	M	SEM	Z *	N	M	SEM	Z *
TEMA-3	37	100.2	1.68	1.140	37	123.9	1.23	0.864
ANS Precision	37	62.8%	2.0%	0.930	37	76.0%	2.5%	0.781
Age (months)	37	51.9	1.63	0.633	37	55.3	1.65	0.786
PPVT-4	24	117.5	3.34	0.429	31	122.4	2.06	0.705

* Kolmogorov-Smirnov Z scores for comparison to normal distribution. No significant deviations were observed.